Zamolodchikov's C function for the multiflavored Schwinger model

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We carry out a path-integral evaluation of the correlation functions of the energy-momentum tensor in the N-flavored Schwinger model using point-splitting regularization. The C function is obtained and found to decrease from N to $N-1$ along a renormalization-group trajectory as expected.

I. INTRODUCTION

Recently, Zamolodchikov devised a C function $C(g)$ of coupling constants such that the function is monotonically decreasing along a renormalization-group (RG) trajectory and, at a fixed point, equals the central charge of a certain conformal field theory. Since the central charge characterizes a conformal field theory to a certain extent, the C function becomes a natural tool to study the relations among different conformal field theories. When two or more fixed points are close to one another, a reliable perturbative calculation of the C function can be performed so that useful information can be obtained on the corresponding theory. Such perturbative approaches have been discussed by several authors.[1-3] One of the most interesting recent developments is the finding of perturbations which leave the system integrable.[4] A nonconformal integrable field theory can thus be constructed. In this respect, it is useful to have examples at our hand in which the evolution of the C function along a RG trajectory can be exactly solved. In this work, we present the calculation of the C function in the N-flavored Schwinger model which is perhaps the most widely investigated integrable system. We choose not to use the Abelian[5] or non-Abelian[6] bosonization methods because we would like to see how point-splitting methods work in calculating correlation functions of the energy-momentum tensor. As far as we know, this has not been attempted before. Direct calculation also provides us with information on how the various fundamental fields contribute to the correlation functions of the energy-momentum tensor which may not be easy to disentangle in the bosonization approach.

The Schwinger model is solvable since the exact fermionic propagator in the external field is known.[7] In Sec. II we review briefly the path-integral evaluation of correlation functions of gauge-invariant quantities in the Schwinger model. In Sec. III we apply the result of Sec. II to the calculation of two-point functions of the energy-momentum tensor. The C function is obtained from this calculation. Extension to the N-flavored case is quite straightforward in the present approach. We present the results in Sec. IV. In Sec. V we summarize and discuss our results.

II. PATH-INTEGRAL COMPUTATION OF THE CORRELATION FUNCTIONS

Let us briefly review the method of computing correlation functions of gauge-invariant quantities. We follow the notation of Ref. 8. The generating functional is

$$Z(J,\eta,\bar\eta)=\int[dA][d\psi][d\bar\psi]\times\exp\left[-S+\int d^2x(A_\mu J^\mu+\bar\eta\psi+\bar\psi\eta)\right], \tag{1}$$

where

$$S=\int d^2x[-\bar\psi(i\partial\!\!\!/+eA\!\!\!/)\psi+\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}]\ . \tag{2}$$

The exact propagator $G(x,y)$ satisfying

$$(i\partial\!\!\!/+eA\!\!\!/)G(x,y)=-\delta(x-y) \tag{3}$$

is given by

$$G(x,y)=e^{i[\phi(x)-\phi(y)]}S(x-y)\ , \tag{4}$$

where

$$\begin{aligned}&\phi(x)=e\frac{1}{\partial^2}\partial\!\!\!/ A\!\!\!/(x),\quad S(x-y)=i\partial\!\!\!/ D(x-y)\ ,\\&\partial^2D(x)=\delta(x)\ .\end{aligned} \tag{5}$$

Carrying out the fermionic path integration, we obtain

$$Z(J,\eta,\bar\eta)=\int[dA]\exp(\bar\eta G\eta-\tfrac{1}{2}A_\mu\Delta^{-1}e^{\mu\nu}A_\nu+J_\mu A^\mu)\ , \tag{6}$$

where

$$\begin{aligned}&\left[-\partial^2+\frac{e^2}{\pi}\right]\Delta(x-y)=\delta(x-y)\ ,\\&e^{\mu\nu}=g^{\mu\nu}-\frac{\partial^\mu\partial^\nu}{\partial^2}\end{aligned} \tag{7}$$

and all the integral signs are suppressed.

