Parity-violating effects of the bound system of a charged fermion and an Abelian dyon with charge $Z_d < Z_d^c$ and for $j \geq |q| + \frac{1}{2}$

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(Received 20 July 1989)

The $\Delta j = 0$ parity-violating electric dipole transitions are discussed and nonvanishing electric dipole moments $\langle d_z \rangle_{qnjm}$ are obtained for the bound system of a charged fermion and an Abelian dyon with electric charge $Z_d < Z_d^c$, a critical value of order α^{-1}, and for total angular momentum $j \geq |q| + \frac{1}{2}$. A new possibility to search for dyons in bound conditions is analyzed.

I. INTRODUCTION

In recent years, the problem of bound states of a fermion with an Abelian monopole[1] or a non-Abelian monopole[2] and dyonic generalizations carrying also an electric charge have been extensively discussed.[3-19] The detailed analysis of such a bound system may lead to the possibility to search for monopoles or dyons in bound conditions. The production of superheavy monopoles and dyons in the early Universe is predicted in unified theories of the strong and electroweak interactions[12] and hence they may remain in the present Universe. Dyons would like to form bound states with a charged fermion. One method of detecting dyons is to find the electromagnetic spectrum, for example, from astronomical observations, of their bound system, and to find their other unusual properties. In Refs. 9, 15, 18, and 19 we obtained wave functions and the energy spectrum of the bound states for a charged fermion and an Abelian dyon with electric charge $Z_d < Z_d^c$ (Ref. 9), a critical value of order α^{-1}, for total angular momentum $j \geq |q| + \frac{1}{2}$. Based on these works we now discuss the effects of parity violation for this system.

In this system spatial parity is violated by the magnetic charge of the dyon.[7,20] The magnetic field of the dyon $\mathbf{H}_D = g\mathbf{r}/r^3$ is transformed to $-\mathbf{H}_D$ under the space reflection P. The invariance of the Dirac equation in the external magnetic field under P requires that the vector potential $\mathbf{A}(\mathbf{x},t)$ transform as[7]

$$P\,\mathbf{A}(\mathbf{x},t)P^{-1} = -\mathbf{A}(-\mathbf{x},t) \ . \tag{1.1}$$

Obviously the transformation of $\mathbf{H}_D$ contradicts (1.1), unless one changes the sign of g by hand under P. This parity violation leads to a modification of the selection rules of the electromagnetic transition for this system. In particular, $\Delta j = 0$ (without changes in the total angular momentum) in the electric dipole transition is allowed. In the hydrogenlike atom, electric dipole transitions are subject to strict selection rules as regards the total angular momentum j and the parity P:

$$|j' - j| \leq 1 \leq j' + j \ , \tag{1.2}$$

$$P'P = -1 \ , \tag{1.3}$$

From $|j'-j| \leq 1$, the selection rules of the total angular momentum j are $\Delta j = 0, \pm 1$; but from (1.3), the parities of the initial (P) and the final (P') states must be opposite, so that the $\Delta j = 0$ transition is forbidden. For the fermion-dyon system, parity is violated, thus the $\Delta j = 0$ electric dipole transitions are allowed. The violation of parity by the magnetic charge allows this system to possess a nonvanishing electric dipole moment also.[7,20]

In Sec. II we will review the main results of the bound system of a fermion and a dyon and fix our notation. In Sec. III we will discuss the electric dipole transition. The detailed results of the $\Delta j = 0$ parity-violating transition are obtained. In Sec. IV we will calculate the electric and magnetic dipole moments of this system. In a paper by Kazama,[7] the electric and the magnetic dipole moments were calculated for the zero-energy bound state of a charged fermion with an extra magnetic moment in the field of a fixed Dirac monopole. But for the nonzero-energy bound states, because the radial functions were not obtained even in an approximate manner, only the dominant term in the limit of very loosely bound states was roughly estimated. Here we obtain the detailed results of a nonvanishing electric dipole moment. Section V is devoted to a summary and discussions. A possibility of searching for the dyon in bound conditions is analyzed.

