Rate and spectrum of $K_L \to \pi^0\gamma\gamma$

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The branching ratio of $K_L \to \pi^0\gamma\gamma$ and the invariant-mass spectrum of the photon pair are calculated taking account of two distinct mechanisms: an amplitude involving $VP\gamma$ couplings and an amplitude involving a pion loop. The two contributions are comparable in rate, but strikingly different in their spectra. We determine the spectrum, including both components, and estimate a branching ratio $(1.8, 3.9)\times 10^{-6}$ for (destructive, constructive) interference.

Experiments devoted to precision measurements of the parameter $|\eta_{00}/\eta_{+-}|$ in $K_L \to 2\pi$ decays are yielding, as a by-product, information on a variety of radiative K decays.[1] One process that is currently the focus of interest is the decay $K_L \to \pi^0\gamma\gamma$ [Fig. 1(a)] which is anticipated to occur with a branching ratio of the order of 10^{-6}, well within the current range of observability. Theoretically, it is possible to conceive of two rather distinct mechanisms for this reaction. The first of these (which we shall refer to as the "pion-loop" mechanism) was calculated a number of years ago,[2] and is based on the transition $K_2 \to \pi^+\pi^-\pi^0$ followed by $\pi^+\pi^-$ annihilation into two photons [Fig. 1(b)]. This mechanism has recently been refined[3] with the help of a theoretical framework called "chiral perturbation theory" that allows one to include kaon loops as well. The rate and spectrum are not very different from those obtained in the simple pion-loop model, which we will adopt in the remaining discussion. A second mechanism[4–6] [which we shall refer to as vector-meson dominance (VMD)] envisages that the transition is mediated by $K_L \to \pi^0$, $K_L \to \eta^0$, and $K_L \to \eta'^0$ vertices, the final $\pi^0\gamma\gamma$ state being reached by a succession of vector-meson-mediated $PV\gamma$ couplings [Fig. 1(c)]. This second mechanism is, for practical purposes, the only one available for the decay $\eta^0 \to \pi^0\gamma\gamma$, so that some idea of the corresponding rate and spectrum in the case of $K_L \to \pi^0\gamma\gamma$ may be obtained from the empirical knowledge of $\eta^0 \to \pi^0\gamma\gamma$ (Ref. 4). A realistic discussion of the reaction $K_L \to \pi^0\gamma\gamma$ requires consideration of both mechanisms and the possible interference between them. Ideally, one would hope that an effective Lagrangian for radiative K decays could be constructed[7] that incorporates the two mechanisms mentioned above as well as satisfying the general symmetry properties dictated by QCD and the $\Delta I = \frac{1}{2}$ rule for the nonleptonic weak interactions. For the present, we will content ourselves with a phenomenological analysis.

Interest in the decay $K_L \to \pi^0\gamma\gamma$ stems, in part, from the repercussions it has on the decay $K_L \to \pi^0 e^+e^-$ which has been seriously discussed as a possible testing ground for CP violation.[8] In the presence of CP noninvariance, the decay $K_L \to \pi^0 e^+e^-$ can proceed through a one-photon intermediate state, with possible contributions from both direct CP violation and the indirect influence of the ϵ impurity in the K_L wave function.[9] However, a competing CP-conserving amplitude is possible through a two-photon intermediate state connected with the decay $K_L \to \pi^0\gamma\gamma$ (Refs. 4–6). The latter, however, is of importance only to the extent that a VMD-type amplitude is present in this decay, since a pion-loop-type contribution produces an amplitude for $K_L \to \pi^0 e^+e^-$ proportional to the electron mass, and hence negligible. Thus a study of $K_L \to \pi^0\gamma\gamma$ is important if one wishes to determine how strongly it will contribute to the reaction $K_L \to \pi^0 e^+e^-$.

