Nonperturbative QED and loop bremsstrahlung of neutral lepton pairs in heavy-ion collisions

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A simplified model of an eikonal representation for the scattering of high-Z ions is formulated, involving an IR approximation to closed fermion loops which themselves carry full radiative corrections. Signals are found for the possible production of "resonances" from such a loop when the impact-parameter-dependent electric field between the ions reaches successive thresholds on the order of $m^2c^{5/2}h^{-3/2}$. Qualitative arguments are given leading to narrow widths ($\sim$30 keV) and a prediction of peak energies in reasonable agreement with the data. An explicit enhancement factor, arising from the computation of low-frequency radiative corrections across the loop in the presence of a sufficiently strong external field, compensates the smallness of the fine-structure constant.

I. INTRODUCTION

Recent experiments[1] indicating that e^+-e^- pairs of well-defined total energy have been produced in the collisions of energetic heavy ions have lead to theoretical speculations[2] concerning the possible existence of a "strong-coupling" (SC) phase of QED. Prompted by these suggestions we have applied the continuum, infrared (IR) method[3] previously defined for other, SC (chiral) problems to the construction of eikonal amplitudes for the scattering and associated production of lepton pairs of a pair of heavy ions. Although the analysis is too complicated to permit at present a full calculation of the model's predictions, certain general features suggest a form of behavior of both scattering and production amplitudes consistent with observed results; and it is these features of the model, together with suggestions for subsequent experimental measurements, which we would like to describe here.

If at all relevant to a SC phase of QED, the analysis of these experiments must involve closed electron loops, and associated radiative corrections, in an essential manner. One imagines virtual, vacuum-polarization loops appearing in the presence of "external" (essentially electric) fields E^{ext}, which are themselves due to the Coulomb fields of the high-Z ions at small distances; and one asks if the existence of such loops can contribute significantly to the production of heavy, virtual objects, constructed from multiple photons emitted from those loops, which produce a final e^+-e^- pair. Each loop is supposed to have its full complement of radiative corrections, with an infinite number of virtual photons exchanged across that loop; each loop is joined to the scattering ions by an infinite number of virtual photons which comprise and define E^{ext}. From the loop one imagines a number of photons emitted, with a possible "resonance" defined as a sum over all numbers of such photons; in photon language, appropriate to the situation when the ions have separated and their electric field in the vicinity of the loop is decreased below an appropriate threshold, such a "resonance" would have a projection onto any number of photons. Those photons can be either virtual (Fig. 1) or real (Fig. 2), and, if virtual, as discussed here, can form a real lepton pair.

Our work was stimulated by the work of Caldi and Chodos,[2] even though the immediate objects of concern, there and here, are somewhat different. They considered an electron Green's function $G_c[A]$ in a constant electric field, and asked if a particular kind of supersymmetry of that $G_c[A]$ could be broken in the limit of an intense field. Their answer was negative; but they commented that it might be different if other than constant fields were used; and that there might be an effect of some relevance if radiative corrections across that $G_c[A]$ were included. Our paper is an attempt to include just those things, but for the log of the fermion determinant, $L[A]$, rather than for $G_c[A]$, and in the context of eikonal scattering and production amplitudes.

That part of the eikonal formulation which describes the exchange of virtual photons between the scattering ions is well known, and can be found in many places.[4] That part of the eikonal formulation dealing with closed fermion loops can be obtained by applying the "IR ap-

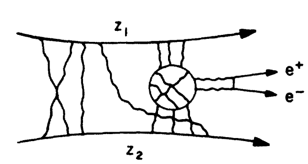

FIG. 1. Pictorial representation of the scattering of a pair of heavy ions ($Z_{1,2}$), and the emission from a closed electron loop of one or more virtual photons which materialize into a lepton pair.

