PHYSICAL REVIEW D VOLUME 41, NUMBER 4 15 FEBRUARY 1990

Infrared divergences and confinement in massive (2+1)-dimensional QED

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(Received 28 June 1989; revised manuscript received 20 September 1989)

In (2+1)-dimensional quantum electrodynamics with massless photons and massive matter fields, it is shown that the mass renormalization of the latter is infrared divergent at one loop. This result remains unchanged at two loops. A simple argument based on a similar divergence of the Coulomb potential leads us to conjecture that charged states are not observable in this model. This argument holds in 1+1 dimensions also.

Quantum field theories containing massless fields are usually plagued by infrared (IR) problems. Such problems are often related to important physical effects. In four-dimensional QCD, for example, the confinement of quarks is believed to be related to some of the IR singularities in the theory. However, the connection between the two in that theory is not completely clear at the moment. Confinement appears to be a long-distance nonperturbative phenomenon,[1] whereas IR divergences are well understood only in perturbation theory which is valid at short distances alone.

It may therefore be useful to examine theories where confinement and IR divergences can both occur in perturbation theory, and the connection between the two may be easier to study. This is the situation in some lower-dimensional superrenormalizable theories, for example, in the Schwinger model which is QED in 1+1 dimensions.[2] Confinement also occurs in non-Abelian gauge theories, perturbatively in 1+1 dimensions[3] and nonperturbatively in 2+1 dimensions.[4] A study of such theories may lead to some additional physical intuition about the properties of the more realistic four-dimensional confining theories.

In this paper, we study (2+1)-dimensional QED with a massless photon and massive matter fields (which could be either fermions or scalars). If the matter fields are also massless, this theory is known to suffer from severe IR divergences which can be cured by an unusual resummation of the perturbative expansions.[5] If the matter fields are massive, then there is still an IR divergence which occurs in a gauge-invariant quantity: namely, the on-shell mass renormalization of the matter fields. We will argue later that this divergence may be related to a physical property of the theory: namely, confinement.

To begin with, we show that the divergence occurs at one loop and is logarithmic in the following sense. If the matter field has a mass m and the photon is massless, we find that the mass shift δm is gauge variant and therefore physically meaningless. To cure this, a small photon mass μ is introduced. One then finds that δm is gauge invariant, but it diverges as $g^2 \ln(m/\mu)$ in the IR limit $\mu \to 0$, where g is the gauge coupling. On going to two loops, a similar logarithmic divergence is found except that the coefficient is of order g^4/m.

On the other hand, the matter fields are probably confined in this theory if the photon is massless. (The latter point is not trivial because a photon mass can be radiatively generated in 2+1 dimensions.[6-10] More on this later.) This claim is made plausible by examining the tree-level Coulomb potential $V(r)$ between two matter particles with equal and opposite charges $\pm g$. The potential also diverges logarithmically as $g^2 \ln \mu r$ as $\mu \to 0$, for a fixed distance r. This divergence is related to that in the mass shift δm. In fact, the two cancel precisely so that the mass of this charge-neutral bound state does not suffer from any IR divergence.[11,12] This observation can be extended to the mass of any other neutral state and even to (1+1)-dimensional QED.

Before proceeding further, we must point out that (2+1)-dimensional QED (QED_3) has certain similarities and differences from both QED_2 and QED_4. (We take all the matter fields to be massive in the sense that there is a bare mass for them in the Lagrangian.) In 1+1 dimensions, the photon always becomes massive. The quantized matter fields are confined; namely, all physical states are neutral. In 3+1 dimensions, the photon remains massless to all orders in perturbation theory if its bare mass is zero. The matter fields are not confined and charge-carrying physical states are observable. In 2+1 dimensions, the photon can become massive radiatively even if its bare mass is zero. In that case, there is no confinement and charged physical states exist. If, however, the photon mass is strictly zero (this can be arranged by an appropriate choice of fermionic matter fields as discussed later), then the matter fields are confined. All physical states must then be neutral.

