Brief Reports

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Axion bremsstrahlung in red giants

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We calculate the bremsstrahlung emission rate of axions from a degenerate but weakly coupled plasma, using a Debye structure factor to take account of the nuclei correlations. This result pertains to axion emission from red giants near the helium flash and thus could be used to make a previous bound precise which involves the suppression of helium ignition in red giants by axionic energy losses.

I. INTRODUCTION

The emission of light particles such as axions or neutrinos can substantially change the pattern of stellar evolution that would be expected otherwise, leading to powerful constraints on the interaction of these particles with matter and radiation. The most widely discussed case of hypothetical particles is that of Nambu-Goldstone bosons of a global symmetry of the fundamental interactions which is broken at some large energy scale. The important case of a chiral unitary symmetry, such as the Peccei-Quinn symmetry in the case of invisible axions,[1] leads to a derivative Nambu-Goldstone coupling with electrons that can be written in pseudoscalar form as

$$\mathcal{L}_{\rm int}=g_a\bar{\psi}_e\gamma_5\psi_e a \ , \tag{1}$$

where a is the Nambu-Goldstone field ("axion"), and g_a is a Yukawa coupling constant which is related to more fundamental parameters of the model such as the symmetry-breaking scale. The most restrictive astrophysical constraints on g_a have been derived by considering axion emission from degenerate stars. A very simple argument involves axion emission from white dwarfs[2] where the observationally established time scale of cooling limits the possible axion losses.

A more intricate and slightly more powerful argument involves the axion emission from the degenerate cores of red giants before the helium flash.[3] A red-giant core can be viewed as a helium white dwarf: it is supported by electron degeneracy pressure with no nuclear energy source in the center, although it is surrounded by a hydrogen-burning shell. Because this shell must support itself by thermal pressure, the gravity at the surface of the core determines the shell temperature and hence the core temperature essentially by the virial theorem. As the core mass grows, its radius shrinks, as appropriate for a degenerate star, leading to an increased density, an increased gravitational potential in the hydrogen-burning shell, and hence to an increased temperature. Eventually, the density and temperature are sufficiently high to ignite helium, taking the star to a new configuration with central helium burning and a hydrogen-burning shell. Stars in this evolutionary phase are identified with horizontal-branch stars in globular clusters and with "clump giants" in open clusters and in the galactic disk population,[4] so that helium is known to actually ignite. Excessive axion losses in a red-giant core in conjunction with its limited thermal conductivity would lead to a temperature profile which decreases from the hydrogen-burning shell to the center enough to delay helium ignition until the hydrogen-burning front reaches the stellar surface. Therefore the star would be taken directly to the white-dwarf stage without ever igniting helium, contrary to the observed existence of horizontal-branch stars and "clump giants."

In white dwarfs and red-giant cores, the dominant axion emission process is bremsstrahlung, $e^-+(A,Z)\rightarrow(A,Z)+e^-+a$, where (A,Z) is a nucleus of charge Ze and mass number A. In Ref. 2 the emission rate was calculated for the relevant degenerate conditions, but ion correlations were neglected, leading to an overestimate of the emission rate from white dwarfs by a factor of about 3. Later, the emission rates were numerically calculated including ion correlations for the case of a strongly coupled ^{12}C plasma,[5] but the much simpler case of a weakly coupled ^{4}He plasma relevant for a red-giant core was ignored. The actual "helium ignition bounds" of Ref. 3 were based on a crude estimate in terms of nondegenerate emission rates. Therefore the most important case in the calculation of the bremsstrahlung rates has so far been ignored, and we set out to remedy this deficiency.

II. CALCULATION OF THE EMISSION RATE

Assuming that the target nuclei are static and heavy, following the usual Feynman rules, and including the effect of Pauli blocking of final states, the energy-loss rate per unit mass is found to be, in natural units where $\hbar = c = k_B = 1$,

$$\epsilon = \frac{4\alpha^2\alpha_a}{\pi^2\rho}\sum_j Z_j^2 n_j \int_{m_e}^{\infty} dE_1 f(T,E_1)\int_{m_e}^{E_1} dE_2[1-f(T,E_2)] \times \int\frac{d\Omega_2}{4\pi}\int\frac{d\Omega_a}{4\pi}\frac{|\mathbf{p}_1||\mathbf{p}_2|E_a^2}{|\mathbf{q}|^4}\left[2E_a^2\frac{p_1p_2-m_e^2+p_a(p_2-p_1)}{(p_1p_a)(p_2p_a)}+2-\frac{p_1p_a}{p_2p_a}-\frac{p_2p_a}{p_1p_a}\right], \tag{2}$$

