Influence of the B^* resonance on $\bar{B}\to\pi e\bar{\nu}_e$

Nathan Isgur
Department of Physics, University of Toronto, Toronto, Canada M5S 1A7

Mark B. Wise
California Institute of Technology, Pasadena, California 91125
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In the chiral limit and as $m_b\to\infty$, the decay $\bar{B}\to\pi^+e\bar{\nu}_e$ would be dominated near zero pionic recoil by the effects of the B^* resonance. We examine the influence of this mechanism on the $\bar{B}\to Xe\bar{\nu}_e$ electron spectrum in the region near the end point which is important for the determination of the Kobayashi-Maskawa matrix element V_{ub}.

In the standard six-quark model the coupling of the W bosons to the quarks has the form

$$\mathcal{L}_{\rm int}=\frac{g_2}{2\sqrt{2}}(\bar{u},\bar{c},\bar{t})\gamma_\mu(1-\gamma_5)V\begin{bmatrix}d\\s\\b\end{bmatrix}W^\mu+\text{H.c.}\ , \quad (1)$$

where V is the 3×3 unitary Kobayashi-Maskawa (KM) matrix[1] that arises from diagonalization of the quark mass matrices. In the standard model, the elements of V are fundamental parameters whose values must be determined by experiment. The semileptonic decays of B mesons provide information on the magnitude of the elements V_{ub} and V_{cb}. The direct comparison of measured "semileptonic" branching rates for $\bar{B}\to X_ce\bar{\nu}_e$ with calculations gives $|V_{cb}|\simeq0.05$. It is possible to get experimental information on the much smaller matrix element $|V_{ub}|$ by examining the electron spectrum in the end-point region. In a decay $\bar{B}\to Xe\bar{\nu}_e$, the maximum electron energy is $E_e^{\max}=(m_B^2-m_X^2)/2m_B$, so semileptonic B decays with electron energies greater than $(m_B^2-m_D^2)/2m_B$ must originate from the $b\to u$ quark transition.

The simplest possible decay that can arise from the $b\to u$ weak transiton is $\bar{B}\to\pi^+e\bar{\nu}_e$ and this decay is responsible for the most energetic electrons. The differential decay rate for $\bar{B}\to\pi^+e\bar{\nu}_e$ has the form[2,3]

$$\frac{d^2\Gamma}{dx\,dy}=|V_{ub}|^2\frac{G_F^2m_B^5}{32\pi^3}|f_+(t)|^2(1-2x)[y_{\max}(x)-y]\ , \quad (2)$$

where

$$y=t/m_B^2=(p_B-p_\pi)^2/m_B^2\ , \quad (3a)$$

$$x=E_e/m_B\ , \quad (3b)$$

and $f_+(t)$ is the hadronic form factor for the $\bar{B}\to\pi^+$ transition

$$\langle\pi^+(p_\pi)|\bar{u}\gamma_\mu b|\bar{B}(p_B)\rangle=f_+(t)(p_B+p_\pi)_\mu+f_-(t)(p_B-p_\pi)_\mu\ . \quad (4)$$

For a fixed electron energy, y varies over the region

$$0\le y\le y_{\max}(x)=\frac{4x(x_m-x)}{1-2x}\ ,$$

where $x_m=(m_B^2-m_\pi^2)/2m_B^2$ is the maximum value of x.

Naively, at the kinematic limit $t=t_m=(m_B-m_\pi)^2$ where the pion is at rest, the form factors $f_\pm(t_m)$ scale with the large b-quark mass in the following way:

$$f_+(t_m)+f_-(t_m)\sim m_b^{-1/2}\ , \quad (5a)$$

$$f_+(t_m)-f_-(t_m)\sim m_b^{+1/2}\ . \quad (5b)$$

(Here logarithms of m_b which arise from perturbative QCD corrections[4,5] have been neglected.) This scaling is deduced by noting that in the heavy-b-quark limit the b quark acts essentially as a static color source in the B meson so that the only dependence of the left-hand side of Eq. (4) on the bottom-quark mass is a factor of $\sqrt{m_b}$, from the normalization of the B-meson state. Indeed, previous estimates of $f_+(t_m)$ and $f_-(t_m)$ using the valence nonrelativistic quark potential model exhibit this behavior. For example, using variational solutions of the Coulomb plus linear potential problem in a harmonic basis,

$$\psi_i^{1S}\simeq\frac{\beta_i^{3/2}}{\pi^{3/4}}e^{-\beta_i^2r^2/2} \quad (6)$$