The basic gauge-invariant quantities are the gauge field $F_{\mu\nu}$ and the string $\bar{\psi}(x_1)\Gamma\exp(-ie\int_{x_1}^{x_2} A_\mu dx^\mu)\psi(x_2)$ where Γ can be 1, γ_5, γ_μ, or $\gamma_\mu\gamma_5$. To calculate the Green's function involving $F_{\mu\nu}$, we only have to introduce source currents $J_{\mu\nu}$ for $F_{\mu\nu}$, i.e., replace the source current J^μ in Eqs. (1) and (6) by $\partial_\nu(J^{\mu\nu}-J^{\nu\mu})$ where the source current $J^{\mu\nu}$ couples to $F_{\mu\nu}$. As to the calculation of the Green's function involving strings, we illustrate the method by computing

$$W_{\mu\alpha}(x_1,x_2;y_1,y_2)=\left\langle\bar{\psi}(x_1)\gamma_\mu\exp\left[-ie\int_{x_1}^{x_2}A_\mu dx^\mu\right]\psi(x_2)\bar{\psi}(y_1)\gamma_\alpha\exp\left[-ie\int_{y_1}^{y_2}A_\mu dx^\mu\right]\psi(y_2)\right\rangle . \tag{8}$$

Using Eqs. (4)–(7), we get

$$W_{\mu\alpha}(x_1,x_2;y_1,y_2) =\{[\mathrm{tr}S(x_2-x_1)\gamma_\mu\cdot][\mathrm{tr}S(y_2-y_1)\gamma_\alpha\cdot] -[\mathrm{tr}S(x_2-y_1)\gamma_\alpha S(y_2-x_1)\gamma_\mu\cdot]\}e^{\Sigma} , \tag{9}$$

where

$$\Sigma(x_1,x_2;y_1,y_2)=\tfrac{1}{2}j_\mu\Delta e^{\mu\nu}j_\nu=\tfrac{1}{2}j_\mu\Delta j^\mu ,$$
$$j_\mu(z)=-ie\left[\int_{x_1}^{x_2}\mathcal{T}^x_{\mu\nu}(z-x)dx^\nu+\int_{y_1}^{y_2}\mathcal{T}^y_{\mu\nu}(z-y)dy^\nu\right] \tag{10}$$

and $\mathcal{T}_{\mu\nu}$ is the kernel

$$\mathcal{T}^x_{\mu\nu}(z-x)=\left\langle z\left|\frac{\bar{\partial}_\nu(\bar{\partial}_\nu-i\partial_\nu\gamma_5^x)}{\partial^2}\right|x\right\rangle , \tag{11}$$

where $e_{\mu\nu}=\bar{\partial}_\mu\bar{\partial}_\nu/\partial^2$, $\bar{\partial}_\mu=\epsilon_{\mu\alpha}\partial^\alpha$. Note that j_μ is conserved. Indeed,

$$\partial^\nu\mathcal{T}_{\mu\nu}=0 . \tag{12}$$

The superscript x in γ_5^x in Eq. (11) indicates that the γ_5 is to be inserted in the trace involving $S(x_2-x_1)$ and similarly for γ_5^y. The dot inside the trace in Eq. (9) indicates the position where such terms are to be inserted. For the second term in $W_{\mu\alpha}$ where only one trace is involved, the superscripts in γ_5^x can be ignored.

It is easy to see how to generalize Eqs. (9) and (10) to the case when an arbitrary number of vector or axial-vector strings are involved. If in addition, there are scalar and pseudoscalar strings in the correlation functions we want to calculate, then additional terms besides $\mathcal{T}_{\mu\nu}$ will appear in j_μ. Evaluating the exponential factor in Eq. (9), we obtain

$$\Sigma(x_1,x_2;y_1,y_2)=-\frac{e^2}{2}\int_{x_1}^{x_2}dx^\tau\int_{x_1}^{x_2}dx'^\lambda\mathcal{P}^{xx}_{\tau\lambda}\Delta(x-x') -\frac{e^2}{2}\int_{x_1}^{x_2}dx^\tau\int_{y_1}^{y_2}dy^\lambda\mathcal{P}^{xy}_{\tau\lambda}\Delta(x-y) +(x\leftrightarrow y) , \tag{13}$$

where

$$\mathcal{P}^{xy}_{\mu\nu}=\frac{(\bar{\partial}_\mu-i\partial_\mu\gamma_5^x)(\bar{\partial}_\nu-i\partial_\nu\gamma_5^y)}{\partial^2} . \tag{14}$$