We begin by discussing fermionic QED_3. The Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\xi}(\partial \cdot A)^2 + \frac{\mu^2}{2}A_\nu^2 + \bar{\psi}(i\partial\!\!\!/ - gA\!\!\!/ - m)\psi \; . \tag{1}$$

A gauge-fixing term proportional to ξ^{-1} has been introduced. Physical quantities cannot depend on the parameter ξ. The fermion mass m can appear with either a positive or a negative sign (this makes a difference to the radiatively generated photon mass). Since $d=3$, the charge g

is dimensionful and we will assume that g^2/m is small. We will also take the $\mu/m \to 0$ limit at the end of all our calculations to extract the desired IR divergences. The photon mass term breaks the ordinary gauge invariance $A_\mu \to A_\mu + \partial_\mu \lambda$ but it has a Becchi-Rouet-Stora-Tyutin (BRST) invariance if one adds Faddeev-Popov ghost fields c and $\bar{c}$ in the form[13]

$$\mathcal{L}_{\rm FP} = \partial_\mu \bar{c} \partial^\mu c - \mu^2 \xi \bar{c} c \ . \tag{2}$$

The fact that these ghost fields are uncoupled to A_μ in the Abelian theory means that one can simply work with Eq. (1) and expect to get gauge-invariant (ξ-independent) expressions for various physical quantities. Note that this introduction of a small gauge-boson mass as an IR regulator does not work in a non-Abelian theory since a mass term is not BRST invariant there and unitarity is ruined.

This trick of adding a small photon mass is also used in QED in $d=4$ in order to calculate the wave-function renormalization Z_2 and the mass renormalization δm for the matter fields.[14] While Z_2 is ξ dependent and IR divergent as $\mu/m \to 0$, δm is ξ independent, and is finite in the IR limit in $d=4$.

To calculate the one-loop expression for δm, we need the photon and fermion propagators $D_{\mu\nu}(k)$ and $S(k)$ from Eq. (1). These are

$$\begin{aligned} D_{\mu\nu}(k) &= \frac{-i}{k^2-\mu^2+i\epsilon}\left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2-\mu^2\xi+i\epsilon}\right] , \\ S(k) &= \frac{i}{\not{k} - m + i\epsilon} \ . \end{aligned} \tag{3}$$

The one-loop mass correction δm is obtained from the fermion self-energy

$$-i\Sigma(p) = (-ig)^2 \int \frac{d^3k}{(2\pi)^3} \gamma^\mu S(p+k) \gamma^\nu D_{\mu\nu}(k) \ . \tag{4}$$

Then Z_2 and δm are given by

$$\begin{aligned} &\Sigma(\not{p}=m) = \delta m \ , \\ &\frac{\partial}{\partial \not{p}} \Sigma(p) \Big|_{\not{p}=m} = Z_2 - 1 \ . \end{aligned} \tag{5}$$

To cancel these, counterterms of the form $(Z_2-1)\bar{\psi}(i\not{\partial}-m)\psi$ and a mass $Z_2 \delta m \bar{\psi}\psi$ must be added to the Lagrangian if the fermion propagator is to continue to have a pole with unit residue at $\not{p}=m$. In other words, if we want the fermion to be an observable particle with renormalized (physical) mass m, its bare (Lagrangian) mass must be $m-\delta m$. Conversely, if the bare mass is kept fixed at m so that a mass counterterm is not added to the Lagrangian, the renormalized mass becomes $m+\delta m$. The second point of view will be adopted later because it proves to be physically more useful.

There is a simple argument to show the ξ independence of δm at one loop. The term in $D_{\mu\nu}(k)$ proportional to $(1-\xi)k_\mu k_\nu B(k^2)$ does not contribute to δm because

$$\gamma^\mu S(p+k)\gamma^\nu k_\mu k_\nu B(k^2) = i\left[\not{k} - (\not{p}-m)\frac{1}{\not{p}+\not{k}-m}\not{k}\right] B(k^2) \ .$$

The first term vanishes upon integrating over k because $B(k^2)$ is an even function of k. The second term vanishes on the mass shell $\not{p}=m$. However, this formal argument for the gauge invariance of δm actually works only if $\mu \neq 0$. If μ is set equal to zero to begin with, IR singularities as $k \to 0$ spoil the argument in $d=3$ (and also in $d=2$). We then find a ξ dependent in δm given by $\delta m_\xi = -\xi g^2/8\pi$ in $d=3$. This was discovered by Deser, Jackiw, and Templeton[6] who then chose to work in the Landau gauge $\xi=0$. We, on the other hand, keep μ nonzero. This makes the mass shift gauge invariant, and we work in the Feynman gauge $\xi=1$ to simplify the computations.

