Limits on a Lorentz- and parity-violating modification of electrodynamics

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The Chern-Simons Lagrangian has been studied previously in (2+1)-dimensional spacetime, where it is both gauge and Lorentz invariant. In 3+1 dimensions, this term couples the dual electromagnetic tensor to an external four-vector. If we take this four-vector to be fixed, the term is gauge invariant but not Lorentz invariant. In this paper, we examine both the theoretical consequences of such a modification and observational limits we can put on its magnitude. The Chern-Simons term would rotate the plane of polarization of radiation from distant galaxies, an effect which is not observed. From the observations we deduce that the magnitude of the vector is $<1.7\times10^{-42}h_0$ GeV, where h_0 is the Hubble constant in units of 100 km sec^{-1} Mpc^{-1}.

I. INTRODUCTION

Gauge and Lorentz invariance are two symmetries of Maxwell's electrodynamics that have come to dominate all fundamental physical theory. They provide physical principles that guide the invention of models describing fundamental phenomena, and their experimental status—within electromagnetism—is well established. The properties of electromagnetic radiation, both in a natural setting and in high-energy accelerators, are precisely described by Lorentz-invariant dynamics. Gauge invariance, interpreted as the masslessness of the photon, is validated by stringent limits on the photon mass.

Experimental tests of such well-established and universal physical ideas are best discussed within a theoretical framework that allows departures to be governed by arbitrary parameters; experimental data then set limits on the magnitude of these symmetry-breaking parameters. Thus, violations of gauge invariance are parametrized by a mass μ for the photon field A_ν. A mass term is hypothesized to modify the electromagnetic Maxwell Lagrange density $\mathcal{L}_{\rm EM}$,

$$\mathcal{L}_{\rm EM}=-\tfrac{1}{4}F_{\nu\lambda}F^{\nu\lambda} \tag{1}$$

so that the photon becomes massive:

$$\mathcal{L}_\mu=-\tfrac{1}{4}F_{\nu\lambda}F^{\nu\lambda}+\frac{\mu^2}{2}A^\nu A_\nu \ . \tag{2}$$

Here $F_{\nu\lambda}$ is the electromagnetic tensor $F_{\nu\lambda}=\partial_\nu A_\lambda-\partial_\lambda A_\nu$, and the field equations in the presence of a conserved current J_ν read

$$\Box A_\nu+\mu^2 A_\nu=4\pi J_\nu,\quad \partial_\nu A^\nu=0 \ , \tag{3}$$

where $\Box$ is the d'Alembertian $\Box=\partial_t^2-\nabla^2$. (We set c equal to unity throughout.) Gauge invariance

$$A_\nu\to A_\nu+\partial_\nu\chi \tag{4}$$

is clearly lost. Geomagnetic data then set the limit[1] $\mu\le 3\times10^{-24}$ GeV; observations of the galactic magnetic field set the more stringent bound[2] of $\mu\le3\times10^{-36}$ GeV; see below.

In this paper we explore the experimental limits on another modification of Maxwell theory, which also involves a mass parameter p_α, but respects gauge invariance—rather, it is Lorentz invariance that is violated.

The modification we consider involves adding to the Maxwell Lagrange density a Chern-Simons term:

$$\mathcal{L}_p=\mathcal{L}_{\rm EM}+\mathcal{L}_{\rm CS} \ . \tag{5}$$

The Chern-Simons term is given by

$$\mathcal{L}_{\rm CS}=-\tfrac{1}{2}p_\alpha A_\beta \tilde F^{\alpha\beta} \ , \tag{6}$$

where $\tilde F^{\alpha\beta}$ is the dual electromagnetic tensor, $\tilde F^{\alpha\beta}=\frac{1}{2}\epsilon^{\alpha\beta\mu\nu}F_{\mu\nu}$. This modification couples the electromagnetic field to an (as yet unspecified) four-vector p_α.

When electromagnetic phenomena are confined to a plane, as in the quantum Hall effect and high-T_c superconductivity, the approximation can be made that no interesting dynamical motion takes place in the direction perpendicular to the plane. Then the external vector p_α may be chosen to lie in that direction as well, and (6) reduces to an unconventional electrodynamic action that is Lorentz and gauge invariant in a three-dimensional spacetime, i.e., boosts in the plane leave dynamics unchanged. It was in this context that the Chern-Simons term was initially investigated as a "topological mass" term for gauge fields in (2+1)-dimensional spacetime.[3] Models in which $\mathcal{L}_{\rm CS}$ is taken to be the entire gauge field action have found application in examinations of the quantum Hall effect[4] and high-T_c superconductivity.[5] Moreover, several purely mathematical applications for $\mathcal{L}_{\rm CS}$ have also been found.[6]

In this paper we shall consider the (3+1)-dimensional case, where considerations of both Lorentz and gauge invariance play a crucial role.

II. PHYSICAL INTERPRETATION

We begin by determining under what conditions $\mathcal{L}_{\mathrm{CS}}$ will be gauge invariant. The variation $\Delta\mathcal{L}_{\mathrm{CS}}$ under the gauge transformation $\Delta A_\alpha = \partial_\alpha \chi$ is

$$\Delta\mathcal{L}_{\mathrm{CS}} = \tfrac{1}{4}\chi \tilde{F}^{\beta\alpha}(\partial_\alpha p_\beta - \partial_\beta p_\alpha) \tag{7}$$

to within a divergence. Gauge invariance requires that Eq. (7) vanish for arbitrary χ.

Equation (7) will vanish if p_α is, in some sense, a constant of nature. In flat spacetime, when $\partial_\alpha p_\beta = 0$ in some frame it will vanish in all frames, $\Delta\mathcal{L}_{\mathrm{CS}} = 0$, and the theory is gauge invariant. The notion of a constant four-vector is distressing, for if p_α couples to observable fields, it would then pick out a preferred direction in spacetime. Specifically, a nonvanishing spatial component $\mathbf{p}$ violates rotational invariance, and a nonvanishing time component p^0 destroys invariance under Lorentz boosts. Since there would be no way to "shield" an experiment from the effects of the Chern-Simons term, Lorentz invariance would be violated. Thus, by putting limits on the magnitude of p_α we are testing Lorentz invariance itself.

