Vector-meson contributions to the processes $\gamma\gamma\to\pi^0\pi^0, \pi^+\pi^-$, $K_L\to\pi^0\gamma\gamma$, and $K^+\to\pi^+\gamma\gamma$

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We implement standard chiral perturbation theory to $O(p^4)$ by adding vector mesons. We show that it leads to a good agreement of the prediction for $\sigma(\gamma\gamma\to\pi^0\pi^0)$ with experiments and the vector mesons unitarize the chiral amplitude in an effective way. We also consider $K_L\to\pi^0\gamma\gamma$, including the effect of vector mesons, and obtain a very distinct spectrum for two photons. Also, their effects on $\gamma\gamma\to\pi^+\pi^-$ and $K^+\to\pi^+\gamma\gamma$ are briefly discussed.

I. INTRODUCTION

The chiral Lagrangian was proposed a long time ago[1] to reproduce the successful results of current algebra,[2] using field-theoretic language and invoking the idea of spontaneous breaking of chiral symmetry.[3] The chiral Lagrangian describes strong (and electromagnetic) interactions of the lowest-lying mesons, which are regarded as Nambu-Goldstone bosons associated with the spontaneously broken chiral symmetry. It incorporates all the symmetries that QCD has, if the QCD anomaly term is suitably taken into account in the form of the Wess-Zumino action, and it has been shown that the chiral Lagrangian of systems of infinitely many mesons is equivalent to QCD in the limit of $N_c\to\infty$, where N_c is the number of colors.[4]

The $SU(2)_L\times SU(2)_R$ chiral Lagrangian was fully investigated up to $O(p^4)$ and shown to describe well the low-energy dynamics of pions.[5] The authors of Ref. 5 extended their derivative expansion of the chiral Lagrangian to the $SU(3)_L\times SU(3)_R$ case,[6] and listed all the relevant terms up to $O(p^4)$. This chiral Lagrangian is good for small m_s.

A difficult question to answer is the valid energy range of the theory. Presumably, the lower the energy is, the better predictions the theory gives. For example, the reaction cross section for the $\gamma\gamma\to\pi^0\pi^0$ process can be calculated unambiguously within the framework of the loop expansion based on the chiral Lagrangian mentioned in the previous paragraph [we call this the standard chiral perturbation theory (ChPT)].[7,8] The theoretical prediction is good at low energy, but the deviation of that prediction from the experimental data[9] starts to rapidly grow above $\sim$500 MeV, and it has been suggested to unitarize the chiral amplitude to get better theoretical results. (See Sec. III for detailed discussions of this subject and comments on other approaches.)

The weak and electromagnetic processes of the kaon system have been studied making use of the standard ChPT in elegant ways,[10] and many interesting results have been obtained. For example, the authors of Ref. 10 obtained the branching ratio for $K_S\to\gamma\gamma$ to be 2.0×10^{-6}, which agrees well with the recently measured value[11] $(2.4\pm1.2)\times10^{-6}$. This approach was used to calculate the branching ratio for $K_L\to\pi^0\gamma\gamma$, which is interesting not only by itself but also for its contribution to the *CP*-conserving part of the $K_L\to\pi^0e^+e^-$ decay. This two-photon process is very small compared to the *CP*-violating one-photon process, when the amplitudes are obtained in ChPT up to $O(p^4)$. Ecker, Pich, and de Rafael[10] noted that the $O(p^6)$ calculation might lead to an enhanced two-photon amplitude if the coefficient of the dimension-6 operators were not ~1. The two-photon process through intermediate vector mesons,[12,13] however, has been shown to be dominant over the two-photon process through chiral loops up to $O(p^4)$ (Ref. 10) and possibly over the *CP*-violating one-photon-exchange process in $K_L\to\pi^0e^+e^-$ decay. This subject has been controversial,[14] and we had better reconsider the validity of the standard ChPT in the presence of electromagnetic couplings.

In this paper, we add an interaction Lagrangian describing the vector-meson coupling to the pion octet and a photon. We rederive the reaction cross section for $\gamma\gamma\to\pi^0\pi^0$ and find that inclusion of the vector mesons suppresses the high-energy behavior of the reaction cross section, so that the theoretical prediction is in good agreement with experiment up to $m_{\gamma\gamma}=950$ MeV.

Motivated by this encouraging result, we reinvestigate $K_L\to\pi^0\gamma\gamma$, and find that the interference between chiral bosons and vector mesons produces a very striking spectrum of two final photons, easily distinguishable from the predictions of other approaches. The branching ratio is enhanced by about a factor 2–4 compared to the predictions of lowest-order ChPT (Ref. 10) and the pion-rescattering model.[15]

At the end of each section, we give short descriptions of the effects of vector mesons on $\gamma\gamma\to\pi^+\pi^-$ and $K^+\to\pi^+\gamma\gamma$ for complete discussions. As easily expected, the effects are screened by direct couplings of a photon to a charged particle.

Before showing our results, we briefly review the standard ChPT with and without weak and electromagnetic interactions, and then discuss the vector-meson coupling to the pion octet and a photon. We follow the notation and normalization conventions of Ref. 10.

II. BASIC SETUP

We start with the lowest-order $SU(3)_L \times SU(3)_R$ chiral Lagrangian

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\text{tr}(\partial_\mu U\, \partial^\mu U^\dagger) + v\,\text{tr}(MU + U^\dagger M) , \quad (1)$$

where $U(x)$ is defined as

$$U(x) = \exp\left[\frac{i}{F_\pi}\sum_{i=1}^{8}\lambda_i\phi_i(x)\right] , \quad (2)$$

$$\frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i\phi_i = \begin{bmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{bmatrix} \quad (3)$$

and F_π is the pion decay constant and is 93.3 MeV in our normalization. M is a quark mass matrix which explicitly breaks chiral symmetry, and $M = \text{diag}(m_u, m_d, m_s)$. v is a vacuum expectation value of $\bar{q}q$ representing spontaneous chiral-symmetry breaking. v relates the quark masses with the meson masses in the following way:

$$v = \frac{F_\pi^2 m_{\pi^+}^2}{2(m_u+m_d)} = \frac{F_\pi^2 m_{K^+}^2}{2(m_u+m_s)} = \frac{F_\pi^2 m_{K^0}^2}{2(m_d+m_s)} . \quad (4)$$

The first term contains kinetic energy terms and the quartic and higher-order couplings of the pion octet. The fact that the SU(3) chiral symmetry is spontaneously broken is embedded in the nonlinear realization of the chiral symmetry.[2] Since $SU(3)_L \times SU(3)_R$ is spontaneously broken into $SU(3)_V$, v develops a nonvanishing value, and the field $U(x)$ transforms under $SU(3)_L \times SU(3)_R$ as

$$U(x) \to LU(x)R^{-1} . \quad (5)$$

The Lagrangian of Eq. (1) gives a satisfactory explanation of $\pi\pi$ scattering, for example, and reproduces the Gell-Mann–Okubo formula.

