Weak-boson production by charm quarks

K. Hagiwara, F. Halzen, and C. S. Kim
Department of Physics, University of Durham, DH1 3LE, Durham, England
and Department of Physics, University of Wisconsin, Madison, Wisconsin 53706
(Received 19 June 1989)

We propose a method to extract the charm structure function of the nucleon $c(x,Q^2=m_W^2)$ from measurements of the rapidity distribution of $W^\pm$ and Z in $p\bar{p}$ collisions. The production of weak intermediate bosons via charm occurs at the few percent level at the Fermilab Tevatron and has in particular a strong influence on the important measurement of the ratio of W and Z events as $c\bar{s}\to W$ is expected to be much larger than $c\bar{c}\to Z$.

The ratio R of $W\to e\nu$ and $Z\to e^+e^-$ events in $p\bar{p}$ collisions measures the product of the ratio of the partial widths

$$[\Gamma(W\to e\nu)/\Gamma(W\to \text{all})]/[\Gamma(Z\to e^+e^-)/\Gamma(Z\to \text{all})]$$

with the ratio of the production cross sections $\sigma(p\bar{p}\to W)/\sigma(p\bar{p}\to Z)$:

$$R=\frac{\Gamma(W\to e\nu)/\Gamma(W)}{\Gamma(Z\to e^+e^-)/\Gamma(Z)}\frac{\sigma_{W^+}+\sigma_{W^-}}{\sigma_Z}\equiv R_\Gamma R_\sigma \ . \quad (1)$$

$R_\Gamma=3.24$ for $m_t>m_W-m_b$ in the standard model with three generations of quarks and leptons. Traditionally[1] measurements of R have been exploited to obtain constraints on R_Γ (e.g., the number of neutrinos) assuming in (1) values of R_σ computed in perturbative QCD. It is well known that the intrinsic limitation in interpreting data on R is associated with our knowledge of the nucleon structure functions entering in the computation of the cross sections in R_σ. Assuming confirmation of the three-generation value of R_Γ and the fact that the W does not decay into $t\bar{b}$, we draw attention to the possibility of exploiting R measurements to make a measurement of the charm structure function of the nucleon $c(x,Q^2=m_W^2)$.

The ambiguity related to the nucleon structure function has two prominent features: the u/d ratio and the charm structure function. At the energy $\sqrt{s}=630$ GeV of the CERN collider the production of W,Z is dominated by valence quarks, therefore symbolically

$$R_\sigma\sim\int\frac{u\bar{d}+d\bar{u}}{u\bar{u}+d\bar{d}}\otimes\frac{\hat{\sigma}(q\bar{q}'\to W)}{\hat{\sigma}(q\bar{q}\to Z)} \ . \quad (2)$$

The ratio of the quark subprocesses has been computed to higher order and is known accurately.[1] The structure functions $u(x),d(x)$ cancel out of the calculation in the limit $u=d$, see Eq. (2). Therefore the precision of a calculation of the relative cross sections for W,Z in $p\bar{p}$ interactions is controlled by our knowledge of $u(x)/d(x)$ in the vicinity[2] of $x=m_W/\sqrt{s}\simeq 0.15$.

At the Fermilab Tevatron energy of $\sqrt{s}=1800$ GeV weak bosons are predominately produced by annihilation of valence and sea quarks as a result of the reduced characteristic x ($=m_W/\sqrt{s}\simeq 0.05$) value at higher energy. The sensitivity to the relatively poorly known u/d ratio becomes less critical.[3] Unfortunately, a different aspect of the structure functions will spoil the precision of relation (1) as a test of the standard model. The $c\bar{s}\to W$ and $c\bar{c}\to Z$ subprocesses contribute significantly to the ratio $\sigma(p\bar{p}\to W^\pm)/\sigma(p\bar{p}\to Z)$ even though $c\bar{s}\to W$ is expected to contribute 5% to the numerator. The reason is that the charm contribution to $p\bar{p}\to Z$ is expected to be much smaller as it is proportional to $c\bar{c}$ and this significantly increases the cross-section ratio. It should be further noted that the charm contribution to the W production in the vicinity of rapidity $y\simeq 0$ can modify the cross section by as much as 15% at $\sqrt{s}=1.8$ TeV. We will show how this can be exploited to determine $c(x,Q^2=m_W^2)$. The same measurement will be difficult at CERN S$p\bar{p}$S energy as R is modified by less than 2% even when we assume an SU(4)-symmetric charm contribution.