The above discussion applies to Minkowski spacetime, but the actual Universe is a curved spacetime manifold. If we define "constant" in a generally covariant way, we require that the covariant derivative vanishes everywhere:

$$\nabla_\mu p_\nu = 0 \ . \tag{8}$$

This expression is generally covariant, while $\partial_\mu p_\nu = 0$ is not. It is well known, however, that a vector cannot have vanishing covariant derivative everywhere on a curved manifold.[7]

The solution to this apparent dilemma is that, because we have introduced p_α as a parameter which violates Lorentz invariance, it is inappropriate to demand general covariance. That is, p_α picks out a preferred direction, and therefore a preferred coördinate frame, in which $\partial_\mu p_\nu = 0$. Once this is true, $\partial_\mu p_\nu - \partial_\nu p_\mu = \nabla_\mu p_\nu - \nabla_\nu p_\mu = 0$ in any frame, and gauge invariance will be maintained.

In fact, the requirement that Eq. (7) vanish is somewhat less restrictive than the statement that $\partial_\mu p_\nu = 0$. For example, $\tilde{F}^{00}$ is identically zero, so a nonvanishing $\partial_0 p_0$ would not violate gauge invariance if $\partial_0 p_k = 0$; thus p_0 may vary with time. Further, the quantity $\partial_\mu p_\nu - \partial_\nu p_\mu$ will vanish identically if p_α is a gradient of a scalar; that is, if $p_\alpha = \partial_\alpha \theta$ for some scalar field θ. However, these cases amount to considering p_α to be a *field*, with dynamics of its own, a possibility we do not consider in this paper.[8]

Observationally, of course, the Universe does exhibit a preferred frame of reference, that of a typical galaxy (with peculiar motions removed). It is only in this frame that the surrounding Universe will appear homogeneous and isotropic. The existence of this frame introduces a preferred time direction and notion of simultaneity. While these observational facts do not necessarily imply that p_α points along this preferred time direction, they do indicate that it would be a natural possibility to consider. In this case p_α is timelike, so

$$p_\alpha p^\alpha \equiv m^2 > 0 \ . \tag{9}$$

For simplicity, we shall frequently work in the rest frame of p_α, where

$$\mathcal{L}_{\mathrm{CS}} = -\frac{m}{2}\mathbf{B}\cdot\mathbf{A} \ . \tag{10}$$

However, when no additional complexity is encountered, we shall retain the full four-vector $p^\alpha = (p^0, \mathbf{p})$. It is evident that nonisotropic effects due to $\mathbf{p} \neq 0$ are attained by moving with a constant velocity relative to the rest frame. Therefore, absolute motion of the observer is detectable, which vividly demonstrates the Lorentz noninvariance of the Chern-Simons term in four dimensions.

In this paper we shall examine the observational effects of the Chern-Simons term to put limits on p_α as a parametrization of violation of Lorentz invariance. Of course, the axial structure of the term also leads to parity violation, as is seen from (10), which involves the scalar product of the axial vector $\mathbf{B}$ and the polar vector $\mathbf{A}$.

III. FIELD EQUATIONS AND SOLUTIONS

The effect of the Chern-Simons term on the field equations is simply to replace the source current four-vector J^ν by $J^\nu + p_\mu \tilde{F}^{\mu\nu}/4\pi$. Thus the field equations which follow from $\mathcal{L}_p$ are

$$\partial_\mu F^{\mu\nu} = 4\pi J^\nu + p_\mu \tilde{F}^{\mu\nu} \tag{11}$$

or, in terms of components,

$$\nabla\cdot\mathbf{E} = 4\pi\rho - \mathbf{p}\cdot\mathbf{B} \ , \tag{12a}$$

$$-\partial_t \mathbf{E} + \nabla\times\mathbf{B} = 4\pi\mathbf{J} - p_0\mathbf{B} + \mathbf{p}\times\mathbf{E} \ . \tag{12b}$$

Of course the homogeneous Maxwell equations that express the field-potential relationship,

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \tag{13}$$

or, in components,

$$\nabla\cdot\mathbf{B} = 0 \ , \tag{14a}$$

$$\partial_t \mathbf{B} + \nabla\times\mathbf{E} = 0 \tag{14b}$$

are not modified. These equations, which reduce to Maxwell's equations when $p_\alpha = 0$, are expressed in terms of field strengths; hence, the Chern-Simons addition preserves gauge invariance.

The energy-momentum tensor for our theory is

$$\Theta_p^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{g^{\mu\nu}}{4}F^{\alpha\beta}F_{\alpha\beta} + \frac{p^\nu}{2}\tilde{F}^{\mu\alpha}A_\alpha \ . \tag{15}$$

In the absence of sources, this is conserved by virtue of the equations of motion; with vanishing J^ν in (11) and (13),

$$\partial_\mu \Theta_p^{\mu\nu} = 0 \ . \tag{16}$$

Sources produce the conventional modification

$$\partial_\mu \Theta_p^{\mu\nu} = 4\pi J_\alpha F^{\alpha\nu} . \tag{17}$$

Note that the Chern-Simons addition, which contributes the last term to (15), renders the energy-momentum tensor nonsymmetric:

$$\Theta_p^{\mu\nu} \neq \Theta_p^{\nu\mu} , \tag{18}$$

which again indicates the absence of Lorentz invariance.