II. RADIAL WAVE FUNCTIONS

As is well known, for the system of a fermion and a Dirac monopole there is the Lipkin-Weisberger-Peshkin (LWP) difficulty[21] in the angular momentum states $j = |q| - \frac{1}{2}$ which shows the singularity of fermion radial wave functions at the origin. To avoid this difficulty, an infinitesimal extra magnetic moment is endowed to the fermion by Kazama and Yang.[5,6] In Ref. 9 we show that for the system of a fermion and Abelian dyon there is also the LWP difficulty in the angular momentum states

$j \geq |q| + \frac{1}{2}$ when the dyon charge Z_d exceeds a critical value Z_d^c. In order to avoid the LWP difficulty, in addition to the Kazama-Yang term $-(\kappa q/2Mr^3)\beta\boldsymbol{\Sigma}\cdot\mathbf{r}$, the term $(\kappa Z Z_d e^2/2Mr^3)\boldsymbol{\gamma}\cdot\mathbf{r}$ should also be considered. Thus the Hamiltonian of the system is

$$H = \boldsymbol{\alpha}\cdot(-i\nabla - Ze\,\mathbf{A}) + \beta M - \lambda/r - (\kappa q/2Mr^3)\beta\boldsymbol{\Sigma}\cdot\mathbf{r} - (\kappa\lambda/2Mr^3)\boldsymbol{\gamma}\cdot\mathbf{r}\ , \quad (2.1)$$

where $\mathbf{A}$ is the vector potential of the dyon. In order to remove the string of singularities, $\mathbf{A}$ is defined in terms of two or more functions in a corresponding number of overlapping regions.[3] $\lambda = ZZ_d e^2$, Z is the electric charge of the fermion which is an integer, and Z_d is the electric charge of the dyon which need not be an integer.

Only for the cases $j \geq |q| + \frac{1}{2}$ and dyon charge $Z_d < Z_d^c$ is there no singularity at the origin, that we can harmlessly take the limit $\kappa \to 0$ in (2.1) and thus reduce (2.1) to

$$H = \boldsymbol{\alpha}\cdot(-i\nabla - Ze\,\mathbf{A}) + \beta M - \lambda/r\ . \quad (2.2)$$

For the states $j \geq |q| + \frac{1}{2}$ there are two types of simultaneous eigensections of $\mathbf{J}^2$, J_Z, and H (Ref. 4):

$$\text{Type A}\quad \psi_{jm}^{(1)} = \frac{1}{r}\begin{bmatrix} h_1(r)\xi_{jm}^{(1)} \\ -ih_2(r)\xi_{jm}^{(2)} \end{bmatrix},\quad j \geq |q| + \tfrac{1}{2}\ , \quad (2.3)$$

$$\text{Type B}\quad \psi_{jm}^{(2)} = \frac{1}{r}\begin{bmatrix} h_3(r)\xi_{jm}^{(2)} \\ -ih_4(r)\xi_{jm}^{(1)} \end{bmatrix},\quad j \geq |q| + \tfrac{1}{2}\ , \quad (2.4)$$

where

$$\xi_{jm}^{(1)} = c_j\phi_{jm}^{(1)} - s_j\phi_{jm}^{(2)}\ , \quad (2.5)$$

$$\xi_{jm}^{(2)} = s_j\phi_{jm}^{(1)} + c_j\phi_{jm}^{(2)}\ , \quad (2.6)$$

$$c_j = q[(2j+1+2q)^{1/2} + (2j+1-2q)^{1/2}]/2|q|(2j+1)^{1/2}\ , \quad (2.7)$$

$$s_j = q[(2j+1+2q)^{1/2} - (2j+1-2q)^{1/2}]/2|q|(2j+1)^{1/2}\ , \quad (2.8)$$

$$\phi_{jm}^{(1)} = \begin{bmatrix} \left[\dfrac{j+m}{2j}\right]^{1/2} Y_{qj-1/2,m-1/2} \\ \left[\dfrac{j-m}{2j}\right]^{1/2} Y_{qj-1/2,m+1/2} \end{bmatrix}, \quad (2.9)$$

$$\phi_{jm}^{(2)} = \begin{bmatrix} -\left[\dfrac{j-m+1}{2j+2}\right]^{1/2} Y_{qj+1/2,m-1/2} \\ \left[\dfrac{j+m+1}{2j+2}\right]^{1/2} Y_{qj+1/2,m+1/2} \end{bmatrix}, \quad (2.10)$$

where $Y_{q,L,M}$ is the monopole harmonic whose basic properties are obtained in Refs. 3 and 22–24.

