Reply to Comment by Roskies

Guo Shuohong and Zheng Weihong
Center of Theoretical Physics, China Center of Advance Science and Technology (World Laboratory), Beijing, People's Republic of China,
Institute of Theoretical Physics, Academia Sinica, Beijing,
*and Department of Physics, Zhongshan University, Guangzhou 510275 China**
(Received 25 April 1989)

The expectation of a negligible term in both the Euclidean path-integral formalism and the Hamiltonian formalism blowing up is illustrated. This is believed to result from the high-energy fluctuation [$O(1/a)$ part] of the lattice which may not affect substantially the low-energy behavior of the theory.

Before answering the problem that $\langle \Delta H \rangle_0$ blows up as $a \to 0$ pointed out by Roskies[1] in a comment on our earlier paper,[2] we illustrate two similar cases in the Euclidean formalism and Kogut-Susskind Hamiltonian, respectively, by the same approach as that of Roskies.

First, let us consider the Kogut-Susskind Hamiltonian H_{KS}:

$$H_{\mathrm{KS}} = \frac{g^2}{2a} \sum_l E_l^2 + \frac{1}{ag^2} \sum_p \mathrm{Tr}(2 - U_p - U_p^\dagger) \equiv H_e + H_m \ . \tag{1}$$

If we add a negligible term ΔH to H_{KS}, and choose ΔH as

$$\Delta H = \frac{g^2}{2} \sum_l E_l^2 = a H_e \ , \tag{2}$$

compared with the color-electric energy H_e of H_{KS}, ΔH is negligible. Let us see the expectation value of ΔH in the state chosen as

$$|\Psi_0\rangle = \exp(R)|0\rangle \ , \tag{3}$$

where $R = (4/3g^4)\sum_p \mathrm{Tr}\, U_p$ for (2+1)-dimensional SU(2) theory:

$$\begin{aligned}\langle \Delta H \rangle_0 &= \langle \Psi_0 | \Delta H | \Psi_0 \rangle / \langle \Psi_0 | \Psi_0 \rangle \\ &= \left\langle 0 \left| e^R \sum_l \frac{g^2}{2} E_l^2 e^R \right| 0 \right\rangle \Big/ \langle 0 | e^{2R} | 0 \rangle \\ &= \left\langle \sum_l \frac{g^2}{2} ([E_l, R][E_l, R] + [E_l, [E_l, R]]) \right\rangle_0 .\end{aligned}$$

By the approach of Roskies, it is easy to show that $\langle \Delta H \rangle_0$ blows up like V/a^3. We can also show that the expectation of aH_m in $|\Psi_0\rangle$ blows up like V/a despite its classical continuum limit being zero. Therefore, we cannot deduce the continuum limit of a lattice quantity from its expectation value.

Second, let us consider the situation in the Euclidean path-integral formalism. The Wilson action is

$$S_W = \frac{1}{g^2} \sum_p \mathrm{Tr}(U_p + U_p^\dagger - 2) \ . \tag{5}$$

Let us check the expectation of a negligible term $\Delta S = aS_W$ in S_W:

$$\langle \Delta S \rangle = \frac{\int [dU_l] a S_W \exp(S_W)}{\int [dU_l] \exp(S_W)} \ . \tag{6}$$

In order to make this quantity exactly integrable, we only consider the (1+1)-dimensional SU(2) theory here. It is easy to show that $\langle \Delta S \rangle$ also blows up like V/a.

From the two simple examples above, we can see that not only in the Hamiltonian formulation, but also in the Euclidean path-integral formulation, there exists the same problem which Roskies pointed out (actually, in our first paper on the exact ground state,[3] we had been aware of this problem). Obviously, this problem is not the result of the subtlety in the difference between Lagrangian path-integral formalism and Hamiltonian formalism which Roskies believed.[1] We believe this problem results from the high-energy $O(1/a)$ part of Hamiltonian or action.[4] In general, any lattice quantity, including the Wilson action, Kogut-Susskind Hamiltonian, and our Hamiltonian with soluble exact ground state, is composed of two different parts: a high-energy part [$O(1/a)$] and low-energy part [less than $O(1/a)$]. When we refer to the continuum limit of a lattice quantity we always mean its classical continuum limit, i.e., the low-energy behavior of that quantity when $a \to 0$. Our theory relies only on the classical limit $\Delta H \to 0$ as $a \to 0$ in the same sense as the Kogut-Susskind Hamiltonian approaches the continuum Hamiltonian when $a \to 0$. The existence of a high-energy part in the Hamiltonian causes the corresponding ground state to contain high-energy fluctuations which differ violently from the continuum theory. These fluctuations may result in $\langle \Delta H \rangle_0$ blowing up as an inverse power of a despite the classical continuum limit of ΔH being zero, but these do not affect the evaluation of the low-energy spectrum. We believe that, as was supported by our variational calculation, when the low-energy state $|\Psi\rangle$ is a smooth function with long-range correlation, then

$$\Delta\epsilon = \langle \Psi | H | \Psi \rangle - \langle \Psi_0 | H | \Psi_0 \rangle \tag{7}$$

may give the correct result.

Formally, the ground state of our Hamiltonian is an independent plaquette. But this does not mean that the theory lacks long-range correlation. The Hamiltonian formulation differs from the Lagrangian formulation in that in addition to the plaquette variables U_p, there are also link variables E_l. When we calculate any dynamical quantity involving E_l, correlations between plaquettes naturally occur and the theory is essentially not an independent plaquette.

We stress that $\langle \Delta H \rangle_0 \neq 0$ is not a defect of our theory. In general, any two different Hamiltonians with the same classical continuum limit will have ground-state energies differing by a divergent amount. The main point that should be clarified is whether these Hamiltonians belong to the same universality class. At present there is no definite answer to this question. We should work out the consequences of different Hamiltonians and compare the results. Our results for $(2+1)$-dimensional SU(2) theory strongly suggest that our Hamiltonian belongs to the same universality class as the Kogut-Susskind Hamiltonian, at least for the non-Abelian case. More work will be done to clarify this point further.

This work was supported in part by the Foundation of Zhongshan University Advance Research Center and by the Science Fund of the Chinese Education Committee.

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*Permanent address.

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