The energy and momentum densities are not gauge invariant:

$$\mathcal{E} \equiv \Theta_p^{00} = \tfrac{1}{2}\mathbf{E}^2 + \tfrac{1}{2}\mathbf{B}^2 + \frac{p^0}{2}\mathbf{B}\cdot\mathbf{A} , \tag{19a}$$

$$\mathcal{P}^i \equiv \Theta_p^{0i} = (\mathbf{E}\times\mathbf{B})^i + \frac{p^i}{2}(\mathbf{B}\cdot\mathbf{A}) . \tag{19b}$$

Rather, under a gauge transformation they change by a total derivative, since

$$\mathbf{B}\cdot\mathbf{A} \rightarrow \mathbf{B}\cdot(\mathbf{A} - \nabla\chi) = \mathbf{B}\cdot\mathbf{A} - \nabla\cdot(\mathbf{B}\chi) .$$

Consequently, the integrals over all space, which define the electromagnetic energy and momentum, are gauge invariant. Indeed, the spatial integrals may be presented as explicitly gauge invariant by spatially nonlocal formulas that make use of the identity

$$\int d\mathbf{r}\, \mathbf{B}(\mathbf{r})\cdot\mathbf{A}(\mathbf{r}) = \int d\mathbf{r}\, \mathbf{B}(\mathbf{r})\cdot \int d\mathbf{r}' \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\nabla\times\mathbf{B}(\mathbf{r}') , \tag{20}$$

which holds provided functions drop off sufficiently rapidly at large distances so that the integration by parts needed to relate the two sides of (20) does not produce surface terms. Thus

$$\begin{aligned} E &= \int d\mathbf{r}\, \mathcal{E}(\mathbf{r}) \\ &= \tfrac{1}{2}\int d\mathbf{r}[\mathbf{E}^2(\mathbf{r}) + \mathbf{B}^2(\mathbf{r})] \\ &\quad + \frac{p^0}{2}\int d\mathbf{r}\, d\mathbf{r}' B^n(\mathbf{r}) K^{nm}(\mathbf{r}-\mathbf{r}') B^m(\mathbf{r}') , \end{aligned} \tag{21a}$$

$$\begin{aligned} \mathbf{P} &= \int d\mathbf{r}\, \mathcal{P}(\mathbf{r}) \\ &= \tfrac{1}{2}\int d\mathbf{r}[\mathbf{E}(\mathbf{r})\times\mathbf{B}(\mathbf{r})] \\ &\quad + \frac{\mathbf{p}}{2}\int d\mathbf{r}\, d\mathbf{r}' B^n(\mathbf{r}) K^{nm}(\mathbf{r}-\mathbf{r}') B^m(\mathbf{r}') , \end{aligned} \tag{21b}$$

where the kernel K^{nm} is given by

$$K^{nm}(\mathbf{r}) = \epsilon^{nmi}\partial_i \frac{1}{4\pi r} . \tag{22}$$

Note that the energy is not positive definite. This is most easily seen from (19a), which implies

$$E = \tfrac{1}{2}\int d\mathbf{r} \left[\mathbf{E}^2 + \left[\mathbf{B} + \frac{p^0}{2}\mathbf{A} \right]^2 \right] - \frac{p_0^2}{8}\int d\mathbf{r}\, \mathbf{A}^2 . \tag{23}$$

Here we encounter the first evidence of an instability in the theory; we shall discuss this in more detail below.

To find wave solutions to the source-free ($\rho = \mathbf{J} = 0$) versions of (11)–(14), we posit the phase-exponential ansatz and get

$$\omega^2\mathbf{E} - k^2\mathbf{E} + (\mathbf{k}\cdot\mathbf{E})\mathbf{k} = i(-p_0\mathbf{k}\times\mathbf{E} + \omega\mathbf{p}\times\mathbf{E}) , \tag{24}$$

where ω is the frequency and $\mathbf{k}$ the wave vector, which form the four-vector $k^\alpha = (\omega, \mathbf{k})$; $k = |\mathbf{k}|$. The corresponding dispersion relation is

$$(k^\alpha k_\alpha)^2 + (k^\alpha k_\alpha)(p^\beta p_\beta) = (k^\alpha p_\alpha)^2 \tag{25}$$

or

$$\omega^2 - k^2 = \pm(p_0 k - \omega p\cos\theta) \left[1 - \frac{p^2\sin^2\theta}{\omega^2 - k^2} \right]^{-1/2} , \tag{26}$$

where θ is the angle between $\mathbf{p}$ and $\mathbf{k}$, $p = |\mathbf{p}|$, and the $+$ and $-$ correspond to right-and left-handed circularly polarized waves, respectively. From this it is clear that introducing p_α has the consequence of splitting the photons into two modes. The waves travel with a group velocity which differs from one in second order in p_α:

$$\frac{\partial\omega}{\partial k} = 1 \pm O(p_\alpha^2) \tag{27}$$

while the phase velocity ω/k already differs from 1 even in the lowest order. That the two polarization modes propagate at different velocities is forceful evidence for violation of Lorentz and parity invariance.

Note that the wave four-vector k^α can become spacelike; i.e., exponentially unstable modes can solve the field equations. This is clearly seen in the p_α rest frame, where (26) implies

$$\omega^2 = k(k \pm m) \tag{28}$$

so ω becomes imaginary for $k < m$. Such runaway solutions do not contradict energy conservation because the two integrals in (23) can each become arbitrarily large while their difference remains a finite, time-independent quantity.

However, runaway, exponentially growing, tachyonic modes need not be excited by well-behaved sources. The modified Maxwell equations imply (for $\mathbf{p} = 0$)

$$\Box\mathbf{E}_T + m\nabla\times\mathbf{E}_T = -4\pi\partial_t\mathbf{J}_T , \tag{29a}$$

$$\Box\mathbf{B} + m\nabla\times\mathbf{B} = 4\pi\nabla\times\mathbf{J}_T , \tag{29b}$$

where $\mathbf{E}_T$ and $\mathbf{J}_T$ are transverse electric field and current, respectively. Equivalently we have, in the Coulomb gauge,

$$\Box\mathbf{A} + m\nabla\times\mathbf{A} = 4\pi\mathbf{J}_T . \tag{30}$$

The form of the fields responding to the source is most appropriately described by constructing the Green's function for (29), i.e., the response function to a disturbance localized by a δ function at the origin in space and time. The Green's function is transverse and is given by

$$G^{ij}(t,\mathbf{r}) = [(\delta^{ij} - \partial_i\partial_j/\nabla^2)\Box + m\epsilon^{ijk}\partial_k] g(t,\mathbf{r}) , \tag{31}$$

where g satisfies

$$\Box^2 g + m^2\nabla^2 g = 4\pi\delta^4(x) . \tag{32}$$

In Fourier space, with $\tilde{g}(\omega,\mathbf{k})=\int dt\, d\mathbf{r}\, e^{i(\omega t-\mathbf{k}\cdot\mathbf{r})}g(t,\mathbf{r})$, this reads

$$[(-\omega^2+k^2)^2-m^2k^2]g=1 \ . \tag{33}$$

From (33) we see that a tachyonic pole arises in g for long wavelengths, $k<m$. A solution which does not grow in time, and reduces to the Lienard-Wiechert formula when $m=0$, is

$$g(t,r)=\frac{2\sin\frac{m}{2}r}{\pi m r}\left\{\theta(t)\int_0^\infty dz\frac{\cos r\left[\frac{m^2}{4}+z^2\right]^{1/2}}{\left[\frac{m^2}{4}+z^2\right]^{1/2}}\sin tz-\tfrac{1}{2}\int_0^{m/2}dz\frac{\cos r\left[\frac{m^2}{4}-z^2\right]^{1/2}}{\left[\frac{m^2}{4}-z^2\right]^{1/2}}e^{-|t|z}\right\} . \tag{34}$$