We begin by writing the general matrix element for $K_L \to \pi^0\gamma\gamma$ (Refs. 2, 3, and 5):

$$M = A(\epsilon\cdot\epsilon' k\cdot k' - \epsilon\cdot k'\epsilon'\cdot k) + B(\epsilon\cdot\epsilon' k\cdot Q k'\cdot Q + k\cdot k'\epsilon\cdot Q\epsilon'\cdot Q - \epsilon\cdot Q\epsilon'\cdot k k'\cdot Q - \epsilon\cdot k' k\cdot Q\epsilon'\cdot Q)/k\cdot k' , \quad (1)$$

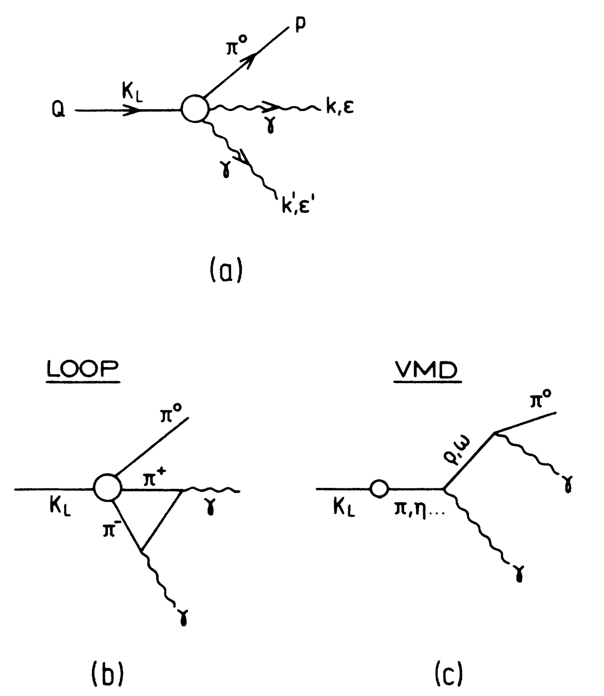

FIG. 1. (a) Definition of momenta in the decay $K_L \to \pi^0\gamma\gamma$, (b) diagram illustrating pion-loop mechanism, and (c) diagram illustrating VMD mechanism.

where the symbols are defined in Fig. 1(a). Here A and B are functions of the two independent invariants which we choose as

$$s=(Q-p)^2=(k+k')^2\ ,\qquad \Delta=t-t'=(Q-k)^2-(Q-k')^2 \tag{2}$$

which can be regarded as the two coordinates of the Dalitz plot. The boundaries of phase space are defined by[2]

$$0<s<(M-\mu)^2,\quad \Delta^2<(s-M^2-\mu^2)^2-4M^2\mu^2\ , \tag{3}$$

M and μ denoting the mass of the K and π meson, respectively.

The pion-loop model has the feature that it contributes only to the A-type matrix element in Eq. (1), and furthermore produces a function A that depends on the variable s only. Explicitly, from Ref. 2, the loop contribution is

$$\begin{aligned} A_{\text{loop}}&=\text{Re}\,A_{\text{loop}}+i\,\text{Im}\,A_{\text{loop}},\quad \text{Im}\,A_{\text{loop}}=2\left[\frac{ge^2}{4\pi}\right]\frac{\mu^2}{s^2}\ln\frac{\sqrt{s}+\sqrt{s-4\mu^2}}{\sqrt{s}-\sqrt{s-4\mu^2}}\Theta(s-4\mu^2)\ ,\\ \text{Re}\,A_{\text{loop}}&=2\left[\frac{1}{4\pi}\right]\left[\frac{ge^2}{4\pi}\right]\left\{\left[\frac{2\mu^2}{s^2}\left[\pi^2-\ln^2\frac{\sqrt{s}+\sqrt{s-4\mu^2}}{\sqrt{s}-\sqrt{s-4\mu^2}}\right]-2\right]\Theta(s-4\mu^2)\right.\\ &\qquad\left.+\frac{8\mu^2}{s^2}\left[\left[\arctan\frac{\sqrt{s}}{\sqrt{4\mu^2-s}}\right]^2-2\right]\Theta(4\mu^2-s)\right\}\ . \end{aligned} \tag{4}$$

Here g is the coupling constant for $K_2\to\pi^+\pi^-\pi^0$ with the numerical value $|g|=0.84\times10^{-6}$ (Ref. 10).

