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Remarks on charge quantization of fermions and bosons

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We reexamine charge quantization in the standard model with and without a right-handed neutrino. We remark that, without the right-handed neutrino, only the standard charges are allowed for Weyl quarks and leptons under the standard group. With it, there are two independent quantized hypercharge and electric-charge assignments and nonquantized charge assignments would result from linear combinations of these two. One more condition is sufficient to recover the unique correct charge quantization.

It has been a long-standing puzzle that the electric charges of elementary particles of nature are quantized. In our recent paper,[1] we showed that this puzzle can be understood within the context of the standard model without going to grand unification. We demonstrated that, in the absence of a Higgs particle, by insisting on the cancellation of the mixed gauge-gravitational anomaly[2] in addition to the well-known triangular gauge anomaly[3] to the standard group, the hypercharges of the 15 Weyl fermion members of a single generation without a right-handed neutrino are uniquely determined as the standard one despite the existence of a "bizarre" assignment.[4,5] As first indicated by Georgi,[6] the requirement of nonvanishing Dirac masses for quarks and leptons, which is achieved by the introduction of a Higgs-doublet boson to spontaneously break the electroweak gauge group in the standard model, obviates the need to invoke the mixed gauge-gravitational anomaly cancellation to fix the quantized hypercharges of the Weyl fermions for the standard group. However, if one introduces a right-handed neutrino into the standard model, the hypercharges are no longer determined by the anomaly-free conditions even if the standard Higgs particle is used.[1,7] In this report, we will first remark that without a right-handed neutrino, the bizarre hypercharge assignment arises from the ambiguity of right-handed quark fields in the anomaly-free conditions and results in a nonchiral fermion[5] and, therefore, for Weyl quarks and leptons, the standard hypercharges are the unique ones which guarantee the correct electric-charge quantization after the electroweak symmetry breaks down to $U(1)_{EM}$. We then show that with the right-handed neutrino, two independent quantized hypercharge and electric-charge assignments are allowed by introducing both the anomaly cancellations and the Dirac mass terms under the standard group. Among the two hypercharge assignments, one can be identified as the standard one in which the right-handed neutrino carries zero charge and the other one is $B-L$. Any other quantized and nonquantized assignments are formed by linear combinations of these two. One more extra condition will uniquely pin down the correct charge quantization.

We begin by considering one generation of quarks and leptons including a right-handed neutrino with the quantum numbers under the standard group as follows:

$$
\begin{array}{lcccc}
 & & SU(3)_C & \times\ SU(2)_L & \times\ SU(1)_Y \\
Q_L = & \begin{pmatrix} u \\ d \end{pmatrix}_L & 3 & 2 & y_1 \\
 & u_R & \bar{3} & 1 & -y_2 \\
 & d_R & \bar{3} & 1 & -y_3 \\
l_L = & \begin{pmatrix} \nu \\ e \end{pmatrix}_L & 1 & 2 & y_4 \\
 & e_R & 1 & 1 & -y_5 \\
 & \nu_R & 1 & 1 & -y_6
\end{array}
\tag{1}
$$

where y_i $(i=1,2,\ldots,6)$ are arbitrary. The cancellation of the triangular anomalies yields the three nontrivial equations

$$2y_1+y_2+y_3=0\ , \tag{2a}$$

$$3y_1+y_4=0\ , \tag{2b}$$

$$6y_1^3+3y_2^3+3y_3^3+2y_4^3+y_5^3+y_6^3=0\ . \tag{2c}$$

Since there are four $SU(2)_L$ doublets in (1), the global Witten SU(2) anomaly[8] is satisfied. The mixed gauge-gravitational anomaly-free condition leads to

$$3(2y_1+y_2+y_3)+2y_4+y_5+y_6=0\ . \tag{3}$$

Clearly, the conditions in Eqs. (2) and (3) are not sufficient to fix y_i since there are six unknown parameters out of five equations [one overall normalization factor of $U(1)_Y$ has been included].

