Nonresonant three-body decays of charmed mesons

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(Received 25 August 1989)

Nonresonant three-body decays of charmed mesons are first studied in the approach of effective $SU(4)\times SU(4)$ chiral Lagrangians. It is pointed out that the predictions of the branching ratios in chiral perturbation theory are in general too small when compared with experiment. However, the experimental results are comprehensible in the general framework of the quark-diagram scheme. The existence of a sizable W-annihilation amplitude, which is evident by the observation of $D_s^+\to(\pi^+\pi^+\pi^-)_{NR}$, is the key toward an understanding of the three-body nonresonant decays of D^+ and D_s^+. The measurement of $D^0\to\bar{K}^0K^+K^-$ and $D^0\to\bar{K}^0\pi^+\pi^-$ indicates that color suppression is not effective in the three-body decay. Based on the quark-diagram analysis, predictions for some other nonresonant modes are given.

I. INTRODUCTION

The three-body decays of the charmed meson are in general dominated by vector-meson resonances. For example, the recent Mark III data[1] reveal that the resonant decay $\bar{K}^0\rho^+$ constitutes about 70% of the $D^+\to\bar{K}^0\pi^+\pi^0$ decay rate, while the $K^-\rho^+$ decay mode accounts for 80% of the $K^-\pi^+\pi^0$ rate. Nonresonant contributions are usually only small fractions of the total $D\to 3P$ decay rate. (A noticeable exception is the $D^+\to K^-\pi^+\pi^+$ mode whose nonresonant contribution is 80%.)

Pseudoscalar-pseudoscalar (PP) and pseudoscalar-vector (PV) two-body decays of the charmed mesons have been studied in great detail both experimentally and theoretically. The gross features of $D\to PP$ and PV data are understandable, at least at the qualitative level, within the framework of the quark-diagram approach[2] and of the vacuum-insertion approximation supplemented with the $1/N_c$ expansion (N_c being the number of colors),[3,4] for example. Now, the question is whether we can describe the nonresonant three-body decay of the charmed meson in the same framework. We will demonstrate in this paper that the qualitative features of the direct $D\to 3P$ data can be well explained by the quark-diagram approach.

The nonresonant $D\to 3P$ amplitudes receive two contributions: the direct weak transition and the pole diagrams which arise from the combination of a two-point weak vertex and a four-point strong vertex (see Fig. 1). Therefore, in order to compute the nonresonant decay rates we need a theory for describing the $DP\to DP$ or $PP\to PP$ strong-interaction scattering at the energies $\sqrt{s}\sim m_D$. Recall that in the case of $K\to 3\pi$ decays, the use of chiral symmetry and PCAC (partial conservation of axial-vector current) enables us to write down the low-energy KK, $K\pi$, and $\pi\pi$ scatterings. Moreover, the $K\to 3\pi$ amplitude can be related to the $K\to 2\pi$ transition via the soft-pion theorem. Unfortunately, there is no such analogous low-energy theorem available in the charm decay; a generalized $SU(4)\times SU(4)$ chiral symmetry does not exist *a priori* since the charm quark is much heavier than the light u, d, and s quarks and since the SU(4) symmetry is not seen in the hadronic spectrum. In spite of the absence of a justified SU(4) chiral symmetry, attempts of using the effective $SU(4)\times SU(4)$ chiral Lagrangians have been made by several authors to calculate the nonresonant decays of the charmed meson.[5–7]

We will show in Sec. II that the theoretical prediction of the nonresonant decay rate in chiral perturbation theory is in general too small when compared with the experiment. The difficulty with the SU(4) chiral Lagrangian is discussed. We then move to the quark-diagram approach in Sec. III. We point out that though the effective chiral Lagrangian fails to describe the main bulk of the nonresonant $D\to 3P$ data, nevertheless it shows that in the limit of SU(3) symmetry the nonresonant decay amplitudes can be expressed in terms of six distinct quark-diagram amplitudes, a basic assertion of the quark-diagram scheme. It becomes clear in this scenario that a sizable W-annihilation diagram, as evidenced by the observation of $D_s^+\to(\pi^+\pi^+\pi^-)_{NR}$, is the key towards an understanding of the nonresonant three-body decays of D^+ and D_s^+. Based on the available data, we can even make further predictions for other decay modes of $D\to 3P$. Sec. IV contains the summary and conclusions.

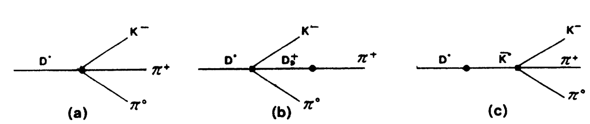

FIG. 1. The direct transition (a) and pole diagrams (b) and (c) for $D\to 3P$.