Solving (2.2)–(2.4), we obtain the energy spectrum[15,18,25]

$$E_{qnj} = M\left[1 + \frac{\lambda^2}{(n+\nu)^2}\right]^{-1/2}, \quad (2.11)$$

where $n = 0,1,2,\ldots$ is the radial quantum number, $\nu = (\mu^2 - \lambda^2)^{1/2} > 0$, $\mu = [(j+\frac{1}{2})^2 - q^2]^{1/2} > 0$, $j \geq |q| + \frac{1}{2}$, $q = Zeg \neq 0$, g is the strength of the magnetic monopole. Dirac quantization sets $eg = N/2 (N = 0, \pm 1, \pm 2, \ldots)$ (Ref. 1). The radial wave functions $R_i^{qnj}(\rho) = 2ph_i^{qnj}(\rho)/\rho$ $(i = 1,2,3,4)$ are

$$\text{Type A}\quad R_{1,2}^{qnj}(\rho) = 4p^2(M \pm E_{qnj})^{1/2} A_1^{qnj} e^{-\rho/2}\rho^{\nu-1} \times \left[F(-n,2\nu+1,\rho) \mp \frac{n}{\mu + (\mu^2+n^2+2n\nu)^{1/2}} \times F(-(n-1),2\nu+1,\rho)\right], \quad (2.12)$$

$$\text{Type B}\quad R_{3,4}^{qnj}(\rho) = 4p^2(M \pm E_{qnj})^{1/2} A_3^{qnj} e^{-\rho/2}\rho^{\nu-1} \times \left[F(-n,2\nu+1,\rho) \pm \frac{n}{\mu - (\mu^2+n^2+2n\nu)^{1/2}} \times F(-(n-1),2\nu+1,\rho)\right], \quad (2.13)$$

where $\rho = 2pr, p = (M^2 - E_{qnj}^2)^{1/2}$. $F(a,b;\rho)$ is the confluent hypergeometric function. When $a = -n$ $(n = 0,1,2,\ldots)$, $F(-n,b;\rho)$ is reduced to a polynomial. The radial wave functions $R_i(\rho)$ satisfy the following normalization condition:

$$\int_0^\infty \sum_{i=1(3)}^{2(4)} |R_i^{qnj}(2pr)|^2 r^2 dr = 1\ . \quad (2.14)$$

The normalization constants $A_{1,3}$ are

$$A_{1,3}^{qnj} = \frac{1}{2\Gamma(2\nu+1)}\left[\frac{n!Mp}{\Gamma(2\nu+n+1)}\left[1 + \frac{n(2\nu+n)}{[\mu \pm (\mu^2+n^2+2n\nu)^{1/2}]^2}\right]\right]^{-1/2}. \quad (2.15)$$

III. PARITY-VIOLATING TRANSITION

The interaction Hamiltonian of the system in an external electromagnetic field is

$$H_I=-iZe\beta\gamma_\mu A_\mu(\mathbf{x},t)\ , \tag{3.1}$$

where the four-potential of the external electromagnetic field is

$$A_\mu(\mathbf{x},t)=\epsilon_\mu A\,\exp[i(\mathbf{k}\cdot\mathbf{x}-\omega t)]\ . \tag{3.2}$$

The polarization four-vector satisfies $\epsilon_\mu k_\mu=0$. In the transverse Coulomb gauge $\epsilon=(\boldsymbol{\epsilon},0)$. When the velocity of the fermion is small compared with that of light, the factor $\exp(i\mathbf{k}\cdot\mathbf{x})$ varies only slightly in the region where the bound-state wave functions of the system are appreciably different from zero, so we can approximate $\exp(i\mathbf{k}\cdot\mathbf{x})\approx 1$ (the electric dipole approximation). When the time-dependent factor $\exp(-i\omega t)$ is separated, H_I is reduced to

$$H_I'=-Ze\boldsymbol{\alpha}\cdot\boldsymbol{\epsilon}A\ . \tag{3.3}$$

In the following we show the detailed calculation of the transition matrix elements for type A; type B can be similarly treated. By (2.3) and (2.12), the electric dipole transition matrix elements are