Let us first study the case of the minimal standard model[1] (i.e., without the right-handed neutrino ν_R). In this case the six unknown parameters reduce to five. Therefore, one expects that the charges are able to be fixed. Indeed, there are only three possible solutions for $(y_1,y_2,\ldots,y_5)$ which are given by[1,4,5]

$$(\tfrac{1}{3}y,-\tfrac{4}{3}y,\tfrac{2}{3}y,-y,2y)\ , \tag{4a}$$

$$(\tfrac{1}{3}y,\tfrac{2}{3}y,-\tfrac{4}{3}y,-y,2y)\ , \tag{4b}$$

$$(0,y,-y,0,0)\ , \tag{4c}$$

where y is a nonzero arbitrary factor and (5c) is referred to as a "bizarre" solution. Normalizing the overall factor to one, i.e., $y=1$, we have three quantized hypercharge assignments:

$$(\tfrac{1}{3},-\tfrac{4}{3},\tfrac{2}{3},-1,2)\ , \tag{5a}$$

$$(\tfrac{1}{3},\tfrac{2}{3},-\tfrac{4}{3},-1,2)\ , \tag{5b}$$

$$(0,1,-1,0,0)\ . \tag{5c}$$

It is expected that the hypercharges Y_a in (5a) and Y_b in (5b) become equivalent when y_2 and y_3 are interchanged since, from the anomaly cancellation alone, one could not distinguish u_R and d_R quark fields. The ambiguous hypercharge assignments for the right-handed quark fields further cause the "bizzare" solution Y_c in (5c) which can be seen from the relation

$$Y_c=\tfrac{1}{2}(Y_b-Y_a)\ . \tag{6}$$

In fact, there is no real ambiguity physically for the right-handed quark hypercharges. Indeed, if quarks and leptons are considered to be chiral fermions, i.e., no mass terms can be generated without breaking the standard $SU(3)_C\times SU(2)_L\times U(1)_Y$ symmetry, only the standard hypercharge assignment in (5a) or (5b) is allowed. Thus the hypercharge assignment in (5c) should not exist. It should be noted that one cannot build models which have more than one U(1) symmetry based on the three hypercharge assignments in (5) or any nontrivial linear combinations among them in addition to $SU(3)_C\times SU(2)_L$ with the 15 standard Weyl states of fermions because $\mathrm{Tr}\,Y_iY_j^2$ $(i,j=a,b,c;\ i\neq j)$ arising from the triangular anomalies on $[U(1)_{Y_i}]^2U(1)_{Y_j}$ are not equal to zero. Therefore, prior to spontaneous symmetry breaking in the standard model, the "anomalies" approach explains the hypercharge quantization.

To study the electric charges of fermions and bosons we assume that the electric charge operator Q of $U(1)_{EM}$ is given by

$$C'Q=\xi' I_{3L}+\zeta' Y\ , \tag{7}$$

where C', ξ', and ζ' are free parameters. If the electron charge is defined to be nonzero, ζ must be nonzero too. Thus we can rewrite Eq. (7) as

$$CQ=\xi I_{3L}+\frac{Y}{2}\ , \tag{8}$$

where C is treated as the overall normalization factor for $U(1)_{EM}$ which shall be determined by the definition of the electron charge. Since quarks and electrons are Dirac fermions which are left-right symmetric under $SU(3)_C\times U(1)_{EM}$, we have

$$Q(u_L)=Q(u_R),\quad Q(d_L)=Q(d_R),\quad Q(e_L)=Q(e_R)\ , \tag{9}$$

which implies that[6]

$$y_1+y_3=-(y_1+y_2)=y_4+y_5=\xi\ . \tag{10}$$

In the standard model, the electroweak symmetry $SU(2)_L\times U(1)_Y$ spontaneously breaks down to $U(1)_{EM}$ by the Higgs mechanism with a Higgs doublet $H(1,2,Y_H)$ under the standard group and the fermion masses are given through the Yukawa couplings L_Y where

$$L_Y=h^u\bar{Q}_L\tilde{H}u_R+h^d\bar{Q}_LHd_R+h^e\bar{l}_LHe_R+\text{H.c.} \tag{11}$$

This equation requires[4,6,7] Eq. (10) with $\xi=Y_H$. Equation (10) can reproduce the mixed gauge-gravitational constraint in Eq. (3) with

$$y=\xi=Y_H\ . \tag{12}$$

Thus, without using the mixed gauge-gravitational constraint in Eq. (3), Eq. (9) or (11) with the triangular anomaly-free conditions in Eq. (2) also yields a unique solution[6] given by (4a).

One notes that the solutions in (4b) and (4c) do not satisfy the condition in (10). This is not surprising because the definition of the Yukawa couplings in (11) has eliminated the ambiguity of the hypercharges between u_R and d_R and, furthermore, Eq. (11) along with Eq. (2) also implies having only chiral fermions under the standard group. Hence both Y_b in (5b) and Y_c in (5c) are not allowed. From Eq. (12) we see that fixing the overall factor y is equivalent to fixing the Higgs hypercharge Y_H.

