Solution to the strong CP problem without an axion

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We present a solution to the strong CP problem using softly broken parity invariance in the context of left-right-symmetric gauge models with a "seesaw" mechanism for quark masses. A distinguishing feature of the model is that the first nonvanishing contribution to $\bar{\theta}$ arises only at the two-loop level, whereas the electric dipole moment of the neutron d_n^e is generated at the one-loop level via weak CP violation. For the W_R mass at the TeV scale, we estimate $d_n^e \approx 10^{-25}-10^{-26}$ e cm and $\bar{\theta} \leq 10^{-12}$.

I. INTRODUCTION

There are many solutions to the so-called CP problem,[1] where in suitable extensions of the standard electroweak model involving new symmetries, one ensures that the strong CP parameter $\bar{\theta}$ is computable and is less than 10^{-9} or so. The $\bar{\theta}$ is defined as

$$\bar{\theta}=\theta+\operatorname{Arg}\operatorname{Det}(M_u M_d) , \tag{1}$$

where $\theta/32\pi^2$ is the coefficient of the $F\tilde{F}$ term in the QCD Lagrangian and M_u and M_d are the mass matrices for the charge $+\frac{2}{3}$ and $-\frac{1}{3}$ quarks, respectively. The general strategy employed in these models is to use symmetries that constrain both the θ term as well as the Arg Det $(M_u M_d)$ term to vanish naturally at the tree level. Since realistic weak-interaction models require these symmetries to be broken, Det $(M_u M_d)$ will in general acquire a finite and computable phase in the loop approximation. If this phase is below the experimental upper limit of 10^{-9} or so (arising from the bounds on the electric dipole moment of the neutron) for a reasonable choice of the electroweak parameters of the theory, one has a solution to the strong CP problem.

If the symmetry is a continuous U(1) symmetry as in the Peccei-Quinn-type solution,[2] one gets a nearly massless pseudoscalar boson, the axion, which has eluded experimental searches to date. On the other hand, if the symmetry is a discrete symmetry, one obtains a solution without an axion. There are a number of such models in the literature.[3] In a recent paper,[4] it has been argued that in the class of models discussed in Ref. 3, the electric dipole moment of the neutron d_n^e arises solely from the strong CP parameter $\bar{\theta}$. This would mean that d_n^e is insensitive to the detailed nature of weak interaction, and cannot therefore be used to discriminate between different models of CP violation.

In this paper, we study a model where the soft breaking of parity symmetry allows a solution to the strong CP problem. While this model is similar in spirit to the class of models discussed in Ref. 3, d_n^e and the $\bar{\theta}$ parameter in this case are independent of each other. In fact, the $\bar{\theta}$ parameter arises only at the two-loop level whereas d_n^e gets a one-loop contribution from weak CP-violating effects leading to a value of $d_n^e \approx 10^{-25}-10^{-26}$ e cm. This "decoupling" of $\bar{\theta}$ and d_n^e is achieved without any fine-tuning of parameters or the imposition of new symmetries. In this sense, the present model is different from the models of Ref. 3.

We use the "seesaw" picture for quark masses introduced recently,[5] where one uses a new set of heavy vectorlike isosinglet quarks P and N with electric charges $\frac{2}{3}$ and $-\frac{1}{3}$, respectively, and lepton E with charge -1 in addition to the usual quarks and leptons. The quark and charged-lepton mass matrices in this case have the "seesaw" form leading to small masses for e,u,d, etc., without severe fine-tuning of the Yukawa couplings. The CP-violation phenomenology of this model was discussed by us in a recent paper,[6] where it was noted that the model provides a solution to the strong CP problem. In this paper, we show by explicit computation that at the one-loop level indeed $\bar{\theta}=0$ whereas $d_n^e \neq 0$. For reasonable values of the parameters of the model we estimate the nonzero two-loop contribution to $\bar{\theta}$ to be at the level of 10^{-12}. The contribution of $\bar{\theta}$ to d_n^e is thus negligible in relation to the pure weak CP-violating contribution.