As another example, let us compute the Green's function

$$U_{\mu\alpha\beta}(x_1,x_2;y_1,y_2)=\left\langle\bar{\psi}(x_1)\gamma_\mu\exp\left[-ie\int_{x_1}^{x_2}A_\mu dx^\mu\right]\psi(x_2)F_{\lambda\alpha}(y_1)F^\lambda_\beta(y_2)\right\rangle . \tag{15}$$

We have

$$U_{\mu\alpha\beta}=-[\mathrm{tr}S(x_2-x_1)\gamma_\mu\cdot]g^{\lambda\sigma}\frac{\delta}{\delta J^{\lambda\alpha}(y_1)}\frac{\delta}{\delta J^{\sigma\beta}(y_2)}e^{\Sigma'}\Bigg|_{J=0} , \tag{16}$$

where

$$\Sigma'=\tfrac{1}{2}j'_\mu\Delta j'^\mu,\quad j'_\mu(z)=-ie\int_{x_1}^{x_2}\mathcal{T}^x_{\mu\nu}(z-x)dx^\nu+\partial^\nu(J_{\mu\nu}-J_{\nu\mu}) . \tag{17}$$

Carrying out the functional differentiation in (16), we obtain

$$U_{\mu\alpha\beta}=-[\mathrm{tr}S(x_2-x_1)\gamma_\mu\cdot][-\partial^2\Delta(y_1-y_2)g_{\alpha\beta}+g^{\lambda\sigma}(j_\sigma\partial_\beta\Delta-j_\beta\partial_\sigma\Delta)(y_2)(j_\lambda\partial_\alpha\Delta-j_\alpha\partial_\lambda\Delta)(y_1)]e^{\Sigma} , \tag{18}$$

where j_μ and $\Sigma(x_1,x_2)$ are the same as given in Eq. (10) but with all the terms involving y_1,y_2 suppressed.

Now we are ready to compute the two-point function of the energy-momentum tensor.

III. TWO-POINT FUNCTION OF THE ENERGY-MOMENTUM TENSOR

The energy-momentum tensor $T_{\mu\nu}$ is

$$T_{\mu\nu}=\tfrac{1}{4}g_{\mu\nu}F_{\lambda\sigma}F^{\lambda\sigma}-F_{\lambda\mu}F^\lambda_\nu +\frac{i}{4}[\bar{\psi}(\gamma_\mu D_\nu+\gamma_\nu D_\mu)\psi-\bar{\psi}(\overleftarrow{D}_\nu\gamma_\mu+\overleftarrow{D}_\mu\gamma_\nu)\psi] , \tag{19}$$

where $D_\mu=\partial_\mu-ieA_\mu$, $\overleftarrow{D}_\mu=\overleftarrow{\partial}_\mu+ieA_\mu$.

We regularize $T_{\mu\nu}$ by point splitting. Hence, to compute the fermionic part of the correlation function, we have only to differentiate $W_{\mu\alpha}$ with respect to x and y and then take the limit $x_2\to x_1, y_2\to y_1$, averaging over all directions. Explicitly, let

$$\hat{T}_{\mu\nu}=\frac{i}{4}[\bar{\psi}(\gamma_\mu D_\nu+\gamma_\nu D_\mu)\psi-\bar{\psi}(\overleftarrow{D}_\nu\gamma_\mu+\overleftarrow{D}_\mu\gamma_\nu)\psi] ,$$

then we have

$$\langle \hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)\rangle = -\frac{1}{16}\left[(\partial_\nu^{x_2}-\partial_\nu^{x_1})(\partial_\beta^{y_2}-\partial_\beta^{y_1})W_{\mu\alpha} + (\mu\leftrightarrow\nu)+(\alpha\leftrightarrow\beta)+\begin{bmatrix}\mu\leftrightarrow\nu\\ \alpha\leftrightarrow\beta\end{bmatrix}\right] \tag{20}$$

where on the right-hand side, we put

$$x_2=x_1+\epsilon,\quad y_2=y_1+\eta,\quad x_1=x,\quad y_1=y \tag{21}$$

and average ϵ,η over all directions.