We now leave Y_H to be arbitrary and study the electric charges for quarks and leptons after the electroweak symmetry $SU(2)_L\times U(1)_Y$ spontaneously breaks down to $U(1)_{EM}$ by the Higgs mechanism. With the definition of $Q(e)=-e$, we find that $C^{-1}=eY_H=e\xi=ey$ or $C^{-1}=e$ for choosing $Y_H=\xi=y=1$. Thus the fermion electric charges are given by

$$Q(u)=\tfrac{2}{3}e,\quad Q(d)=-\tfrac{1}{3}e,\quad Q(e)=-e,\quad Q(\nu_L)=0\ . \tag{13}$$

From Eq. (8), we see that the electric charges for photon, the standard Higgs (H^0) and the standard gauge bosons (Z^0 and $W^\pm$) (Ref. 9) are

$$Q(\gamma)\equiv Q(H^0)\equiv Q(Z^0)\equiv 0,\quad Q(W^\pm)=\pm e\ . \tag{14}$$

The electric charges in (13) and (14) are precisely the standard one. Therefore, in the context of the standard model, the charge quantization for both fermions and bosons can be understood without going to grand unification theories (GUT's).

We now study the case with the right-handed neutrino. We shall solve Eqs. (2) and (3) in terms of y_1 and y_6. For $y_1\neq 0$, we first set that $y_1=\alpha$ and $y_6=3\alpha\beta$ where α and β are arbitrary parameters. We find that there are only two solutions for $(y_1,y_2,\ldots,y_6)$:

$$(\tfrac{1}{3},-\tfrac{4}{3}+\beta,\tfrac{2}{3}-\beta,-1,2-\beta,\beta)3\alpha\ , \tag{15a}$$

$$(\tfrac{1}{3},\tfrac{2}{3},-\beta,-\tfrac{4}{3}+\beta,-1,2-\beta,\beta)3\alpha\ . \tag{15b}$$

Like the case without ν_R, the values in (15a) and (15b) become equivalent when y_2 and y_3 are interchanged. It is straightforward to show that the solutions that arise from interchanging the hypercharges y_5 and y_6 of the right-handed leptonic fields in (15) are also allowed. This gives two more solutions:

$$(\tfrac{1}{3},-\tfrac{4}{3}+\beta,\tfrac{2}{3}-\beta,-1,\beta,2-\beta)3\alpha\ , \tag{16a}$$

$$(\tfrac{1}{3},\tfrac{2}{3}-\beta,-\tfrac{4}{3}+\beta,-1,\beta,2-\beta)3\alpha\ . \tag{16b}$$

Actually, (16a) and (16b) can be derived by substituting $2-\beta$ for β in (15b) and (15a), respectively. It is interesting to note that there is only one solution for $\beta=1$ case, i.e.,

$$Y_{15a}=Y_{15b}=Y_{16a}=Y_{16b}=3\alpha(\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},-1,1,1)$$
$$\equiv 3\alpha Y_{B-L}\ , \tag{17}$$

in which there is no ambiguity of the hypercharge assignments for the right-handed fields. For $y_1=0$, we get

$$(0,\gamma,-\gamma,0,-\delta,\delta)\ , \tag{18}$$

where γ and δ are arbitrary. Again, as the case without ν_R in (5c), the solution in (18) may be viewed as a result of the ambiguities of the hypercharge definitions between the right-handed quark and lepton fields in (15) and (16) when $\beta\neq 1$. To see this, we write

$$Y_R^q=\frac{1}{6\alpha(1-\beta)}(Y_{15a}-Y_{15b})$$
$$=\frac{1}{6\alpha(1-\beta)}(Y_{16a}-Y_{16b})$$
$$=(0,-1,1,0,0,0)\ , \tag{19a}$$

$$Y_R^l=\frac{1}{6\alpha(1-\beta)}(Y_{15a}-Y_{16a})$$
$$=\frac{1}{6\alpha(1-\beta)}(Y_{15b}-Y_{16b})$$
$$=(0,0,0,0,1,-1) \tag{19b}$$