Formally the solution to these problems is straightforward. Experiments measure three independent cross sections, $\sigma_{W^-}(y)$, $\sigma_{W^+}(y)$, and $\sigma_Z(y)$ as a function of the weak-boson rapidity. (This assumes, of course, that R_Γ is given by its standard model value with three generations and $m_t>m_W-m_{b'}$.) Because of K factors and experimental normalization ambiguities we can achieve much better precision by concentrating on the two independent cross-section ratios which we choose to be

$$A(y)\equiv\frac{\sigma_A(y)}{\sigma_S(y)} \ , \quad (3)$$

$$B(y)\equiv\frac{\sigma_S(y)}{\sigma_Z(y)} \ . \quad (4)$$

Here

$$\sigma_S(y)\equiv[\sigma_{W^+}(y)+\sigma_{W^-}(y)] \quad (5)$$

and

$$\sigma_A(y)\equiv[\sigma_{W^+}(y)-\sigma_{W^-}(y)] \ . \quad (6)$$

The asymmetry $A(y)$ determines the ratio[3] u/d. This measurement can be done independently at the S$p\bar{p}$S and

 ©1990 The American Physical Society

the Tevatron. We will show that $B(y)$ determines c/s at Tevatron energies. The usual R ratio counting the relative number of $W^{\pm}$ and Z events is related to $B(y)$ by

$$R = R_{\Gamma} \frac{\int \sigma_S(y)dy}{\int \sigma_Z(y)dy} . \tag{7}$$

The role of $A(y)$ in determining u/d has been studied in detail in Ref. 3. Symbolically

$$A(y) = \frac{u(x_1)-d(x_1)}{u(x_1)+d(x_1)} - \frac{u(x_2)-d(x_2)}{u(x_2)+d(x_2)} . \tag{8}$$

Analyses relying on deep-inelastic scattering measurements[4,5] of F_2^n/F_2^p have been performed. They yield,[1,3] for the ratio of $p\bar{p} \to W$ and $p\bar{p} \to Z$ events,

$$R_\sigma = \begin{cases} 3.33 \pm 0.03 \text{ (BCDMS)} , \\ 3.41 \pm 0.04 \text{ (EMC)} \end{cases} \tag{9}$$

at $\sqrt{s} = 630$ GeV and

$$R_\sigma > 3.11 \tag{10}$$

at $\sqrt{s} = 1800$ GeV. The inequality sign comes from the fact that we omitted the charm structure function thus calculating the lower limit for R_σ.

We here concentrate on the issue of controlling the large ambiguity associated with our poor knowledge of $c(x)$ in relating the number of W-to-Z events to the quantity of interest R_Γ in Eq. (1). It is straightforward to calculate $B(y)$ in the QCD-improved parton model to leading order. We find the following expression useful to appreciate the order of magnitude of the charm contribution to W,Z production:

$$B(y) = \frac{\sigma_S(y)}{\sigma_Z(y)} \simeq 2 \left[\frac{m_W^2}{m_Z^2} \right] \frac{\Delta_{v\cdot v}^S + [u_v(x_1)S(x_2)+S(x_1)u_v(x_2)] + 2(1+\zeta)S(x_1)S(x_2)}{\Delta_{v\cdot v}^Z + [g_u^2 + g_d^2(1-x_0)r][u_v(x_1)S(x_2)+S(x_1)u_v(x_2)] + 2[2g_d^2 + g_u^2(1+\zeta^2)]S(x_1)S(x_2)} \tag{11}$$