$$\begin{aligned}(H_I')_{qn'j'm',njm}&=\int d^3x\,\psi^{(1)\dagger}_{qn'j'm'}H_I'\psi^{(1)}_{qnjm}\\&=16iZeAp^2_{qn'j'}p^2_{qnj}A_1^{qn'j'}A_1^{qnj}(I_1^{qn'j',nj}\Sigma_{12}^{j'm',jm}-I_2^{qn'j',nj}\Sigma_{21}^{j'm',jm})\ ,\end{aligned} \tag{3.4}$$

where the radial integrals $I_{1,2}$ are

$$\begin{aligned}I_{1,2}^{qn'j',nj}&=(M\pm E_{qn'j'})^{1/2}(M\mp E_{qnj})^{1/2}\\&\quad\times\left[I_{n',n}\pm\frac{n}{\mu+(\mu^2+n^2+2n\nu)^{1/2}}I_{n',n-1}\mp\frac{n'}{\mu'+(\mu'^2+n'^2+2n'\nu')^{1/2}}I_{n'-1,n}\right.\\&\quad\left.-\frac{n'n}{[\mu'+(\mu'^2+n'^2+2n'\nu')^{1/2}][\mu+(\mu^2+n^2+2n\nu)^{1/2}]}I_{n'-1,n-1}\right]\ ,\end{aligned} \tag{3.5}$$

$$\begin{aligned}I_{n',n}&=\int_0^\infty r^2dr\,\exp[-(P_{qn'j'}+P_{qnj})r](2p_{qn'j'}r)^{\nu'-1}(2p_{qnj}r)^{\nu-1}\\&\quad\times F(-n',2\nu'+1;2p_{qn'j'}r)F(-n,2\nu+1;2p_{qnj}r)\\&=\sum_{l'=0}^{n'}\sum_{l=0}^{n}C^{n'}_{2\nu'+1,l'}C^{n}_{2\nu+1,l}\frac{(2p_{qn'j'})^{\nu'+l'-1}(2p_{qnj})^{\nu+l-1}}{(p_{qn'j'}+p_{qnj})^{\nu'+\nu+l'+l+1}}\Gamma(\nu'+\nu+l'+l+1)\ ,\end{aligned} \tag{3.6}$$

$$C^n_{2\nu+1,l}=\frac{(-1)^l n(n-1)\cdots(n-l+1)}{l!(2\nu+1)(2\nu+2)\cdots(2\nu+l)},\quad C^n_{2\nu+1,0}\equiv 1\ . \tag{3.7}$$

The normalization constant A_1^{qnj} is given by (2.15), and the angular integrals Σ_{ij} are

$$\Sigma_{12}^{j'm',jm}=\int d\Omega\,\xi^{(1)\dagger}_{j'm'}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\xi^{(2)}_{jm}=c_{j'}s_j(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{11}^{j'm',jm}+c_{j'}c_j(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{12}^{j'm',jm}-s_{j'}s_j(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{21}^{j'm',jm}-s_{j'}c_j(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{22}^{j'm',jm}\ , \tag{3.8}$$

$$\Sigma_{21}^{j'm',jm}=\int d\Omega\,\xi^{(2)\dagger}_{j'm'}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\xi^{(1)}_{jm}=s_{j'}c_j(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{11}^{j'm',jm}-s_{j'}s_j(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{12}^{j'm',jm}+c_{j'}c_j(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{21}^{j'm',jm}-c_{j'}s_j(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{22}^{j'm',jm}\ , \tag{3.9}$$

$$(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{kl}^{j'm',jm}=\int d\Omega\,\phi^{(k)\dagger}_{j'm'}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\phi^{(l)}_{jm}\quad(k,l=1,2)\ . \tag{3.10}$$