Similarly, we have

$$\langle \hat{T}_{\mu\nu}(x)F_{\lambda\alpha}(y_1)F^\lambda_\beta(y_2)\rangle = \frac{i}{4}[(\partial_\nu^{x_2}-\partial_\nu^{x_1})U_{\mu\alpha\beta}+(\mu\leftrightarrow\nu)] . \tag{22}$$

From this, we can easily obtain $\langle \hat{T}_{\mu\nu}(x)T^B_{\alpha\beta}(y)\rangle$, where $T^B_{\alpha\beta}=\frac{1}{4}g_{\alpha\beta}F_{\lambda\sigma}F^{\lambda\sigma}-F_{\lambda\alpha}F^\lambda_\beta$.

To facilitate the point-splitting procedure, we need to expand Σ in powers of $\epsilon=x_2-x_1$ and $\eta=y_2-y_1$ to fourth order. Since Σ is path dependent, we shall choose the straight paths $x=x_1+t\epsilon$, $y=y_1+t'y$ with $0\le t,t'\le 1$. Choosing straight paths and averaging over all directions is the conventional prescription for point splitting. With this convention, we obtain

$$\Sigma(x_1,x_2,y_1,y_2) = -\frac{e^2}{2}\int_0^1 dt\int_0^1 dt'[\epsilon^\tau\epsilon^\lambda\mathcal{P}^{xx}_{\tau\lambda}\Delta((t-t')\epsilon)+\eta^\tau\eta^\lambda\mathcal{P}^{yy}_{\tau\lambda}\Delta((t-t')\eta)] - e^2\int_0^1 dt\int_0^1 dt'\epsilon^\tau\eta^\lambda\mathcal{P}^{xy}_{\tau\lambda}\Delta(x_1-y_1+t\epsilon-t'\eta) \tag{23}$$

so that, to fourth order in ϵ and η,

$$\begin{aligned}\Sigma = -\frac{e^2}{2}[&\epsilon^\tau\epsilon^\lambda\mathcal{P}^{xx}_{\tau\lambda}\Delta(0)+\eta^\tau\eta^\lambda\mathcal{P}^{yy}_{\tau\lambda}\Delta(0)+2\epsilon^\tau\eta^\lambda\mathcal{P}^{xy}_{\tau\lambda}\Delta(x_1-y_1)+\epsilon^\sigma\epsilon^\tau\eta^\lambda\partial_\sigma\mathcal{P}^{xy}_{\tau\lambda}\Delta(x_1-y_1)-\epsilon^\tau\eta^\lambda\eta^\sigma\partial_\sigma\mathcal{P}^{xy}_{\tau\lambda}\Delta(x_1-y_1)\\ &+\tfrac{1}{12}\epsilon^\kappa\epsilon^\sigma\epsilon^\tau\epsilon^\lambda\partial_\kappa\partial_\sigma\mathcal{P}^{xx}_{\tau\lambda}\Delta(0)+\tfrac{1}{12}\eta^\kappa\eta^\sigma\eta^\tau\eta^\lambda\partial_\kappa\partial_\sigma\mathcal{P}^{yy}_{\tau\lambda}\Delta(0)\\ &+\tfrac{1}{3}\epsilon^\kappa\epsilon^\sigma\epsilon^\tau\eta^\lambda\partial_\kappa\partial_\sigma\mathcal{P}^{xy}_{\tau\lambda}\Delta(x_1-y_1)+\tfrac{1}{3}\eta^\kappa\eta^\sigma\epsilon^\tau\eta^\lambda\partial_\kappa\partial_\sigma\mathcal{P}^{xy}_{\tau\lambda}\Delta(x_1-y_1)-\tfrac{1}{2}\epsilon^\kappa\eta^\sigma\epsilon^\tau\eta^\lambda\partial_\kappa\partial_\sigma\mathcal{P}^{xy}_{\tau\lambda}\Delta(x_1-y_1)] .\end{aligned} \tag{24}$$