II. CHIRAL PERTURBATION THEORY

Since the framework of SU(4) chiral perturbation theory has been discussed in detail in Ref. 7, we will just recapitulate the main points here. There are two weak operators responsible for nonleptonic charmed meson decay: namely, $\Theta^{(20)}$ and $\Theta^{(84)}$, which transform as **20** and **84** representations, respectively, of SU(4). The effective Lagrangian thus has the form

$$\mathcal{L}=c_1\Theta^{(20)}+c_2\Theta^{(84)}\ , \tag{2.1}$$

where the coupling constants c_1 and c_2 can be determined from the measured $D\to\bar{K}\pi$ rates. We remind ourselves that the $\Delta S=1$ effective weak Lagrangian has only one unknown coupling constant since it is dominated by the octet representation of SU(3) owing to the $\Delta I=\frac{1}{2}$ rule. The weak operators $\Theta^{(20)}$ and $\Theta^{(84)}$ are obtained from the corresponding quark weak currents by making the replacement

$$\bar{q}_i\gamma_\mu(1-\gamma_5)q_j\to\frac{i}{2}f^2(L_\mu)_{ij}\ , \tag{2.2}$$

where $L_\mu=(\partial_\mu U)U^\dagger$ is an SU(4) singlet, f is a meson decay constant, $U=\exp(2i\phi/f)$, and $\phi=\phi^a\lambda^a/\sqrt{2}$. Therefore, the lowest-order effective SU(4)×SU(4) chiral Lagrangian for $\Delta C=1$ transitions reads

$$\begin{aligned}\mathcal{L}^{\Delta C=1}=\frac{G_F}{2\sqrt{2}}\{&\cos^2\theta_C[(c_1+c_2)L_{\mu 21}L^{\mu}_{43}-(c_1-c_2)L_{\mu 23}L^{\mu}_{41}]\\&+\sin\theta_C\cos\theta_C[(c_1+c_2)L_{\mu 13}L^{\mu}_{43}-L_{\mu 21}L^{\mu}_{42}-(c_1-c_2)L_{\mu 23}L^{\mu}_{41}-L_{\mu 22}L^{\mu}_{41}]\\&-\sin^2\theta_C[(c_1+c_2)L_{\mu 31}L^{\mu}_{42}+(c_1-c_2)L_{\mu 32}L^{\mu}_{41}]\}\ .\end{aligned} \tag{2.3}$$

The unknown coupling constants of the **20** and **84** weak operators can be determined from the experimentally observed $D\to\bar{K}\pi$ rates. The branching ratios of $D\to\bar{K}\pi$ measured by the Mark III Collaboration[8] are given by

$$\begin{aligned}&B(D^0\to K^-\pi^+)=(4.2\pm0.4\pm0.4)\%,\quad B(D^0\to\bar{K}^0\pi^0)=(1.9\pm0.4\pm0.2)\%\ ,\\&B(D^+\to\bar{K}^0\pi^+)=(3.5\pm0.5\pm0.4)\%\ .\end{aligned} \tag{2.4}$$

Taking into account final-state interactions, namely, $\delta_{1/2}-\delta_{3/2}\equiv(77\pm11)^\circ$ (Ref. 1), we obtain[9]

$$c_1=1.67,\quad c_2=0.49\ , \tag{2.5}$$

where use of the charm lifetimes[10]

$$\begin{aligned}&\tau(D^+)=(10.9\pm0.39)\times10^{-13}\ \text{sec},\quad \tau(D^0)=(4.22\pm0.13)\times10^{-13}\ \text{sec}\ ,\\&\tau(D_s^+)=(4.70\pm0.45)\times10^{-13}\ \text{sec}\end{aligned} \tag{2.6}$$

has been made.

Finally, an effective chiral Lagrangian for strong interactions is also needed for computing the three-body decays of the charmed meson; it is given by

$$\mathcal{L}_s=\frac{f^2}{8}\text{Tr}(\partial_\mu U\partial^\mu U^\dagger)+\frac{f^2}{8}\text{Tr}(MU+MU^\dagger) \tag{2.7}$$

with $M_{ij}=0$ for $i\neq j$ and

$$\pi^2=M_{11}=M_{22},\quad K^2=\tfrac{1}{2}(M_{11}+M_{33}),\quad D^2=\tfrac{1}{2}(M_{11}+M_{44}),\quad D_s^2=\tfrac{1}{2}(M_{33}+M_{44})\ , \tag{2.8}$$

where meson masses are denoted by the particle symbols.