The VMD contribution has been treated in Refs. 4–6, most explicitly by Morozumi and Iwasaki[5] whose notation we adopt. If we ignore the mass difference of ρ and ω, the VMD model produces an amplitude containing both of the invariant forms in Eq. (1), with functions A and B given by

$$\begin{aligned} A_{\text{VMD}}&=-\frac{G}{2}\left[\frac{M^2+t}{t-M_V^2}+\frac{M^2+t'}{t'-M_V^2}\right]\ ,\\ B_{\text{VMD}}&=-\frac{G}{2}\left[\frac{s}{t-M_V^2}+\frac{s}{t'-M_V^2}\right]\ . \end{aligned} \tag{5}$$

Here G is a coupling constant that involves the $\omega\pi\gamma$ coupling, the K-π, K-η, K-η' vertices and the η-η' mixing angle, and has been estimated[5,11] to be

$$G=\sum_{V=\omega,\rho}G_V=-0.33\times10^{-7}M^{-2}\ . \tag{6}$$

[In Eq. (5) M_V denotes the mass of the ρ or ω.] Thus the full amplitude for $K_L\to\pi^0\gamma\gamma$ is characterized by

$$A=A_{\text{loop}}+A_{\text{VMD}},\quad B=B_{\text{VMD}}\ . \tag{7}$$

The form factor A clearly involves an interference between the two mechanisms, i.e., between the real part of A_{loop} and the real amplitude A_{VMD}. Pending a dynamical clarification of the relative sign of g and G, we shall consider both constructive and destructive interference of the two components. The formula for the differential decay rate is[4,5]

$$\frac{d\Gamma}{ds\,d\Delta}=\frac{1}{2^{11}\pi^3M^3}\left[|As-BM^2|^2+\left[\frac{B}{s}\right]^2(M^2\mu^2-tt')^2\right], \tag{8}$$

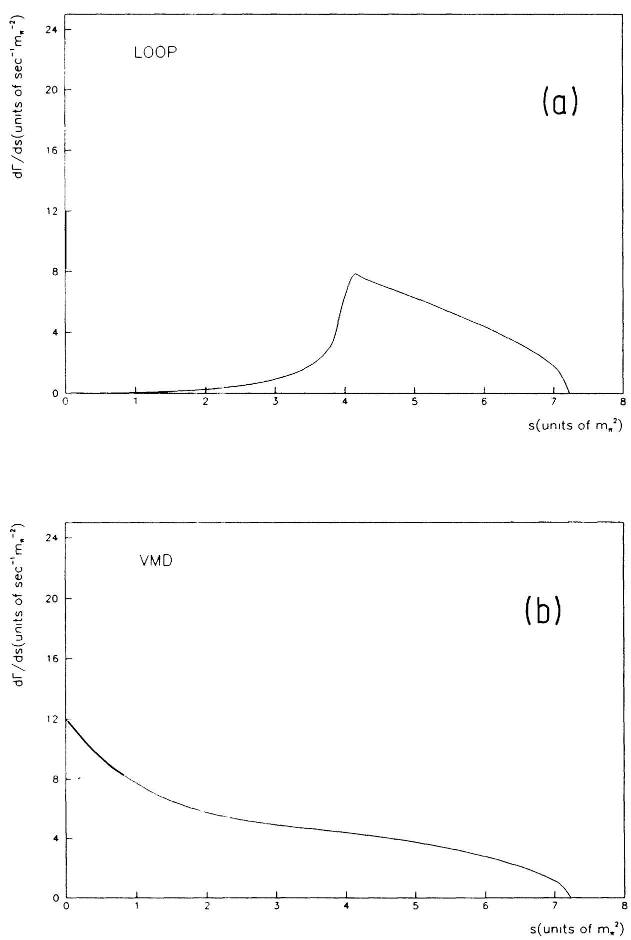

FIG. 2. Differential decay rate as function of invariant mass of photon pair. (a) Loop mechanism alone and (b) VMD mechanism alone.

where

$$tt'=\tfrac{1}{4}[(M^2+\mu^2-s)^2-\Delta^2] \ . \tag{9}$$

The invariant-mass spectrum of the photon pair is

$$\begin{aligned} \frac{d\Gamma}{ds} &= \int_{-\Delta_0}^{+\Delta_0} d\Delta \frac{d\Gamma}{d\Delta\, ds} \ , \\ \Delta_0 &\equiv [(M^2+\mu^2-s)^2-4M^2\mu^2]^{1/2} \ . \end{aligned} \tag{10}$$

We now proceed to summarize our results.