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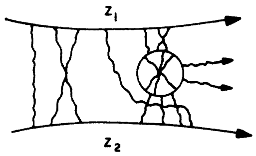

FIG. 2. The same as in Fig. 1, except that the photons are real.

proximation" to QED_4, in direct analogy with the techniques used[3] for the $\langle \bar{\psi}\psi \rangle$ problem in QED_2 and QCD_2. (That is, all possible numbers of virtual photons are included, with a continuous spectrum of frequencies less than or on the order of the lepton mass of the loop, leading to an explicit, finite, gauge-invariant result.) Upon combining these two aspects of the eikonal formulation, one here meets an awkward and practical difficulty: it does not seem possible to perform the necessary integration over the loop coordinate x as long as one retains all powers of E^{ext}. In order to proceed in a nonperturbative way, we have adopted an averaging procedure which—although it precludes the possibility of direct kinematical statements—still contains enough structure to permit reasonable inferences to be drawn concerning the properties of this eikonal system.

The averaging procedure replaces the initial eikonal forms whose "external" fields acting on the loop contain dependence on the ion coordinates x_1, x_2 as well as on the loop coordinate x, with a factorized model in which the "external" field acting on the loop is taken, realistically, as the classical, impact-parameter-dependent, transverse electric field existing between the two ions: $E^{\text{ext}}=E(b) \approx b^{-2}$, with $\mathbf{b}=\mathbf{x}_1-\mathbf{x}_2$ denoting the transverse, interion separation. We consider a closed-electron loop defined in this background field, including all soft radiative corrections across the loop and an arbitrary number of virtual photons emitted from the loop, and take this as one part of a factorized model of production, assuming that it will eventually be justified by a better calculation.

II. CALCULATIONS

Applying the IR method (fully documented in Ref. 3) to this closed loop of QED_4, one obtains a representation for $L[A]$ which resembles Schwinger's solution[5] for (that most IR of all fields) a constant $F_{\mu\nu}$, except that in this case the $F_{\mu\nu}$ which appear in the integrand of his proper time (τ) representation can themselves be dependent on τ, as well as on the loop coordinate x, according to the replacement (for each μ,ν component)

$$F \rightarrow F_{\text{IR}}(x)=\int d^4y\, f(x-y)F(y)\ ,$$

with (in Euclidean space) $f(z)=(\mu_c^2/4\pi)^2\exp[-(z\mu_c/2)^2]$. One has the option of choosing μ_c as proportional to $1/\tau, \mu_c=c/\sqrt{\tau}$, or more simply as a constant on the order of the mass of the fermion loop, which specifies[6] the upper limit of the soft momentum region: $\mu_c=cm$ where $c\sim 1$. Just as for a constant of integration, or a subtraction constant in a dispersion relation, c must be specified by some method external to this estimation; in $(QED)_2$, for example, equating the result of the IR method (using $\mu_c=cm$) for the quenched, chiral limit of $\langle \bar{\psi}\psi \rangle$ to the known, exact result[7] yields $c=(2\pi)^{1/2}\exp(H)$, where H is Euler's constant. For simplicity and convenience we set here $c=(8\pi)^{1/2}$.

The result of this IR method is to provide the explicit, if approximate, representation:

$$L[A]=(i/8\pi^2)\int d^4x \int d\tau\, \tau^{-3}\exp(-\tau m^2)\mathcal{F}_\tau((F_{\text{IR}})^2/4, F_{\text{IR}}\,{}^*F_{\text{IR}}/4)\ , \quad (1)$$

with $\mathcal{F}_\tau$ given by Schwinger's renormalized expression

$$\mathcal{F}_\tau(\alpha,\beta)=(e\tau)^2\beta\coth(e\tau X_r)\cot(e\tau X_i)-1-\tfrac{2}{3}(e\tau)^2\alpha\ , \quad (2)$$

where

$$X_r=(1/\sqrt{2})[(\alpha+i\beta)^{1/2}+(\alpha-i\beta)^{1/2}],\quad X_i=(-i/\sqrt{2})[(\alpha+i\beta)^{1/2}-(\alpha-i\beta)^{1/2}]\ ,$$

and $\mathcal{F}_\tau(0,0)=0$. Here, $F^2/4$ and $F^*F/4$ are the two invariant quantities $(B^2-E^2)/2$ and $B\cdot E$, respectively, with ${}^*F_{\mu\nu}=(i/2)\epsilon_{\mu\nu\sigma\lambda}F_{\sigma\lambda}$. For our calculation, $F\rightarrow(F+F^{\text{ext}})_{\text{IR}}$, with the quantum fluctuations of the loop expressed in terms of F.