By using (2.9), (2.10), and the basic properties of the monopole harmonic,[3,22–24] we obtain

$$\begin{aligned}(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{11}^{j'm',jm}=&\ \delta_{j'j}\delta_{m'm}j^{-1}m\epsilon_3+\delta_{j'j}\delta_{m',m+1}(2j)^{-1}(j+m+1)^{1/2}(j-m)^{1/2}(\epsilon_1-i\epsilon_2)\\&+\delta_{j'j}\delta_{m',m-1}(2j)^{-1}(j-m+1)^{1/2}(j+m)^{1/2}(\epsilon_1+i\epsilon_2)\ ,\end{aligned} \tag{3.11}$$

$$\begin{aligned}(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{12}^{j'm',jm}=&-\delta_{j',j+1}\delta_{m'm}(j+1)^{-1}(j+m+1)^{1/2}(j-m+1)^{1/2}\epsilon_3\\&+\delta_{j',j+1}\delta_{m',m+1}(2j+2)^{-1}(j+m+2)^{1/2}(j+m+1)^{1/2}(\epsilon_1-i\epsilon_2)\\&-\delta_{j',j+1}\delta_{m',m-1}(2j+2)^{-1}(j-m+2)^{1/2}(j-m+1)^{1/2}(\epsilon_1+i\epsilon_2)\ ,\end{aligned} \tag{3.12}$$

$$\begin{aligned}(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{21}^{j'm',jm}=&-\delta_{j',j-1}\delta_{m'm}j^{-1}(j-m)^{1/2}(j+m)^{1/2}\epsilon_3-\delta_{j',j-1}\delta_{m',m+1}(2j)^{-1}(j-m)(\epsilon_1-i\epsilon_2)\\&+\delta_{j',j-1}\delta_{m',m-1}(2j)^{-1}(j+m)(\epsilon_1+i\epsilon_2)\ ,\end{aligned} \tag{3.13}$$

$$\begin{aligned}(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})_{22}^{j'm',jm}=&-\delta_{j'j}\delta_{m'm}(j+1)^{-1}m\epsilon_3-\delta_{j'j}\delta_{m',m+1}(2j+2)^{-1}(j-m)^{1/2}(j+m+1)^{1/2}(\epsilon_1-i\epsilon_2)\\&-\delta_{j'j}\delta_{m',m-1}(2j+2)^{-1}(j+m)^{1/2}(j-m+1)^{1/2}(\epsilon_1+i\epsilon_2)\ .\end{aligned} \tag{3.14}$$

Equations (3.4) and (3.8)–(3.14) show that the selection rules of the electric dipole transition are

$$\Delta j=0,\pm 1,\quad \Delta m=0,\pm 1\ . \tag{3.15}$$

In the above the most important feature is the new selection rules $\Delta j=0$ (without changes in the total angular momentum) parity-violating transition. When $j'=j$, we have $\mu'=\mu, \nu'=\nu$. Thus the $\Delta j=0$ transition matrix elements are

$$(H_I')_{qn'jm,njm}=i16ZeAp_{qn'j}^2p_{qnj}^2A_1^{qn'j}A_1^{qnj}\times\frac{qm\epsilon_3}{j(j+1)}(I_1^{qn'j,nj}-I_2^{qn'j,nj})\ , \tag{3.16}$$

$$(H_I')_{qn'j(m\pm1),njm}=i16ZeAp_{qn'j}^2p_{qnj}^2A_1^{qn'j}A_1^{qnj}\times\frac{q(j\pm m+1)^{1/2}(j\mp m)^{1/2}(\epsilon_1\mp i\epsilon_2)}{2j(j+1)}\times(I_1^{qn'j,nj}-I_2^{qn'j,nj})\ , \tag{3.17}$$

where $I_{1,2}$ are given by (3.5) for $j'=j,\mu'=\mu,\nu'=\nu$.

In order to obtain some impression about the magnitude of $(H_I')_{qn'jm,njm}$ we consider the case with $q=\frac{1}{2}$ (in this case the dyon carries one Dirac unit of pole strength), $j=|q|+\frac{1}{2}$, $n=1$, $n'=0$, $Z=-1$, $Z_d=+1$. Thus we have $|\lambda|\ll 1$. Up to α^2 order, from (3.16) and (3.17) we have

$$(H_I')_{1/2\,01m,11m}=iC_1A\epsilon_3 m\ ,$$

$$(H_I')_{1/2\,01(m\pm1),11m}=iC_2A(\epsilon_1\mp i\epsilon_2)(2\pm m)^{1/2}(1\mp m)^{1/2}\ ,$$

where C_1 and C_2 are numbers of order 10^{-2}. In the above estimation the asymptotic formula of the Γ function

$$\Gamma(az+b)\sim\sqrt{2\pi}\,e^{-az}(az)^{az+b-1/2}\quad(|\arg z|<\pi, a>0)$$

has been used.