Armed with the effective chiral Lagrangians (2.3) and (2.7), it is straightforward (though tedious) to compute the nonresonant $D\to3P$ decays. Experimentally, only ten such channels have been measured thus far. In the following we give the theoretical amplitudes for those ten decay modes:

$$\begin{aligned}&A(D^+\to K^-\pi^+\pi^+)=-a\left[(c_1+c_2)(s_1-2\pi^2)\frac{2D_s^2}{D_s^2-\pi^2}+(c_1-c_2)(D_s^2+3\pi^2-3s_1)\right]\ ,\\&A(D^+\to\bar{K}^0\pi^+\pi^0)=\frac{1}{\sqrt{2}}a\left[(c_1+c_2)(s_1-2\pi^2)\frac{D_s^2}{D_s^2-\pi^2}+(c_1-c_2)\left[-s_1+s_3-(s_2-s_3)\frac{D^2}{D^2-K^2}\right]\right]\ ,\\&A(D^+\to\pi^-\pi^+\pi^+)=b[2(c_1+c_2)(s_1-2\pi^2)+(c_1-c_2)(D^2+3\pi^2-3s_1)]\ ,\\&A(D^+\to\pi^+K^+K^-)=b\left[(c_1+c_2)\left[\pi^2-D^2+(D^2+2K^2-s_3)\frac{D^2+K^2}{D^2-\pi^2}-K^2\frac{D^2+D_s^2}{D^2-\pi^2}\right]+(c_1-c_2)(s_3-s_2)\right]\ ,\end{aligned}$$

$$\begin{aligned}
A(D^0\to\bar K^0\pi^+\pi^-)&=a\left[(c_1+c_2)(2\pi^2-s_1)+\frac{D_s^2}{D_s^2-\pi^2}+(c_1-c_2)\left[s_2\frac{D^2}{D^2-K^2}-s_3\frac{K^2}{D^2-K^2}-\pi^2\right]\right],\\
A(D^0-K^-\pi^+\pi^0)&=\frac{1}{\sqrt2}a\left[(c_1+c_2)(2\pi^2-s_1)\frac{D_s^2}{D_s^2-\pi^2}+(c_1-c_2)\left[s_1-s_2+(s_2-s_3)\frac{K^2}{D^2-K^2}\right]\right],\\
A(D^0\to\bar K^0K^+K^-)&=a[(c_1+c_2)(s_1-s_2)+(c_1-c_2)(s_2-K^2)],\\
A(D^0\to\pi^-\pi^+\pi^0)&=\frac{1}{\sqrt2}b\left[(c_1+c_2)\left[D^2+3\pi^2-3s_2+\frac{\pi^2}{D^2-\pi^2}(s_1-s_3)\right]\right.\\
&\qquad\left.-(c_1-c_2)\left[\pi^2+s_1-2s_2+\frac{\pi^2}{D^2-\pi^2}(s_1-s_3)\right]\right],\\
A(D_s^+\to K^-K^+\pi^+)&=a[(c_1+c_2)(-s_1+K^2)+(c_1-c_2)(s_1-s_2)],\\
A(D_s^+\to\pi^-\pi^+\pi^+)&=-a(c_1+c_2)\frac{\pi^2}{D_s^2-\pi^2}(D_s^2+\pi^2-s_1),
\end{aligned}\tag{2.9}$$

where $a=(G_F/2\sqrt2)\cos^2\theta_C$, $b=(G_F/2\sqrt2)\sin\theta_C\cos\theta_C$, $s_i=(p_D-p_i)^2$, and p_i is the four-momentum of the ith meson. The calculation of D^+ and D^0 three-body decays in chiral perturbation theory was already done in Ref. 7. Nevertheless, these results are included in Eq. (2.9) for the purpose of completeness and for the later purpose of discussion.

After integrating the amplitude squared over all phase space, we obtain the branching ratios of nonresonant $D\to3P$ decays as exhibited in Table I. It is evident that the chiral-Lagrangian predictions for $D^+\to\bar K^0\pi^+\pi^0$, $\pi^+\pi^+\pi^-$, and $D_s^+\to K^-K^+\pi^+$ are somewhat surprisingly in agreement with experiment although SU(4) chiral symmetry is not expected to work well. The predicted nonresonant rates for other channels are in general too small by 1 order of magnitude when compared to data. The channel $D_s^+\to(\pi^+\pi^+\pi^-)_{\rm NR}$ is prohibited since its amplitude goes as $m_\pi^2/m_{D_s}^2$. We shall see in the next section that the decay can proceed only via the W-annihilation mechanism. This means that nonspectator diagrams (W exchange or W annihilation) are predicted to be zero in chiral perturbation theory, a result not observed experimentally.

Apart from the above-mentioned problem as we are going to elaborate on in Sec. III, there exists an intrinsic difficulty with the use of SU(4)×SU(4) chiral symmetry. Suppose the chiral-symmetry-breaking scale (which is also the scale of the higher-order Lagrangian terms) is given by $\Lambda_\chi\approx2\sqrt2\pi f$ (Ref. 13). It is well known that $f\approx f_\pi$ and hence $\Lambda_\chi\approx1$ GeV (f_π being normalized to 132 MeV) in the chiral-SU(3) case. It is not clear what is the scale of Λ_χ for SU(4) symmetry. If Λ_χ is around 1 GeV, then the use of SU(4) chiral perturbation theory will become meaningless since higher-order terms, which are of order p^2/Λ_χ^2, are larger than the leading ones. However, if f is of the same order as the decay constant f_D or f_{D_s}, which is estimated to be in the range of 190–200 MeV in recent lattice calculations,[14] then $\Lambda_\chi\approx1.6-1.7$ GeV in the SU(4) case. Even so, contributions to $D\to(PPP)_{\rm NR}$ decays from higher-order chiral Lagrangians are still not substantially suppressed relative to the leading terms. This indicates that a sensible calculation of SU(4) chiral perturbation theory should include higher-order contributions; this explains why the nonresonant decay rates predicted by the lowest-order chiral Lagrangian are in general too small.