The above Lagrangian $\mathcal{L}_2$ is not renormalizable and we need to add counterterms when we perform the one-loop corrections to the tree amplitudes obtained from $\mathcal{L}_2$. The one-loop counterterms give a new Lagrangian $\mathcal{L}_4$ of $O(p^4)$, obtained in Ref. 6. This Lagrangian $\mathcal{L}_4$ contains ten parameters which cannot be determined by chiral symmetry alone, and should be determined by experiments.

The electromagnetic interactions can be easily accommodated by gauging the U(1) symmetry of the $\mathcal{L}_2$, or simply replacing the plain derivative ∂_μ by a covariant derivative D_μ which acts as on the $U(x)$ as

$$D_\mu U = \partial_\mu U - ieA_\mu[Q,U] , \quad (6)$$

where Q is the electric-charge operator,

$$Q = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} . \quad (7)$$

The weak interactions are introduced as a current-current interaction. In the following discussions, we need only the $\Delta S=1$ part of the weak interactions. So, we concentrate only on the $\Delta S=1$ part. The $V-A$ Noether current for $\mathcal{L}_2$ is

$$L_\mu = iF_\pi^2 U\,\partial_\mu U^\dagger . \quad (8)$$

By including a term which transforms as $8_L \otimes 1_R$ under chiral-SU(3) transformations (octet enhancement), we can write the weak-interaction part as

$$\mathcal{L}_{2,\Delta S=1} = g_8(L_\mu L^\mu)_{23} + \text{H.c.} + \text{higher-order terms} . \quad (9)$$

We determine $|g_8| \simeq 5.1$ from $K \to \pi\pi$ decays.

Now, we list the terms of $O(p^4)$ in the total Lagrangian describing strong, weak, and electromagnetic interactions of pions and kaons which are relevant to our discussions of $\gamma\gamma \to \pi^0\pi^0$ and $K_L \to \pi^0\gamma\gamma$. For $\gamma\gamma \to \pi^0\pi^0$, we need

$$\mathcal{L}_4 = -ieL_9[\text{tr}(F_{\mu\nu}D^\mu U\, D^\nu U^\dagger) + \text{tr}(F_{\mu\nu}D^\mu U^\dagger D^\nu U)] + e^2 L_{10}\text{tr}(F_{\mu\nu}U\,F^{\mu\nu}U^\dagger) . \quad (10)$$

For $K_L \to \pi^0\gamma\gamma$, we need

$$\mathcal{L}_{4,\Delta S=1}^{\text{em}} = -\frac{ieg_8}{2F_\pi^2}F^{\mu\nu}[\omega_1\text{tr}(Q\lambda_{6-i7}L_\mu L_\nu) + \omega_2\text{tr}(QL_\mu\lambda_{6-i7}L_\nu)] + \frac{e^2F_\pi^2}{2}g_8\omega_4 F^{\mu\nu}F_{\mu\nu}\text{tr}(\lambda_{6-i7}QUQU^\dagger) + \text{H.c.} \quad (11)$$

The above Lagrangian is the base of the standard ChPT up to $O(p^4)$. To this chiral Lagrangian, we add an interaction Lagrangian describing the vector-meson coupling to the pion octet and a photon assuming Zweig's rule and C and P conservation.[14,16] The lowest-order couplings are supposed to be given by

$$\mathcal{L}_{V\pi\gamma} = \frac{e}{\Lambda}\bar{a}\,\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}\text{tr}(\pi\{Q,\partial^\alpha V^\beta\}) , \quad (12)$$

$$\pi = \frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i\phi_i , \quad (13)$$

where $\bar{a}=1.27$ is fixed by the electromagnetic decays of the vector mesons, with $\Lambda=1$ GeV, e.g., from $\omega\to\pi^0\gamma$. This term gives $O(p^6)$ contributions to the two-photon process we are going to discuss below. So, it should be considered with the $O(p^6)$ terms in the chiral Lagrangian, in principle. The relevant $O(p^6)$ terms are

$$\frac{G_8\omega_1}{(16\pi^2F_\pi^2)^2}e^2F^{\mu\rho}F^{\nu}_{\rho}\mathrm{tr}(Q\lambda_{6-i7}\{L_\mu,L_\nu\}UQU^\dagger)+\mathrm{H.c.}\ , \tag{14}$$

$$\frac{G_8\omega_2}{(16\pi^2F_\pi^2)^2}e^2F^{\mu\rho}F^{\nu}_{\rho}\mathrm{tr}(UQU^\dagger\lambda_{6-i7}U\partial_{\{\mu}U^\dagger)\mathrm{tr}(QU\partial_{\nu\}}U^\dagger)+\mathrm{H.c.} \tag{15}$$

Here, $G_8=G_Fg_8\sin\theta_C/\sqrt{2}$, and θ_C is the Cabibbo angle. The parameters $(4\pi)^2\omega_1$ and $(4\pi)^2\omega_2$ would be ~ 1 from dimensional analysis. The point is that the contributions of the vector mesons to two-photon processes, which are $O(p^6)$, are comparable to that of the one-loop contributions from ChPT which is $O(p^4)$. The $O(p^6)$ terms in the chiral Lagrangian shown in Eqs. (14) and (15) can be safely neglected, as discussed in Ref. 10.

This sounds a bit disturbing, because the systematic derivative expansions of the chiral Lagrangian seems to work well in many examples. Our impression is that we better be careful when we consider any processes of pions and kaons involving electromagnetic interactions above $\sim$400 MeV and that we may not be able to simply discard the role of the vector mesons even in the energy range of the kaon mass. This comes from our calculations of the reaction cross section for the $\gamma\gamma\to\pi^0\pi^0$ process. We are still lacking some systematic way to incorporate the effect of the vector mesons into dynamics of the lowest-lying pseudoscalar mesons.

We use the above Lagrangians to study the two processes mentioned before. The reason we are looking at those processes is that the leading-order contributions to those processes in standard ChPT are $O(p^4)$, so that we can see the effects of the vector mesons, which are $O(p^6)$. If we consider, for example, the $\gamma\gamma\to\pi^+\pi^-$ process, the leading-order contribution comes from the $O(p^2)$ part of the chiral Lagrangian because the charged pions can directly couple to the photons without any loops. This may be an obstacle for us to see clean effects of the vector mesons, especially in experiments. For other process such as $K^+\to\pi^+\gamma\gamma$, the situation is worse than in $\gamma\gamma\to\pi^+\pi^-$, because the $O(p^4)$ chiral amplitude contains an arbitrary parameter which has not been fixed yet.[10] Therefore, we cannot clearly talk about the effect of the vector mesons.