This solution is noncausal, in the sense that the second term in parentheses, present when $m\neq 0$, acts even at $t<0$, before the δ-function disturbance has occurred. A causal solution, vanishing for $t<0$, would of course possess exponential growth in time—it would be a runaway solution.

The potential instability manifests itself once again in the existence of nonsingular, static solutions to the source-free equations. To find these we set sources and time derivatives to zero in (11)–(14) and find (for $\mathbf{p}=0$) that the divergence and curl of $\mathbf{E}$ vanish, hence so does $\mathbf{E}$, while $\mathbf{B}$ is a divergence-free vector field satisfying

$$\nabla\times\mathbf{B}=-m\mathbf{B} \ , \tag{35a}$$

which also implies

$$(\nabla^2+m^2)\mathbf{B}=0 \ . \tag{35b}$$

Equations (35) have arisen previously in magnetohydrodynamics.[9] There they are obtained within conventional electrodynamics, in the presence of static, neutral sources ($\rho=0$, $\nabla\cdot\mathbf{J}=0$), when the condition is imposed that $\mathbf{J}$ is proportional to $\mathbf{B}$, so that (35a) is equivalent to Ampère's law. Here, Eqs. (35) arise in the static, source-free case when Maxwell theory is modified by the Chern-Simons term.

The solution to (35) is most easily presented in terms of the vector potential $\mathbf{A}$ whose curl is $\mathbf{B}$. One verifies that with $\mathbf{A}$ given by

$$\mathbf{A}=\nabla\times\mathbf{a}-m\mathbf{a} \ , \tag{36a}$$

where $\mathbf{a}$ is any triplet of fields satisfying

$$(\nabla^2+m^2)\mathbf{a}=0 \tag{36b}$$

then $\mathbf{B}=\nabla\times\mathbf{A}$ solves (35). For example, $\mathbf{a}$ could have a single nonvanishing component equal to $(\sin mr)/r$. This space-filling magnetic field makes stationary the magnetic energy, defined as the energy in (21a) with vanishing electric field:

$$E_M=\tfrac{1}{2}\int d\mathbf{r}\,\mathbf{B}^2(\mathbf{r}) +\frac{p^0}{2}\int d\mathbf{r}\,d\mathbf{r}'B^n(\mathbf{r})K^{nm}(\mathbf{r}-\mathbf{r}')B^m(\mathbf{r}') \ . \tag{37}$$

It is seen that the vanishing of $\delta E_M/\delta\mathbf{A}(\mathbf{r})$ implies (35); also, E_M vanishes when evaluated on a solution to (35).

Note that the Chern-Simons mass term enters (35b) and (36b) as an imaginary mass, in the sense that a conventional mass arising, for example, as in (2) and (3), leads to the static equation $(-\nabla^2+\mu^2)A_\mu=0$, which has no regular solutions. In the Chern-Simons case μ^2 is replaced by $-m^2$, and regular solutions do exist.

IV. GEOMAGNETIC CONSTRAINTS

Just as for the gauge-noninvariant modification (2) and (3), geomagnetic data limit the magnitude of the Chern-Simons modification, but not as effectively as astrophysical data (discussed below). Static fields arising from stationary, neutral sources ($\rho=0$, $\nabla\cdot\mathbf{J}=0$) obey equations which follow from (11)–(14) with $\mathbf{p}$ set to zero, so that $p_0=m$:

$$\nabla\cdot\mathbf{B}=0 \ , \tag{38a}$$

$$\nabla\times\mathbf{E}=0 \ , \tag{38b}$$

$$\nabla\cdot\mathbf{E}=0 \ , \tag{38c}$$

$$\nabla\times\mathbf{B}=4\pi\mathbf{J}-m\mathbf{B} \ . \tag{38d}$$

Evidently, the magnetic field satisfies

$$(-\nabla^2-m^2)\mathbf{B}=4\pi\nabla\times\mathbf{J}-4\pi m\mathbf{J} \ . \tag{39}$$

Again we see the mass parameter m entering as an imaginary mass. The solution to (39) is

$$\mathbf{B}(\mathbf{r})=\int d\mathbf{r}'\frac{\cos m|\mathbf{r}-\mathbf{r}'|}{|\mathbf{r}-\mathbf{r}'|}[\nabla\times\mathbf{J}(\mathbf{r}')-m\mathbf{J}(\mathbf{r}')]+\mathbf{B}_0 \ , \tag{40}$$

where $\mathbf{B}_0$ is the space-filling solution to the homogeneous equations (35) and (36).