TABLE I. Quark-diagram amplitudes and branching ratios for nonresonant $D\to3P$ decays; $\mathcal{E}_{d-b}\equiv\mathcal{E}_d-\mathcal{E}_b,\mathcal{E}_{s-b}\equiv\mathcal{E}_s-\mathcal{E}_b$. Because of the complications, final-state interactions in three-body decay are not taken into account at this stage.

Decay mode	Amplitude	$(B)_{\rm expt}$ (%)	$(B)_{\rm theor}$ (%) in chiral perturbation theory
$D^0\to\bar K^0\pi^+\pi^-$	$V_{ud}V_{cs}^*(\mathcal{A}+\mathcal{B}_2+\mathcal{C})$	$2.1\pm0.3\pm0.7$[a]	0.13
$\to K^-\pi^+\pi^0$	$V_{ud}V_{cs}^*\frac{1}{\sqrt2}(\mathcal{A}+\mathcal{B}_1)$	$1.2\pm0.2\pm0.6$[a]	0.18
$\to\bar K^0K^+K^-$	$V_{ud}V_{cs}^*(\mathcal{B}_2+\mathcal{C})$	$0.85^{+0.27+0.19}_{-0.24-0.18}$[a]	0.02
$\to\bar K^0\bar K^0K^0$	$V_{ud}V_{cs}^*\sqrt2(\mathcal{C})$		
$\to K^-\pi^+\eta_8$	$V_{ud}V_{cs}^*\frac{1}{\sqrt6}(-\mathcal{A}+\mathcal{B}_1)$		

TABLE I. (*Continued.*)