where $\alpha_a \equiv g_a^2/4\pi$, the sum is extended over all species of nuclei, the index 1 refers to the incoming, 2 to the outgoing electron, $f(T,E)$ is the electron phase-space distribution, $f = (e^{(E-E_F)/T}+1)^{-1}$ for the degenerate case, $\mathbf{q} \equiv \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_a$ is the momentum transfer to the nucleus, and the axion energy is $E_a = E_1 - E_2$. If the electrons are very degenerate, the energy integrals can be evaluated analytically. Moreover, all electron momenta are close to the Fermi momentum: $|\mathbf{p}_1| \approx |\mathbf{p}_2| \approx p_F$. Using the notation $\beta_F \equiv p_F/E_F$ and c_{12} for the cosine of the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$, etc., and considering only one nuclear species so that $\rho = A m_u n_{\rm nuc}$ with the atomic mass unit $m_u = 1.66\times10^{-24}$ g, we find

$$\epsilon = \frac{\pi^2}{15}\frac{Z^2\alpha^2\alpha_a}{A}\frac{T^4}{m_e^2 m_u}F\ , \tag{3}$$

where

$$F = \frac{m_e^2}{m_e^2+p_F^2} \times \int\frac{d\Omega_2}{4\pi}\int\frac{d\Omega_a}{4\pi}\frac{2(1-c_{12})-(c_{1a}-c_{2a})^2}{(1-\beta_F c_{1a})(1-\beta_F c_{2a})}\frac{4p_F^4}{|\mathbf{q}|^4}\ . \tag{4}$$

Numerically,

$$\epsilon = 1.08\times10^{17}\,{\rm erg\,g^{-1}s^{-1}}\alpha_a(Z^2/A)T_8^4 F\ ,$$

where $T_8 = T/10^8$ K and for a helium plasma, $Z^2/A = 1$.

In a degenerate plasma, the electric fields of the nuclei are screened because of the polarizability of the degenerate electron gas. Hence the Coulomb propagator, $|\mathbf{q}|^{-2}$, must be replaced by $(|\mathbf{q}|^2+k_{\rm TF}^2)^{-1}$, appropriate for an exponentially screened electric field with the Thomas-Fermi screening scale,[6] $k_{\rm TF} = (4\alpha p_F E_F/\pi)^{1/2}$. Moreover, the scattering amplitudes from different nuclei interfere and since the positions of the nuclei are correlated because of their electromagnetic interaction, the interference terms do not average to zero. Therefore one must include the static ionic structure factor, $S(\mathbf{q})$, leading to

$$\frac{1}{|\mathbf{q}|^4} \to \frac{S(\mathbf{q})}{(|\mathbf{q}|^2+k_{\rm TF}^2)^2}\ . \tag{5}$$

In Ref. 5, the emission rate was numerically calculated for degenerate ^{12}C, using this full expression and numerical results for $S(\mathbf{q})$.

The correlation of the ions is governed by a parameter which measures the ratio of the Coulomb interaction energy between ions and their thermal kinetic energy: $\Gamma \equiv \alpha Z^2/aT$ where the "ion sphere radius" is defined by virtue of $n_{\rm nuc} = (4\pi a^3/3)^{-1}$. For $\Gamma \gtrsim 1$, the plasma is strongly coupled and for $\Gamma \gtrsim 168$, the ions arrange themselves in a body-centered cubic lattice.[7] Taking a central density of 1.8×10^6 g cm^{-3} as an example for a typical white dwarf, a composition of pure ^{12}C, and temperatures in the range 10^6–10^7 K we find $\Gamma = 433$–43.3, the larger value corresponding to the smaller temperature. Hence the plasma is, indeed, strongly coupled. Taking the core of a red giant with a density of about 10^6 g cm^{-3} as another example, taking pure ^{4}He and a temperature of 10^8 K, we find $\Gamma = 0.57$ so that the plasma is weakly coupled. This is the major difference between the state of the plasma in red-giant core and a typical white dwarf.

For a weakly coupled plasma with degenerate electrons, the static structure factor is given by the Debye-Hückel formula

$$S(\mathbf{q}) = \frac{|\mathbf{q}|^2}{|\mathbf{q}|^2+k_{\rm DH}^2} = \frac{|a\mathbf{q}|^2}{|a\mathbf{q}|^2+3\Gamma}\ , \tag{6}$$

where the screening scale for ions of charge Ze is $k_{\rm DH}^2 = 4\pi Z^2\alpha n_{\rm nuc}/T$. In Fig. 1 we show $S(\mathbf{q})$ as obtained from a Monte Carlo calculation[8] for $\Gamma = 2$, 10, and 100 (solid lines) and compare it with the corresponding Debye formula (dashed lines). For $\Gamma \lesssim 1$, the Debye result gives a reasonable approximation while for a strongly coupled plasma it would be completely misleading. We note that for the bremsstrahlung process the maximum momentum transfer is $|\mathbf{q}_{\max}| = 2p_F$ so that $|a\mathbf{q}_{\max}| = (Z18\pi)^{1/3}$ which is 4.84 for helium with $Z = 2$.