Before closing this section, we give a general form of the amplitude for two-photon processes. The kinematical variables are defined as follow: k_1 and k_2 are four-momenta of the incoming or outgoing photons, P is the four-momentum of one of the final pions in $\gamma\gamma\to\pi\pi$, and the four-momentum of the initial kaon in $K\to\pi\gamma\gamma$, respectively. In case all the particles are on the mass shell, Bose symmetry and gauge invariance require the general form for the two-photon process to be given by[10]

$$\mathcal{A}(\gamma\gamma\to\pi\pi)=\epsilon_{1\mu}\epsilon_{2\nu}T^{\mu\nu}(k_1,k_2,P)\ , \tag{16}$$

and

$$\mathcal{A}(K\to\pi\gamma\gamma)=\epsilon^*_{1\mu}\epsilon^*_{2\nu}T^{\mu\nu}(k_1,k_2,P)\ , \tag{17}$$

$$\begin{aligned}T_{\mu\nu}(k_1,k_2,P)=&A(P\cdot k_1,P\cdot k_2)(k_{2\mu}k_{1\nu}-k_1\cdot k_2g_{\mu\nu})\\&+B(P\cdot k_1,P\cdot k_2)\left[\frac{P\cdot k_1P\cdot k_2}{k_1\cdot k_2}g_{\mu\nu}+P_\mu P_\nu-\frac{P\cdot k_1}{k_1\cdot k_2}k_{2\mu}P_\nu-\frac{P\cdot k_2}{k_1\cdot k_2}k_{1\nu}P_\mu\right]\\&+C(P\cdot k_1,P\cdot k_2)\epsilon_{\mu\nu\rho\sigma}k^{1\rho}k^{2\sigma}\ ,\end{aligned} \tag{18}$$

where $A(P\cdot k_1,P\cdot k_2)$, $B(P\cdot k_1,P\cdot k_2)$, and $C(P\cdot k_1,P\cdot k_2)$ are symmetric functions with respect to their two arguments. We note that the form factors A, B, and C describe two-photon states in total angular momentum $J=0$, 2, and 1, respectively.

Note that only A and B amplitudes contribute to $\gamma\gamma\to\pi\pi$ because parity is conserved in the strong and electromagnetic interactions. In the limit of exact CP symmetry, only A and B are relevant to $K_L\to\pi^0\gamma\gamma$. For $K^+\to\pi^+\gamma\gamma$, all the three amplitudes, A, B, and C should be included.

III. $\gamma\gamma\to\pi^0\pi^0,\pi^+\pi^-$

Two-photon production of pions, i.e., $\gamma\gamma\to\pi^+\pi^-$ and $\gamma\gamma\to\pi^0\pi^0$, has been measured up to $m_{\gamma\gamma}\sim 2$ GeV at e^+e^- colliders.[9] For the $\gamma\gamma\to\pi^0\pi^0$ process, the experimental data in the energy range between 500 MeV and 1 GeV are very different from the prediction of the standard ChPT up to $O(p^4)$ (Ref. 9). To cure this disaster, the authors of Refs. 7 and 8 suggested unitarization of the chiral amplitude by hand. This kind of unitarization, however, was done a long time ago and has been revived to explain the experimental data.[17] The latter approach takes the rescattering of pions into account, and gives a good account of the energy dependences of the reaction cross sections for $\gamma\gamma\to\pi^+\pi^-$ and $\gamma\gamma\to\pi^0\pi^0$ up to some polynomial ambiguities in the amplitude, which can be regarded as negligible without loss of consistency with other experiments such as $K_S\to\gamma\gamma$.

Here, we suggest another explanation for the smooth behavior of the reaction cross section for $\gamma\gamma\to\pi^0\pi^0$ above $\sim$400 MeV. We include the vector-meson contributions to this process as well as the lowest-order contributions from ChPT. The reason that we consider $\gamma\gamma\to\pi^0\pi^0$ was discussed in the previous section. The interaction Lagrangian for the vector-meson coupling to the pion octet and a photon and the general structure of the invariant matrix element for a two-photon process were discussed in Sec. II.

The chiral Lagrangian, Eq. (1) with ∂_μ replaced by D_μ, gives four different Feynman diagrams [see Figs. 1(a)–1(d).] They produce two photons in a $J=0$ final state, so Eq. (1) contributes only to the A part in the invariant amplitude in Eq. (18), and the amplitude $A_{\rm ch}$ depends only on $s=m_{\gamma\gamma}^2$. (k's are the four-momenta of photons, and p's are those of pions.) Since we are working above the pion threshold, $A_{\rm ch}$ can be written as

$$A_{\rm ch}=A_{\rm ch}^{(\pi)}+A_{\rm ch}^{(K)}\ , \tag{19}$$

$$A_{\rm ch}^{(\pi)}=\frac{e^2}{16\pi^2F_\pi^2}\frac{4(s-m_{\pi^+}^2)}{s}F\left[\frac{s}{m_{\pi^+}^2}\right]\ , \tag{20}$$

$$A_{\rm ch}^{(K)}=\frac{e^2}{16\pi^2F_\pi^2}F\left[\frac{s}{m_{K^+}^2}\right]\ , \tag{21}$$

where

$$F(z)=\begin{cases}1-\dfrac{4}{z}\left[\arcsin\left[\dfrac{\sqrt{z}}{2}\right]\right]^2 & (z\le 4)\ ,\\[2ex] 1+\dfrac{1}{z}\left[\ln\left[\dfrac{1+\sqrt{1-4/z}}{1-\sqrt{1-4/z}}\right]-i\pi\right]^2 & (z\ge 4)\ .\end{cases} \tag{22}$$

For simplicity, we will consider the energy range below the kaon pair threshold, i.e., $\sqrt{s}\sim$900 MeV.

The vector mesons give another set of diagrams shown in Fig. 1(e). Here, the internal vector meson is either ρ^0 or ω. These diagrams produce two photons in both $J=0$ and 2 amplitudes, i.e., both A_V and B_V are nonvanishing. Therefore, angular dependence in the differential cross section occurs. The invariant amplitude arising from vector-meson exchange can be expressed as

$$A_V=-\sum_{V=\rho^0,\omega}\frac{\tilde{G}_V}{2}\left[\frac{m_{\pi^+}^2+t}{t-m_V^2}+\frac{m_{\pi^+}^2+u}{u-m_V^2}\right]\ , \tag{23}$$

$$B_V=-\sum_{V=\rho^0,\omega}\tilde{G}_V\frac{s}{2}\left[\frac{1}{t-m_V^2}+\frac{1}{u-m_V^2}\right]\ , \tag{24}$$

where $\tilde{G}_{\rho^0}=\frac{1}{9}g_{\omega\pi^0\gamma}^2$, $\tilde{G}_\omega=g_{\omega\pi^0\gamma}^2$, and $g_{\omega\pi^0\gamma}=7.70\times10^{-4}$ MeV^{-1}, which is fixed from the radiative decay of the ω meson,[12–14,16] $\omega\to\pi^0\gamma$. ($g_{\omega\pi^0\gamma}=2e\bar{a}/\Lambda$.) In this case, $C_{\rm ch}=C_V=0$. Here, we note that there is no sign ambiguity in Eqs. (23) and (24).