Decay mode	Amplitude	$(B)_{\rm expt}$ (%)	$(B)_{\rm theor}$ (%) in chiral perturbation theory
$\to K^-\pi^+\eta_0$	$V_{ud}V_{cs}^*\frac{1}{\sqrt{3}}(2\mathcal{A}+\mathcal{B}_1+3\mathcal{C})$		
$\to \pi^+\pi^-\pi^0$	$V_{ud}V_{cd}^*\frac{1}{\sqrt{2}}(\mathcal{B}_1-\mathcal{C}+\mathcal{E}_{d-b})+V_{us}V_{cs}^*\frac{1}{\sqrt{2}}(\mathcal{E}_{s-b})$		0.04
$\to \pi^+\pi^-K^0$	$V_{ud}V_{cd}^*(\mathcal{B}_2)$		
$\to K^+\bar{K}^0\pi^-$	$V_{ud}V_{cd}^*(\mathcal{C}+\mathcal{E}_{d-b})+V_{us}V_{cs}^*(\mathcal{A}+\mathcal{C}+\mathcal{E}_{s-b})$		
$\to K^+K^-\pi^0$	$V_{ud}V_{cd}^*\frac{1}{\sqrt{2}}(2\mathcal{E}_{d-b})+V_{us}V_{cs}^*\frac{1}{\sqrt{2}}(\mathcal{A}+\mathcal{B}_1+\mathcal{C}+2\mathcal{E}_{s-b})$		
$\to K^-K^0\pi^+$	$V_{ud}V_{cd}^*(\mathcal{A}+\mathcal{C}+\mathcal{E}_{d-b})+V_{us}V_{cs}^*(\mathcal{C}+\mathcal{E}_{s-b})$		
$\to K^0\bar{K}^0\pi^0$	$V_{ud}V_{cd}^*\frac{1}{\sqrt{2}}(\mathcal{B}_1+2\mathcal{C})+V_{us}V_{cs}^*\frac{1}{\sqrt{2}}(\mathcal{B}_1-\mathcal{C})$		
$\to K^+K^-K^0$	$V_{ud}V_{cd}^*(\mathcal{B}_2)$		
$D^+\to \bar{K}^0\pi^+\pi^0$	$V_{ud}V_{cs}^*\frac{1}{\sqrt{2}}(-\mathcal{A}_1-\mathcal{B}_1)$	1.3±0.7±0.9[a]	0.76
$\to K^-\pi^+\pi^+$	$V_{ud}V_{cs}^*\sqrt{2}(\mathcal{A}+\mathcal{B}_1)$	7.2±0.6±1.8[a]	1.71
$\to K^+\bar{K}^0\bar{K}^0$	$V_{ud}V_{cs}^*\sqrt{2}(\mathcal{B}_2)$		
$\to \bar{K}^0\pi^+\eta_8$	$V_{ud}V_{cs}^*\frac{1}{\sqrt{6}}(-\mathcal{A}-\mathcal{B}_1+2\mathcal{B}_2)$		
$\to \bar{K}^0\pi^+\eta_0$	$V_{ud}V_{cs}^*\frac{1}{\sqrt{3}}(2\mathcal{A}-2\mathcal{B}_1+2\mathcal{B}_2)$		
$\to \pi^+\pi^+\pi^-$	$V_{ud}V_{cd}^*\sqrt{2}(\mathcal{A}+\mathcal{B}_1+\mathcal{D}+\mathcal{E}_{d-b})+V_{us}V_{cs}^*\sqrt{2}(\mathcal{E}_{s-b})$	0.25±0.07±0.02[a]	0.15
$\to \pi^+\pi^0\pi^0$	$V_{ud}V_{cd}^*\frac{1}{\sqrt{2}}(\mathcal{A}+\mathcal{B}_1+\mathcal{D}+\mathcal{E}_{d-b})+V_{us}V_{cs}^*\frac{1}{\sqrt{2}}(\mathcal{E}_{s-b})$		
$\to K^-K^+\pi^+$	$V_{ud}V_{cd}^*(\mathcal{D}+\mathcal{E}_{d-b})+V_{us}V_{cs}^*(\mathcal{A}+\mathcal{B}_1+\mathcal{E}_{d-b})$	0.54±0.25±0.09[a] 0.45±0.07±0.09[b] 0.56±0.18[c]	0.02
$\to K^+K^0+\pi^0$	$V_{ud}V_{cd}^*\frac{1}{\sqrt{2}}(-\mathcal{B}_2)+V_{us}V_{cs}^*\frac{1}{\sqrt{2}}(-\mathcal{A})$		
$\to K^0\bar{K}^0\pi^+$	$V_{ud}V_{cd}^*(\mathcal{A}+\mathcal{B}_1+\mathcal{D}+\mathcal{E}_{d-b})+V_{us}V_{cs}^*(\mathcal{B}_1+\mathcal{E}_{s-b})$		
$D_s^+\to K^-K^+\pi^+$	$V_{ud}V_{cs}^*(\mathcal{A}+\mathcal{B}_1+\mathcal{D})$	0.81±0.25±0.25[b,d] 3.4±1.1[c,d]	0.42
$\to K^+\bar{K}^0\pi^0$	$V_{ud}V_{cs}^*\frac{1}{\sqrt{2}}(-\mathcal{B}_1+\mathcal{B}_2)$		
$\to K^0\bar{K}^0\pi^+$	$V_{ud}V_{cs}^*(\mathcal{A}+\mathcal{B}_2+\mathcal{D})$		
$\to K^+\bar{K}^0\eta_8$	$V_{ud}V_{cs}^*\frac{1}{\sqrt{6}}(-\mathcal{B}_1-\mathcal{B}_2)$		
$\to K^+\bar{K}^0\eta_0$	$V_{ud}V_{cs}^*\frac{1}{\sqrt{3}}(2\mathcal{B}_1+2\mathcal{B}_2+3\mathcal{D})$		
$\to \pi^-\pi^+\pi^+$	$V_{ud}V_{cs}^*\sqrt{2}(\mathcal{D})$	1.0±0.3±0.1[b]	5×10^{-5}
$\to \pi^+\pi^0\pi^0$	$V_{ud}V_{cs}^*\frac{1}{\sqrt{2}}(\mathcal{D})$		
$\to K^+K^+K^-$	$V_{ud}V_{cd}^*\sqrt{2}(\mathcal{E}_{d-b})+V_{us}V_{cs}^*\sqrt{2}(\mathcal{A}+\mathcal{B}_1+\mathcal{D}+\mathcal{E}_{s-b})$		
$\to K^+K^0\bar{K}^0$	$V_{ud}V_{cd}^*(\mathcal{B}_1+\mathcal{E}_{d-b})+V_{us}V_{cs}^*(\mathcal{A}+\mathcal{B}_1+\mathcal{D}+\mathcal{E}_{s-b})$		
$\to K^+\pi^+\pi^-$	$V_{ud}V_{cd}^*(\mathcal{A}+\mathcal{B}_1+\mathcal{E}_{d-b})+V_{cs}^*(\mathcal{D}+\mathcal{E}_{s-b})$		
$\to K^0\pi^+\pi^0$	$V_{ud}V_{cd}^*\frac{1}{\sqrt{2}}(-\mathcal{A}-\mathcal{B}_2)$		
$\to K^+\pi^0\pi^0$	$V_{ud}V_{cd}^*\frac{1}{\sqrt{2}}(\mathcal{B}_1-\mathcal{B}_2+\mathcal{E}_{d-b})+V_{us}V_{cs}^*\frac{1}{\sqrt{2}}(\mathcal{D}+\mathcal{E}_{s-b})$		

[a]Mark III Collaboration (Ref. 8).
[b]E691 Collaboration (Ref. 11).
[c]ACCMOR Collaboration (Ref. 12).
[d]$B(D_s^+\to\phi\pi^+)=0.035$ has been assumed.