with

$$\begin{aligned} g_u^2 &= \tfrac{1}{2}(1 - \tfrac{8}{3}\sin^2\theta_W + \tfrac{32}{9}\sin^4\theta_W) , \\ g_d^2 &= \tfrac{1}{2}(1 - \tfrac{4}{3}\sin^2\theta_W + \tfrac{8}{9}\sin^4\theta_W) , \\ \Delta_{v\cdot v}^S &\simeq r u_v(x_0)u_v(x_0) - (1-r)u_v(x_0)S(x_0) , \\ \Delta_{v\cdot v}^Z &\simeq [g_u^2 + g_d^2(1-2x_0)r^2]u_v(x_0)u_v(x_0) , \\ \zeta(x) &\equiv \frac{2c(x)}{\bar{u}(x)+\bar{d}(x)} \simeq \frac{c(x)}{S(x)} , \end{aligned} \tag{12}$$

and we take $m_W = 80.8$ GeV, $m_Z = 92.1$ GeV, and $\sin^2\theta_W = 0.23$. In Eqs. (11) and (12) we assumed universal SU(3)-symmetric sea-quark distributions $S(x) = \bar{u}(x) = \bar{d}(x) = s(x) = \bar{s}(x)$ at Q^2 of order m_W^2. We isolated the terms $\Delta_{v\cdot v}^S$ and $\Delta_{v\cdot v}^Z$ describing valence-valence annihilation. They are approximately constant over the range $|y| < 1$. The ratio $r(x) = [d_v(x)/u_v(x)]/(1-x)$ can be determined[3] from $A(y)$. It is to a large extent independent[6] of Q^2 and roughly equal to 0.5. In Eq. (11) x_0 represents the x value corresponding to $y = 0$. Among the two remaining terms in Eq. (11), the valence-sea annihilation terms populate the high $|y|$ region, whereas the sea-sea annihilation terms contribute mainly to central production. [For instance, using EHLQ 1 structure functions,[7] typical values are $\Delta_{v\cdot v}^S \simeq 29$ and $\Delta_{v\cdot v}^Z \simeq 27$. The valence-sea fusion term, which is about 61 (21) for $W(Z)$ at $y = 0$, grows with increasing $|y|$ to a value of 74 (25) at $|y| = 1$. The sea-sea annihilation term has a value of about 30 (15) for $W(Z)$ at $y = 0$ and decreases with increasing $|y|$ to about 19 (9) at $|y| = 1$.] The charm contribution can be as large as 16% for W production at $y = 0$ in the SU(4) limit ($\zeta = 1$), whereas its contribution to Z production is at most 8%. Though $2s/(\bar{u}+\bar{d}) < 1$ for lower-Q^2 deep-inelastic scattering experiments,[8] this asymmetry in the sea has disappeared by QCD evolution at $Q^2 = m_W^2$. For instance, in the EHLQ 1 parametrization, $2s/(\bar{u}+\bar{d})$ is $0.4 \sim 0.5$ at $Q^2 = 10$ GeV^2 and reaches a value of 0.8–0.9 at $Q^2 = m_W^2$. This makes the analysis in terms of the ζ variable practical.