II. DESCRIPTION OF THE MODEL

The model is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ with quark doublets $Q \equiv (u,d)$ and lepton doublets $\Psi \equiv (\nu,e)$ assigned in a left-right-symmetric manner.[7] In addition, the model includes vector-like isosinglet heavy quarks P and N (with charges $\frac{2}{3}$ and $-\frac{1}{3}$), and leptons E (charge -1) (one per generation). The fermionic spectrum thus consists of (suppressing generation indices)

$$\begin{aligned}
&Q_L(2,1,1/3), Q_R(1,2,1/3) ,\\
&\Psi_L(1,1,-1), \Psi_R(1,2,-1) ,\\
&P_{L,R}(1,1,4/3) ,\\
&N_{L,R}(1,1,-2/3) ,\\
&E_{L,R}(1,1,-2) .
\end{aligned} \tag{2}$$

The Higgs sector of the model is extremely simple and consists only of a pair of left-right-symmetric doublets $\chi_L(2,1,1)$ and $\chi_R(1,2,1)$ with the following Higgs potential:

$$V=-(\mu_L^2\chi_L^\dagger\chi_L+\mu_R^2\chi_R^\dagger\chi_R)+\frac{\lambda_1}{2}[(\chi_L^\dagger\chi_L)^2+(\chi_R^\dagger\chi_R)^2]+\lambda_2(\chi_L^\dagger\chi_L)(\chi_R^\dagger\chi_R)\ . \quad (3)$$

Note that the potential is invariant with respect to parity operation under which $\chi_L\leftrightarrow\chi_R$, except for the scalar mass terms which break this symmetry softly.[8] The minimum of the potential corresponds to

$$\langle\chi_L^0\rangle=v_L,\quad \langle\chi_R^0\rangle=v_R\ , \quad (4)$$

where

$$v_L^2=\frac{\lambda_2\mu_R^2-\lambda_1\mu_L^2}{\lambda_2^2-\lambda_1^2},\quad v_R^2=\frac{\lambda_2\mu_L^2-\lambda_1\mu_R^2}{\lambda_2^2-\lambda_1^2}\ . \quad (5)$$

Choosing $\mu_R\ge\mu_L$, we guarantee $v_R\ge v_L$ which in turn implies that the right-handed charged-current effects are suppressed. It is also worth pointing out here that there are only two physical Higgs scalars in this model: $\sigma_L\equiv\sqrt{2}\,\mathrm{Re}\chi_L^0$ and $\sigma_R\equiv\sqrt{3}\,\mathrm{Re}\chi_R^0$. These two states mix at the tree level. The mass matrix is given by

$$\begin{bmatrix} 2\lambda_1 v_L^2 & 2\lambda_2 v_L v_R \\ 2\lambda_2 v_L v_R & 2\lambda_1 v_R^2 \end{bmatrix}\ . \quad (6)$$

The σ_L-σ_R mixing angle ζ is

$$\tan 2\zeta=\frac{2\lambda_2 v_L v_R}{\lambda_1(v_R^2-v_L^2)}\ . \quad (7)$$

The masses of the physical scalars are

$$M_{\sigma_R}^2\simeq 2\lambda_1 v_R^2,\quad M_{\sigma_L}^2\simeq 2\lambda_1\left[1-\frac{\lambda_2^2}{\lambda_1^2}\right]v_L^2\ll M_{\sigma_R}^2\ . \quad (8)$$

In the gauge sector, there is no mixing between the charged gauge bosons $W_L^\pm$ and $W_R^\pm$ at the tree level. Their masses are given by $gv_L/\sqrt{2}$ and $gv_R/\sqrt{2}$, respectively, so that $M_{W_L}\ll M_{W_R}$. The neutral gauge bosons W_{3L}, W_{3R}, and B mix at the tree level. The mass eigenstates A, Z_1, and Z_2 are

$$\begin{aligned} A&=\sin\theta_W(W_{3L}+W_{3R})+\sqrt{\cos 2\theta_W}B\ ,\\ Z_1&=\cos\alpha Z_L+\sin\alpha Z_R\ ,\\ Z_2&=-\sin\alpha Z_L+\cos\alpha Z_R\ , \end{aligned} \quad (9)$$

with masses

$$M_A=0,\quad M_{Z_1}\simeq M_{W_L}/\cos\theta_W,\quad M_{Z_2}\simeq M_{W_R}\cos\theta_W/\sqrt{\cos 2\theta_W}\ . \quad (10)$$