This is to be contrasted with the corresponding solution when the mass term violates gauge invariance:

$$\mathbf{B}(\mathbf{r})=\int d\mathbf{r}'\frac{e^{-\mu|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\nabla\times\mathbf{J}(\mathbf{r}') \ . \tag{41}$$

The geomagnetic constraint on μ is determined by expanding the integrand of (41) in powers of r'/r and keeping the dipole term:

$$\mathbf{B}(\mathbf{r})=\frac{e^{-\mu r}}{r^3}\{[3(\hat{\mathbf{r}}\cdot\mathbf{D})\hat{\mathbf{r}}-\mathbf{D}](1+\mu r+\tfrac{1}{3}\mu^2r^2) -\tfrac{2}{3}\mu^2r^2\mathbf{D}\} \ , \tag{42}$$

where

$$\mathbf{D}=\tfrac{1}{2}\int d\mathbf{r}\,\mathbf{r}\times\mathbf{J}(\mathbf{r}) \tag{43}$$

is the dipole moment. The first term in the curly brackets is also present in the conventional theory; evaluated at $r=R_\oplus$, the radius of Earth, the effect of finite μ cannot be separated from a redefinition of $\mathbf{D}$. (Spherical harmonics of the dipole field arising in the conventional theory, $[3(\hat{\mathbf{r}}\cdot\mathbf{D})\hat{\mathbf{r}}-\mathbf{D}]/r^3$, are now known[10] through thirteenth order and degree.) The second term proportional to the dipole moment is constant over the surface of Earth (unlike a dipole field or any higher-order spherical harmonic) and in principle can be used to deduce μ. Unfortunately, currents in the magnetosphere can mimic the same effect, leading Goldhaber and Nieto[1] to conclude that the best that can be said is

$$\tfrac{2}{3}\mu^2\frac{D}{R_\oplus^3}<0.0012\ \text{G}\ , \tag{44}$$

whence

$$\mu\leq 3\times 10^{-24}\ \text{GeV}\ . \tag{45}$$

A similar analysis of (40) is hampered by our lack of any information about $\mathbf{B}_0$. Setting it (arbitrarily) to zero, which certainly weakens any conclusions about m, and expanding the remainder of (40) in powers of m leaves

$$\mathbf{B}=\frac{1}{r^3}[3(\hat{\mathbf{r}}\cdot\mathbf{D})\hat{\mathbf{r}}-\mathbf{D}]+\frac{1}{r^3}(mr)(\hat{\mathbf{r}}\times\mathbf{D})\ . \tag{46}$$

The first term is conventional; the second is the first nonvanishing correction in the expansion of our modified theory in powers of m. It represents an azimuthal field

$$B_\phi=-(mr)\frac{D}{r^3}\cos\theta\ , \tag{47}$$

where θ is the angle from the dipole axis. Because the conventional theory assumes that $\nabla\times\mathbf{B}=0$ outside Earth, such a term cannot then occur, because $\oint\mathbf{B}\cdot d\mathbf{s}$ vanishes for solenoidal magnetic fields. It is readily shown that a field of the form (47) is generated in the conventional theory by a uniform current density J_z along the dipole axis equal to $(m/2\pi)(D/r^3)$.

If the effect of nonvanishing m is to be sought in azimuthal magnetic fields, the effect of nonvanishing axial-vector currents must be removed first. It is known[11] that worldwide, an upward surface current of 1500 A generated by thunderstorms is canceled in time scales of minutes by a fair weather downward conduction current of the same magnitude. It is likely that the net current through either magnetic hemisphere is a small fraction of 1500 amperes. However, an extreme upper limit on the associated equatorial azimuthal magnetic field can be found by assuming that all of the thunderstorms are located in one magnetic hemisphere. In this case I_z would be 750 amperes, and the associated azimuthal field would be

$$|B_\phi|=\frac{I_z(\text{A})}{5r}=2.3\times 10^{-7}\ \text{G}\ . \tag{48}$$

In fact, it has not yet been possible to detect such small fields because, as explained by Langel,[10] surface magnetic fields generated by time-variable currents in the ionosphere and/or magnetosphere make it difficult to measure Earth's field with a precision better than $\sim 10^{-4}$ G. Since D/r^3 is 0.3 G, if we assume that net azimuthal components larger than 10^{-4} G would have been noticed,[12] (47) indicates that $mr\lesssim 3\times 10^{-4}$, so

$$m\lesssim 5\times 10^{-13}\ \text{cm}^{-1}=6\times 10^{-26}\ \text{GeV}\ . \tag{49}$$

Fortunately, much more definite information can be obtained from astrophysical data, which we now examine.

V. ASTROPHYSICAL TESTS

Returning now to our plane-wave solutions, we observe that if a plane polarized wave is considered as the superposition of two circularly polarized modes of opposite handedness, the different velocities of the two modes causes a rotation of the polarization as the wave travels through space. As in the Faraday effect, this is due to the changing phase relation between the two modes. The angle by which the plane of polarization rotates is half of the difference in phase. Since we expect p_α to be small, we can expand the dispersion relation (26) in powers of p_α to obtain, to first order,

$$k=\omega\mp\tfrac{1}{2}(p_0-p\cos\theta)\ . \tag{50}$$

As the change in phase of a circularly polarized mode traveling a distance L is $\phi=kL$, the polarization vector rotates by

$$\Delta\phi=\tfrac{1}{2}(\phi_L-\phi_R)=-\tfrac{1}{2}(p_0-p\cos\theta)L\ . \tag{51}$$

Note that this result is independent of the wavelength.

It is this effect, the rotation of the polarization vector, which is amenable to observational constraint.

Evidently, the best chance of detecting a small p_α is to observe objects at the largest possible distance L. Radio galaxies and quasars at distances comparable to the Hubble length ($=1/H_0\sim 10^{10}$ light year) provide ideal cases to search for the predicted rotation. The polarization of the synchrotron radiation of these objects has been extensively studied.[13] The objects themselves are often elongated along one axis, so that one may define an "intrinsic position angle" ψ from observation. If the polarization angle of the radiation at the source were aligned either parallel or perpendicular to ψ, then the difference between the polarization as measured on Earth and that at the source could, in principle, be determined. Since models indicate that the polarization at the source is due to the presence of a magnetic field $\mathbf{B}$ aligned either parallel or perpendicular to the elongation of the source, such an expectation might not be unrealistic.

To determine if the polarization at the source is correlated with the intrinsic position angle, Faraday rotation (due to the radiation moving through an intervening magnetized plasma) must first be removed. This can be done by using the fact that Faraday rotation is proportional to the square of the wavelength. The process of removing Faraday rotation is well understood.[14] This leaves us with an observed polarization angle χ, which would be rotated from the polarization angle at the source only by the effect of the Chern-Simons term. Note from Eq. (51) that, unlike Faraday rotation, the predicted

TABLE I. The source designations are 3CR catalogue numbers. p_{max} is the maximum polarization, χ is the angle of polarization, ψ is the intrinsic position angle, and z is the redshift. "z ref" is the reference from which we obtained z; "a" means Burbidge and Crowne (Ref. 17); "b" means Spinrad (Ref. 18); and "ab" means both.