(a) In Fig. 2(a) we show the invariant-mass spectrum of the photons following from the pion loop alone ($G=0$). Notable is the dominance of large-s values. The integrated rate is 19.0 s^{-1} corresponding to a branching ratio 0.99×10^{-6}. The absorptive and dispersive parts of the rate are 10.8 and 8.2 s^{-1}, respectively.

(b) Figure 2(b) shows the spectrum in the case of the VMD mechanism alone ($g=0$). The contrast with the loop model is striking, especially the enhancement at low invariant masses. The integrated rate is 35.8 s^{-1}, corresponding to a branching ratio 1.86×10^{-6}.

(c) Figure 3(a) shows the spectrum when both contributions are taken into account with a relative sign corresponding to constructive interference. The integrated rate is 75.2 s^{-1}, corresponding to a branching ratio 3.9×10^{-6}.

(d) Finally, Fig. 3(b) shows the result when the two contributions interfere destructively. The integrated rate is 34.5 s^{-1} and the branching ratio 1.79×10^{-6}.

On the basis of statements (c) and (d) above, we would

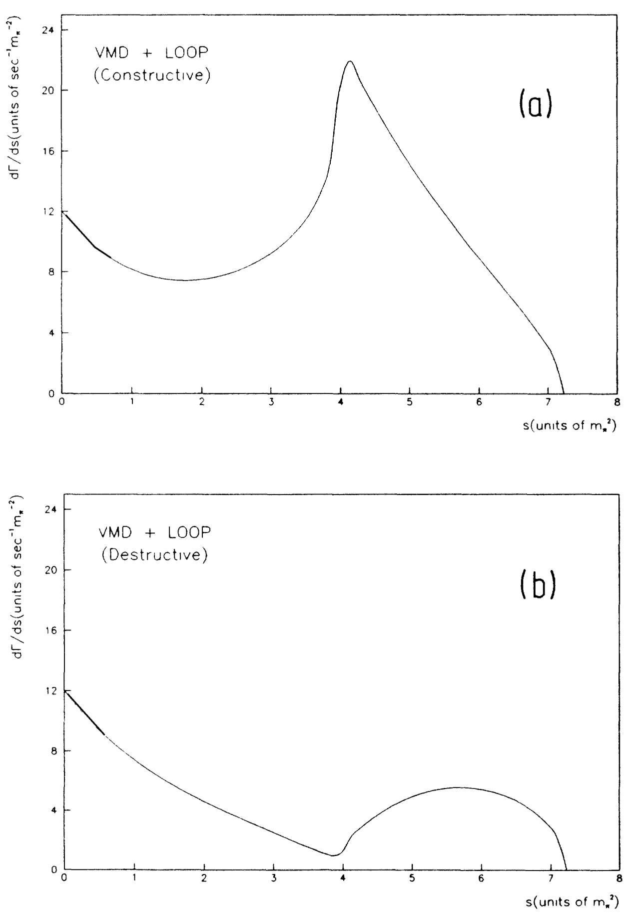

FIG. 3. Differential decay rate including both loop and VMD contributions. (a) Constructive interference and (b) destructive interference.