Here Z_L and Z_R are defined as

$$\begin{aligned} Z_L&=\cos\theta_W W_{3L}-\sin\theta_W\tan\theta_W W_{3R}-\tan\theta_W\sqrt{\cos 2\theta_W}B\ ,\\ Z_R&=\sqrt{\cos 2\theta_W}\sec\theta_W W_{3R}-\tan\theta_W B\ . \end{aligned} \quad (11)$$

The weak mixing angle θ_W is related to the gauge couplings via

$$e=\frac{g}{\csc\theta_W}=\frac{g'}{\sqrt{\sec 2\theta_W}}\ , \quad (12)$$

and the angle α is given by

$$\tan 2\alpha=\frac{2v_L^2\sec^2\theta_W\sqrt{\sec 2\theta_W}}{v_R^2\cot^2\theta_W\sec 2\theta_W+v_L^2(\tan^2\theta_W\sec 2\theta_W-\csc^2\theta_W\sec^2\theta_W)}\ . \quad (13)$$

Note that $\alpha\approx(v_L^2/v_R^2)\ll 1$.

In order to discuss the fermion masses and the question of *CP* violation, we now give the most general Yukawa interaction of the model:

$$\mathcal{L}_Y=\bar{Q}_L h_u\tilde{\chi}_L P_R+\bar{Q}_L h_d\chi_L N_R+\bar{\Psi}_L h_e\chi_L E_R+L\leftrightarrow R+\bar{P}_L M_P P_R+\bar{N}_L M_N N_R+\bar{E}_L M_E E_R+\mathrm{H.c.} \quad (14)$$

Here M_i, $i=P,N,E$ are chosen to be complex non-Hermitian matrices whereas $h_{u,d,e}$ are arbitrary real matrices in generation space. Thus the model has both soft *P* and *CP* violation. We may remark here that even if *CP* violation is hard, the model provides a solution to the strong *CP* problem. (Parity invariance alone is sufficient to set $\theta=0$). In order to be more general, we shall keep the matrices $h_{u,d}$ of Eq. (7) to be complex in the subsequent sections. Of course, the case of soft *CP* violation will be recovered by identifying $h^\dagger$ with h^T.

The present model is a more economical version of the model in Ref. 4 since the parity-odd singlet scalar field has been omitted. This has two interesting implications: (i) Since parity invariance is now broken softly, rather than spontaneously, the model is free from cosmological domain wall problem without recourse to inflation or grand unification; (ii) a solution to the strong *CP* problem can be achieved with a low right-handed scale.

III. $\bar{\theta}$ AT TREE LEVEL

We use the fact that parity is only a softly broken symmetry in order to set $\theta=0$. The quark mass matrices that follow from Eq. (7) after spontaneous symmetry breaking are

$$M_u^{(0)}=\begin{bmatrix} 0 & h_u v_L \\ h_u^\dagger v_R & M_P \end{bmatrix},$$
$$M_d^{(0)}=\begin{bmatrix} 0 & h_d v_L \\ h_d^\dagger v_R & M_N \end{bmatrix}. \quad (15)$$

It is clear that $\mathrm{Det}(M_u^{(0)})$ and $\mathrm{Det}(M_d^{(0)})$ are separately real and therefore we have $\bar{\theta}=0$ at the tree level.

IV. $\bar{\theta}$ AND d_n^e AT THE ONE-LOOP LEVEL

In this section, we prove by an explicit computation that $\bar{\theta}=0$ in the one-loop approximation. On the other hand, d_n^e will be shown to receive nonzero contribution at this level.