for $i=\pi$, B, Ref. 3 finds

$$f_+^{\rm QM}(t_m)=\left[\frac{2m_d}{m_d+m_b}\right]^{1/2}\left[\frac{\beta_B\beta_\pi}{\beta_{B\pi}^2}\right]^{3/2}\times\left[1+\frac{m_b-m_d}{2m_d}-\frac{1}{8}\left[\frac{m_b^2-m_d^2}{m_bm_d}\right]\frac{\beta_B^2}{\beta_{B\pi}^2}\right] \quad (7a)$$

and

$$f_-^{\rm QM}(t_m)=\left[\frac{2m_d}{m_d+m_b}\right]^{1/2}\left[\frac{\beta_B\beta_\pi}{\beta_{B\pi}^2}\right]^{3/2}\times\left[1-\frac{m_b+3m_d}{2m_d}+\frac{1}{8}\frac{(m_b+3m_d)(m_b+m_d)}{m_bm_d}\frac{\beta_B^2}{\beta_{B\pi}^2}\right]\ , \quad (7b)$$

where $\beta_{B\pi}^2 = 1/2(\beta_B^2 + \beta_\pi^2)$.

The form factors $f_\pm^{\rm QM}(t)$ fall off as t decreases from its maximum value t_m. This t dependence arises because in the rest frame of the B meson momentum must be transferred to the light "spectator quark" in order for the final state to consist of a single pion, and as a result, as a function of $(t_m - t)/m_B$ the slope is of order a typical hadronic length scale ~ 1 GeV^{-1}. In the approximation to the quark potential model mentioned above, the dependence on $(t_m - t)$ is exponential

$$f_\pm^{\rm QM}(t) = f_\pm^{\rm QM}(t_m) \exp\left[-\frac{m_d}{8(m_d + m_b)} \frac{t_m - t}{\kappa^2 \beta_{B\pi}^2} \right] . \quad (8)$$

Here $\kappa \simeq 0.7$ is a phenomenological factor that is included in an attempt to correct for some relativistic effects. With this dependence on $(t_m - t)$, the $\bar{B} \to \pi^+ e \bar{\nu}_e$ decay is dominated by soft recoils and Eqs. (2) and (5) then imply that $B(\bar{B} \to \pi^+ e \bar{\nu}_e) \sim m_b^{-3}$. [Actually, we do not expect this exponential behavior to be appropriate for large $(t_m - t)$; see Ref. 6.]

The purpose of this paper is to show that, because the B^* and B states become degenerate in the large-m_b limit, there is a B^* contribution to $f_+(t)$ that previous quark-model-type estimates [e.g., Eq. (7a)] and the naive arguments leading to Eqs. (5) have not adequately taken into account. We will see that this B^* pole contribution gives rise to an $f_+^{B^*}$ that at threshold behaves as

$$f_+^{B^*}(t_m) \sim m_b^{3/2} \quad (9)$$

for $m_\pi = 0$, and hence would in this limit dominate $\bar{B} \to \pi^+ e^- \bar{\nu}_e$ near $t = t_m$. We will show as well that, as a result of a rapidly varying form factor, this contribution would not dominate the total rate for $\bar{B} \to \pi^+ e \bar{\nu}_e$ even for $m_b \to \infty$.

In the limit of very large m_b (compared with a typical hadronic scale ~ 1 GeV) the bottom quark in both the B and B^* mesons acts as a static color source[5] with its spin degree of freedom decoupled. Therefore, in this limit these two mesons, which differ only in the orientation of the light-quark spin relative to that of the heavy quark, become degenerate. The residual mass splitting arises from a spin interaction that is suppressed by a power of m_b, so, in the large-m_b limit,

$$m_{B^*}^2 - m_B^2 \sim 1 \ {\rm GeV}^2 . \quad (10)$$

(Again logarithmic dependence on m_b, which arises from perturbative strong-interaction effects, has been neglected.) The B^*-B mass splitting can be deduced by scaling the measured D^*-D mass splitting:

$$m_{B^*} - m_B = (m_c/m_b)(m_{D^*} - m_D) \simeq 50 \ {\rm MeV} . \quad (11)$$

Figure 1 depicts the B^* pole contribution to the $\bar{B} \to \pi^+ e \bar{\nu}_e$ decay amplitude. It depends on the amplitude for the vector current to annihilate a B^* resonance,

$$\langle 0 | \bar{u} \gamma_\mu b | \bar{B}^*(p^*, \lambda) \rangle = f_{B^*} \epsilon_\mu(p^*, \lambda) , \quad (12)$$

and on the amplitude for the effective Hamiltonian density for pair creation $\mathcal{H}_{\rm eff}^{\rm PC}$ to cause a $\bar{B} \to \bar{B}^* \pi$ transition