and thus we have

$$Y_{18}=-(\gamma Y_R^q+\delta Y_R^l)\ . \tag{20}$$

Especially, for $\gamma=\delta=1$, we have

$$Y_{18}=-(Y_R^q+Y_R^l)=-(0,-1,1,0,1,-1)$$
$$\equiv -Y_R\ . \tag{21}$$

We now use the Yukawa couplings in (11) to see whether we can fix the free parameters in (15) or (16) and (18). Clearly, only one of the four solutions, for example, (15a), in (15) and (16) is allowed by (11). From Eqs. (10) and (12), one finds

$$(1-\beta)3\alpha=Y_H\equiv(1-Y_H')3\alpha \tag{22}$$

for (15a) and

$$\gamma=\delta=-Y_H \tag{23}$$

for (18), respectively. Thus, the free parameters in (15a) and (18) are related to the definitions of the normalization of $U(1)_Y$ and the hypercharge Y_H of the Higgs doublet. If we choose the overall normalization factor $\alpha=\frac{1}{3}$ so that $1-\beta=Y_H\equiv 1-Y_H'$ in (15a) and $Y_H=-1$ in (16), we get

$$Y_a=(\tfrac{1}{3},-\tfrac{4}{3}+Y_H',\tfrac{2}{3}-Y_H',-1,2-Y_H',Y_H')\ , \tag{24a}$$

$$Y_b=-Y_R=(0,1,-1,0,-1,1)\ , \tag{24b}$$

respectively. We remark that, in fact, it is sufficient to derive (24) by the triangular anomaly-free conditions in (2) and the conditions in (10) given by the Dirac mass terms or the Yukawa couplings while the mixed gauge-gravitational anomaly-free condition in (3) is automatically satisfied once (10) is introduced.

The charges Y_a in (24a) contain a free parameter Y_H' for the right-handed fermion fields but the hypercharges for the left-handed fermions are quantized just like the case without ν_R in (5a) whereas Y_b is quantized. Actually, we can write (24a) and (24b) as

$$Y_a=Y_{SD}+Y_H'Y_R=(1+Y_H')Y_{SD}-Y_H'Y_{B-L}\ , \tag{25a}$$

$$Y_b=-Y_R=-Y_{SD}+Y_{B-L}\ , \tag{25b}$$

where $Y_{SD}=(\frac{1}{3},-\frac{4}{3},\frac{3}{3},-1,2,0)$ are the charges in (24a) by taking $Y_H'=0$ which are the standard ones given in (5a) plus zero hypercharge for right-handed neutrino. This indicated that all the hypercharge assignments are composed of Y_{SD} and Y_{B-L}. Therefore, there are only two possible independent quantized hypercharge assignments. Clearly, any linear combination of these two quantized hypercharge assignments is also a possible one: for instance,

$$Y_{xy}=xY_{SD}+yY_{B-L}\ , \tag{26}$$

where x and y are arbitrary parameters. Hence, Y_{xy} may not be quantized.

It is interesting to note that the introduction of the right-handed neutrino allows one to have multi-U(1) symmetries in addition to $SU(3)_C$ and $SU(2)_L$. This is possible because the anomalies related to $\mathrm{Tr}\,Y_i^2Y_j$ are zero where $Y_{i,j}$ are arbitrary linear combinations of Y_{SD} and Y_{B-L}. Moreover, for independent charge assignments, it is straightforward to show from Eqs. (15)–(18) that the number of multi-U(1) symmetries cannot exceed two.

We now check the electric charges without fixing α and Y_H. Using Eqs. (8) and (12), the electric charges of the fermions and gauge bosons by the definition of electric charge of electron, i.e., $Q(e)=-e$, are as follows: for the hypercharge assignments in (15a),

$$Q(u)=\frac{4/3-Y_H'}{2-Y_H'}e,\quad Q(d)=\frac{-2/3+Y_H'}{2-Y_H'}e\ ,$$
$$Q(e)=-e,\quad Q(\nu)=\frac{-Y_H'}{2-Y_H'}e\ , \tag{27a}$$

$$Q(\gamma)\equiv Q(H^0)\equiv Q(Z^0)\equiv 0,\quad Q(W^\pm)=\pm\frac{2-2Y_H'}{2-Y_H'}e \tag{27b}$$

with

$$C^{-1}\frac{2e}{3\alpha(2-Y_H')}\ , \tag{28}$$

where we have excluded the case with $Y_H'=2$ since it gives zero charge for the electron and, for the case in (16),

$$Q(u)=-Q(d)=-Q(e)=Q(\nu)=e\ , \tag{29a}$$

$$Q(\gamma)\equiv Q(H^0)\equiv Q(Z^0)\equiv 0,\quad Q(W^{\pm})=\pm 2e \tag{29b}$$