No.	Source	p_{max}	χ	ψ	z	z ref
1	0017+15	18	34	143	2.012	*b*
3	0031+39	2	74	143	1.351	*b*
3	0033+18	6.5	80	171	1.469	*b*
4	0034−01	6	49	144	0.0730	*ab*
5	0038+32	6	118	30	0.4820	*ab*
6	0040+51	1.8	61	105	0.350	*b*
7	0048+50	6	25	104	0.937	*b*
8	0104+32	3.8	31	160	0.0167	*ab*
9	0105+72	7	134	47	0.181	*b*
10	0106+13	9.5	92	19	0.0595	*ab*
11	0107+31	8	4	78	0.689	*b*
12	0114−47	15	123	155	0.1460	*a*
13	0123+32	7.5	33	139	0.794	*b*
14	0128+06	12	95	4	0.66	*b*
15	0132+37	17	160	67	0.4373	*ab*
16	0133+20	5.5	38	32	0.425	*b*
17	0145+53	3.5	151	21	0.2854	*b*
18	0152+43	5	67	24	0.8274	*b*
19	0154+28	6.5	179	96	0.2400	*a*
20	0214−48	17	82	178	0.0640	*a*
21	0220+39	14	168	99	1.176	*b*
22	0221+27	3.5	62	178	0.3102	*ab*
23	0229+34	6.5	85	173	1.238	*b*
24	0300+16	17	113	110	0.0324	*ab*
25	0307+16	9	28	106	0.2559	*ab*
26	0325+02	6	81	65	0.0302	*ab*
27	0336−35	10.5	116	40	0.0049	*a*
28	0356+10	7.5	46	27	0.0306	*ab*
29	0404+03	5	49	129	0.0886	*ab*
30	0404+42	12	74	160	0.33	*b*
31	0410+11	6	60	143	0.3056	*ab*
32	0427−53	6	175	79	0.0392	*a*
33	0433+29	7	195	137	0.2177	*ab*
34	0453+22	9	18	132	0.2140	*ab*
35	0459+35	5	141	108	0.2775	*b*
36	0511+00	4	87	88	0.1273	*ab*
37	0518−45	3.2	95	104	0.0350	*a*
38	0605+48	6.5	37	57	0.2771	*ab*
39	0610+26	7	13	99	0.5804	*b*
40	0618−37	13	72	90	0.0326	*a*
41	0640+23	10	65	120	0.29	*b*
42	0651+54	10.5	85	101	0.2384	*ab*
43	0659+25	7.5	95	16	0.5191	*b*
44	0702+74	7	52	20	0.2920	*ab*
45	0710+11	10.5	16	59	0.768	*b*
46	0711+14	15	136	68	0.920	*b*
47	0724−01	2	153	171	0.22	*b*
48	0725+14	3.8	67	116	1.382	*b*
49	0733+70	5.5	176	112	0.994	*b*
50	0734+80	3	65	157	0.1178	*ab*
51	0742+02	9	56	160	0.3500	*ab*
52	0755+37	12	113	115	0.0433	*a*
53	0802+24	5	72	121	0.0598	*ab*
54	0809+48	3.3	157	24	0.871	*b*
55	0818+47	5	166	8	0.1301	*ab*
56	0819+06	10	95	38	0.0815	*ab*
57	0824+29	6	34	157	0.4580	*ab*

TABLE I. (*Continued*).

No.	Source	p_{max}	χ	ψ	z	z ref
58	0835+58	13	74	15	1.534	*b*
59	0838+13	4	20	100	0.684	*b*
60	0855+14	4	51	136	1.049	*b*
61	0903+16	0	76	134	0.411	*b*
62	0905+38	8.5	15	104	0.8975	*b*
63	0917+45	5.5	155	35	0.1744	*ab*
64	0927+36	7.5	18	43	1.157	*b*
65	0936+36	8	79	164	0.1368	*ab*
66	0938+39	3.5	46	16	0.1075	*ab*
67	0941+10	9	67	144	0.823	*b*
68	0945+07	6	162	86	0.0861	*ab*
69	0947+14	8	108	9	0.5524	*b*
70	0958+29	4	179	67	0.1848	*ab*
71	1030+58	6	118	167	0.4280	*ab*
72	1040+12	11	22	100	1.029	*b*
73	1056+43	6	12	69	0.7489	*b*
74	1100+77	4	7	95	0.311	*b*
75	1108+35	14	99	104	1.10	*b*
76	1137+66	3	41	111	0.6563	*b*
77	1140+22	2.3	89	55	0.366	*b*
78	1142+19	10	129	20	0.0208	*ab*
79	1142+31	6.5	41	105	0.8110	*ab*
80	1147+13	4.5	149	76	1.140	*b*
81	1157+73	3	148	93	0.97	*b*
82	1158+31	0	83	19	0.3620	*ab*
83	1203+64	4	77	160	0.3710	*ab*
84	1206+43	11	41	37	1.400	*b*
85	1216−10	7	113	165	0.0875	*a*
86	1216+06	12	93	85	0.0073	*ab*
87	1218+33	11	91	160	1.519	*b*
88	1222+13	7.5	147	3	0.0031	*ab*
89	1226+02	2.7	154	43	0.158	*b*
90	1228+12	4	20	111	0.0043	*ab*
91	1232+21	19	165	66	0.4220	*ab*
92	1233+16	0	24	96	0.0784	*a*
93	1241+16	3	124	167	0.557	*b*
94	1251+27	5.5	7	166	0.0857	*ab*
95	1252−12	7	14	71	0.0145	*a*
96	1254+47	12	34	92	0.996	*b*
97	1258+40	7.5	38	124	1.659	*b*
98	1308+27	14	20	100	0.2394	*ab*
99	1319+42	8.5	82	78	0.0794	*ab*
100	1330+02	4.5	145	93	0.2159	*ab*
101	1343+50	5.5	6	108	0.9674	*b*
102	1350+31	1.8	74	127	0.0452	*ab*
103	1358−11	11	121	125	0.0250	*a*
104	1414+11	16	36	37	0.0237	*ab*
105	1420+19	2	96	132	0.2700	*ab*
106	1425−01	7	120	83	0.3080	*ab*
107	1441+52	5	36	97	0.1410	*ab*
108	1502+26	5	37	163	0.0540	*ab*
109	1508+08	2.6	4	59	0.4610	*ab*
110	1511+26	7	118	10	0.1083	*ab*
111	1514+00	8	161	133	0.0530	*a*
112	1522+54	6	92	47	0.192	*b*
113	1529+24	11.5	0	116	0.0960	*ab*
114	1545+21	7.5	130	22	0.264	*b*
115	1547+21	6	153	73	1.2063	*b*
116	1549+62	8	79	123	0.860	*b*
117	1550+20	9	11	99	0.0895	*b*