To simplify further we stress that the forward divergence of the Coulomb denominator is mostly cut off by the ion correlation effect because $k_{\rm DH} \gg k_{\rm TF}$ so that we may neglect $k_{\rm TF}$ entirely. Also, the momentum transfer can be approximated by $|\mathbf{q}|^2 \approx |\mathbf{p}_1-\mathbf{p}_2|^2 \approx 2p_F^2(1-c_{12})$. Finally we consider a nonrelativistic approximation where we may use $\beta_F = 0$ in Eq. (4) leading to

$$F = \frac{m_e^2}{m_e^2+p_F^2}\frac{2}{3}\ln\left[\frac{8p_F^2+k_{\rm DH}^2}{k_{\rm DH}^2}\right]\ . \tag{7}$$

An exact calculation would only slightly change the argument of the logarithm. For a helium plasma with $\rho = 10^6$

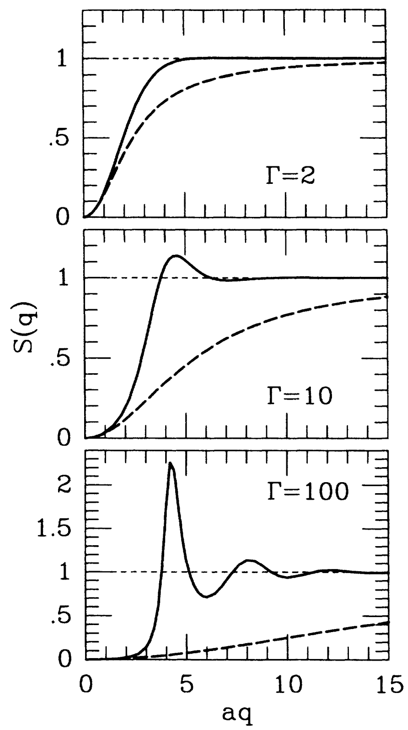

FIG. 1. Static structure factor for a one-component plasma as a function of number, **q**, expressed in units of the inverse ion sphere radius a^{-1}. The solid lines are from the Monte Carlo calculations of Ref. 8, the dashed lines represent the Debye approximation, Eq. (6). The maximum momentum transfer for axion bremsstrahlung is $|a\mathbf{q}_{\max}|=2ap_F=(Z18\pi)^{1/3}$, which is 4.84 for helium with $Z=2$.

$\mathrm{g\,cm^{-3}}$ and $T=10^8$ K, we find $p_F=409$ keV and $k_{\rm DH}=222$ keV whence $F=1.4$.

In order to appreciate the importance of the ion correlations we also quote the result that one obtains if one uses $S(\mathbf{q})=1$, keeps $k_{\rm TF}$, and uses the same approximations,

$$F=\frac{m_e^2}{m_e^2+p_F^2}\,\frac{2}{3}\left[\ln\left(\frac{4p_F^2+k_{\rm TF}^2}{k_{\rm TF}^2}\right)-\frac{4p_F^2}{4p_F^2+k_{\rm TF}^2}\right]. \tag{8}$$

For the same plasma conditions, $k_{\rm TF}=50$ keV, so that $F=1.9$.

It is also instructive to compare our result with that for a nondegenerate plasma[9,10] where screening or correlations are of little importance. If there is only one species of nuclei, one finds[10]

$$\epsilon=\frac{128}{45}\left(\frac{2}{\pi}\right)^{1/2}\frac{\alpha^2\alpha_a}{m_e m_u}n_e\left(\frac{T}{m_e}\right)^{5/2}\frac{1}{A}\left(Z^2+\frac{Z}{2^{1/2}}\right), \tag{9}$$

where the term quadratic in Z represents scattering on nuclei and the linear term represents scattering on electrons.

III. CONCLUSIONS

The "helium ignition bound" of Ref. 3 was based on the nondegenerate bremsstrahlung rate, somewhat arbitrarily suppressed by a Boltzmann factor $e^{-\omega_0/T}$ with the plasma frequency ω_0. For a helium plasma with $\rho=10^6$ $\mathrm{g\,cm^{-3}}$ and $T=10^8$ K this procedure overestimates the emission rate by a factor of about 4. Hence we estimate that, including our emission rate in a stellar-evolution code, would lead to a constraint on the Yukawa coupling (or on the Peccei-Quinn scale) of axions a factor of about 2 less restrictive than was claimed in Ref. 3.

A rigorous calculation of the emission rates of light particles from stellar plasmas is generally difficult, and sometimes intractable, because of the importance of many-body effects. We have provided a rigorous treatment of the emission rate which enters an astrophysical argument which provides the most restrictive constraint on the coupling of light bosons to electrons.

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