In the context of monopole-fermion states for $j_{\min}=|q|-\frac{1}{2}>0$. $j_{\min}\to j_{\min}$ transitions have been discussed in Ref. 26 also.

IV. DIPOLE MOMENTS

The electric dipole moment $\mathbf{d}=e\mathbf{r}$ of this system can be represented by the total angular momentum

$$\mathbf{J}=\mathbf{r}\times(\mathbf{p}-Ze\,\mathbf{A})-q\mathbf{r}/r+\tfrac{1}{2}\mathbf{\Sigma} \tag{4.1}$$

as

$$\mathbf{d}=\frac{e}{j(j+1)}(-qr+\tfrac{1}{2}\mathbf{\Sigma}\cdot\mathbf{r})\mathbf{J}\ . \tag{4.2}$$

The only nonvanishing expectation value of $\mathbf{d}$ is the z component

$$\langle d_z\rangle_{qnjm}=\int d^3x\,\psi_{qnjm}^{(1)\dagger}d_z\psi_{qnjm}^{(1)}\ . \tag{4.3}$$

In the following, we show the results for type A. By using (2.3) and (2.12), we obtain

$$\begin{aligned}\langle d_z\rangle_{qnjm}&=-\frac{eqm}{j(j+1)}\int_0^\infty dr\,r^3\{[R_1^{qnj}(2pr)]^2+[R_2^{qnj}(2pr)]^2\}\\&=-\frac{2eqm}{j(j+1)}(A_1^{qnj})^2\left[M\sum_{l,l'=0}^{n}C_{2\nu+1,l}^nC_{2\nu+1,l'}^n+\frac{Mn^2}{[\mu+(\mu^2+n^2+2n\nu)^{1/2}]^2}\sum_{l,l'=0}^{n-1}C_{2\nu+1,l}^{n-1}C_{2\nu+1,l'}^{n-1}\right.\\&\qquad\left.-\frac{2En}{\mu+(\mu^2+n^2+2n\nu)^{1/2}}\sum_{l=0}^{n}\sum_{l'=0}^{n-1}C_{2\nu+1,l}^{n}C_{2\nu+1,l'}^{n-1}\right]\Gamma(2\nu+l+l'+2)\ ,\end{aligned} \tag{4.4}$$

where $C_{2\nu+1,l}^n$ is given by (3.7).

For the $n=0$ case, $E_{q0j}=M\nu/\mu, p=M\lambda/\mu$, Eq. (4.4) is reduced to

$$\langle d_z\rangle_{q0jm}=-\frac{eqm}{2j(j+1)}\frac{\mu}{\lambda M\Gamma(2\nu+1)}\ . \tag{4.5}$$

Taking $q=\frac{1}{2}, j=1$ from (4.15) we obtain $\langle d_z\rangle_{(1/2)01m}\sim-10(M^{-1}e)m$. The magnetic dipole moment of the system is

$$\boldsymbol{\mu}=\boldsymbol{\mu}_0+\boldsymbol{\mu}_1,\quad \boldsymbol{\mu}_0=\frac{e}{2}\mathbf{r}\times\boldsymbol{\alpha},\quad \boldsymbol{\mu}_1=\frac{\kappa e}{2M}\beta\mathbf{\Sigma}\ . \tag{4.6}$$

As in the electric-dipole-moment case, we only need to consider the z component. For μ_{0z}, we have

$$\begin{aligned}\langle\mu_{0z}\rangle_{qnjm}&=\int d^3x\,\psi_{qnjm}^{(1)\dagger}\mu_{0z}\psi_{qnjm}^{(1)}\\&=-(ie/2)\int dr\,r^2R_1^{qnj}(2pr)R_2^{qnj}(2pr)\int d\Omega[\xi_{jm}^{(1)\dagger}(\mathbf{r}\times\boldsymbol{\sigma})_z\xi_{jm}^{(2)}-\xi_{jm}^{(2)\dagger}(\mathbf{r}\times\boldsymbol{\sigma})_z\xi_{jm}^{(1)}]\ .\end{aligned} \tag{4.7}$$