We next calculate $B(y)$ as a function of the strength of the charm sea ζ from the full expression of Eq. (4) without making the simplifying assumptions that lead to Eq. (10). The calculation reproduces all the features of the simplified expressions. Our calculation in terms of W rapidity cannot be directly compared with experiment because the longitudinal momentum of the neutrino is not measured and as a consequence the W rapidity cannot be reconstructed. Only its transverse component is determined by the missing energy measurement. While the rapidity of the Z can be fully reconstructed from the measured rapidity of the e^- and e^+, the rapidity of the W

$$y_\pm = \tfrac{1}{2} \ln \frac{E_e + p_{eZ} + p_{\nu T} e^{y_{\nu\pm}}}{E_e - p_{eZ} + p_{\nu T} e^{-y_{\nu\pm}}} \tag{13}$$

is only reconstructed up to a twofold ambiguity[9] in terms of the electron momentum $(E_e, \mathbf{p}_e)$ and $p_{\nu T}$. In Eq. (13),

$$y_{\nu\pm} = y_e \pm \ln[1 + \delta + \sqrt{\delta(2+\delta)}] , \tag{14}$$

with

$$\delta = \max \left[0, \frac{M_W^2 - M_{WT}^2}{2p_{eT}p_{\nu T}} \right] . \tag{15}$$

Here M_{WT} is the transverse mass of the e,ν. Given the ambiguity in the sign defining y_ν, one can introduce as a definite and experimentally accessible variable, e.g., y_+

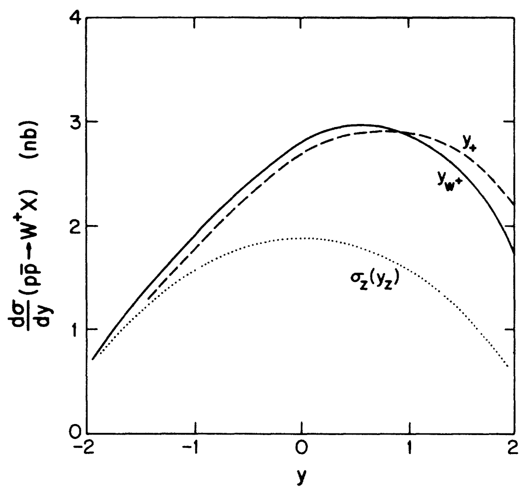

FIG. 1. Rapidity distribution of W^+ and Z produced in $\sqrt{s}=1800$ GeV $p\bar{p}$ collision according to EHLQ 1 structure functions (Ref. 7). Also shown is the W^+ distribution in the variable y_+ defined in Eq. (13) and below. We show the rapidity distribution of the Z for comparison.

or y_- themselves or whichever has the smaller magnitude ($y_{\min}$). In Fig. 1 we compare the rapidity distribution of the W^+ using the experimentally accessible variable y_+ with the true (and unmeasurable) rapidity distribution. The rapidity distribution of the Z is shown for comparison. It turns out that y_+ (y_-) closely traces the rapidity distribution of W^+ (W^-). We therefore redefine $\sigma_S(y)$ and replace it by the experimental observable

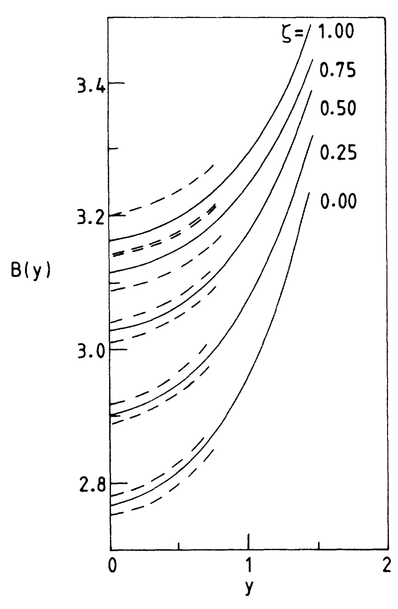

FIG. 2. The quantity $B(y)=\sigma_S(y)/\sigma_Z(y)=[\sigma_{W^+}(y_+)+\sigma_{W^-}(y_-)]/\sigma_Z(y)$ at $\sqrt{s}=1800$ GeV for different values of the charm content of the nucleon parametrized in terms of $\zeta\equiv 2c/(\bar{u}+\bar{d})$. Dashed lines represent our estimate of the systematic errors associated with the nucleon structure functions other than charm; see text. Errors associated with the values of m_W, m_Z, and $\sin^2\theta_W=0.23\pm0.005$ have not been included; they should not exceed 1%.