III. QUARK-DIAGRAM APPROACH

In this section we will apply the quark diagram scheme to analyze the data in a phenomenological way, and to relate the previous chiral perturbation theory in this framework. It has been established that[14,15,2] all meson nonleptonic weak decays can be expressed in terms of six quark diagrams: $\mathcal{A}$, the external W-emission diagram; $\mathcal{B}$, the internal W-emission diagram; $\mathcal{C}$, the W-exchange diagram; $\mathcal{D}$, the W-annihilation diagram; $\mathcal{E}$, the horizontal W-loop diagram; and $\mathcal{F}$, the vertical W-loop diagram. These quark diagrams are specific and well-defined physical quantities. They are classified according to the topology of first-order weak interactions, *but all QCD strong-interaction effects are included.* Such scheme has been applied to study the two-body charmed-meson decays and new predictions have even made;[2] it presently offers the least model-dependent way of analyzing the experimental results and making predictions.

The quark-diagram amplitudes of some $D\to(PPP)_{\rm NR}$ decay modes are given in Table I. The amplitude $\mathcal{B}_1$ is referred to the internal W-emission diagram in which the quark-antiquark pair is created on the side of the spectator quark, while $\mathcal{B}_2$ denotes the case when $q\bar{q}$ is created along the charm-quark line. The use of quark-diagram scheme for three-body decays of the charmed meson has two complications which do not exist in the analysis of exclusive two-body decays. First of all, quark-diagram amplitudes include not only direct weak transitions but also the pole diagrams mentioned in the previous section. Second, the three-body quark-diagram amplitudes are in general momentum dependent even when all external particles are on the mass shell. This means that unless its momentum dependence is known, the quark-diagram amplitude of $D\to(PPP)_{\rm NR}$ cannot be simply determined from experiment without making further assumptions. Moreover, the momentum dependence of each quark-diagram amplitude may vary from channel to channel.

To ensure that the quark-diagram scheme is applicable to $D\to 3P$ even in the presence of pole contributions, we have checked that in the SU(3) limit, which is the starting point of this scenario, all nonresonant amplitudes given by Eq. (2.9) in chiral perturbation theory can recast in terms of the quark-diagram amplitudes[16]

$$\mathcal{A}=(c_1+c_2)(2\pi^2-s_1)\frac{D^2}{D^2-\pi^2},\quad \mathcal{B}_1=\mathcal{B}_2=0\ ,$$

$$\mathcal{C}=(c_1-c_2)(s_2-\pi^2)\ , \tag{3.1}$$

$$\mathcal{D}=-(c_1+c_2)\frac{\pi^2}{D^2-\pi^2}(D^2+\pi^2-s_1)\ ,$$

This gives an indication that although the effective chiral Lagrangian fails to reproduce the main bulk of the experimental results for nonresonant decays, it demonstrates specifically the validity of the quark-diagram approach. In the presence of SU(3) breaking, the amplitudes $\mathcal{B}_1$ and $\mathcal{B}_2$ are no longer vanishing. For example, we find from Eq. (2.9) that

$$\mathcal{B}_1=(c_1-c_2)\left[s_1+s_2-2s_3+(s_2-s_3)\frac{K^2}{D^2-K^2}\right]\ , \tag{3.2}$$

for $D^+\to\bar{K}^0\pi^+\pi^0$, whereas

$$\mathcal{B}_1=(c_1-c_2)\left[s_1-s_2+(s_2-s_3)\frac{K^2}{D^2-K^2}\right]\ , \tag{3.3}$$

for $D^0\to K^-\pi^+\pi^0$. Obviously, $\mathcal{B}_1$ in $D^+\to\bar{K}^0\pi^+\pi^0$ has a momentum dependence different from that in $D^0\to K^-\pi^+\pi^0$. To simplify the ensuing discussions, we will nevertheless assume that after the phase-space integration, each quark-diagram amplitude behaves the same from channel to channel.