Let us first turn to the computation of $\bar{\theta}$. For this purpose, we have to calculate the one-loop radiative corrections to the up- and down-quark mass matrices of Eq. (15). We find it convenient to work in the weak basis, rather than the mass eigenbasis. Thus we treat the mass matrices as part of the interaction Lagrangian. Let the radiatively corrected up quark mass matrix be

$$M_u=M_u^{(0)}(1+C) . \quad (16)$$

Then using the fact that the determinant of $M_u^{(0)}$ is real, one can express

$$\begin{aligned}\bar{\theta}&=\mathrm{Arg\,Det}(1+C)\\ &=\mathrm{Arg}e^{\mathrm{Tr}\ln(1+C)}\\ &=\mathrm{Im\,Tr}\ln(1+C) . \end{aligned} \quad (17)$$

Using the loop expansion for C,

$$C=C_1+C_2+\cdots , \quad (18)$$

where the subscripts 1,2,... denote the number of loops, we have

$$\bar{\theta}=\mathrm{Im\,Tr}C_1+\mathrm{Im\,Tr}(C_2-\tfrac{1}{2}C_1^2)+\cdots . \quad (19)$$

Similar arguments apply for the down-quark mass matrix as well. In what follows, we shall show that $\mathrm{Tr}C_1$ is real, so that $\bar{\theta}$ vanishes up to the one-loop level.

The one-loop radiative corrections to the up-quark mass matrix arise through neutral scalar and neutral-gauge-boson exchange. The relevant diagrams are shown in Fig. 1. Since the charged gauge bosons $W_L^\pm$ and $W_R^\pm$ do not mix at the tree level, there is no contribution from their exchange to one-loop order. Let us denote the one-loop radiative correction to the mass matrix by

$$\delta M_u=\begin{bmatrix} \delta M_{LL}^u & \delta M_{LH}^u \\ \delta M_{HL}^u & \delta M_{HH}^u \end{bmatrix}. \quad (20)$$

Then $\bar{\theta}$ to this order is given by

$$\bar{\theta}=\mathrm{Im\,Tr}\left[-\frac{1}{v_L v_R}\delta M_{LL}^u(h_u^\dagger)^{-1}Mph_u^{-1}+\frac{1}{v_L}\delta M_{LH}^u h_u^{-1}+\frac{1}{v_R}\delta M_{HL}^u(h_u^\dagger)^{-1}\right]. \quad (21)$$

There is a similar contribution from the down-quark sector as well. Note that to one-loop order δM_{HH} does not enter into the equation for $\bar{\theta}$. This means that we need not compute the diagrams in Fig. 1 for the moment.

Consider the contribution from Fig. 1(a). Since we are treating the mass as part of the interaction, the cross on the internal fermion line stands for all possible tree-level diagrams with an initial P_L and in all P_R states. Hence