One must calculate those fluctuations in Euclidean space, by continuing $\mathbf{E}$ (in the relativistic notation of Ref. 5) to imaginary values; or, effectively, by removing the factor of (i) in the definition of ${}^*F$ of (2). Then, the $\mathcal{F}_\tau$ of (1) may be rewritten as

$$\int d\alpha \int d\beta\, \mathcal{F}_\tau(\alpha,i\beta)\int d\omega \int d\xi (2\pi)^{-2}\exp[-i(\alpha\omega+\beta\xi)]\exp[i\omega(F_{\text{IR}}+F_{\text{IR}}^{\text{ext}})^2/4+i\xi(F_{\text{IR}}+F_{\text{IR}}^{\text{ext}})^*(F_{\text{IR}}+F_{\text{IR}}^{\text{ext}})/4]\ , \quad (3)$$

where all four integrals from from $-\infty$ to $+\infty$. One then performs the Gaussian functional integration (corresponding to photon fluctuations across the loop) over this F_{IR} and ${}^*F_{\text{IR}}$ dependence, using

$$i(\omega F^2/4+\xi F^*F/4)=(i/2)\int\int A_\mu(u)K_{\mu\nu}(u,v)A_\nu(v)$$

and the definition

$$K_{\mu\nu}(u,v)\equiv\omega[\delta_{\mu\nu}\partial_\lambda f(u-x)\partial_\lambda f(v-x)-\partial_\nu f(u-x)\partial_\mu f(v-x)]-\xi\epsilon_{\mu\nu\sigma\lambda}\partial_\sigma f(u-x)\partial_\lambda f(v-x)\ .$$

There results

$$\mathcal{W}\exp\left[(i/2)\int A^{\rm ext}\mathcal{M}A^{\rm ext}\right], \tag{4}$$

with

$$\mathcal{W}=(1-i\Lambda\omega_+/2)^{-3/2}(1-\Lambda\omega_-/2)^{-3/2},$$
$$\mathcal{M}_{\mu\nu}(u,v)=\tfrac{1}{2}(K_{\mu\nu}-H_{\mu\nu})(1-i\Lambda\omega_-/2)^{-1}+\tfrac{1}{2}(K_{\mu\nu}+H_{\mu\nu})(1-i\Lambda\omega_+/2)^{-1}, \tag{5}$$

and where $\omega_+=\omega+\xi$, $\omega_-=\omega-\xi$, $\Lambda=(\mu_c^2/8\pi)^2=m^4$, and $H_{\mu\nu}$ is the same as $K_{\mu\nu}$ with ω and ξ interchanged. Note that (4) is, at it must be, a gauge-invariant expression, since $\partial_\mu^u\mathcal{M}_{\mu\nu}(u,v)=\partial_\nu^v\mathcal{M}_{\mu\nu}(u,v)=0$. One may also note that in the limit when all radiative corrections across the loop are discarded, $\Lambda=0$, (4) and (3) reproduce the $\mathcal{F}_\tau$ of (1).

Although the integrations $\int d\omega_+\int d\omega_-$ are not difficult to perform, as given below, and those over α,β are nicely convergent, integration over the loop coordinate x is awkward if all powers of $A^{\rm ext}$ are retained, as appropriate for a SC calculation, for that x dependence appears in every $\mathcal{M}_{\mu\nu}$ of (5). [For any such finite-order $A^{\rm ext}$ calculation, it is easy to see that the range of the α,β variables is limited by $0\le\alpha\le\infty$, $-\alpha\le\beta\le+\alpha$, restrictions which correspond to the Euclidean conditions $(B\pm E)^2\ge0, B^2+E^2\ge0$; and we shall adopt these limits below.]