TABLE I. (*Continued*).

No.	Source	$p_{\max}$	χ	ψ	z	z ref
118	1559+02	9	160	90	0.1039	*ab*
119	1602−63	14	112	120	0.0591	*a*
120	1609+66	4.5	131	62	0.5490	*ab*
121	1615+32	9	122	17	0.1515	*b*
122	1618+17	6.3	44	125	0.555	*b*
123	1622+23	6.5	132	32	0.927	*b*
124	1625+27	24	143	49	0.448	*b*
125	1627+23	12.5	177	90	0.7754	*b*
126	1641+17	2.5	172	71	0.1610	*ab*
127	1658+47	4.8	39	145	0.2050	*ab*
128	1709+46	8	64	167	0.8057	*ab*
129	1717−00	6.8	93	88	0.0304	*ab*
130	1723+51	8.8	131	161	1.079	*b*
131	1826+74	4.5	114	164	0.256	*b*
132	1828+48	1.8	15	142	0.691	*b*
133	1832+47	5	71	4	0.1605	*ab*
134	1836+17	2	28	15	0.0170	*ab*
135	1845+79	7.5	18	145	0.0561	*ab*
136	1949+02	6.5	45	82	0.0590	*ab*
137	2014−55	10	108	155	0.0606	*a*
138	2018+29	5.4	0	129	0.2480	*b*
139	2019+09	8.5	172	112	0.4690	*ab*
140	2040−26	9.2	160	158	0.0403	*a*
141	2058−28	0	57	169	0.0377	*a*
142	2104−25	3	175	136	0.0370	*a*
143	2104+76	10	51	140	0.572	*b*
144	2117+60	5.5	160	35	0.1016	*ab*
145	2130−53	3.1	77	130	0.0763	*a*
146	2141+27	8	169	172	0.2145	*ab*
147	2203+29	14	0	148	0.707	*b*
148	2211−17	0	176	164	0.1530	*a*
149	2212+13	11.5	148	65	0.0263	*b*
150	2221−02	8.8	103	170	0.0562	*ab*
151	2229+39	8	106	179	0.0171	*ab*
152	2243+39	11	3	80	0.0811	*ab*
153	2247+11	12	54	30	0.0243	*a*
154	2252+12	4	145	45	0.5427	*b*
155	2310+05	4.2	49	72	0.2890	*ab*
156	2318+23	3	101	36	0.2700	*ab*
157	2335+26	7	21	123	0.0293	*ab*
158	2345+18	7	114	159	0.6320	*a*
159	2353+79	11	85	172	1.336	*b*
160	2356−61	8.5	19	133	0.0963	*a*

effect is independent of wavelength.

Clarke, Kronberg, Simard-Normandin,[15] and Haves and Conway[16] investigated the correlation between the observed polarization angle χ (corrected for Faraday rotation) and the observed position angle ψ, and found that there is a large peak at $\chi-\psi\approx 90^\circ$ and a smaller one at $\chi-\psi\approx 0^\circ$, a result which can be explained by models of the sources. The correlation holds most strongly for sources with the larger values of integrated polarization considered as a function of wavelength. However, there appears to be no study in the literature of the relationship between $\chi-\psi$ and distance L or redshift z.

We have searched the catalogues of radio galaxies of Burbidge and Crowne[17] and Spinrad *et al.*[18] to determine the redshifts of the galaxies with known $\chi-\psi$ in Clarke, Kronberg, Simard-Normandin;[15] the data are listed in Table I. There are a total of 160 sources whose values of both z and $\chi-\psi$ have been determined; however, since the polarization angles of those with very low polarizations seem to correlate with position angle weakly, if at all, we have chosen to work only with those 116 galaxies with polarizations $\geq 5\%$.

To determine the theoretical phase shift due to p_a, we take the distance traveled along the light path to be

$$L = t \ . \tag{52}$$

This is an approximation because the two modes will be moving at slightly different velocities, but that is a higher-order effect. In an $\Omega=1$ (flat), matter-dominated universe, the time elapsed since an object at redshift z emitted the light we observe is given by

$$t = t_0[1-(1+z)^{-3/2}] \ , \tag{53}$$

where

$$t_0 = \frac{2}{3H_0} \tag{54}$$

is the present age of the Universe and H_0 is the Hubble constant, $(\dot{R}/R)_0$. Thus, Eqs. (51)–(53) give

$$\Delta\phi = -\frac{p_0 - p\cos\theta}{3H_0}[1-(1+z)^{-3/2}] \ . \tag{55}$$

Note that for sources with large values of z, the predicted angle of rotation is of order p/H_0, offering the chance to quantify p_α for values as low as $H_0 \sim 10^{-10}y^{-1} \sim 3\times10^{-18}\,\text{sec}^{-1}$.

In Fig. 1 we have plotted $\chi-\psi$ vs z for the 116 sources with polarization $\geq 5\%$. Several features of the data are immediately apparent. First, most of the points are clustered around $\chi-\psi\approx 90°$, with others around $\chi-\psi\approx 0°$, in agreement with the findings of Clarke, Kronberg, Simard-Normandin.[15] For us this is an important result, because it gives us confidence both that the polarization vectors of the sources are aligned either parallel or perpendicular to their position angles, and that the process of removing Faraday rotation is successful. The second obvious feature is that the clustering seems to hold for all redshifts. If p_α were large enough to make $\Delta\phi$ several radians, then we would expect the correlation between χ and ψ to become weaker as z increases.