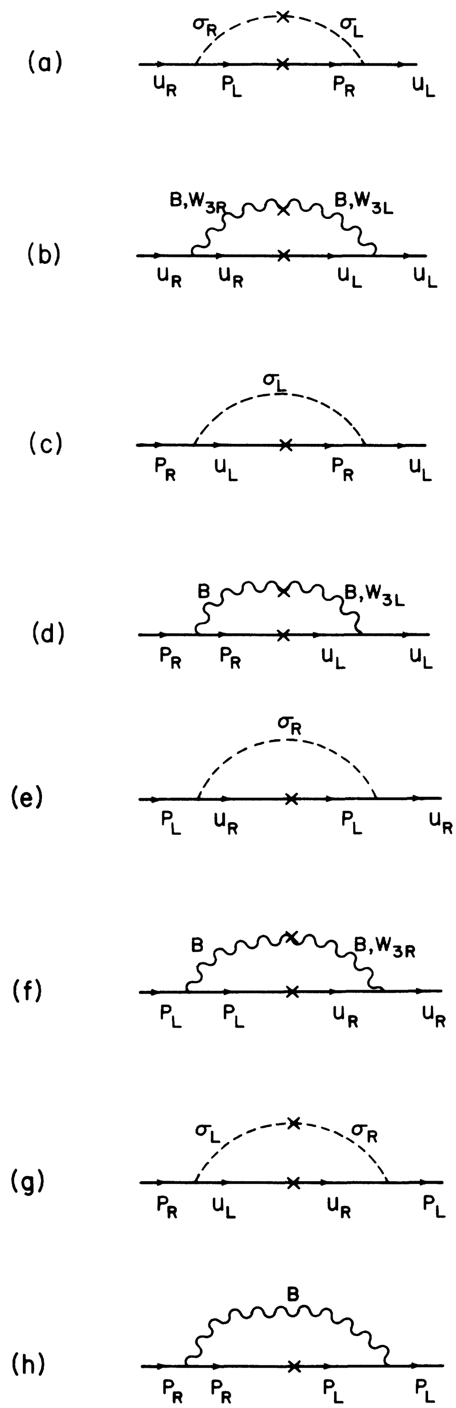

FIG. 1. One-loop radiative corrections to the up-quark mass matrix. The cross on the internal fermion line stands for all possible mass insertions.

our first task is to sum all such tree diagrams at nonzero external momentum. This can be done in the following manner. Let $F_L \equiv (u,P)_L$ and $F_R \equiv (u,P)_R$. The mass matrix $M_u^{(0)}$ of Eq. (15) corresponds to a term $\bar{F}_L M_u^{(0)} F_R$ in the Lagrangian. The full tree-level propagator with all possible mass insertions is then

$$\bar{F}_R \left[M_u^{(0)\dagger} \frac{k^2}{k^2 - M_u^{(0)} M_u^{(0)\dagger}} \right] F_L \ . \tag{22}$$

The propagators relevant for Fig. 1 can be obtained from the above by suitable projection operators. For this purpose, let us define

$$(M_u^{(0)} M_u^{(0)\dagger} - k^2)^{-1} \equiv \begin{bmatrix} X(k^2) & Y(k^2) \\ Y^\dagger(k^2) & Z(k^2) \end{bmatrix} \tag{23}$$

with $X = X^\dagger$, $Z = Z^\dagger$. Then by ordinary matrix inversion, X, Y, Z are determined by

$$\begin{aligned} &(v_R^2 h_u^\dagger h_u + M_P M_P^\dagger - K^2) Y^\dagger = -v_L M_P h_u^\dagger X \ , \\ &v_L h_u h_u^\dagger X + h_u M_P^\dagger Y^\dagger = \frac{1}{v_L}(I + k^2 X) \ , \\ &Y = -v_L H h_u M_P^\dagger Z \ , \end{aligned} \tag{24}$$

where

$$H = (v_L^2 h_u h_u^\dagger - k^2)^{-1} = H^\dagger \ . \tag{25}$$

Here I is an identity matrix in the generation space. From Eqs. (22)–(24) it follows that the interaction corresponding to the cross on the internal fermion lines of Fig. 1 can be read off from

$$\begin{aligned} -\mathcal{L}_{\text{eff}}^{\text{tree}} = \bar{P}_R &\left[\frac{k^4}{v_L} h_u^{-1} Y(k^2) \right] P_L + \bar{u}_R [k^2 h_u v_R Z(k^2)] P \\ &+ \bar{P}_R \left[\frac{k^2}{v_L} h_u^{-1} [I + k^2 X(k^2)] \right] u_L \\ &+ \bar{u}_R [k^2 h_u v_R Y^\dagger(k^2)] + \text{H.c.} \end{aligned} \tag{26}$$

Now we are ready to evaluate the one-loop radiative corrections to the mass matrices. Consider the scalar exchange of Fig. 1(a). The amplitude is given by

$$\delta M_{LL}^u(\text{Higgs}) = \int \frac{d^4k}{(2\pi)^4} h_u \frac{1}{v_L} h_u^{-1} \times \frac{k^2 Y(k^2) h_u^\dagger \lambda_2 v_L v_R}{[(p-k)^2 - M_{\sigma_L}^2][(p-k)^2 - M_{\sigma_R}^2]} \ , \tag{27}$$

Its contribution to $\bar{\theta}$ is then [see Eq. (21)]

$$-\text{Im Tr} \left[\frac{\lambda_2}{v_L} \int \frac{d^4k}{(2\pi)^4} \times \frac{k^2 Y(k^2) M_P h_u^{-1}}{[(p-k)^2 - M_{\sigma_L}^2][(p-k)^2 - M_{\sigma_R}^2]} \right] . \tag{28}$$

One can evaluate the trace before performing the momentum integration. Using Eqs. (14) and (25), one has

$$\text{Tr}[Y(k^2) M_P h_u^{-1}] = -v_L \text{Tr}[(h_u^\dagger h_u v_L^2 - k^2)^{-1} M_P^\dagger Z(k^2) M_P] \ . \tag{29}$$

Since the right-hand side is the product of two Hermitian matrices, its trace is real. Hence we conclude that the contribution from Fig. 1(a) to $\bar{\theta}$ is zero.