On calculating the one-loop mass correction, denoted by δm_1, one finds that it diverges in the IR limit $\mu/m \to 0$. The divergent term is given by

$$\delta m_1 = \frac{g^2}{4\pi} \ln \frac{m}{\mu} \ . \tag{6}$$

For a fixed bare mass m, the renormalized mass, therefore, goes to infinity in the IR limit. (A similar observation has been made while analyzing some other theories as well.[15,16])

These calculations can be repeated for a complex scalar particle whose Lagrangian is

$$\mathcal{L} = (\partial_\mu - igA_\mu)\phi^*(\partial^\mu + igA^\mu)\phi - m^2\phi^*\phi \ . \tag{7}$$

Just as for fermions, one finds that δm is ξ dependent if $\mu=0$. In fact, $\delta m_\xi = -\xi g^2/8\pi$ again. With nonzero μ, δm is ξ independent and the infrared divergent piece of it is found to be the same as in Eq. (6) as $\mu/m \to 0$. (Unlike the fermionic case, the scalar mass shift is also ultraviolet divergent. This divergence is irrelevant in the present discussion.)

We have extended the calculation of mass shift to two loops, denoted by δm_2. For technical simplicity, we consider only fermionic QED. A two-loop computation is important for the following reason. The one-loop mass shift δm_1 in Eq. (6) becomes significant compared to m only if $\mu/m \lesssim \exp[-4\pi(m/g^2)]$, which is tiny even for modestly small values of the parameter $g^2/4\pi m$. At such a small value of μ/m, the actual perturbative expansion parameter for the on-shell higher-loop IR divergences is, on dimensional grounds, g^2/μ which is certainly not small.

This point is borne out by looking at the one-loop fermion gauge vertex $-ig\Gamma^\mu(p,k)$, where p and k are the incoming fermion and photon momenta, respectively. At the tree level, $\Gamma^\mu = \gamma^\mu$. At one loop and in the Feynman gauge, $\Gamma^\mu(\not{p}=m, k=0)$ diverges as $-(g^2/4\pi\mu)[1-(\mu/m)\ln(m/\mu)]$ which is much more divergent than δm_1. A Ward identity then implies that the IR-divergent part of $Z_2 - 1$ is given by

$$Z_2 - 1 = \frac{g^2}{4\pi\mu}\left[1 - \frac{\mu}{m}\ln\frac{m}{\mu}\right] \ . \tag{8}$$

To cancel this divergence, we add a vertex counterterm to the Lagrangian equal to $-g(Z_2-1)\bar{\psi}A\!\!\!/\psi$.

We now compute the various two-loop contributions to δm_2 shown in Fig. 1. The IR divergences come from regions in momentum space where one or both photon momenta go to zero. One must also add diagrams containing insertions of the various counterterms arising from one-loop corrections to the fermion and photon propagators and the fermion-gauge vertex. Several cancellations occur, and the IR-divergent part of $\delta m=\delta m_1+\delta m_2$ is finally obtained as

$$\delta m=\frac{g^2}{4\pi}\left[1+O\left[\frac{g^2}{m}\right]\right]\ln\frac{m}{\mu}\,, \tag{9}$$

where the $O(g^2/m)$ terms are infrared finite. In short, the two-loop on-shell self-energy $\Sigma_2(p\!\!\!/=m)$ diverges as $(1/\mu)\ln(m/\mu)$ and $[\ln(m/\mu)]^2$, but these divergences cancel with those of Z_2 (at one loop) to leave behind a mass shift which is only logarithmically divergent as in one loop.