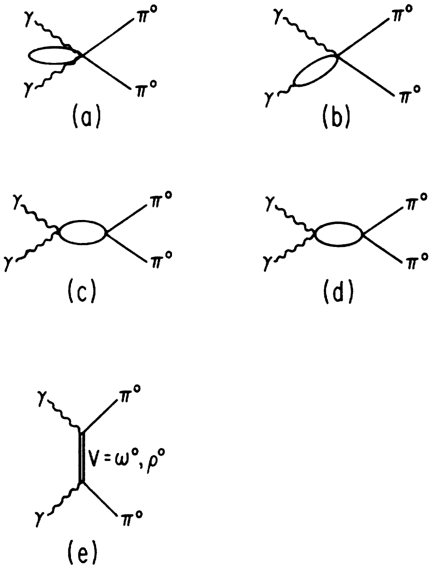

FIG. 1. Feynman diagrams relevant for $\gamma\gamma\to\pi^0\pi^0$.

Here s, t, and u are the Mandelstam variables. In our case,

$$\begin{aligned}s&=(k_1+k_2)^2=2k_1\cdot k_2\ ,\\ t&=(k_1-p_1)^2=m_{\pi^+}^2-2k_1\cdot p_1\ ,\\ u&=(k_1-p_2)^2=m_{\pi^+}^2-2k_1\cdot p_2\ .\end{aligned} \tag{25}$$

To obtain the total cross section, we add all the amplitudes obtained above, take the absolute square of the sum of the amplitudes, average over the polarizations of the initial photons, and integrate over phase space. The final form is

$$\sigma(s)=\frac{1/2}{128\pi s^2}\int_{t_0}^{t_1}dt\left[|As-m_{\pi^+}^2B|^2+\frac{|B|^2}{s^2}(m_{\pi^+}^4-tu)^2\right]\ , \tag{26}$$

$$t_{0,1}=-\tfrac{1}{2}\{(s-2m_{\pi^+}^2)\mp[s(s-4m_{\pi^+}^2)]^{1/2}\}\ . \tag{27}$$

The result is shown in Fig. 2(a) where it is compared with the predictions based on ChPT and the vector-meson-dominance model. It is compared in Fig. 2(b) with the experimental data. We can see that inclusion of the vector mesons modifies the ChPT prediction in a significant way above $\sqrt{s}\sim$400 MeV, and makes it unitary in a simple way. The theoretical prediction agrees with the data up to $\sqrt{s}\sim$900 MeV except near threshold. This seems to be reasonable, since we included the particles with masses around the energy range we are interested in. It should improve the unitarity of ChPT.

Different approaches show different threshold behavior for pion production. Ref. 17 gives $(3+\pi^2/4)^2$, Ref. 8 gives $(-1+\pi^2/4)^2$, and ours gives $(-1+\pi^2/4+0.028+0.095)^2$, with a common factor

$$\frac{\alpha^2}{256\pi^3 s F_\pi^4}\left[1-\frac{4m_{\pi^+}^2}{s}\right]^{1/2}(s-m_{\pi^+}^2)^2 . \tag{28}$$

$(-1+\pi^2/4)$ comes from the pion loop, 0.028 from the kaon loop, and 0.095 from the vector mesons in our case. Note that even at the threshold for two-pion production, the vector mesons give a larger enhancement than the kaon does. Actually, the ω meson gives a more dominant contribution to the electromagnetic processes of the chiral mesons than the ρ meson.

It would be interesting to apply our method to the photoproduction of charged pions, i.e., $\gamma\gamma\to\pi^+\pi^-$. In this case, direct couplings of the charged pions to the photons give the leading contributions, and the chiral loop and the vector meson give the next-order contributions. Borrowing the chiral amplitude for $\gamma\gamma\to\pi^+\pi^-$ from Ref. 7 and adding the vector-meson contribution, we get

$$A_{\rm ch}=\frac{-4(a-1)e^2}{s} , \tag{29}$$

$$B_{\rm ch}=4e^2\left[\frac{1}{m_{\pi^+}^2-t}+\frac{1}{m_{\pi^+}^2-u}\right] , \tag{30}$$

$$A_V=-\frac{\tilde{G}_\rho}{2}\left[\frac{m_{\pi^+}^2+t}{t-m_\rho^2}+\frac{m_{\pi^+}^2+u}{u-m_\rho^2}\right] , \tag{31}$$

$$B_V=-\tilde{G}_\rho\frac{s}{2}\left[\frac{1}{t-m_\rho^2}+\frac{1}{u-m_\rho^2}\right] , \tag{32}$$

where

$$a=1+\frac{2s}{F_\pi^2}(L_9^r+L_{10}^r)-\frac{s}{32\pi^2F_\pi^2}\left[F\left[\frac{s}{m_{\pi^+}^2}\right]+\frac{1}{2}F\left[\frac{s}{m_{K^+}^2}\right]\right] , \tag{33}$$

$$L_9^r+L_{10}^r=1.4\times10^{-3} \tag{34}$$

from $\pi\to e\nu\gamma$ (Refs. 7 and 10). Note that only ρ^+ contributes to this process. The reaction cross section is given by Eqs. (26) and (27) again, and the resulting reaction cross section for $\gamma\gamma\to\pi^+\pi^-$ is shown in Fig. 3.