Consider next the gauge contribution Fig. 1(b). Proceeding as before, we extract the matrix structure of the diagram. This is exactly the same as of Fig. 1 (a), namely, $Y(k^2) h_u^\dagger$. Therefore the contribution to $\bar{\theta}$ from Fig. 1(b) is also zero.

Figures 1(c)–1(f) represent correction to the off-diagonal entries of the up-quark mass matrix. Figures 1(c) and 1(d) have the matrix structure

$$\begin{aligned} &\text{Fig. 1(c)}: \ [I + k^2 X(k^2)] h_u \ , \\ &\text{Fig. 1(d)}: \ [I + k^2 X(k^2)](h_u^\dagger)^{-1} \ . \end{aligned} \tag{30}$$

After multiplying by h_u^{-1}, the relevant trace for $\bar{\theta}$ is seen to involve [from Eq. (21)] $I + k^2 X$ and $(I + k^2 X)(h_u h_u^\dagger)^{-1}$. Both of these traces are real since X is a Hermitian matrix.

Finally, the contribution from Fig. 1(e) is proportional to $h_u^\dagger h_u Z(k^2) h_u^\dagger$ and Fig. 1(f) to $Z(k^2) h_u^\dagger$. Again we see that their contribution to $\bar{\theta}$ is zero. This completes the proof that $\bar{\theta} = 0$ to the one-loop order.

Turning now to the neutron electric dipole moment d_n^e, we see that once an external photon line is attached to the loop diagram of Fig. 1, they represent a potential contribution to d_n^e, if the $(\delta M_{u,d})_{11}$ is complex. The matrix structure of these diagrams is the same as in Eqs. (27)–(30). It is clear that, for arbitrary complex $M_{P,N}$, $(\delta M_{u,d})_{11}$ is indeed complex, thereby leading to a nonvanishing d_n^e. The crucial difference here is that unlike in the computation of $\bar{\theta}$, there is no tracing involved. The dominant contribution to d_n^e arises from Figs. 1(a) and 1(b). The contribution from Fig. 1(a) is estimated to be

$$\begin{aligned} d_n^e &\approx e \frac{h_u h_u^\dagger}{16\pi^2 M_P} \lambda_2 \left[\frac{v_L}{v_R} \right] \ln \left[\frac{M_P^2}{M_{\sigma_L}^2} \right] \sin\delta \\ &\approx \frac{e \lambda_2 m_q}{16\pi^2 v_R^2} \ln \left[\frac{M_P^2}{M_{\sigma_L}^2} \right] \sin\delta \approx 10^{-25} \lambda_2 \left[\frac{10\,\text{TeV}}{v_R} \right]^2 e\ \text{cm} \ . \end{aligned} \tag{31}$$

Here we assumed that $v_R \approx M_P$ and the CP phase $\sin\delta \approx 1$. The contribution from Fig. 1(b) is similar, except that λ_2 is replaced by g'^2. We see that $d_n^e \approx 10^{-25}$ -20^{-26} e cm. This value is in the accessible range of the next generation of experiments.