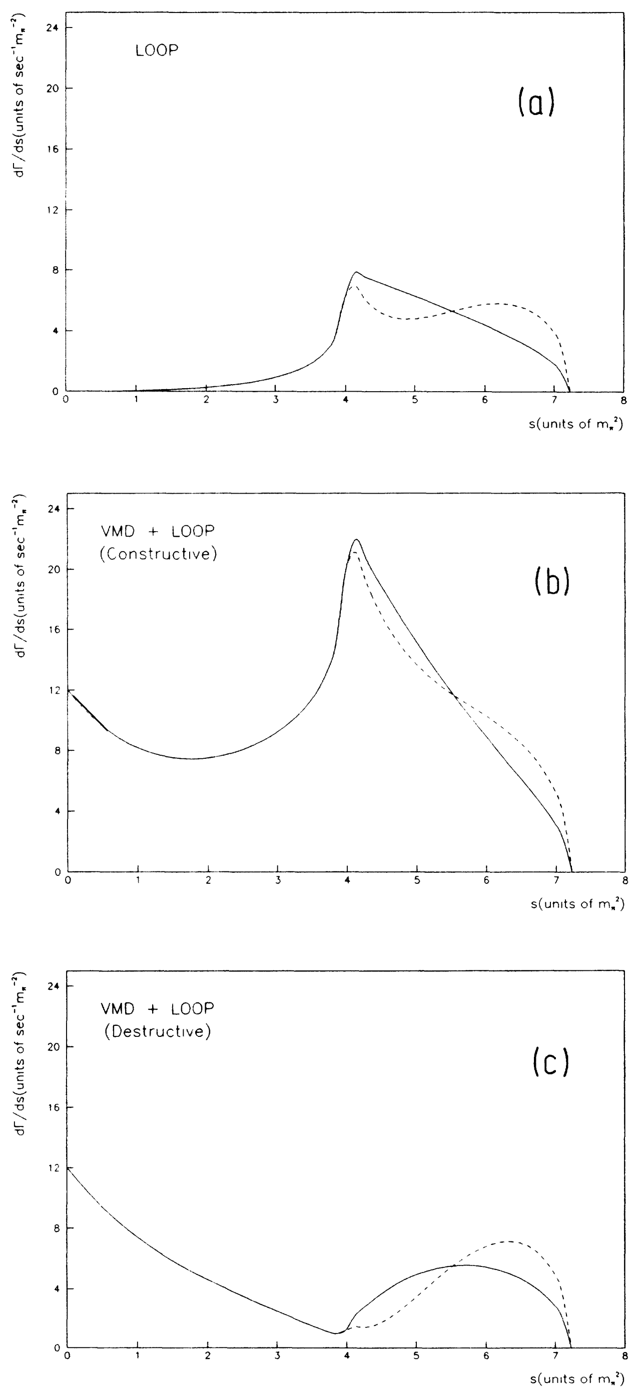

FIG. 4. Modification produced by inclusion of energy dependence in the Dalitz plot of $K_L \rightarrow \pi^+\pi^-\pi^0$ (dashed curves). (a) Effect on loop spectrum, (b) effect on complete spectrum, with constructive interference, and (c) effect on complete spectrum with destructive interference.