As we can expect, the role of the vector meson is negligible because of the direct coupling of the charged pions to the photons. Our result is consistent with the predictions based on standard ChPT (Ref. 7) and the pion-rescattering model.[17]

Another interesting thing to look at is the process $K_S\to\gamma\gamma$. The ChPT (Ref. 10) and the pion-rescattering model[17] tell us that its branching ratio is 2.0×10^{-6}, and

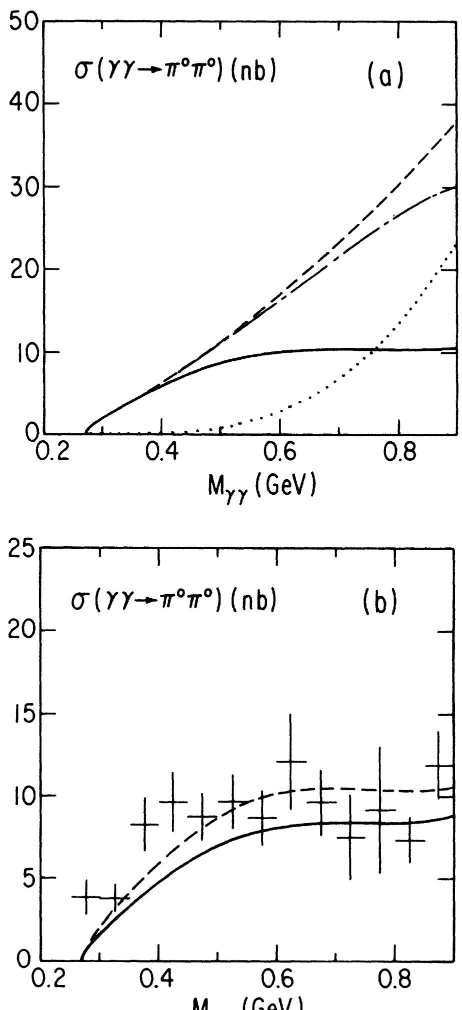

FIG. 2. The reaction cross section for $\gamma\gamma\to\pi^0\pi^0$. (a) the comparison of various predictions: ChPT with the pion loop only (dashed), ChPT with both pion and kaon loops (dash-dotted), vector meson only (dotted), both the chiral loops and the vector mesons (solid); (b) comparison of the data $|\cos\theta_{\rm c.m.}|\le0.8$ with our predictions: full solid angle (dashed) and $|\cos\theta_{\rm c.m.}|\le0.8$ (solid).

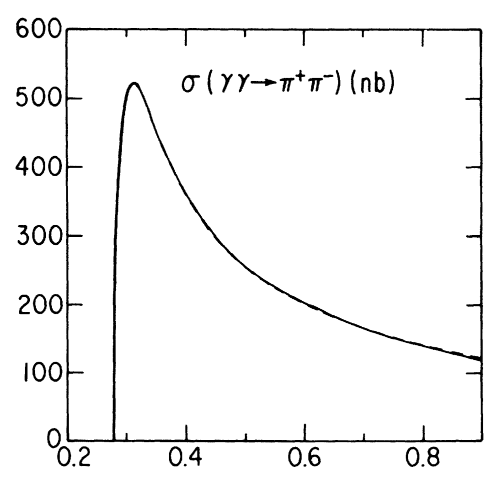

FIG. 3. The reaction cross section for $\gamma\gamma\to\pi^+\pi^-$: prediction by ChPT up to $O(p^4)$ (dashed) and our result (solid).

it agrees very well with the measured value,[11] $(2.4\pm1.2)\times10^{-6}$. Indeed, the charged-meson loop gives the leading contribution and our investigation of $\gamma\gamma\to\pi^+\pi^-$ suggests that the role of the vector mesons should be negligible in this case, too. In fact, the term B_V cannot come in because of angular momentum conservation. Note that B_V is a $J=2$ amplitude, and that K_S is spinless. Also, we have to include other possible intermediate states such as $K_S\to\pi^+\pi^-$ or K^+K^-, in which charged mesons exchange ρ or K^* with subsequent emission of two photons. We expect that such contributions will not be large due to cancellation between various intermediate states, but we need more work to get a definite conclusion.

IV. $K_L\to\pi^0\gamma\gamma$ AND $K^+\to\pi^+\gamma\gamma$

Recently, the $K_L\to\pi^0e^+e^-$ decay mode has drawn much attention from both theoreticians and experimentalists.[18] The reason is that it may be a place where we can observe direct CP violation. That process can arise only through two-photon exchange $[O(G_F\alpha^2)]$ in the limit of exact CP symmetry. In the real world, CP symmetry is slightly violated, and a one-photon intermediate state can contribute to the $K_L\to\pi^0e^+e^-$ decay mode. This one-photon-exchange process $[O(\epsilon G_F\alpha)]$ is CP violating. Because the CP-violating parameter ϵ is $\sim10^{-3}$, it is not certain which of the one- and two-photon-exchange diagrams is more important in the actual decay process. If the CP-violating part is appreciably larger than the other, then we can observe direct CP violation, which is quite different from indirect CP violation in the $K_L\to2\pi$ decay, which occurs through the mixing of K_L and K_S.

The standard chiral perturbation theory predicts that the CP-conserving amplitude is much smaller than the CP-violating amplitude up to $O(p^4)$ (Ref. 10). But some other groups have claimed that the vector mesons gave a large contribution to the two-photon process.[12,13] To settle this issue, it will be helpful to study the $K_L\to\pi^0\gamma\gamma$ decay mode. The branching ratio of this decay mode has been calculated by Ecker, Pich, and de Rafael[10] using ChPT up to $O(p^4)$ without any ambiguities. It has been also calculated using the pion-rescattering model,[15] and the result is rather consistent with the prediction based on ChPT. The two approaches predict that $B(K_L\to\pi^0\gamma\gamma)=6.3$ and 7.5×10^{-7}, respectively. This is smaller than the prediction based on the vector-meson-dominance model, which is about 10^{-6} (Refs. 12 and 13).

The experiment to measure $K_L\to\pi^0\gamma\gamma$ decay is very difficult because of the huge background from $K_L\to3\pi$ followed by the successive decays of π^0 into photons, and we have only an upper limit of the branching ratio. Recently, the sensitivity of searches for $K_L\to\pi^0\gamma\gamma$ has been improved, and we can have a better upper limit on the branching ratio.[19] The measurement of the branching ratio and the spectrum of two photons will be important to see how large the vector-meson contributions are.

In this section, we investigate the spectrum of two final photons and the branching ratio of the decay mode $K_L\to\pi^0\gamma\gamma$, including both chiral loops and vector mesons. The chiral bosons give the following amplitude corresponding to the Feynman diagrams shown in Figs. 4(a)–4(d):

$$A_{\rm ch}=\frac{G_8\alpha}{\pi}\left[\left(1-\frac{m_{\pi^+}^2}{s}\right)F\left(\frac{s}{m_{\pi^+}^2}\right)-\left(1-\frac{m_{K^+}^2}{s}-\frac{m_{\pi^+}^2}{s}\right)F\left(\frac{s}{m_{K^+}^2}\right)\right], \quad (35)$$

where $|G_8|\alpha m_{K_L}^2/\pi=0.524\times10^{-8}$, and

$$s=(P_K-p_{\pi_0})^2=(k_1+k_2)^2=2k_1\cdot k_2\ ,$$
$$t=(P_K-k_1)^2=m_{K^0}^2-2P\cdot k_1\ , \quad (36)$$
$$u=(P_K-k_2)^2=m_{K^0}^2-2P\cdot k_2\ .$$

Here, the function $F(z)$ is the same as Eq. (22) which appeared in the discussion of the $\gamma\gamma\to\pi^0\pi^0$ process.