It is now straightforward to obtain, from Eqs. (9), (18), (20), and (22),

$$\begin{aligned}\langle \hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)\rangle = &-\tfrac{1}{4}(D_{\mu\nu}D_{\alpha\beta}-D_{\bar{\mu}\nu}D_{\bar{\alpha}\beta}-D_{\mu\nu\beta}D_\alpha+D_{\bar{\mu}\nu\beta}D_{\bar{\alpha}}+\text{perm})\\ &+\tfrac{1}{2}e^2[(D_{\bar{\mu}}D_{\bar{\alpha}}-D_\mu D_\alpha)(H_{\bar{\nu}\bar{\beta}}-H_{\nu\beta})-(D_\mu D_{\bar{\alpha}}+D_{\bar{\mu}}D_\alpha)(H_{\nu\bar{\beta}}+H_{\bar{\nu}\beta})+\text{perm}]\\ &+\frac{e^4}{8\pi^2}(H_{\bar{\mu}\bar{\alpha}}H_{\bar{\nu}\bar{\beta}}+H_{\mu\alpha}H_{\nu\beta}-H_{\bar{\mu}\alpha}H_{\bar{\nu}\beta}-H_{\mu\bar{\alpha}}H_{\nu\bar{\beta}}+\text{perm}) ,\end{aligned} \tag{25}$$

$$\langle \hat{T}_{\mu\nu}(x)F_{\lambda\alpha}F^\lambda_\beta(y)\rangle = -\frac{e^2}{\pi}(\Delta_{\bar{\mu}}\Delta_{\bar{\nu}}-\Delta_\mu\Delta_\nu)g_{\alpha\beta} , \tag{26}$$

where

$$D_{\mu\nu}=\partial_\mu\partial_\nu D(x-y),\quad D_{\bar{\mu}}=\epsilon_{\mu\alpha}\partial^\alpha D(x-y) ,\qquad H_{\mu\nu}=\frac{\partial_\mu\partial_\nu}{\partial^2}\Delta(x-y),\quad \text{etc.} , \tag{27}$$

and "perm" means $\mu\leftrightarrow\nu$, $\alpha\leftrightarrow\beta$, and both $\mu\leftrightarrow\nu$ and $\alpha\leftrightarrow\beta$.

From (26), we get

$$\langle \hat{T}_{\mu\nu}(x)T^B_{\alpha\beta}(y)\rangle = \frac{e^2}{2\pi}(\Delta_{\bar{\mu}}\Delta_{\bar{\nu}}-\Delta_\mu\Delta_\nu)g_{\alpha\beta} . \tag{28}$$

We also have

$$\langle T^B_{\mu\nu}(x)T^B_{\alpha\beta}(y)\rangle = \tfrac{1}{2}(\partial^2\Delta)^2 g_{\mu\nu}g_{\alpha\beta} . \tag{29}$$

The three lines in Eq. (25) correspond to the contributions from Feynman diagrams of the type Figs. 1(a), 1(b), and 1(c), respectively, while Figs. 2 and 3 represent the Feynman diagrams that contribute to (28) and (29), respectively.

The correlation function of the energy-momentum tensor $\langle T_{\mu\nu}(x)T_{\alpha\beta}(y)\rangle$ is obtained from (25), (28), and (29) by

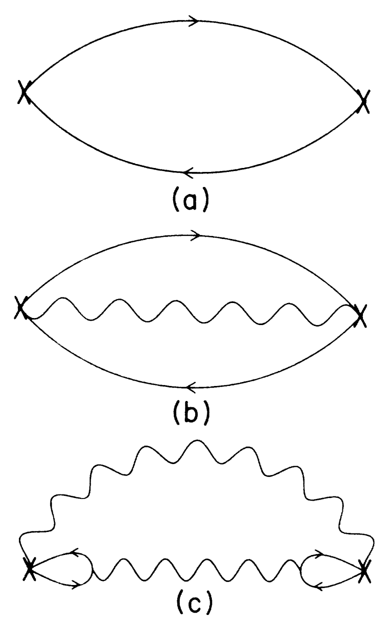

FIG. 1. Representative Feynman diagrams contributing to the correlation function $\langle \hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)\rangle$.