Let us first focus on the decay modes $\bar{K}^0\pi^+\pi^0$ and $\bar{K}^-\pi^+\pi^+$ of D^+. From Table I we expect $4\Gamma(D^+\to\bar{K}^0\pi^+\pi^0)\simeq\Gamma(D^+\to K^-\pi^+\pi^+)$ from the quark-diagram scheme. Data are consistent with this result. Next turn to $D^+\to K^-K^+\pi^+$ and $\pi^+\pi^+\pi^-$. Because the available phase space for the final state $K^-K^+\pi^+$ is about three times smaller than that of $\pi^+\pi^+\pi^-$, it is naively expected that $\Gamma(D^+\to K^-K^+\pi^+)\approx\frac{1}{6}\Gamma(D^+\to\pi^+\pi^+\pi^-)$. Indeed, this is also the result obtained in chiral perturbation theory (see Table I). Experimentally, the $K^-K^+\pi^+$ mode has however a larger branching ratio than $\pi^+\pi^+\pi^-$, more precisely, $\Gamma(D^+\to K^-\pi^+\pi^+)\approx 2\Gamma(D^+\to\pi^+\pi^+\pi^-)$. From Table I it is clear that this seemingly surprising experimental result can be explained only if the W-annihilation diagram is nonvanishing. (The penguin diagram is negligible in the charm decay owing to the good approximation $V_{us}V_{cs}^*\simeq V_{ud}V_{cd}^*$). Furthermore, the W-annihilation diagram must exist in such a way that it contributes constructively to $D^+\to K^-K^+\pi^+$ but destructively to the $\pi^+\pi^+\pi^-$ channel. *A nonvanishing W-annihilation amplitude* $\mathcal{D}$ *is evidenced by the observation of* $D_s^+\to(\pi^+\pi^+\pi^-)_{\rm NR}$ *since it can only proceed through the W-annihilation mechanism.* We will turn to this crucial point shortly.

The decay rate of $D_s^+\to K^-K^+\pi^+$ and $D^0\to K^-\pi^+\pi^0$ is also understandable within the framework of the quark-diagram approach. Since the phase space of $D^+\to\pi^+\pi^+\pi^-$ is about two times of the size of $D_s^+\to\pi^+K^+K^-$, it follows that

$$B(D_s^+\to K^-K^+\pi^+)_{\rm NR}\simeq B(D^+\to\pi^+\pi^+\pi^-)_{\rm NR}\left[\frac{\cos\theta_C}{\sin\theta_C}\right]^2\times\left[\frac{4.7}{10.9}\right]\times\frac{1}{2}\times\frac{1}{2}$$

$$=(0.50\pm0.15)\%\ , \tag{3.4}$$

in agreement with the E691 measurement,[11] $(0.81\pm0.25\pm0.25)\%$. Our prediction (3.4) also indicates that the branching ratio $(3.4\pm1.1)\%$ measured by the ACCMOR Collaboration[12] is probably too large. For the branching fraction of $D^0\to K^-\pi^+\pi^0$, it is easily seen that

$$B(D^0 \to K^-\pi^+\pi^0)_{NR} \approx B(D^+ \to K^-\pi^+\pi^+)_{NR} \times \tfrac{1}{2} \times \tfrac{1}{4} = (0.9 \pm 0.2)\% , \quad (3.5)$$

which is also consistent with the Mark III result,[1] $(1.2\pm0.6)\%$.

Now we make a more quantitative analysis. Assuming that the phase space of each channel is dominated by the momentum-independent terms, we find that the relative experimental branching ratios of $D^+ \to K^-\pi^+\pi^+$, $\bar{K}^0\pi^+\pi^0$, $D_s^+ \to K^-K^+\pi^+$, $\pi^+\pi^+\pi^-$, and $D^0 \to K^+\pi^+\pi^0$, can be satisfactorily explained provided that[17]

$$\frac{\mathcal{D}}{\mathcal{A}+\mathcal{B}_1} \simeq -0.38 . \quad (3.6)$$

The existence of a sizable W-annihilation contribution is thus the key towards an understanding of the three-body nonresonant decay of D^+ and D_s^+. Recall that the nonspectator diagram (W exchange or W annihilation) is usually argued to be negligible due to helicity suppression. Evidently, this helicity suppression mechanism must be vitiated by some nonperturbative effects, presumably the soft-gluon corrections.[18] The quark-diagram analysis of the exclusive two-body decays of the charmed meson also reveals the evidence of the nonspectator contribution in the decay of $D \to PP$ and VP (Ref. 2).

Before proceeding it is worth mentioning that it has been argued in the past that the W-annihilation diagram should not play an essential role in D_s^+ decay due to the small rate of $D_s^+ \to \rho^0\pi^+$. (This decay mode can only proceed through W annihilation. The current upper bound is $B(D_s^+ \to \rho^0\pi^+) < 0.77\%$ [ARGUS (Ref. 19)] and $< 0.28\%$ [E691 (Ref. 11)].) However, there are two W-annihilation terms which contribute with opposite signs to $D_s^+ \to \rho^0\pi^+$ (Ref. 20):

$$A(D_s^+ \to \rho^0\pi^+) = \frac{1}{\sqrt{2}} V_{cs} V_{ud}^* (\mathcal{D}' - \mathcal{D}) . \quad (3.7)$$

As a consequence, the significant W-annihilation contribution does not conflict with the low rate of $D_s^+ \to \rho^0\pi^+$ due to a possible cancellation between $\mathcal{D}'$ and $\mathcal{D}$.