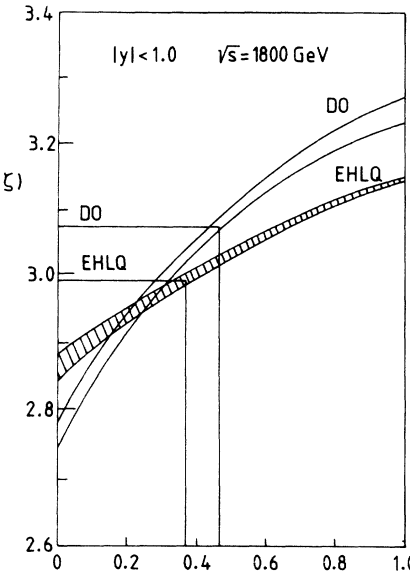

FIG. 3. Integrated cross-section ratio $B=\int_{|y|<1}\sigma_S(y)/\int_{|y|<1}\sigma_Z(y)$ as a function of the relative strength of the charm structure functions $\zeta\equiv 2c/(\bar{u}+\bar{d})$.

$$\sigma_S(y)=\sigma_{W^+}(y=y_+)+\sigma_{W^-}(y=y_-) . \tag{16}$$

$B(y)$ is redefined accordingly.

The virtue of this distribution is that it is a purely experimental observable depending only on the charged-lepton momentum and the missing p_T, while preserving much of the intuitive properties of the W rapidity distribution. We have checked that smearing due to the finite width and p_T of W and the experimental resolution should not affect our conclusions. They should, of course, be accounted for in a quantitative analysis. In Fig. 2 we show the ratio $B(y)$ as a function of y for fixed values of ζ. The ratio ζ can be x dependent. Although

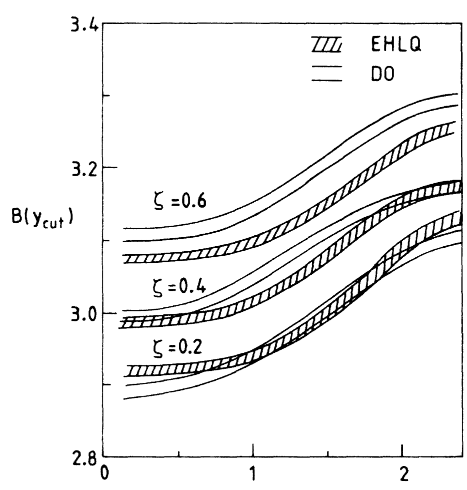

FIG. 4. Dependence of the integrated cross-section ratio B on the cutoff in y (y_{cut}) for some representative values of ζ.

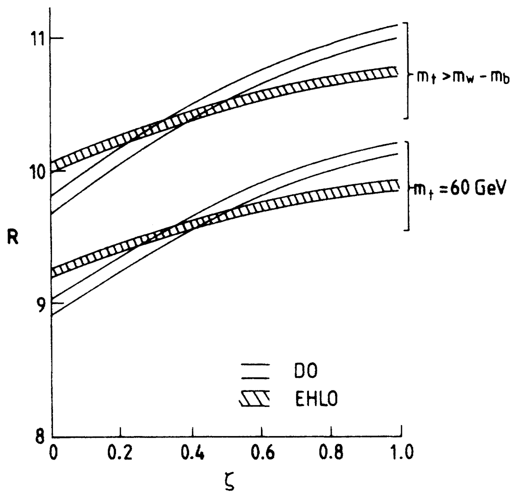

FIG. 5. Calculation of the ratio R of $W \rightarrow e\nu$ to $Z \rightarrow e^+e^-$ for three generations of quarks and leptons as a function of the strength ζ of the charm structure function. The result is shown for $m_t = 60$ GeV and becomes independent of m_t for $m_t > m_W - m_b$. The calculation is performed for two sets of structure functions (Refs. 7 and 10), while allowing for the u_v/d_v ratio to vary as described in the text.