If we replace the $A^{\rm ext}$ by potentials corresponding to a constant $F_{\mu\nu}$, the situation is somewhat simplified, because the x dependence disappears from all parts of the integrand. Physically, the field existing between the incoming ions is essentially electric, as least for moderate v/c values. Its value is certainly b dependent, and we shall call it $E(b)$. It would then not be physically absurd to use $A_\mu^{\rm ext}(z)=-\frac{1}{2}z_\nu F_{\mu\nu}^0$, where only the electric field E_0 of $F_{\mu\nu}^0$ is nonzero, and where $E_0^2\approx E^2(b)$; in that case, (4) contributes to the ω integrals the amount

$$\mathcal{W}\exp[-i(E_0^2/4)\Omega], \tag{6}$$

with $\Omega=[\omega_+(1-i\Lambda\omega_+/2)^{-1}+\omega_-(1-i\Lambda\omega_-/2)^{-1}]$.

In the computation of the ω integrals it is important to remember to return to Minkowski space, $F_0^2\to-2[E(b)]^2$ (otherwise the new, α- and β-dependent oscillations found below will be replaced by nonoscillatory integrands). A change of variables, $\omega_{1,2}=\omega'_{1,2}-2i/\Lambda$, and a straightforward integration permits $(2\pi)^{-2}$ times the $\int d\omega_+\int d\omega_-$ of (6) to be given by

$$(\Lambda/\pi\rho)\sin[(\mathcal{A}_+\rho)^{1/2}]\sin[(\mathcal{A}_-\rho)^{1/2}]\times\exp[(\rho/2)-2(\alpha/\Lambda)], \tag{7}$$

where $\mathcal{A}_\pm=\alpha_\pm/\Lambda, \alpha_\pm=\alpha\pm\beta$, and $\rho=8[E(b)]^2/\Lambda$. Finally, (7) is to be multiplied by $\mathcal{F}_\tau(\alpha,i\beta)$ and inserted under the integrals $\int d\alpha\int d\beta$, with the range of the α,β integrations as described after Eq. (5).

If one virtual photon is emitted from the loop one finds the possibility of a sine and a cosine term, as well as a product of sine terms in the expression corresponding to (7). If two photons are emitted from the loop—and this might seem to be the more intuitively appealing case, for ease of balancing back-to-back momenta in a better calculation—one finds in addition the third possibility of a product of cosine factors in the equation corresponding to (7). Larger numbers of emitted photons produce combinations of these possibilities. The appearance of the lepton pair is the last step in the inelastic process, with all of these photons converting into the pair.

III. MASSES AND WIDTHS

We have now performed all computations necessary for the extraction of those qualitative properties contained within the model. We first comment that contributions to the integrals $\int d\alpha\int d\beta$ of (7), multiplied by the smoothly varying $\mathcal{F}_\tau(\alpha,i\beta)$, really occur only for those "points of enhancement" in the integrand when the oscillating functions of (7) are in phase. The main contributions to the α,β integrals will come when the arguments of the sine functions differ by π times an integer n, and when those arguments are themselves given by $(N+\frac{1}{2})\pi$, where N is a different integer. When this occurs, the values of α and β will be given by

$$\alpha=\pi^2(\Lambda/\rho)[(N+n+\tfrac{1}{2})^2+(N+\tfrac{1}{2})^2], \tag{8}$$

$$\beta=\pi^2(\Lambda/\rho)[(N+n+\tfrac{1}{2})^2-(N+\tfrac{1}{2})^2]. \tag{9}$$

Because of the built-in exponential cutoff, $\exp(-2\alpha/\Lambda)$ of (7), only values of $\alpha\le\Lambda/2$ are going to contribute appreciably to the integrals; and from (8), this means that $\rho\ge2\pi^2[(N+n+\frac{1}{2})^2+(N+\frac{1}{2})^2]$. There is therefore a minimum value of ρ, or $E(b)$, which must be reached before (8) can be satisfied, $\rho_{\rm min}=\pi^2$, or $E(b)_{\rm min}=\pi m^2/\sqrt{8}$. As $E(b)$ becomes larger, more and more values of N and n become visible, there are more $\sin^2(y)$-type peaks in the integrand as one sweeps through contributing α,β values, with the values of $\mathcal{F}_\tau$ evaluated at those α,β acting as the coefficients of expansion in this form of approximation to the integrals.