with

$$C^{-1}=\frac{2e}{Y_H}\ . \tag{30}$$

The normalization factor C^{-1} in Eqs. (28) and (30) justify the free choices for the parameters α in (22) and Y_H in (23). The electric charges in (29) are quantized but ruled out by experiments while those in (27) are not fixed if Y'_H is arbitrary. From (27), it is interesting to see that the electric charge of the proton is quantized and independent of the choice of the parameter Y'_H, which is equal to e, i.e.,

$$Q(p=uud)=e=-Q(e)\ , \tag{31}$$

whereas for the neutron one has

$$Q(n=udd)=-Q(\nu)\ . \tag{32}$$

For the pions, one finds that

$$Q(\pi^0)=0,\quad Q(\pi^{\pm})=Q(W^{\pm})\ . \tag{33}$$

So no neutral spin-$\frac{1}{2}$ baryons and leptons are allowed unless $Y'_H=0$ even for three families of quarks and leptons. Thus, the existence of a neutral spin-$\frac{1}{2}$ baryon or lepton will recover charge quantization. In fact, this is a necessary and sufficient condition to have the correct charge quantization.

Since the charges in (27) involve only one free parameters, any extra condition will fix all the charges. For instance, if

$$Q(\nu)=0\ , \tag{34}$$

which could be derived from having a Majorana mass term for neutrino,[7,10] one fixes $Y'_H=0$ and therefore (24a) and (27) become the standard hypercharges in (5a) with $Y(\nu_R)=0$ and electric charges in (13) and (14), respectively. The zero electric charge for the neutrino in (34) also excludes the possibility of the charges in (24b) and (29) and $B-L$ charges in (17). Thus, the requirement of (34) uniquely determines the electric charges and so the hypercharges as the standard ones. This is possible because the standard fermions (no ν_R), which are Weyl fermions under the standard group, possess a unique set of hypercharges and electric charges. Therefore, imposing only the neutrino as a Majorana particle, regardless of the model once there are 16 states of fermions, will lead to the correct charge quantization. This justifies the speculation by Babu and Mahopatra based on a class of models.[10] However, it should be clear that it is only a sufficient condition to have the correct charge quantization. Nature may not allow such a Majorana mass term since it has not been seen yet. Moreover, imposing a Majorana mass term for ν_R is as good as assuming zero electric charge for ν_R since there is no reason why the other color-weak singlet e_R could not have such a Majorana mass term.

From (27) and (31) we see that if we know one of the charged boson (spin 0 or 1) charges, all the other charges will also be fixed. For example, if $Q(W^{\pm})=\pm e$, one finds the correct charge quantization for all the fermions. We note that for Y_{B-L} which corresponds to $Y'_H=1$ in (24a), to satisfy Eq. (9) or (10), $\xi=0$ in Eq. (9) is needed, i.e., $Q\sim(B-L)$. Thus, all the physical bosons have zero electric charges, i.e.,

$$Q(\gamma)\equiv Q(H^0)\equiv Q(Z^0)\equiv 0,\quad Q(W^{\pm})=0 \tag{35}$$

but no neutral fermions, i.e.,

$$Q(u)=Q(d)=\tfrac{1}{3}e,\quad Q(e)=Q(\nu)=-e \tag{36}$$

from (27). This may also give us an indication why nature chooses $Y_{\rm SD}$ instead of Y_{B-L} as hypercharges.

Finally, we remark that both Y_a ($Y'_H=0$) and Y_b hypercharge assignments in (24) will result in the existence of nonchiral fermions which can form mass terms (Dirac or Majorana) without breaking the standard group symmetry. This implies that it is impossible to protect all the fermions being Weyl states by the standard group. To have 16 chiral fermion states, one requires Y_a with $Y'_H\neq 0$, for example, $Y_a(\alpha=\frac{1}{3},\ Y'_H=1)=Y_{B-L}$, which are ruled out by experiments. The other possibility is to extend the standard gauge group. Clearly, the minimal gauge group contains two U(1) symmetries in addition to $SU(3)_C$ and $SU(2)_L$. For example, one can have models with $SU(3)_C\times SU(2)_L\times U(1)_{Y_1}\times U(1)_{Y_2}$ gauge group where $Y_{1,2}$ are some linear combinations of $Y_{\rm SD}$ and Y_{B-L}.

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