The vector meson ρ^0 or ω generate another set of diagrams shown in Fig. 4(e), and the corresponding amplitudes are

$$A_V=-\sum_{V=\rho^0,\omega}G_V\left[\frac{P\cdot(P-k_2)}{(P-k_2)^2-m_V^2}+\frac{P\cdot(P-k_1)}{(P-k_1)^2-m_V^2}\right], \quad (37)$$

$$B_V=-\sum_{V=\rho^0,\omega}G_V\left[\frac{k_1\cdot k_2}{(P-k_2)^2-m_V^2}+\frac{k_1\cdot k_2}{(P-k_1)^2-m_V^2}\right], \quad (38)$$

or, in terms of the Mandelstam variables,

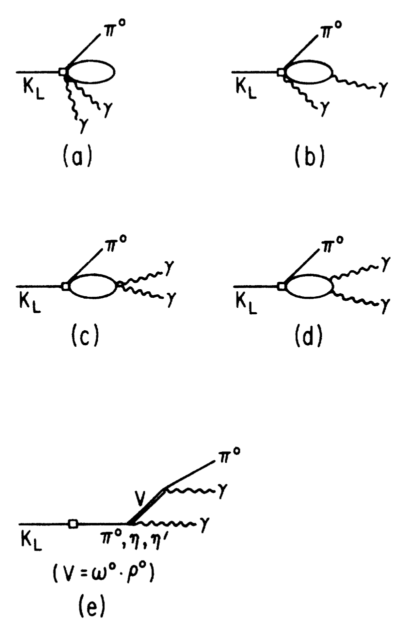

FIG. 4. Feynman diagrams relevant for $K_L\to\pi^0\gamma\gamma$.

$$A_V=-\sum_{V=\rho^0,\omega} G_V\left[1+\frac{m_{K_L}^2+m_V^2}{2}\times\left[\frac{1}{t-m_V^2}+\frac{1}{u-m_V^2}\right]\right], \quad (39)$$

$$B_V=-\sum_{V=\rho^0,\omega}\frac{G_V}{2}s\left[\frac{1}{t-m_V^2}+\frac{1}{u-m_V^2}\right]. \quad (40)$$

The coupling constants, G_V's, are of the form

$$G_\rho m_{K_L}^2=g_{\omega\pi^0\gamma}^2\langle\pi^0|H_w|K_2\rangle\left[\frac{1}{9}\frac{1}{1-r_\pi^2}+\tfrac{1}{9}(\sqrt{3}\cos\theta_p-\sqrt{6}\sin\theta_p)\left[\frac{1}{\sqrt{3}}\cos\theta_p+\frac{4}{\sqrt{6}}\sin\theta_p\right]\frac{1}{1-r_\eta^2}+\tfrac{1}{9}(\sqrt{3}\sin\theta_p+\sqrt{6}\cos\theta_p)\left[\frac{1}{\sqrt{3}}\cos\theta_p-\frac{4}{\sqrt{6}}\sin\theta_p\right]\frac{1}{1-r_{\eta'}^2}\right], \quad (41)$$

$$G_\omega m_{K_L}^2=g_{\omega\pi^0\gamma}^2\langle\pi^0|H_w|K_2\rangle\left[\frac{1}{1-r_\pi^2}+\tfrac{1}{9}(\sqrt{3}\cos\theta_p-\sqrt{6}\sin\theta_p)\left[\frac{1}{\sqrt{3}}\cos\theta_p+\frac{4}{\sqrt{6}}\sin\theta_p\right]\frac{1}{1-r_\eta^2}+\tfrac{1}{9}(\sqrt{3}\sin\theta_p+\sqrt{6}\cos\theta_p)\left[\frac{1}{\sqrt{3}}\cos\theta_p-\frac{4}{\sqrt{6}}\sin\theta_p\right]\frac{1}{1-r_{\eta'}^2}\right], \quad (42)$$

where $r_P^2=m_P^2/m_{K_L}^2$, and we used[20]

$$\langle\pi^0|H_w|K_2\rangle=\sqrt{3}\langle\eta_8|H_w|K_2\rangle=-\frac{\sqrt{6}}{4}\langle\eta_0|H_w|K_2\rangle\ . \quad (43)$$

Some uncertainty comes in through the matrix element, $\langle\pi^0|H_w|K_2\rangle$. There are discrepancies in its value in the literature. In this paper, we use two values for it: -4.03×10^{-2} MeV^2 and -2.88×10^{-2} MeV^2. The former was obtained through PCAC (partial conservation of axial-vector current) and the soft-pion reduction, while the latter was obtained by properly implementing unitarity.[21] The mixing angle θ_p is taken to be $-20°$. Then, the couplings G_V's are fixed to be $G_\rho m_{K_L}^2=(-0.63,-0.45)\times10^{-8}$ and $G_\omega m_{K_L}^2=(-2.92,-2.09)\times10^{-8}$, for $\langle\pi^0|H_\omega|K_2\rangle=(-4.03,-2.88)\times10^{-2}$ MeV^2, respectively.

Also, note that the sign of the G_8 is not fixed from the measurement of the $K\to2\pi$. Therefore, we have two possibilities for choice of the sign of G_8, even in the limit of exact CP symmetry. Since G_8 is determined from $K\to2\pi$, it has a small imaginary part in it which corresponds to CP violation in the kaon system, if G_8 is assumed to be almost real. Here, we assume exact CP symmetry, so that G_8 is purely real.

The decay rate $\Gamma(K_L\to\pi^0\gamma\gamma)$ is obtained by integrating the following expression over s:

$$\frac{d\Gamma}{ds}=\frac{1/2}{256m_{K_L}^3\pi^3}\int_{t_0}^{t_1}dt\left[|As-m_{K_L}^2B|^2+\frac{|B|^2}{s^2}(m_{\pi^0}^2m_{K_L}^2-tu)^2\right], \quad (44)$$

where the limits of the integration are

$$t_0\le t\le t_1,\quad 0\le s\le(m_{K_L}-m_{\pi^0})^2\ , \quad (45)$$

$$t_{1,0}=\tfrac{1}{2}\{(m_{K_L}^2+m_{\pi^0}^2-s)\pm[(m_{K_L}^2+m_{\pi^0}^2-s)^2-4m_{\pi^0}^2m_{K_L}^2]^{1/2}\}\ . \quad (46)$$

Here, $A=A_{\rm ch}+A_V$ and $B=B_V$. Note that $d\Gamma/ds$ gives the spectrum of two photons.