To see this in a clear way, suppose one were to discard all contributions except those which occur when the oscillatory factors are in resonance; then, one could speak of a sequence of "peak contributions" to the value of the integral. More realistically, one has a smooth background on which peaks of the *integrand* are superimposed, corresponding to a series of thresholds for the values of the *integrals,* changes which appear abruptly as $E(b)$ is changed. Because the model masks the detailed kinematics, we do not know with precision how these thresholds are converted into peaks of the production amplitude, although the general method of converting thresholds in a b distribution into peaks of a p_t distribution is clear: One may always expect peaks when oscillating functions are folded into thresholds. The difficulty here is that the widths of the outpeak peaks can be much larger than the experimental widths one is trying to reproduce, and one must be careful to isolate different effects even though a detailed specification of what is to be calculated is missing.

For example, that part of the ions' scattering amplitude dependent on such loop thresholds $H(b)$ would require the evaluation of $\int d^2b \exp(iqb)H(b)$, where q denotes an ion's momentum transfer, and all other factors are suppressed. If $H(b)$ is represented by $\sum_n C_n \theta(b_n - b)$, which neglects the width (of the derivative) of the threshold distribution, then this integral is given by $2\pi \sum_n C_n b_n J_1(qb_n)/q$, which displays "peaks" of the Bessel function, but with widths far too large to be relevant here. However, consider this on a finer scale, including the widths of the threshold distributions; if one calculates an amplitude associated with lepton emission involving, e.g., the p_t taken away by the positron, then one will need to consider something of the qualitative form $\int d^2b \exp(ip_t b) dH/db$, which will be sensitive to the widths that define the threshold distribution, as well as to the separation of the threshold peaks. Until one can handle the detailed kinematics in a better fashion than permitted by the model approximations, one can only make such vague arguments; but the association of p_t peaks with the threshold b dependence, itself correlated with the integers n and N, can still be true. It is adopted here as an assumption of this kinematically incomplete model.

It is most attractive that these peaks become visible only when $E(b)$ is sufficiently large; and that any perturbative development in powers of $E(b)$ would ruin the effect completely. The physical origin of the peaks is a delicate interplay, or interference, between the intense background field and the low-frequency radiative corrections across the loop; each of these contributing factors requires a summation over an infinite number of virtual photons, and is therefore nonperturbative in character. A semiclassical picture of just what happens when a closed loop appears in the neighborhood of the scattering ions, and why the radiative corrections across the loop are so important, has been given elsewhere.[8]

In the case of (7), one expects that significant contributions to the α,β integrals will appear whenever $E(b)/m^2$ reaches a "threshold" given by $(\pi/2)[(n+\frac{1}{2})^2+(l+\frac{1}{2})^2]^{1/2}$ where l and n are integers. If one photon is emitted from the loop there will be significant contributions as above and also when $E(b)/m^2$ reaches $(\pi/2)[(n+\frac{1}{2})^2+l^2]^{1/2}$; when two or more photons are emitted, one expects significant contributions at the above values and also when $E(b)/m^2$ reaches $(\pi/2)(n^2+l^2)^{1/2}$, where both integers cannot be zero simultaneously. This latter situation will be used to illustrate the arguments which follow.

Intuitively, because longitudinal electric fields average to zero, one expects significant dependence on transverse coordinates only, on the impact parameter b, and on the transverse momenta p_t of each of the final leptons. By considering only transverse momenta in what follows we do mean to imply that longitudinal momenta cannot enter; rather, we are going to adopt an indirect method of estimating those enhancements to the continuous lepton distributions—their peak energies and widths—which are reasonable when longitudinal momenta are much less than transverse, and we rather expect this to be the predominant case. Since $E(b) \approx b^{-2}$, one expects enhancements to occur whenever mb decreases to values $\approx (n^2+l^2)^{-1/4}$; and because the p should be correlated with b, at those enhancements one might expect for each lepton, in the c.m., a value of $p/m \approx (l^2+n^2)^{1/4}$. In the final c.m. (decay of the supposed resonance of mass M, or, preferably, the immediate) formation of the lepton pair of invariant mass M, one has $M^2=4\mathcal{E}^2=4(m^2+p^2)$, where $\mathcal{E}(p)$ denotes the energy of each lepton. We may therefore expect an enhancement of the production amplitude when

$$M^2=M_A^2(n,l)=(2m)^2+\xi_A(n^2+l^2)^{1/2}\ , \tag{10}$$

where ξ_A is a constant on the order of m^2 which would appear automatically in a better calculation. Two other (heavier) families [$M_B(n,l)$ and $M_C(n,l)$ with different values of ξ] are also possible, corresponding to the use of $[(n+\frac{1}{2})^2+l^2]^{1/2}$ or $[(n+\frac{1}{2})^2+(l+\frac{1}{2})^2]^{1/2}$ in (10).