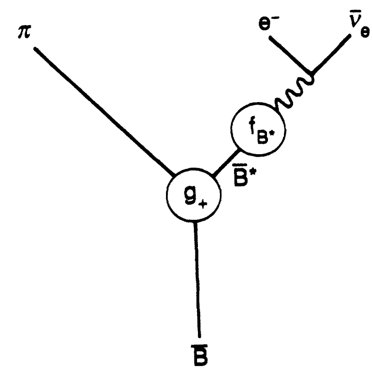

FIG. 1. The B^* pole contribution to $\bar{B} \to \pi^+ e \bar{\nu}_e$.

$$\langle \bar{B}^*(p^*, \lambda) \pi(p_\pi) | \mathcal{H}_{\rm eff}^{\rm PC} | \bar{B}(p_B) \rangle = \epsilon_\mu^*(p^*, \lambda)[g_+(p_B + p_\pi)^\mu + g_-(p_B - p_\pi)^\mu] . \quad (13)$$

Combining these vertices and the B^* propagator gives

$$f_+^{B^*}(t) = \frac{g_+(t) f_{B^*}}{m_{B^*}^2 - t} = \frac{g_+(t) f_B^*}{(t_m - t) + m_{B^*}^2 - (m_B - m_\pi)^2} . \quad (14)$$

Using the scaling arguments introduced earlier, we can deduce that in the large-m_b limit $g_+(t_m) \sim m_b$ and $f_B^* \sim m_b^{1/2}$ so, neglecting the pion mass, Eqs. (10) and (14) imply that $f_+^{B^*}(t_m)$ is of order $m_b^{3/2}$, as earlier claimed. Furthermore, since only the B^* resonance becomes degenerate with the B in this limit, only the B^* pole contribution is of this order. All other contributions to $f_+(t_m)$ are of order $m_b^{1/2}$ in accord with Eq. (5). As a function of $(t_m - t)/m_b$, $g(t)$ has, as we will see, a slope that is governed by the hadronic scale, but (still neglecting the pion mass) the denominator of Eq. (14) is a much more rapidly varying function of $(t_m - t)/m_b$ with a slope of order $m_b/(1 \ {\rm GeV})^2$. As a result [see Eq. (2)], although the B^* would be dominant at t_m, its contribution to the total $\bar{B} \to \pi^+ e \bar{\nu}_e$ branching ratio behaves in the same way as the valence-quark model.

We now turn to a calculation of f_{B^*} and $g_+(t)$ in order to quantify the importance of the B^* pole contribution. In a pair-creation model[7,8] where $\mathcal{H}^{\rm PC}$ is proportional to $\bar{u}u$ we find using the nonrelativistic quark model that

$$g_+(t_m) = \frac{g(m_b + m_d)}{(\beta_B^2 + 2\beta_\pi^2)^{3/2}} \left[1 - \frac{2\beta_\pi^2}{2\beta_\pi^2 + \beta_B^2} \frac{m_d}{m_d + m_b} \right] , \quad (15a)$$

$$g_-(t_m) = \frac{g(m_b + m_d)}{(\beta_B^2 + 2\beta_\pi^2)^{3/2}} \left[\frac{\beta_B^2 - 2\beta_\pi^2}{2\beta_\pi^2 + \beta_B^2} - \frac{2\beta_\pi^2}{2\beta_\pi^2 + \beta_B^2} \frac{m_d}{m_d + m_b} \right] , \quad (15b)$$

and that

$$g_\pm(t)=g_\pm(t_m)\exp\left\{-\frac{m_d}{2(m_b+m_d)}\frac{t_m-t}{\kappa^2}\left[\frac{1}{\beta_B^2}\left[\frac{m_b}{m_b+m_d}\right]^2+\frac{1}{2\beta_\pi^2+\beta_B^2}\left[\frac{m_d}{m_b+m_d}\right]^2\right]\right\} . \tag{16}$$

(Here g is a factor into which we have absorbed some dependence on m_d, β_π, etc., but which is independent of the heavy-quark mass.) The rate for $K^*\to K\pi$ is determined by an analogous form factor which follows from the replacements $m_b\to m_s$ and $\beta_B\to\beta_K$ in Eqs. (15) and (16), so using the measured $K^*\to K\pi$ rate, we find (with the numerical values for the β's and quark masses of Ref. 3) that

$$|g_+(t_m)|=32 . \tag{17}$$

Similarly, using quark-model expressions[9] for the decay constants of vector mesons gives

$$f_{B^*}=0.7\ \text{GeV}^2 . \tag{18}$$

With these values Eq. (14) gives $f_+^{B^*}(t_m)\simeq 11$. Note that the valence-nonrelativistic-quark-model prediction of Eq. (5a) is $f_+^{\text{QM}}(t_m)\simeq 2$ so the B^* pole contribution dominates the value of f_+ at threshold in accord with the arguments made above based on the large-m_b limit. Integrating Eq. (2), with $f_+=f_+^{B^*}$ gives a $\bar{B}\to\pi^+e^-\bar{\nu}_e$ rate $|V_{ub}|^2 0.28\times10^{12}$ sec^{-1}, while setting f_+ to the valence-quark-model value f_+^{QM} gives a rate of $|V_{ub}|^2 0.21\times10^{13}$ sec^{-1}. Figure 2 shows the electron spectra resulting from these two calculations.