this dependence is expected to be weak, it can be revealed also by confronting the curves in Fig. 2 with data. The wavy lines illustrate the errors associated with the other quark structure functions. These were evaluated by contrasting the structure functions of Refs. 7 and 10, set 1 for both. In order to find the errors in our analysis, we extracted the structure function for valence d quarks d_v from F_2^n/F_2^p as measured by BCDMS (Ref. 4) and EMC (Ref. 5), with input values for valence u quarks u_v and sea $(\bar{u}, \bar{d}, s)$ distributions from DO 1 and EHLQ 1 structure functions, respectively.[7,10] We thus obtain extreme descriptions of the deep-inelastic scattering data, respectively, over- and underestimating the values of F_{2n}/F_{2p} which constitute the crucial experimental input into the analysis. As we can see from Fig. 6 of Ref. 3 (Berger *et al.*), those structure functions bracket the BCDMS and EMC data on F_2^n/F_2^p. At $Q^2 \simeq 10$ GeV^2 the DO 1 and EHLQ 1 analyses yield $2s/(\bar{u}+\bar{d}) \simeq 1.0$ and 0.5, respectively. These extreme values also cover the 10–15% normalization uncertainty[11] of the sea structure functions as determined from deep-inelastic scattering experiments. The original DO 1 and EHLQ 1 charm structure functions predict $\zeta \simeq 0.5$ and 0.4 at $Q^2 \simeq m_W^2$, respectively, leading to $R_\sigma \simeq 3.30$ and 3.22.

It should be emphasized however that the determination of $c(x, Q^2 \simeq m_W^2)$ via QCD evolution of low-Q^2 data cannot be trusted as the value depends critically on the choice of Q_0^2 where one initiates the evolution of the $g \rightarrow c\bar{c}$ cascade. The only safe assumption is that the evolution should start at Q^2 of order m_c^2. Duke and Owens[10] evolve charm above a value $Q_0^2 > 4$ GeV^2 which is independent of x. Eichten *et al.*[7] adopted the Glück, Hoffman, and Reya[12] approach, where the threshold mimics the physical threshold behavior of leptoproduction and hence the threshold depends on both Q^2 and x. This may not be appropriate for hadroproduction as the change from space to timelike Q^2 values affects the phase space for heavy-particle production.[13] The only data[14] relevant to this issue are taken too close to threshold to resolve this ambiguity. Naive perturbation theory might not apply in the low-Q^2 range of these measurements. Other methods to experimentally probe the charm structure of the nucleon have been suggested.[15]

We anticipate that measurement of the ratio $B(y)$ and B, integrated over $|y| < 1$ (shown in Figs. 2 and 3), will in the end constitute the most sensitive probe of the charm structure of the nucleon. The data probe large Q^2 values, thus avoiding the ambiguities previously alluded to. With this in mind, we include in Fig. 3 the integrated value of $B(|y|<1)$ as a function of ζ, using the error analysis previously described. The values of $B(|y|<1)$ for the original DO 1 and EHLQ 1 charm structure functions are obtained from Fig. 3 for $\zeta = 0.5$ and 0.4, respectively. These represent the values of $\zeta \simeq c(x,Q^2)/S(x,Q^2)$ at $Q^2 \simeq m_W^2$ for the two sets of structure functions. In Fig. 4 we show the dependence of $B(y)$ on the cutoff in rapidity y_{cut} for some representative values of ζ.