Let us compare the output of (10) with the coincidence experiments, which find three, narrow, c.m. e^+-e^- peaks at 1.64, 1.76, and 1.84 MeV. If the constant ξ_A is fixed by identifying $M_A(1,0)$ with 1.64, then the next highest state is $M_A(1,1)=1.84$, in perfect agreement with the third peak. Presumably, the 1.76 corresponds to the lowest member of one of the other families (most probably B). In fact, the most recent GSI experiments reported at Moriond suggest[9] that of the three measured states, the highest- and the lowest-energy peaks correspond to lepton pairs emitted at 180° (that is, back-to-back in the ions' c.m.); and that the intermediate-energy pair is emitted preferentially forward, at about 90°. These experimental results fit very nicely with the multifamily aspect of this model, suggesting that the "quantum numbers" n and N may have a significance deeper than that of their appearance in these crude, model kinematics. Our prediction for the mass value of the next higher-mass lepton pair to be emitted at 180° is then just the next member of the A family, $M_A(0,2)=2.08$ MeV.

It should be noted that (10) represents a "boson-type" formula, for M^2 rather than for M. For large values of the integers l or n, it has the familiar form of a Chew-Frautschi plot, of a linearly rising M^2 curve.

A qualitative idea of the widths generated by such integrand enhancements can also follow from this analysis. From (7) it is easy to ask for those variations of E, and hence of b, within the oscillating factors which are equivalent to variations of α. A peak of α will occur when, say, $\alpha=\pi^2(n^2+l^2)$. Holding l constant, the next peak will occur at $\alpha=\pi^2[(n+1)^2+l^2]$, and the distance between successive peaks is given by $\Delta\alpha=\pi^2(2n+1)$. The distance between the peak and the next zero, however, is given by $\alpha(n+\frac{1}{2},l)-\alpha(n,l)=\delta\alpha=\pi^2(n+\frac{1}{4})$.

The variations of E, or of b, equivalent to these variations of α, can be inferred by requiring that the appropriate variations of $[\alpha E^2(b)]$ vanish. If δb is that change of b corresponding to $\delta\alpha$, then one has $\delta\alpha/\delta b=4\alpha/b$. Similarly, if Δb is that distance corresponding to $\Delta\alpha$, then $\Delta\alpha/\Delta b=4\alpha/b$. Comparing these, it is apparent that $\delta\alpha/\Delta\alpha=\delta b/\Delta b$. If we now switch to transverse momentum p, where one expects $\delta b/\Delta b=\delta p/\Delta p$, one can (finally) write $\delta p=\Delta p(\delta\alpha/\Delta\alpha)$, where Δp denotes the difference in p values of two neighboring peaks and δp is a measure of the half-width of one of those peaks.

As calculated above, the estimate $\delta\alpha/\Delta\alpha$ is purely geometrical, $\delta\alpha/\Delta\alpha=\frac{1}{2}(n+\frac{1}{4})/(n+\frac{1}{2})<\frac{1}{2}$, while a typical, experimental Δp is on the order of one-tenth of an MeV. To be specific we use the e^+-e^- data, where the separation of Δp of the first two states (corresponding to $l=1$, $n=0$ and $l=1$, $n=1$) is 0.124 MeV/c. For $n=0$, which should correspond to the first state, one then has $\delta p=0.031$ MeV/c. The half-width of the kinetic-energy peak is given by $\delta\mathcal{E}\approx\{p/[m+\mathcal{E}(p)]\}\delta p$, which works out to be about 24 keV. The same calculation done for the noncoincidence e^+ data gives a rough estimate of 31 keV for the width of the first state. These are, of course, only crude estimates; but they do seem to be of the same order of magnitude as the narrow widths seen experimentally.