V. TWO-LOOP CONTRIBUTION TO $\bar{\theta}$

In this section we turn to the two-loop contribution to $\bar{\theta}$. Our aim here is not to compute $\bar{\theta}$ exactly, we shall be content with an order of magnitude estimate. At the two-loop level, $\bar{\theta}$ receives two types of contribution as can be seen from Eq. (19). One type arises from the two-loop diagrams. A few typical diagrams are shown in Fig. 2. The other is the square of one-loop contributions. In the notation of Eq. (20), $\bar{\theta}_{2\,\text{loop}}$ can be written as

$$\bar{\theta}_{2\,\text{loop}}=\text{Im Tr}\Bigg[-\frac{1}{v_L v_R}\delta M_{LL}^{(2)}(h^\dagger)^{-1}Mh^{-1}+\frac{1}{v_L}\delta M_{LH}^{(2)}h^{-1}+\frac{1}{v_R}\delta M_{HL}^{(2)}(h^\dagger)^{-1}-\frac{1}{2v_L^2v_R^2}[\delta M_{LL}^{(1)}(h^\dagger)^{-1}Mh^{-1}]^2$$
$$-\frac{1}{2v_R^2}[\delta M_{HL}^{(1)}(h^\dagger)^{-1}]^2-\frac{1}{2v_L^2}(\delta M_{LH}^{(1)}h^{-1})^2+\frac{1}{v_L^2v_R}\delta M_{LH}^{(1)}h^{-1}\delta M_{LL}^{(1)}(h^\dagger)^{-1}Mh^{-1}$$
$$+\frac{1}{v_L v_R^2}\delta M_{LL}(h^\dagger)^{-1}\delta M_{HL}^{(1)}(h^\dagger)^{-1}Mh^{-1}-\frac{1}{v_L v_R}\delta M_{LL}^{(1)}(h^\dagger)^{-1}\delta M_{HH}^{(1)}h^{-1}\Bigg]\,, \tag{32}$$

where the subscripts u,P have been dropped for brevity. The superscripts (1) and (2) refer to one and two loops, respectively.

Consider first the diagram Fig. 2(a). Its contribution to $\bar{\theta}$ can be estimated to be (for maximal CP phase)

$$\bar{\theta}(\text{Fig.}\,2(\text{a}))\approx\left[\frac{h_t^2}{16\pi^2}\right]^2\left[\frac{\lambda_2 v_L}{v_R}\right]^2\phi^2\,. \tag{33}$$

Here we assumed that the dominant contribution comes from the top-quark exchange. ϕ is a typical mixing parameter involving the third generation. Its appearance can be understood as follows: If only the top-quark contribution is retained, the amplitude of Fig. 2(a) relevant to $\bar{\theta}$ becomes real. Therefore, either two of the Yukawa couplings should belong to second generation, or one should include third generation mixing into the others. This is parametrized by ϕ. For $(v_L/v_R)\approx10^{-2}$, $\phi\approx10^{-2}$, $\lambda_2\approx10^{-1}$, we estimate this contribution to $\bar{\theta}$ to be 10^{-14}.

The contribution from Figs. 2(b) and 2(c) to $\bar{\theta}$ are comparable to that from Fig. 2(a). This statement warrants some explanation. Consider Fig. 2(b) first. At first sight it might appear that there is no $(v_L/v_R)^2$ suppression for this diagram. However, this conclusion is incorrect. If in Fig. 2(b) σ_L is replaced by the longitudinal component of Z_L, the amplitude will be exactly the same, except for an overall sign difference and the replacement of the propagator $(k^2-M_{\sigma_L}^2)^{-1}$ by $(k^2-M_{Z_L}^2)^{-1}$. When the two are combined, there is a suppression factor $M_{\sigma_L}^2/v_R^2$ or $M_{Z_L}^2/v_R^2$. This can also be seen (perhaps more vividly) in the 't Hooft–Feynman gauge, where one includes the contribution from the unphysical neutral scalar bosons as well.

As for the diagram in Fig. 2(c), a straightforward computation yields a suppression factor $(v_L/v_R)^2$. This can be seen by rewriting the interaction Eq. (26) to leading order (i.e., neglecting terms of order v_L^2/v_R^2):

$$\mathcal{L}_{\text{eff}}^{\text{tree}}=\bar{P}_R\left[M_P^\dagger\frac{k^2}{k^2-h_u^\dagger h_u v_R^2-M_PM_P^\dagger}\right]P_L+\bar{u}_R\left[hv_R\frac{k^2}{k^2-h_u^\dagger h_u v_R^2-M_PM_P^\dagger}\right]P_L$$
$$+\bar{P}_R\left[1+M_P^\dagger\frac{1}{k^2-h_u^\dagger h_u v_R^2-M_PM_P^\dagger}M_P\right](h_u^\dagger v_L)v_L$$
$$+\bar{u}_R\left[h_u\frac{1}{k^2-h_u^\dagger h_u v_R^2-M_PM_P^\dagger}M_Ph_u^\dagger v_Lv_R\right]u_L+O\left[\frac{v_L^2}{v_R^2}\right]+\text{H.c.} \tag{34}$$

Using the above approximation, it is not difficult to show that the products of one-loop terms all have the suppression of v_L^2/v_R^2.