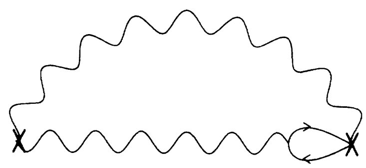

FIG. 2. Representative Feynman diagram contributing to the correlation function $\langle \hat{T}_{\mu\nu}(x)T^B_{\alpha\beta}(y)\rangle$.

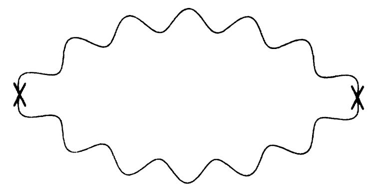

FIG. 3. Representative Feynman diagram contributing to the correlation function $\langle T^B_{\mu\nu}(x)T^B_{\alpha\beta}(y)\rangle$.

$$\langle T_{\mu\nu}(x)T_{\alpha\beta}(y)\rangle=\langle \hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)\rangle+\langle \hat{T}_{\mu\nu}(x)T^B_{\alpha\beta}(y)\rangle +\langle T^B_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)\rangle+\langle T^B_{\mu\nu}(x)T^B_{\alpha\beta}(y)\rangle \ . \tag{30}$$

Using rotational invariance, the two-point function of the energy-momentum tensor can be written as

$$x^4\langle T_{\mu\nu}(x)T_{\alpha\beta}(0)\rangle=\sum_{a=1}^{5} f_a(\hat{e}_a)_{\mu\nu\alpha\beta} \ , \tag{31}$$

where f_a, $a=1,\ldots,5$, are functions of $\tau=\ln\sqrt{(x/R)^2}$ and

$$\hat{e}_{1\mu\nu\alpha\beta}=g_{\mu\nu}g_{\alpha\beta}, \quad \hat{e}_{2\mu\nu\alpha\beta}=g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha} \ ,$$
$$\hat{e}_{3\mu\nu\alpha\beta}=g_{\mu\nu}\frac{x_\alpha x_\beta}{x^2}+g_{\alpha\beta}\frac{x_\mu x_\nu}{x^2} \ , \tag{32}$$
$$\hat{e}_{4\mu\nu\alpha\beta}=g_{\mu\alpha}\frac{x_\nu x_\beta}{x^2}+\text{perm}, \quad \hat{e}_{5\mu\nu\alpha\beta}=\frac{1}{x^4}x_\mu x_\nu x_\alpha x_\beta \ .$$

In two dimensions, $\hat{e}_a$'s are not independent. We have the identity

$$2\hat{e}_1+\hat{e}_4=\hat{e}_2+2\hat{e}_3 \ . \tag{33}$$

Using this relation, we can choose $f_4=0$. Energy-momentum conservation requires the following differential relations to hold:

$$f_1'+f_3'=4f_1+3f_3 \ , \tag{34}$$
$$f_2'=4f_2-f_3 \ , \tag{35}$$
$$f_3'+f_5'=3(2f_3+f_5) \ , \tag{36}$$

where a prime means differentiating with respect to τ.

Let $\Theta=g_{\mu\nu}T^{\mu\nu}$, then we have from Eqs. (31) and (34)–(36) that

$$x^4\langle\Theta(x)\Theta(0)\rangle=4(f_1+f_2+f_3)+f_5 =(f_1+f_2+\tfrac{4}{3}f_3+\tfrac{1}{3}f_5)' \ . \tag{37}$$

The C function is defined by[1]

$$C=-6\pi^2(f_1+f_2+\tfrac{4}{3}f_3+\tfrac{1}{3}f_5) \ . \tag{38}$$

Rewriting Eqs. (25), (28), and (29) in terms of form factors, we get

$$x^4\langle\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(0)\rangle=(D')^2(3\hat{e}_1-\hat{e}_2-4\hat{e}_3+8\hat{e}_5)+D'(H''-2H')(3\hat{e}_1-\hat{e}_2-4\hat{e}_3+8\hat{e}_5) -(H'')^2(-\tfrac{1}{2}\hat{e}_1+\hat{e}_3-2\hat{e}_5)-H'(H''-H')(3\hat{e}_1-\hat{e}_2-4\hat{e}_3+8\hat{e}_5) \ , \tag{39}$$

$$x^4[\langle\hat{T}_{\mu\nu}(x)T^B_{\alpha\beta}(0)\rangle+\langle T^B_{\mu\nu}(x)\hat{T}_{\alpha\beta}(0)\rangle]=m^2x^2(\Delta')^2(\hat{e}_1-\hat{e}_3) \ , \tag{40}$$

$$x^4\langle T^B_{\mu\nu}(x)T^B_{\alpha\beta}(0)\rangle=\tfrac{1}{2}m^4x^4\Delta^2\hat{e}_1, \quad m^2=\frac{e^2}{\pi} \ . \tag{41}$$