It will be noted that we have tried to avoid the use of the word "resonance," with its connotation of a small width and long lifetime. If the natural time scale of the virtual resonance is (at least) on the order of the inverse of its mass, then it is easy to show that this object can last long enough for the transverse $E(b)$ field to fall to a very small fraction of its closest-impact value. But, then, the decay of such a resonance should not be correlated with b, and the lepton momentum correlations leading to sharp values of the total energy of the system would not be operative. The sharp peaks could result from those situations in which a resonance decays rapidly enough so that the resulting lepton production is sensitive to the b dependence of the scattering ions; and in this case, there is probably not too much significance to the idea of a "resonance."

IV. MAGNITUDES

One must face the criticism leveled at any QED closed-loop models used to explain the sharp e^+e^- production, which has recently been given by Peccia, Sola, and Wetterich[10] (PSW). They noted that in calculating the effect of a loop in external fields, however strong, there is always a multiplicative factor of the fine-structure constant (FSC), which will reduce to experimental insignificance the size of any term so calculated. One must therefore ask if this difficulty is present here—even after radiative corrections across the loop are taken into account—or if there is some other mechanism which comes into play.

To be sure, the radiative corrections included here lead to a form different from that found in the usual Schwinger loop in an external field only, in the sense that integrations over the $\alpha=F^2/4$ and $\beta=F^*F/4$ variables are now required. One can trace the e^2 dependence in these formulas and come to the conclusion that—in spite of the difficulty of performing a legitimate expansion in powers of e^2—it is still true that the multiplicative PSW FSC is going to be present.

However, there is an additional multiplicative factor, a "special enhancement," which appears when low-frequency quantum fluctuations across the loop are calculated in the presence of a strong external field. In the present case, this enhancement is given by the factor $\exp(\rho/2)$ of (7), which multiplies integrals of order unity, along with the PSW FSC. In the overlap region of interest here, this term takes on its smallest value when $E(b)_{\min}=\pi m^2/\sqrt{8}$, thereby contributing the amount $\exp(\rho/2)\rightarrow\exp(\pi^2/2)\approx139$, nicely compensating the PSW FSC. Higher n,l overlaps contribute larger amounts, which serve to remove the additional FSC factors associated with multiple-photon production and subsequent conversion to a lepton pair.

This very pleasant feature removes the overt PSW objection to this model, so that *LB* is not automatically ruled out as a possible mechanism for lepton production. In more general terms, this type of effect can be of considerable importance in future studies: Radiative corrections performed in the presence of intense fields need not be small. (Since this is basically a low-frequency, or IR effect, lattice calculations which search for similar quantities should be very careful to estimate or eliminate finite-size effects.) Also, if the effect of a single such loop turns out to be significant, then it could be important to estimate the nonlinear effects of many loops.

V. SUMMARY AND CRITIQUE

In this paper we have tried to suggest the possible importance of *LB* as a mechanism for the sharp lepton peaks seen in the GSI experiments; and we have done this while tied to the crude kinematics forced upon us by our inability to perform one final loop integration in a suitably nonperturbative manner. Summing over infinite numbers of a continuous spectrum of low-frequency photons is not exactly a trivial matter, but this nonperturbative computation can be carried through almost completely because of techniques developed in previous chiral-symmetry-breaking studies.[3] The predictions that have been extracted from the present analysis seem to agree with the experiments whenever comparisons can be made, in the matter of masses, families, and widths. Perhaps the most extraordinary part of the computations are the magnitudes found; and if no errors have been made, if the results are really true, this calculation should have an importance beyond that of the present physical problem.

But that is not all. The most recent experiments reported at Moriond[9] contain two other bits of relevant information, which have a bearing on this model, as well as on other attempts to explain the GSI peaks. These findings are as follows.

(i) The peaks are dependent on incident beam energy. With a slight change on the order of 0.1 MeV/nucleon of the incident ion, either the highest- or the lowest-energy peak may be made to disappear.

(ii) The peaks show a definite dependence on the momentum transfer of the scattered ion.