What can we learn from the decays $D^0 \to \bar{K}^0\pi^+\pi^-$ and $\bar{K}^0K^+K^-$? From our past experience with $D \to \bar{K}\pi$, we know that final-state interactions (FSI's) are important for D^0 decay (at least for exclusive two-body decays).[2] Unfortunately, at this stage we cannot make any concrete estimate of FSI's for $D \to 3P$. Nevertheless, it is instructive to fit the experimentally measured rates of $(\bar{K}^0\pi^+\pi^-)_{NR}$ and $(\bar{K}^0K^+K^-)_{NR}$ without considering FSI's. By doing this, we find

$$\mathcal{B}_2 + \mathcal{C} \approx \mathcal{B}_1 \approx -0.6\mathcal{A} . \quad (3.8)$$

This implies that the color-mismatched quark diagrams $\mathcal{B}$ and $\mathcal{C}$ are neither color nor QCD-correction suppressed, contrary to the naive expectation. This phenomenon of color nonsuppression for the quark-diagram amplitudes $\mathcal{B}$ and $\mathcal{C}$ was also known to two-body charm decays for a long time.[2] The measurement of nonresonant decays $D^0 \to \bar{K}^0\bar{K}^0K^0$ and $D^+ \to \bar{K}^0\bar{K}^0K^+$ will enable us to determine the quark-diagram amplitudes $\mathcal{C}$ and $\mathcal{B}_2$, respectively.

From the quark-diagram expressions we can make the following predictions (after taking into account the phase-space integration):

$$\begin{aligned} B(D^+ \to K^+\bar{K}^0\bar{K}^0)_{NR} &\approx 15B(D^0 \to \pi^+\pi^-K^0)_{NR} \\ &\approx 90B(D^0 \to K^+K^-K^0)_{NR} , \\ B(D^+ \to \pi^+\pi^+\pi^-)_{NR} &\approx 4B(D^+ \to \pi^+\pi^0\pi^0)_{NR} . \end{aligned} \quad (3.9)$$

With Eqs. (3.6) and (3.7) we can further predict that

$$\begin{aligned} B(D^+ \to K^0\bar{K}^0\pi^+)_{NR} &\approx 3B(D^+ \to K^-K^+\pi^+)_{NR} , \\ B(D_s^+ \to K^+K^0\bar{K}^0)_{NR} &\approx 6B(D_s^+ \to K^+K^+K^-)_{NR} . \end{aligned} \quad (3.10)$$

It is important to measure these nonresonant decays listed above to test the quark-diagram approach.

IV. SUMMARY AND CONCLUSIONS

We have studied in this paper the nonresonant three-body decays of the charmed meson in two different frameworks: chiral perturbation theory and the quark-diagram approach. The basic assertion of the quark-diagram scheme that in the SU(3) limit all nonresonant $D \to 3P$ decays can be written in terms of six distinct quark-diagram amplitudes to first order in weak interactions and to all orders in strong interactions (e.g., the pole diagram) is confirmed by the chiral-Lagrangian calculations.

From the quark-diagram-scheme analysis of the data, it is clear that there are two crucial ingredients necessary for an understanding of nonresonant charm decays: a sizable W-annihilation diagram, as is evident by the observation of $D_s^+ \to (\pi^+\pi^+\pi^-)_{NR}$, and color nonsuppression for the color-mismatched quark diagrams. The chiral-Lagrangian approach fails to explain the main bulk of $D \to 3P$ data because it predicts vanishing nonspectator diagrams (i.e., $\mathcal{C}$ and $\mathcal{D}$ in the notation of the quark-diagram scheme) and too small spectator amplitudes (i.e., quark-diagram amplitudes $\mathcal{A}$ and $\mathcal{B}$). The latter has to do with the fact that the chiral-symmetry-breaking scale of the SU(4)×SU(4) effect Lagrangian is about 1.7 GeV and hence high-order contributions are as important as the leading terms. Therefore, the lowest-order chiral-Lagrangian prediction of nonresonant decay rates is in general too small. We wish to emphasize that if the scale of the higher-order Lagrangian terms is of order 1 GeV, then the use of SU(4) chiral Lagrangian will become meaningless. Even if the SU(4) chiral-symmetry-breaking scale is of order 1.6 GeV, a sensible chiral-Lagrangian calculation should include higher-order contributions. This explains why the prediction of nonresonant decay rate in lowest-order chiral Lagrangian turns out too small.

We have suggested the measurements of some channels in order to help determining the individual quark-diagram amplitudes. Moreover, from the presently available data, we can even make some further predictions on the branching ratios of several other nonresonant decay modes. In short, the $D \to (PPP)_{\rm NR}$ decays are comprehensible in the quark-diagram scheme in which the nonspectator diagram is sizable and color suppression is alleviated.

ACKNOWLEDGMENTS

This work was partially supported by the U.S. Department of Energy and the National Science Council of Taiwan. One of the authors (L.L.C.) would like to acknowledge an INCORR grant from the California Coordinate Committee for Nonlinear Science and the Center for Nonlinear Science, Los Alamos Laboratory, which enabled her to carry out this research, and to thank the Center for Nonlinear Science for their warm hospitality.

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