Finally, we discuss the relevance of a determination of $c(x, Q^2 = m_W^2)$ in terms of the ratio R. Some illustrative calculations of the ratio R of $W \rightarrow e\nu$ and $Z \rightarrow e^+e^-$ events are shown in Fig. 5. The figure shows the predicted value of R as a function of ζ for three generations of quarks and leptons using DO 1 and EHLQ 1 structure functions. The calculation is shown for $m_t = 60$ GeV and $m_t > m_W - m_b$. R becomes independent of m_t for $m_t > m_W - m_b$ as W and Z can no longer decay into top. The uncertainty from structure functions can be judged by comparing two calculations in Fig. 5 which use our extreme choices of structure functions as input. Notice that the full range of ζ from 0 to 1 covers almost one unit in R or two neutrino types as[3] $\Delta R \simeq 0.5$ for $\Delta N_\nu = 1$.

In conclusion, we have shown that neutrino counting at the Tevatron will have to face the issue of what the precise charm structure of the nucleon is. We propose $B(y) = [\sigma_{W^+}(y_+) + \sigma_{W^-}(y_-)]/\sigma_Z(y)$ as the most sensitive measure of $\zeta = 2c/(\bar{u}+\bar{d}) \simeq c/S$. e.g., for three neutrinos and $m_t > m_W - m_b$ we predict $R \simeq 10$, 10.5, and 11 for $\zeta \simeq 0$, 0.5, and 1. This level of precision should be within reach of the Tevatron experiments.

One of us (C.S.K) would like to thank the members of the theory group of DESY, where part of this work was completed, for their kind hospitality. K.H. wishes to thank A. D. Martin for discussions on charm-quark distributions. This research was supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U.S. Department of Energy under Contract No. DE-AC02-76ER00881. The work of K.H. and C.S.K. was supported in part by the Science and Engineering Research Council, United Kingdom.

[1]For recent reviews, see F. Halzen, C. S. Kim, and S. Pakvasa, Int. J. Mod. Phys. A **4**, 753 (1989); F. Halzen, C. S. Kim, and J. R. Cudell, *ibid.* **3**, 1051 (1988), and references therein.
[2]A. D. Martin, R. G. Roberts, and W. J. Stirling, Phys. Lett. B **189**, 220 (1987); F. Halzen, C. S. Kim, and S. Willenbrock, Phys. Rev. D **37**, 229 (1988).
[3]E. L. Berger, F. Halzen, C. S. Kim, and S. Willenbrock, Phys. Rev. D **40**, 83 (1989); C. S. Kim and F. Halzen, Durham Report No. DTP/88/48 (unpublished).
[4]BCDMS Collaboration, A. Milztajn, Ph.D. thesis, Université Paris XI.
[5]European Muon Collaboration (EMC), J. J. Aubert *et al.*, Nucl. Phys. **B293**, 740 (1987).
[6]BEBC Collaboration, D. Allasia *et al.*, Z. Phys. C **28**, 321 (1985).
[7]E. Eichten, I. Hinchliffe, K. D. Lane, and C. Quigg, Rev. Mod. Phys. **56**, 579 (1984).
[8]CDHS Collaboration, H. Abramowicz *et al.*, Z. Phys. C **13**, 199 (1982); **17**, 283 (1983).
[9]See, e.g., J. Cortes, K. Hagiwara, and F. Herzog, Nucl. Phys. **B278**, 26 (1986).
[10]D. Duke and J. Owens, Phys. Rev. D **30**, 49 (1984).
[11]M. Diemoz, F. Ferroni, E. Longo, and G. Martinelli, Z. Phys. C **39**, 21 (1988).
[12]M. Glück, E. Hoffman, and E. Reya, Z. Phys. C **13**, 119 (1982).
[13]V. Barger, K. Hagiwara, S. Jacobs, and J. Woodside, Phys. Rev. D **33**, 57 (1986).
[14]EMC, J. J. Aubert *et al.*, Nucl. Phys. **B213**, 31 (1983); BFP Collaboration, A. R. Clark *et al.*, Phys. Rev. Lett. **45**, 1465 (1980).
[15]R. S. Fletcher, F. Halzen, and E. Zas, Phys. Lett. B **221**, 403 (1989).