These two items suggest that theoretical models of a "new neutral particle," or of "excitations of a new phase of QED" are now tenable only with extreme prejudice; but they leave open the possibility that an appropriate generalization of a "resonance production" model,[11] or of the present *LB* model may turn out to be correct. The formulation of the latter is still sufficiently crude to mask the kinematical details necessary for an explanation of (i), although such dependence can easily be there in principle. But, as for any bremsstrahlung model, the essence of *LB* is its dependence upon the scattering ions' impact parameter, or momentum transfer. It is very satisfying that this experimental effect has now been seen; and it is hoped that more detailed correlations of peaks with ion momentum transfer will be measured in the future.

On the other side of the ledger, there are two criticisms which have been raised, and which are not simple to answer. These are the following.

(iii) If *LB* is to be taken seriously, and lepton production is to be correlated with details of the ions' scattering, how can the peaks remain sharp? If the lepton pair is produced in the vicinity of the ions, should not the strong ionic fields distort the leptons' motion and broaden their peak distributions?

(iv) Consider only the scattering process in which the loop appears, without the emission of virtual photons which convert to the lepton pair. If the magnitude of that loop is large, then one might expect the magnitude of the absorptive part of the loop (describing the production of a lepton pair in the nonresonant, or background situation) to be large also. But potential theory estimates of the background agree almost perfectly with what is measured; and if so, how can the loop magnitude be as large as claimed?

These are questions that must be faced, even though they cannot yet be answered by detailed kinematical arguments. For (iii), the best that one can now do is to realize that those pairs which are emitted with zero (or very small) total momentum may be produced at respectable distances—that is, atomic distances— from the scattering center; and in such cases the peak distortion would be on the order of or less than the widths. [In contrast, a pair having a total energy $\approx 3m$ should materialize in a time on the order of $(3m)^{-1}$. For example, one would expect the two-photon virtual state of Fig. 1 to last for a time on the order of m^{-1}.] Pairs which are produced at closer distances can blend into the background; but those that materialize far away should not be affected. Without explicit calculation this argument is no more than a possibility; but it *is* that.

Question (iv) can be "answered" by realizing that the absorptive part of the loop of Fig. 1, with its two leptons on their mass shell, contains IR divergences, which must be removed in the standard, Bloch-Nordsieck manner. (There are no IR divergences in the closed loop.) That absorptive part of the loop corresponding to the potential-theory estimate, in which a lepton pair with no real photons is emitted, is strictly zero because of the IR-divergent damping of virtual photons, and hence makes no contribution to the cross section. But this question raises another, and deeper, question of principle.

In QED, unitarity requires a specification of the minimum energy resolution of photon detectors, etc., quantities which do not appear in the potential-theory background estimates. What is the form that QED unitarity takes in the presence of intense external fields, and just how are the IR divergences removed when such fields are present? Such a question must first be posed, and answered, if one is going to use unitarity to guess the size of production amplitudes. If it is true that the form of the ordinary result of unitarity is preserved for the "states" corresponding to the interference peaks in intense fields—that is, where sums over all real and virtual photons conspire to remove the IR divergences, leaving a finite factor of maximum magnitude $\sim 20\%$ which multiplies the potential theory estimate—then our computation is either wrong or incomplete. But, at present, one has simply no idea of the details and the results following the removal of IR divergences in the presence of intense fields, treated nonperturbatively. If the existence of such electric fields can produce new states, then the expression of unitarity may need to include sums over such states. As suggested by the magnitudes obtained here, by a comparison with experimental data, and by the intuitive expectation that the probability for lepton pair production in large electric fields can be large, conventional unitarity prescriptions may be valid for the nonresonant background, while special enhancements can appear for the interference peaks which are themselves due to the presence of the strong fields. Again, a detailed answer waits upon better calculation.

In summary, the present, crude calculations of *LB* seem to be in accord with experiment in those cases where it is possible to make a comparison, agreements which suggest the possibility that this *LB* mechanism may be physically relevant. The large magnitudes found in these estimates of the interference terms are spectacular, and perhaps "too good to be true;" but, it should be realized, the nonperturbative world of intense fields may have a logic all its own.

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