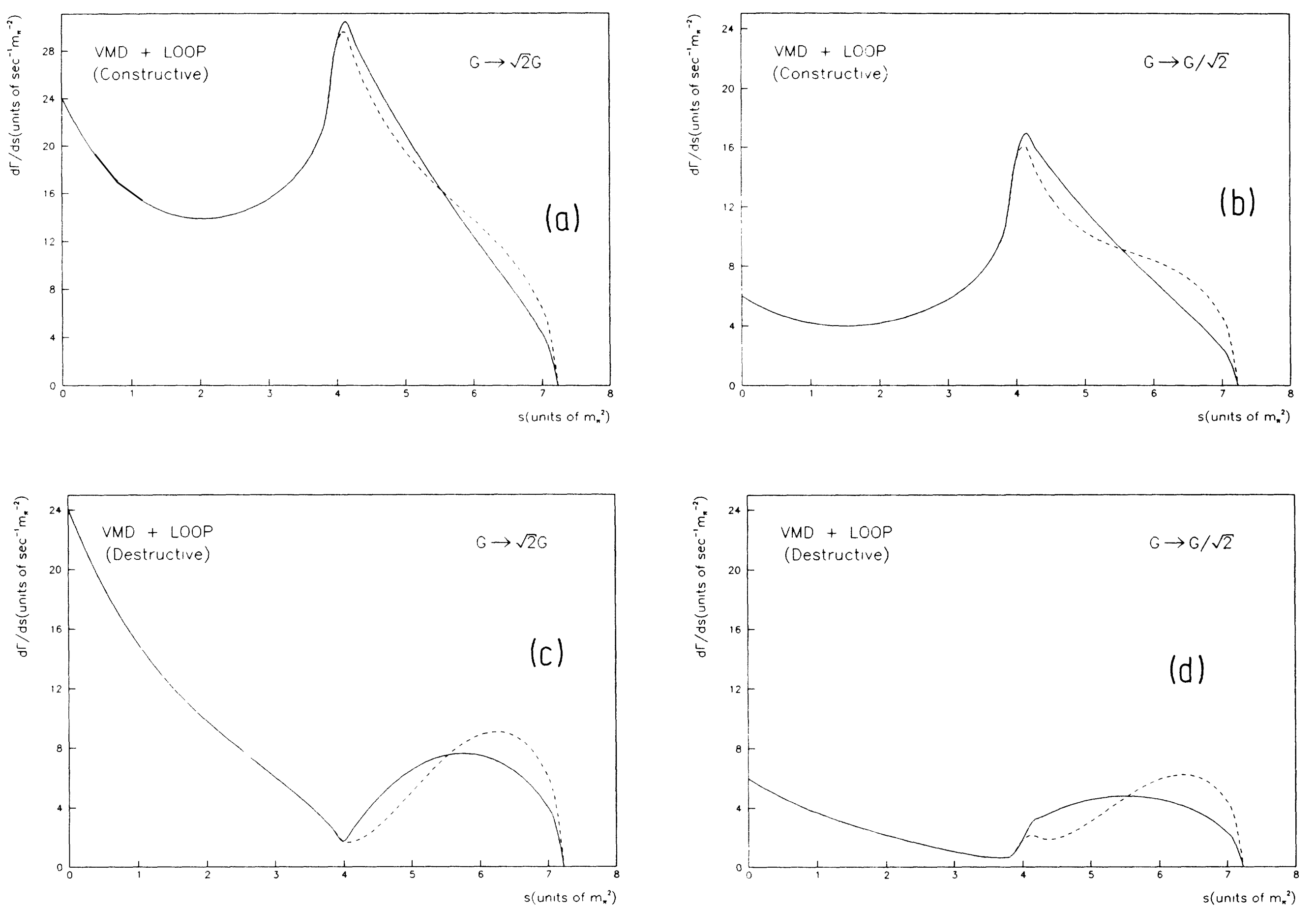

FIG. 5. Effect of changing the VMD coupling constant G by a factor $\sqrt{2}$, for the case of constructive interference [(a) and (b)] and destructive interference [(c) and (d)].

conclude that the branching ratio of $K_L\to\pi^0\gamma\gamma$ has a probable value 3.9×10^{-6}, with a spectrum resembling that shown in Fig. 3(a), or a value 1.8×10^{-6} with a spectrum resembling Fig. 3(b). To obtain an idea of the uncertainty in these predictions, we have considered some reasonable modifications in the underlying model. With respect to the loop contribution, one refinement is to calculate the absorptive part of the $K_L\to\pi^0\gamma\gamma$ amplitude using a matrix element for $K_L\to\pi^+\pi^-\pi^0$ that is not simply a constant, but includes the small energy dependence of the Dalitz plot.[12] This has the effect of changing the shape of the $\gamma\gamma$ invariant-mass spectrum slightly (Fig. 4), while changing the rate by only about 5%. One can also envisage inclusion of the strong $\pi^+\pi^-$ s-wave interaction in the calculation of $\pi^+\pi^-\to\gamma\gamma$. However, the success of the simple pion-loop model in explaining the measured rate of $K_S\to\gamma\gamma$ suggests that no dramatic changes will occur.[13] Similarly, the VMD component of the amplitude has an uncertainty associated with the estimate of the K-π, K-η, and K-η' vertices, particularly as regards the effects of SU(3) breaking. On the assumption that the VMD-induced rate is uncertain by a factor 2, we have varied the parameter G by a factor $\sqrt{2}$ (i.e., $G\to\sqrt{2}G$ and $G\to G/\sqrt{2}$), and examined the effects on the rate and spectrum of $K_L\to\pi^0\gamma\gamma$. These are shown in Fig. 5. Relative to the unmodified rate, the branching ratio changes as follows.