We note that

$$D'=\frac{1}{2\pi}, \quad H'=D'+\Delta', \quad H''=D''+\Delta''=m^2x^2\Delta \ . \tag{42}$$

Using Eq. (42), we can show that, as expected, $\langle T_{\mu\nu}(x)T_{\alpha\beta}(0)\rangle$ given by Eqs. (39)–(41) is the same as that of a free massive scalar field theory with mass m. It is nevertheless interesting to see how the contributions from free massless fermions, which is given in the first line of Eqs. (25) and (39), is canceled at large x while becoming dominant at small x.

The correlation function of the trace of the energy-momentum tensor is

$$x^4\langle\Theta(x)\Theta(0)\rangle=x^4\langle\Theta^B(x)\Theta^B(0)\rangle=2m^4x^4\Delta^2 \tag{43}$$

and the C function defined in (38) is

$$C=2\pi^2[2(\Delta')^2+m^2x^2(\Delta')^2-2\Delta'\Delta''-(\Delta'')^2] \ . \tag{44}$$

Putting $x=R$ and defining the dimensionless coupling $\bar{e}$ by[9,10]

$$mR=\bar{e} \tag{45}$$

the C function becomes a function of $\bar{e}$ only and is given by

$$C(\bar{e})=\frac{\bar{e}^2}{2}[(2+\bar{e}^2)K_1(\bar{e})^2+2\bar{e}K_0(\bar{e})K_1(\bar{e})-\bar{e}^2K_0(\bar{e})^2] \ , \tag{46}$$

where K_0,K_1 are modified Bessel functions. It is straightforward to show that[1]

$$\beta(\bar{e})\frac{dc}{d\bar{e}}=-6\pi^2x^4\langle\Theta(x)\Theta(0)\rangle|_{x=R}=-3\bar{e}^4K_0^2\ , \tag{47}$$

where the β function follows from (45):

$$\beta(\bar{e})=\bar{e}\ . \tag{48}$$

The asymptotic expansions of $C(\bar{e})$ are

$$C(\bar{e})=1+\tfrac{3}{4}\bar{e}^2+O(\bar{e}^4\ln^2\bar{e}),\quad \bar{e}\to 0\ ,$$
$$=\tfrac{3}{4}\pi\bar{e}^2e^{-2\bar{e}}\left[1+\frac{3}{4\bar{e}}+\cdots\right],\quad \bar{e}\to\infty\ . \tag{49}$$

This is in accordance with our expectation that $c=0$ at $\bar{e}=\infty$ (no massless excitations) and $c=1$ at $\bar{e}=0$ (free massless Dirac fermion).

IV. MULTIFLAVORED CASE

The extension of the N-flavored case is almost trivial in the present case. The fermionic determinant gets a factor of N so that the mass term in Eq. (7) $m^2=e^2/\pi$ should be replaced by Nm^2. In $W_{\mu\alpha}$ Eq. (9), Σ remains unchanged, while the contribution to $\langle\hat{T}_{\mu\nu}\hat{T}_{\alpha\beta}\rangle$ arising from the first term of Eq. (9) gets a factor of N^2 due to the two fermionc traces and those arising from the second term of (9) gets a factor of N due to the single fermionic trace. Similarly, the contribution to $\hat{T}_{\mu\nu}T^B_{\alpha\beta}$ arising from (18) gets a factor of N. These translate into the following changes on Eqs. (25), (26), and (39)–(42). We have to multiply by N for the first and second lines and by N^2 for the last line on the right-hand side of (25). The right-hand side of (26) gets a factor of N. In Eqs. (40)–(42), we have to replace m^2 by Nm^2. Finally, the right-hand side of the first line of Eq. (39) gets a factor of N.