After numerical integration with the above parameters chosen, we find the spectra shown in Fig. 5, which shows predictions of both the ChPT (dashed) and the vector-meson-dominance model (dotted) to be compared with our results (solid). (The spectra shown in Fig. 5 are for the second choice of the parameters. For the first choice, the shapes remain almost the same with a slightly different scale.) (See Table I.) First of all, note that three predictions give very different spectra. In particular, vector mesons give rise mainly to two photons with low $m_{\gamma\gamma}^2$. On the contrary, the chiral mesons produce two photons with relatively higher $m_{\gamma\gamma}^2$. The latter corresponds to soft-pion production. From Fig. 5(a) and 5(b), it should be clear that the measurement of the two-photon spectrum at low $m_{\gamma\gamma}^2$ can settle the issue of whether or not the vector mesons are important in $K_L\to\pi^0\gamma\gamma$ and $K_L\to\pi^0e^+e^-$. However, note that the two-photon spectrum for low $m_{\gamma\gamma}^2$ is rather insensitive to the sign of G_8. The branching ratio for this decay process is $(3.6,2.3)\times10^{-6}$ for positive G_8, and $(1.9,1.1)\times10^{-6}$ for negative G_8, for $\langle\pi^0|H_w|K_2\rangle=(-4.03,-2.88)\times10^{-2}$ MeV^2. The ChPT and the pion-rescattering model predict the branching ratio to be 6.3 and 7.5×10^{-7}, so that the vector mesons give a considerably enhanced branching ratio. If the branching ratio and the spectrum of two final photons are measured more accurately, we can tell how important a role the vector mesons play in $K_L\to\pi^0\gamma\gamma$. The present upper limit to the branching ratio is 2.7×10^{-6} assuming the spectrum follows the ChPT prediction, and 4.4×10^{-6} assuming the spectrum follows phase space.[19]

We conclude that the measurement of the spectrum of

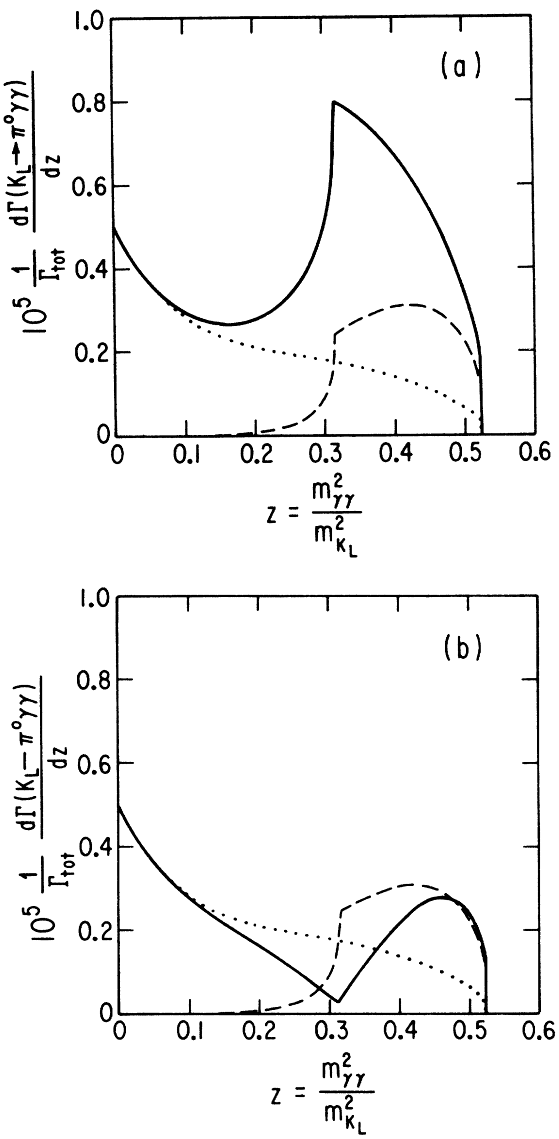

FIG. 5. The spectrum of two photons emerging from $K_L \to \pi^0\gamma\gamma$: (a) positive G_8, (b) negative G_8, with the predictions by ChPT (dashed) and VDM (dotted).

two photons from $K_L \to \pi^0\gamma\gamma$ is important to know the importance of the vector mesons in that decay and $K_L \to \pi^0 e^+ e^-$. Even though we have indirect evidence for their important role from $\gamma\gamma \to \pi^0\pi^0$, we have to await more refined measurements of the spectrum of two photons and the branching ratio for more direct and definite conclusions.

TABLE I. Comparison of three predictions for $10^6 B(K_L \to \pi^0\gamma\gamma)$ for two different values of $\langle \pi^0 | H_w | K_2 \rangle$: (a) -4.03×10^{-2} MeV^2, (b) -2.88×10^{-2} MeV^2. (See also Fig. 5.)

	Positive G_8	Negative G_8
(a)	3.6	1.9
(b)	2.3	1.1
Ref. 22	3.9	1.8

Finally, we consider $K^+ \to \pi^+\gamma\gamma$. This process was fully discussed in Ref. 10 in the framework of the standard ChPT. The chiral amplitude for this process contains a constant which has not been fixed yet by any experiment, so that the prediction of the branching ratio is not definite. Here, we content ourselves with observing the change of the spectrum of two final photons by adding the vector-meson contribution to the chiral amplitude.

The ChPT yields A_{ch} and C_{ch} amplitudes,[10] while the vector mesons generate A_V and B_V amplitudes. The results are

$$A_{ch} = \frac{G_8\alpha}{2\pi}\left[\left(\frac{m_\pi^2}{s} - \frac{m_{K^+}^2}{s} - 1\right)F\left(\frac{s}{m_{\pi^+}^2}\right) + \left(\frac{m_{K^+}^2}{s} - 1 - \frac{m_\pi^2}{s}\right)F\left(\frac{s}{m_{K^+}^2}\right) + \hat{c}\right], \quad (47)$$

$$A_V = -G_\rho\left[1 + \frac{m_{K^+}^2 + m_\rho^2}{2}\left(\frac{1}{t-m_\rho^2} + \frac{1}{u-m_\rho^2}\right)\right], \quad (48)$$

$$B_V = -\frac{G_\rho}{2}s\left[\frac{1}{t-m_\rho^2} + \frac{1}{u-m_\rho^2}\right], \quad (49)$$

$$C_{ch} = \frac{G_8\alpha}{\pi}\left[\frac{s-m_{\pi^0}^2}{s-m_{\pi^0}^2 + i m_{\pi^0}\Gamma_{\pi^0}} - \frac{s - \dfrac{2m_{K^+}^2 + m_{\pi^0}^2}{3m_{K^+}^2}}{s-m_\eta^2}\right], \quad (50)$$

where

$$G_\rho m_{K^+}^2 = g_{\omega\pi^0\gamma}^2 \langle \pi^+ | H_w | K^+ \rangle \frac{1}{9}\frac{1}{1-r_\pi^2} . \quad (51)$$

Note that only ρ^+ contributes to this process. If we use $\langle \pi^+ | H_w | K^+ \rangle = -\langle \pi^0 | H_w | K_2 \rangle = 2.88\times10^{-2}$, we get $G_\rho m_{K^+}^2 = 0.20\times10^{-8}$. For different choice of $\langle \pi^0 | H_w | K_2 \rangle$, the branching ratio varies within 5%, so we restrict ourselves to the above choice of $\langle \pi^0 | H_w | K_2 \rangle$. Here, $\hat{c}$ is a constant of ~ 1 arising from the $O(p^4)$ chiral Lagrangian and has not been fixed yet by any experiments. We take $\hat{c}$ equal to 0, 4, and -4, as chosen in Ref. 10.