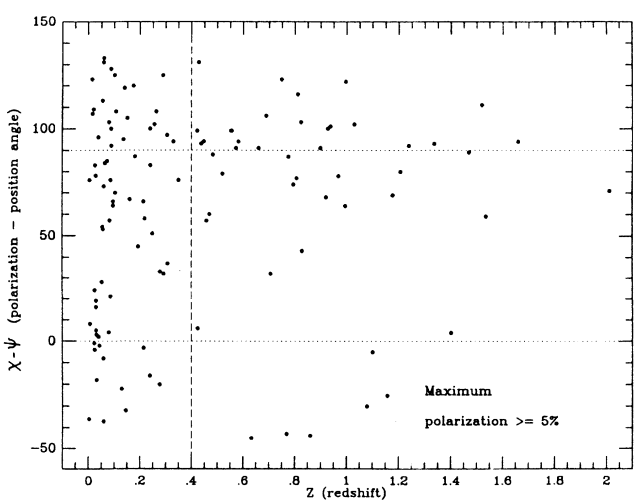

FIG. 1. The polarization angle χ minus the position angle ψ of the galaxies with maximum polarizations $p_{\max}=5\%$, plotted vs redshift z. It is clear that the data are grouped around the horizontal lines at 0° and 90°, even at large redshift. The vertical line at $z=0.4$ indicates the point beyond which we searched for deviation from these values.

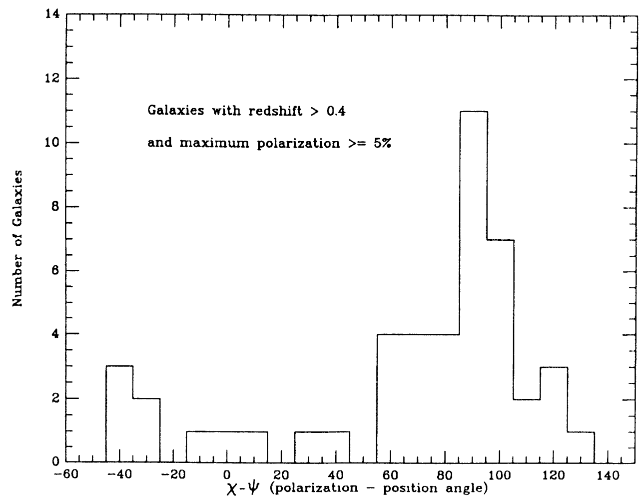

FIG. 2. A histogram of number of galaxies vs $\chi-\psi$, in bins of 10°, for those sources with $p_{\max} > =5\%$ and $z>0.4$. The peak near 90° is obvious.

Since $\Delta\phi$ is evidently small, the functional form of Eq. (55) implies that $\Delta\phi$ approaches a limiting value for large z. Therefore, we have grouped all of the data for which $z\geq 0.4$ to search for any deviation from $\chi-\psi=90°$. A histogram of the number of points (with $z\geq 0.4$) at a given $\chi-\psi$ appears as Fig. 2. The peak near 90° is clear. There is no peak, however, at 0°; this may be due to a selection effect; because sources with $\chi-\psi\approx 0°$ are known to be predominantly low-luminosity objects, they are underrepresented in a sample of very distant objects having $z\geq 0.4$. Taking only points with $45°\leq\chi-\psi\leq 135°$, we obtain a mean for the sample of

$$\overline{\chi-\psi} = 90.4° \pm 3.0° \ , \tag{56}$$

where the errors are standard errors of the mean. Thus, $\Delta\phi\leq 6.0°$ (at the 95% confidence level) at $z=0.4$.

We put this result into Eq. (55) to obtain

$$p_0 - p\cos\theta \leq 1.7\times10^{-42}h_0 \ \text{GeV} \ , \tag{57}$$

where $h_0 = H_0/100\ \text{km}\,\text{sec}^{-1}\ \text{Mpc}^{-1}$. Current observations indicate that $0.5 \lesssim h_0 \lesssim 1.0$. If p is small compared to p_0, then $p_0 - p\cos\theta \approx m$. To illustrate the precision of this test, we can compare the effectiveness of p_α as a parametrization of deviations from Lorentz invariance to that of μ, the mass of the photon, as a measure of departures from gauge invariance. Chibisov[2] uses the Galactic magnetic field to set

$$\mu \leq 3\times10^{-36}\ \text{GeV} \ . \tag{58}$$

Thus, as such measures go, we are able to put very stringent bounds on the magnitude of p_α as a Lorentz-invariance-violating quantity.

VI. SUMMARY AND CONCLUSIONS

We have considered the introduction of a Chern-Simons term

$$\mathcal{L}_{\rm CS} = -\tfrac{1}{2} p_\alpha A_\beta \tilde{F}^{\alpha\beta}$$

to the Lagrange density of classical electromagnetism in $3+1$ spacetime dimensions. This term, which couples the electromagnetic field to a "constant" external four-vector p_α, violates Lorentz invariance and parity, while preserving gauge invariance. The theory possesses an instability at long wavelengths which involves exponentially growing modes, but it is found that these do not violate energy conservation. Exponential growth is absent in noncausal solutions. The instability also gives rise to space-filling magnetic fields, familiar in magnetohydrodynamics.

As with the introduction of a mass for the photon, geomagnetic data can put limits on the Chern-Simons parameter $m = (p_\alpha p^\alpha)^{1/2}$: namely,

$$m \lesssim 6 \times 10^{-26}\ \text{GeV} .$$

However, a more stringent bound comes from examining the correlation between observed position angles and polarization angles of distant radio galaxies; this astrophysical test yields

$$m \lesssim 1.7 \times 10^{-42} h_0\ \text{GeV} .$$

Effects of nonzero m can appear only at distances greater than the associated Compton wavelength, which with the above limit is essentially the distance to the horizon. Thus the question whether m vanishes is answered positively by the astrophysical data, since a nonzero value satisfying the above inequality would lead to phenomena outside the horizon, which cannot be observed.

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