(a) Constructive interference:

$$B(K_L\to\pi^0\gamma\gamma)=3.9\times10^{-6}\to\begin{cases}6.2\times10^{-6}(G\to\sqrt{2}G)\ ,\\ 2.7\times10^{-6}(G\to G/\sqrt{2})\ .\end{cases}$$

(b) Destructive interference:

$$B(K_L\to\pi^0\gamma\gamma)=1.79\times10^{-6}\to\begin{cases}3.2\times10^{-6}(G\to\sqrt{2}G)\ ,\\ 1.17\times10^{-6}(G\to G\sqrt{2})\ .\end{cases}$$

In conclusion, the decay $K_L\to\pi^0\gamma\gamma$ should exhibit, in the invariant-mass spectrum of the photon pair, evidence for two distinct dynamical mechanisms: a pion-loop-induced contribution which emphasizes large invariant masses $s>4\mu^2$, and a VMD component that peaks at low-s values. The absolute rate as well as the shape of the spectrum depend on whether the two components interfere constructively or destructively (Fig. 3). For the central values of the parameter range considered by us, the branching ratio is 3.9×10^{-6} (constructive interference) or 1.8×10^{-6} (destructive interference). Measurement of the invariant-mass spectrum would be valuable in providing a measure of the VMD contribution to $K_L\to\pi^0\gamma\gamma$, which in turn would fix the *CP*-conserving 2γ contribution to $K_L \to \pi^0 e^+ e^-$. In addition, the shape of the spectrum might provide a clue to the sign of the interference between the two components. (Indeed, with sufficient statistics, it may be possible to determine the parameter g/G by a fit to the data.) This information would be a useful constraint on dynamical models of radiative K decays.

Note added. A new experimental limit on $K_L \to \pi^0\gamma\gamma$ has just appeared in print.[14] The result is $B(K_L \to \pi^0\gamma\gamma) < 2.7\times 10^{-6}$, if the photon mass spectrum is assumed to be that given by chiral perturbation theory,[3] and $B(K_L \to \pi^0\gamma\gamma) < 4.4\times 10^{-6}$ if the spectrum follows phase space. These results are compatible with the present paper, if one keeps in view the spectral shape shown in Fig. 3. A theoretical analysis of $K_L \to \pi^0\gamma\gamma$, very similar to that in this paper, has recently appeared in a report by Ko.[15]

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[11]The ingredients that go into this estimate are the value of the $K_2 - \pi^0$ vertex ($\langle \pi^0 | H_W | K_2 \rangle = -3.71\times 10^{-2}$ MeV^2), the ratio $\langle \pi^0|H_W|K_2\rangle : \langle \eta_8|H_W|K_2\rangle : \langle \eta_1|H_W|K_2\rangle = 1 : \sqrt{3} : -\sqrt{6}/4$, the η-η' mixing angle ($\Theta = -20°$), and the quark-model relation between the $PV\gamma$ couplings, the normalization being provided by the $\omega \to \pi\gamma$ width. The parameters G_ρ and G_ω are determined to be $G_\rho M^2 = -0.58\times 10^{-8}$ and $G_\omega M^2 = -2.7\times 10^{-8}$.

[12]The absorptive part of the differential decay rate $d\Gamma/ds$ gets multiplied by a factor

$$1 + g\frac{s - s_0}{\mu^2} + h\left[\frac{s - s_0}{\mu^2}\right]^2$$

with $g = 0.67$, $h = 0.079$, and $s_0 = \frac{1}{3}M^2 + \mu^2$. See Devlin and Dickey (Ref. 10) and Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).

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