In order to calculate (4.7), we need the following formulas of the monopole harmonics[3,22–24] and the $3j$ symbols:[27]

$$e^{\pm i\varphi}\sin\theta=\mp\sqrt{8\pi/3}\,Y_{01,\pm1}(\theta,\varphi),\quad \cos\theta=\sqrt{4\pi/3}\,Y_{010}\ , \tag{4.8}$$

$$Y^*_{qjm}(\theta,\varphi)=(-1)^q Y_{-qj,-m}(\theta,\varphi)\int d\Omega\, Y_{q_1 j_1 m_1} Y_{q_2 j_2 m_2} Y_{q_3 j_3 m_3} \tag{4.9}$$

$$=(-1)^{j_1+j_2+j_3}\left[\frac{(2j_1+1)(2j_2+1)(2j_3+1)}{4\pi}\right]^{1/2}\begin{bmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3\end{bmatrix}\begin{bmatrix} j_1 & j_2 & j_3 \\ q_1 & q_2 & q_3\end{bmatrix}, \tag{4.10}$$

$$\begin{bmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3\end{bmatrix}=\begin{bmatrix} j_3 & j_1 & j_2 \\ m_3 & m_1 & m_2\end{bmatrix}=\begin{bmatrix} j_2 & j_3 & j_1 \\ m_2 & m_3 & m_1\end{bmatrix}, \tag{4.11}$$

$$\begin{bmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3\end{bmatrix}=(-1)^{-j_1-j_2-j_3}\begin{bmatrix} j_2 & j_1 & j_3 \\ m_2 & m_1 & m_3\end{bmatrix}, \tag{4.12}$$

$$\begin{bmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3\end{bmatrix}=(-1)^{j_1+j_2+j_3}\begin{bmatrix} j_1 & j_2 & j_3 \\ -m_1 & -m_2 & -m_3\end{bmatrix}. \tag{4.13}$$

Using (4.8)–(4.13), it is easy to show that

$$\langle \mu_{0z}\rangle=0 . \tag{4.14}$$

For μ_{1z}, we have

$$\langle \mu_{1z}\rangle_{qnjm}=\frac{\kappa e}{2M}\langle \beta\Sigma_z\rangle_{qnjm}$$

$$=\frac{\kappa e}{2M}\frac{m}{2j(j+1)}\left[2\mu+\frac{E_{qnj}}{M}\right]. \tag{4.15}$$

For the $n=0$ case, Eq. (4.15) is reduced to

$$\langle \mu_{1z}\rangle_{q0jm}=\frac{\kappa e}{2M}\frac{m}{2j(j+1)}\left[2\mu+\frac{\nu}{\mu}\right]. \tag{4.16}$$

Taking $q=\frac{1}{2}$, $j=1$, from (4.16) we obtain $\langle \mu_{1z}\rangle_{1/2\,01m}=0.48(\kappa M^{-1}e)m$.

V. A POSSIBLE APPROACH TO SEARCH FOR DYONS IN BOUND CONDITIONS

The discovery of monopoles would have far-reaching consequences. Their existence has been invoked in the explanation of the phenomenon of electric charge quantization.[1] The production of superheavy monopoles in the early Universe is predicted in unified theories of the strong and electroweak interactions[12] and its detection is one of the few experimental handles for these theories.

The search for monopoles or if they carry also an electric charge, dyons, has turned up negative up to now. A summary can be found in a recent review article.[28] In different experiments, different properties have been assumed for the monopoles.[28,29] Some of the assumptions involved are (1) electromagnetic induction, (2) energy losses, (3) scintillation signature, (4) catalysis of proton decay, and (5) trapping and extraction. Monopoles could be trapped in ferromagnetic domains by an image force of the order of 10 eV/Å. The trapped monopoles are supposed to be wrecked out of the material by large magnetic fields.

Because of the importance of these objects, it may be worthwhile to explore other means of establishing their existence. One promising approach of searching for monopoles or dyons is to detect the electromagnetic spectrum of their bound system and to detect their unusual properties, in particular, the parity-violating effects.