Vector-meson pole diagrams have often been invoked[10] to describe the t dependence of f_+ form factors (the B^* in $B\to\pi$, the D_s^* in $D\to K$, the K^* in $K\to\pi$, etc.). We should emphasize that our picture is a very different one. Note in particular the contrast with Ref. 11, in which the t dependence associated with the B^* propagator alone [i.e., $(1-t/m_{B^*}^2)^{-1}$] is used to extrapolate from $t=0$ to t_m. In our picture, the B^* contribution plays a role only very near t_m since it is strongly suppressed by the soft hadronic vertex g_+ when $(t_m-t)/t_m \gtrsim m_d/m_b$. (Note that the use of vector-meson-dominated form factors is not at all fundamental to the model of Ref. 11: The model predicts form factors at $t=0$ and makes this ansatz for their t dependence.)

This observation naturally introduces the issue of the relationship between the vector-meson pole and the valence-quark-model contributions to f_+. It was pointed out in Ref. 3 that there is a basic mismatch between vector-meson dominance and the quark model that can be easily seen by considering the example of the elastic form factor of the η_c. This system clearly has, as $m_c\to\infty$, a radius $r\sim(m_c\alpha_s)^{-1}$, while a vector-meson pole would lead to a form factor with $r\sim m_c^{-1}$. We believe that the resolution of the mismatch lies in considering the effects of anomalous thresholds[12] which ruin the usual argument that the form factors will be controlled by the t-channel vector spectral function. If so, then B^* effects of the type considered here are not dual to the valence-quark-model form factors (or, *a fortiori*, to the free quark decay model[3]) and must be added as a distinct coherent contribution to heavy-quark decay near t_m.

Both the calculated valence-quark-model and B^* contributions to f_+ have considerable uncertainties. For example, in Ref. 3, the branching ratio of $\bar{B}\to\pi^+e^-\bar{\nu}_e$ from f_+^{QM} was estimated to be uncertain by a factor of about 2. It is clear from Fig. 2 that the inaccuracy of the quark-model predictions for this branching ratio associated with the omitted B^* contribution is smaller than this. However, it is interesting that in the end-point region the B^* contribution could substantially affect the rate. In view of the potential importance of this process in the region used for extracting V_{ub}, this possibility deserves further study.

Note added in proof. We are investigating the relative sign of f_+^{QM} and f_+^{B*} to determine whether they interfere constructively or destructively in the $B^*-\pi e\bar{\nu}_e$ rates. However, since the B^* contribution is concentrated in a very small corner of the Dalitz plot near $t=t_m$, the resulting electron spectrum is in any event reasonably close to the incoherent sum of the two spectra shown in Fig. 2: explicit calculations give an interference term which in the end-point region of the electron spectrum is about 50% of that sum. We are grateful to Lincoln Wolfenstein for bringing this point to our attention, and to Daryl Scora for help in calculating the magnitude of the interference effect.

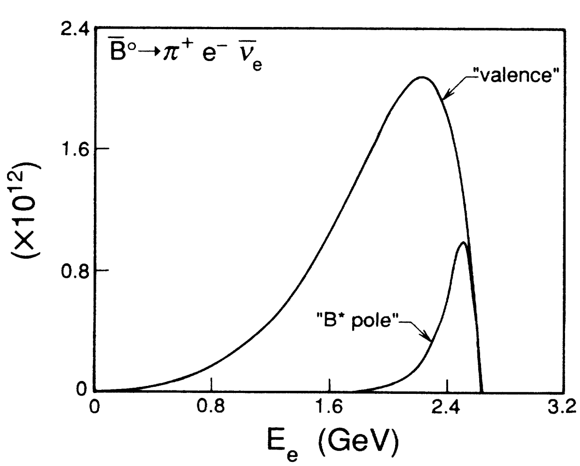

FIG. 2. Rates for $\bar{B}\to\pi^+e\bar{\nu}_e$ from the B^* pole contribution and the valence-nonrelativistic-quark model. For the decay $\bar{B}^-\to\pi^0e\bar{\nu}_e$ the rates are $\frac{1}{2}$ those presented here.

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