Figure 2(d) is somewhat different from all the diagrams considered so far in that it brings in parameters of the down-quark mass matrix into the up sector (and vice versa). The relevant trace is complex, and we estimate

$$\bar{\theta}\approx\left[\frac{h_t^2}{16\pi^2}\right]^2\left[\frac{v_L}{v_R}\right]\phi^2\,. \tag{35}$$

Again, we saturated the Yukawa couplings with the heaviest available quark, namely, the top quark, and allowed for its mixing with the other generations. It is clear that this contribution dominates $\bar{\theta}$. For $\phi \approx 10^{-2}$, $M_t \approx M_{W_L}$, $v_L/v_R \approx 10^{-2}$, we obtain $\bar{\theta} \approx 10^{-12}$. Its contribution to d_n^e is clearly smaller than the pure weak *CP*-violating contribution by two to three orders of magnitude.

The suppression factor v_L/v_R appearing in $\bar{\theta}$ is not hard to understand. In the limit $v_R \to \infty$, $M_{P,N} \to \infty$ ($v_R/M_{P,N}$ fixed), the model reduces to the standard model with Kobayashi-Maskawa-type *CP* violation, where it is well known[9] that $\bar{\theta}$ receives no contribution up to two-loop order. Therefore the two-loop contribution we have estimated should involve a v_L/v_R suppression since it should vanish in the above limit.

VI. CONCLUSIONS

We have presented a very simple extension of the standard model using the left-right-symmetric gauge group with "seesaw" masses for charged fermions which solves the strong *CP* problem. The neutron electric dipole moment in this model arises dominantly from weak *CP*-violating effects at the one-loop level and is in the experimentally interesting range of 10^{-25}–10^{-26} e cm. $\bar{\theta}$ on the other hand, arises only at the two-loop level and is estimated to be $\approx 10^{-12}$. Other interesting phenomenological aspects of the model, such as *CP* violation,[5] small neutrino masses,[10] etc., have been studied elsewhere. It has been shown in Ref. 5 that the model realistically accounts for the observed *CP* violation in the *K*-meson system even in the limit of a decoupled third generation. Neutrinos are predicted to be Dirac particles with their small mass arising at the two-loop level[10] via diagrams analogous to Fig. 2(d). In contrast with most of the models of Ref. 3, the present model does not require any special symmetry (discrete or continuous) or a high-mass scale in order to solve the strong *CP* problem.

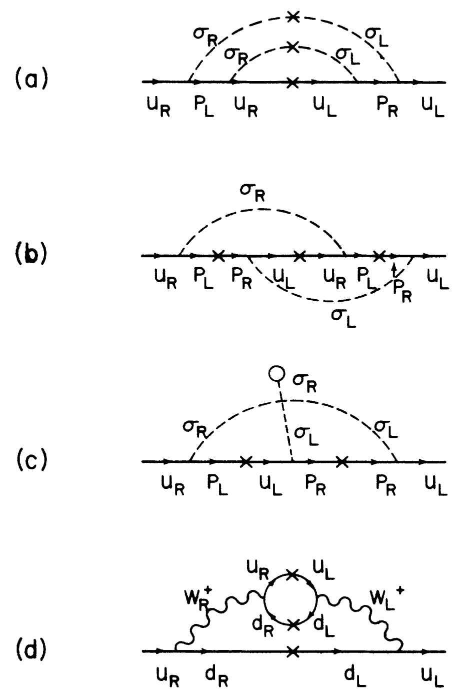

FIG. 2. Typical two-loop diagrams contributing to the up-quark mass matrix.

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