The energy spectrum (2.11) is hydrogenlike, but it is quite different from the ordinary hydrogenlike one. This is because (1) When q takes half-integer values, then the total angular momentum j of this system takes integer values. This leads to a new series energy spectrum which does not exist in the ordinary hydrogenlike atom. (2) When q takes integer, then j takes half-integer values, which is similar to the ordinary hydrogen-like-atom case. But the energy level E_D, comparing with the energy level E_H of the hydrogenlike atom,

$$E_H=M\left[1+\frac{(Ze^2)^2}{\{n+[(j+\frac{1}{2})^2-(Ze^2)^2]^{1/2}\}^2}\right]^{-1/2}, \tag{5.1}$$

is shifted. Now we consider the shift amount. Consider the dyon charge $Z_d=+1$ (Ref. 30). Take $Z=-1$, $|q|=1, j=\frac{3}{2}$. For the $n=1$ energy level, from (2.11) and (5.1), we have $(E_D-E_H)/M\sim-10^{-2}\alpha^2$. On the other hand, comparing $\Delta E=E(n'=1)-E(n=0)$, we have $\Delta E_D-\Delta E_H\sim 10^{-2}\alpha^2 M$. Notice that these differences can be measured by present experimental techniques.

If dyons were produced in the early Universe, formed bound states with charged fermions and remain in the present Universe, then the electromagnetic spectrum according to (2.11) of these bound system may be recorded from astronomical observations, where they have been accumulated for some long period of time. We suggest to compare (2.11) with the unexplained astronomical spectrum.[31]

In conclusion, we note that because of the nonvanishing electric dipole moment $\langle d_z\rangle_{qnjm}$, it is possible to design a special electric field $\mathbf{E}$, for example, some electric well to trap the fermion-dyon bound system through $-\langle \mathbf{d}\cdot\mathbf{E}\rangle_{qnjm}$. These bound systems either come from cosmic rays or are extracted, for example, from ferromagnetic material by high magnetic field. Furthermore, in such devices it is possible to check whether the trapped objects are fermion-dyon bound system by comparing the absorption spectrum with, for example, the hydrogen-like atomic one. We would like to emphasize that the $\Delta j=0$ parity-violating electric dipole transition is the most outstanding feature of the fermion-dyon bound system which distinguishes it from the hydrogen-like atom.

The analysis of the energy spectrum (2.11) and the parity-violating properties (3.16), (3.17), and (4.5) of the fermion-dyon bound system may lead to the promising approach to search for dyons in bound condition. Of course, any attempt to detect monopoles or dyons is a challenging enterprise because if they remain in the present Universe they are surely rare.

ACKNOWLEDGMENTS

The author would like to thank the Theoretical Physics Institute, University of Minnesota for its hospitality and financial support. He would like to thank S. Gasiorowicz, P. H. Frampton, and L. McLerran for helpful discussions. This work was supported in part by the National Natural Science Foundation of China under Grant No. PHY-1880126 and by Shanxi Natural Science Foundation.

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$$\frac{M-E}{M}=\frac{\lambda^2}{2(n+\mu)^2}\left[1+\frac{\lambda^2}{n+\mu}\left[\frac{1}{\mu}-\frac{3}{4(n+\mu)}\right]\right].$$

Reference 8 presents analytic approximate results for dyon-fermion binding energies and the corresponding bound-state wave functions for angular momentum $j \geq |q|+\frac{1}{2}$ with the Kazama-Yang term. But their results are valid only in the limit of weak binding, $M-E \ll M$, and for small charges $ZZ_d e^2 \ll 1$. It is easy to check from the last paper of Ref. 8, taking the limit $\kappa \rightarrow 0$, that their result coincides with our Eq. (2.11) to leading order in λ (just the first term of the above equation) whereas our formula is valid without a restriction to $\lambda \ll 1$.

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[30]For the non-Abelian case, the possible charges of the stable dyon are those of a half-fermion, cf. C. G. Callan (Ref. 13).

[31]There are some astronomical spectra recorded at the Kitt Peak Observatory which cannot be explained by the atomic or molecular spectrum. Chang Yu-fan, University of Arizona (private communication).