The correlation function of the energy-momentum tensor $\langle T_{\mu\nu}(x)T_{\alpha\beta}(0)\rangle$ coincides with that of a free massive scalar field with mass $\sqrt{N}\,m$ plus $N-1$ free mass fermion fields.

It follows from Eqs. (38) and (39) that the contribution of the $N-1$ massless fermions to the C function is just $N-1$. If we define

$$\sqrt{N}mR=\bar{e} \tag{50}$$

for the N-flavored case, then the C function is given by Eq. (46) plus $N-1$:

$$C(\bar{e})=\frac{\bar{e}^2}{2}[(2+\bar{e}^2)K_1(\bar{e})^2+2\bar{e}K_0(\bar{e})K_1(\bar{e})-\bar{e}^2K_0(\bar{e})^2]+N-1\ . \tag{51}$$

Since the $N-1$ massless free fermions do not contribute to $\langle\Theta(x)\Theta(0)\rangle$, Eq. (47) remains valid for the multiflavored case.

V. DISCUSSIONS

Our results indicate that the N-flavored Schwinger model interpolates between the two conformal field theories: in the ultraviolet limit, it approaches a theory of N free massless fermions, while in the infrared limit, it approaches a theory of $N-1$ free massless fermions. These two theories possess quite different symmetries. It would be interesting to see how the primary fields of the two theories are related by the renormalization-group flow.

If we apply Abelian bosonization on the N-flavored Schwinger model,[11] we would conclude that the theory contains a massive scalar and $N-1$ massless scalars. The direct calculation shows that the two-point correlation functions of the energy-momentum tensor indeed agrees with those of such a theory.

One may also apply non-Abelian bosonization.[12] In this case, the N-flavored free massless theory is equivalent to the $k=1$ U(N) Wess-Zumino-Witten (WZW) model.[6] The U(1) and SU(N) part decoupled. When the U(1) symmetry is gauged, we obtain a free massive scalar with the decoupled SU(N) WZW model. Our calculation suggests that this last model is equivalent to a theory of free massless fermions with $N-1$ flavor. This is known to be true for the $N=2$ case.[13]

Even though one could have expected the final results from bosonization, it is nice to see how it comes out from a direct evaluation by path integrals. Moreover, the mechanism by which the massless fermions disappear from the physical spectrum is made clear from our direct computation. From Eq. (6), we see that after the fermionic degrees of freedom are integrated out, the longitudinal part of A_μ decoupled by the requirement of gauge invariance. The propagator for the transverse part is $(\partial_{\bar{\mu}}\partial_{\bar{\nu}}/\partial^2)\Delta$. We see that the massive propagator is accompanied by a massless ghost due to the projection operator which projects out the transverse degrees of freedom. This is easy to understand since there are no transverse degrees of freedom for gauge field in 1+1 dimension. Were it not for the mass generation due to charge screening, the ghost would cancel the transverse part of A_μ completely. However, because of mass generation, the cancellation happens only in the ultraviolet limit and the massless ghost appears in the long-distance limit. The ghost attaches itself to the massless fermion and prevents it from being seen in the asymptotic state. In the correlation function $\langle\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)\rangle$ given in Eq. (25), we see that $H_{\mu\nu}=(\partial_\mu\partial_\nu/\partial^2)\Delta$ appears and in the long-distance limit, the ghost part from $H_{\mu\nu}$ cancels the free fermion contribution such that only a contact term is left. Since the ghost is a flavor singlet, it cannot prevent the flavored massless fermion from being seen in the multiflavored case. In the correlation functions of singlet physical observable such as $\langle\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)\rangle$, the ghost effectively cancels one degree of freedom as in the U(1) case.

In summary, we have carried out a path-integral evaluation of correlation functions of the energy-momentum tensor in the N-flavored Schwinger model using point-splitting regularization. The results agree with the expectations from Abelian and non-Abelian bosonization methods. The C function is obtained from the correlation functions. It decreases from N to $N-1$ along renormalization-group trajectory. Our method can be applied to other interesting models such as the N-flavored chiral Schwinger model. These are currently under investigation.

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