The decay rate $\Gamma(K^+ \to \pi^+\gamma\gamma)$ is obtained by integrating the following expression over s:

TABLE II. $10^7 B(K^+ \to \pi^+\gamma\gamma)$. (See also Fig. 6.)

	ChPT only (dashed)	ChPT plus VDM Positive G_8 (solid)	ChPT plus VDM Negative G_8 (dotted)
$\hat{c}=0$	5.8	5.6	6.0
$\hat{c}=4$	17.6	17.0	18.2
$\hat{c}=-4$	4.4	4.6	4.3

$$\frac{d\Gamma}{ds}=\frac{1/2}{256m_{K^+}^3\pi^3}\int_{t_0}^{t_1}dt\left[|As-m_{K^+}^2B|^2+\frac{|B|^2}{s^2}(m_{\pi^+}^2m_{K^+}^2-tu)^2+|C|^2\right], \tag{52}$$

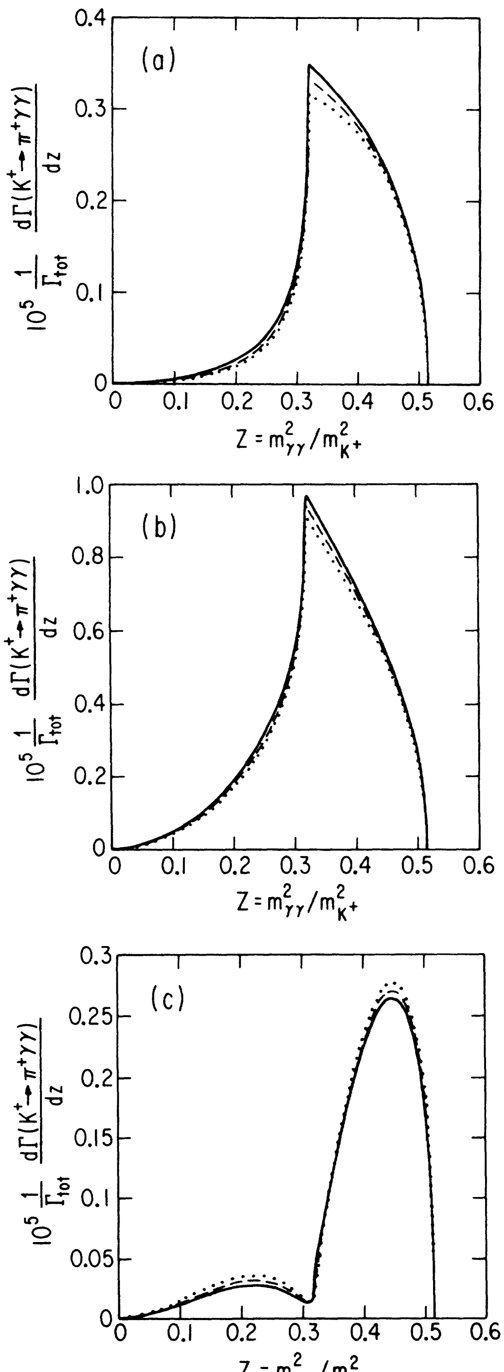

FIG. 6. The spectrum of two photons emerging from $K^+\to\pi^+\gamma\gamma$. ChPT only (dashed), our approaches with positive G_8 (solid) and negative G_8 (dotted). (a) $\hat{c}=0$, (b) $\hat{c}=4$, and (c) $\hat{c}=-4$.

Note that the C term does not interfere with A and B terms because of the Bose symmetry of two photons. The limits of the integration are

$$t_0\le t\le t_1,\quad 0\le s\le(m_{K^+}-m_{\pi^+})^2\ , \tag{53}$$

$$t_{1,0}=\tfrac{1}{2}\{(m_{K^+}^2+m_{\pi^+}^2-s)\pm[(m_{K^+}^2+m_{\pi^+}^2-s)^2-4m_{\pi^+}^2m_{K^+}^2]^{1/2}\}\ . \tag{54}$$

The branching ratio is shown in Table II, and it is around 0.4 to 1.8×10^{-6}, which is below the current upper limit 8×10^{-6} quoted in the Particle Data Group Book. Because of uncertainties in $\hat{c}$ and the relative phase of G_8 and G_ρ, it is very difficult to tell the effect of the vector meson from that of the chiral mesons. The low $m_{\gamma\gamma}$ part of the two-photon spectrum shown in Fig. 6 can be used to do that. However, it is a challenging task to the experimentalists, as in the case of $K_L\to\pi^0\gamma\gamma$. This decay mode is being measured along with other rare-kaon decays.

In conclusion, we find that vector mesons affect the chiral meson dynamics in the presence of electromagnetic coupling even at the energy range around m_K, and above it. They control the bad high-energy behavior of the reaction cross section for $\gamma\gamma\to\pi^0\pi^0$, and give a good fit to the data. Also, they give an interesting spectrum of two photons, and an enhanced branching ratio for the $K_L\to\pi^0\gamma\gamma$ decay process. They give a subleading contributions to $\gamma\gamma\to\pi^+\pi^-$ and $K^+\to\pi^+\gamma\gamma$. It is still to be understood how the vector mesons can be included in the standard ChPT in a systematic way.

Note added. While I was completing this work, I received a paper by Sehgal[22] on the rate and the spectrum of $K_L\to\pi^0\gamma\gamma$, which was substantially overlapping with the last section of my present work. His results, 17.9 and 39×10^{-7}, should be compared to our results (1.9,1.1) and $(3.6,2.3)\times10^{-6}$. (See Table I.) The difference is due to our inclusion of a kaon loop as well as a pion loop, and a different choice of $\langle\pi^0|H_w|K_2\rangle$ and G_8. If we did not include the kaon loop in the chiral amplitude as in Ref. 22, we would get (3.5,2.2) and $(2.0,1.2)\times10^{-6}$. Note that we still have a slight difference in the branching ratio due to a different choice of $\langle\pi^0|H_w|K_2\rangle$ and a different choice of G_8. We used the G_8 which has been fixed[10] from $K\to2\pi$, while the author of Ref. 22 fixed the G_8 from $K\to3\pi$.

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