We have not investigated in detail the IR divergence of δm at higher than two loops. By power counting it is clear that the divergent terms in the wave-function renormalization Z_2 diverge as powers of g^2/μ and $\ln(m/\mu)$. (It may be possible to sum up these divergences in Z_2 by the methods of Ref. 5.) However, the infrared behavior of Z_2 is not related to that of δm. Guided by the two-loop result, I believe that the n-loop mass shift δm_n is only as divergent as $m(g^2/m)^n\ln(m/\mu)$ due to a nontrivial cancellation of stronger divergences between $\Sigma_n(p\!\!\!/=m)$ and $(Z_2)_{n-1}$. This statement can probably be proven by using the Schwinger-Dyson integral equation for the fermion self-energy. We will not pursue this here.

So far, we have shown that if the matter quanta are to be observable with a finite physical mass, the bare mass must be IR divergent. We now adopt the opposite point of view. The bare mass is fixed at the finite value m and the renormalized mass is IR divergent. This has the following benefit. Consider the nonrelativistic Coulomb potential produced by a single photon exchanged between a fermion and an antifermion. This equals

$$V(r)\doteq-g^2\int\frac{d^2k}{(2\pi)^2}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^2+\mu^2}\,. \tag{10}$$

If r is kept fixed, and μ is taken to zero, this diverges as $(g^2/2\pi)\ln\mu r$. The Hamiltonian of this system is given by (ignoring the kinetic energy terms which have no IR divergences to this order in g^2) $2m_R+(g^2/2\pi)\ln\mu r$ where the renormalized mass $m_R=m+\delta m=m+(g^2/4\pi)\ln(m/\mu)$. We thus obtain $2m+(g^2/2\pi)\ln mr$ which is free of any IR divergence. Thus, while the renormalized mass of a single fermion suffers from an IR problem, the mass of a neutral pair does not.[11,12] Note that although the elementary charged quanta are not observable, their bare masses have a physical meaning because they set the scale of the bound-state masses.

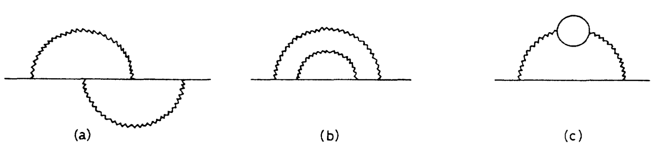

FIG. 1. Two-loop contributions to the fermion self-energy.

This leads us to conjecture that only neutral states will be physically observable in this model because their masses are not IR divergent. The above argument can be easily generalized to the case of a state consisting of N particles with finite bare masses and gauge couplings (m_i,g_i), where $i=1,2,\ldots,N$. The renormalized mass of particle i (which may be a fermion or a scalar) will exceed its bare mass by the IR-divergent term $-(g_i^2/4\pi)\ln\mu$. The tree-level potential between particles i and j will have an IR divergent term $-(g_ig_j/2\pi)\ln\mu$. Adding everything up, the Hamiltonian of the total system has the divergent term $-(1/4\pi)(\sum_i g_i)^2\ln\mu$ which vanishes only for neutral states with charge $Q\equiv\sum_i g_i=0$.

It would be interesting, but difficult, to check if the higher-loop IR divergences are also proportional to powers of Q and, therefore, vanish for $Q=0$ states. This can only be true, if order by order in perturbation theory, there are certain correlations between the IR divergences present in the self-energies of the various elementary charged particles and those present in the two-body (and many-body) potentials.[16]

To summarize, the bare masses of the elementary charged quanta are fixed at certain values which are not IR divergent. However, these quanta (and charged combinations of them) are postulated to be unobservable because their renormalized masses suffer from IR problems. [A somewhat similar situation exists in four-dimensional QCD where the bare- (current-) quark masses are not IR divergent and are related to physical quantities such as the color-neutral pseudoscalar masses.] Note that the IR problem is not directly related to the behavior of the Coulomb potential at large distances. In Eq. (7), r does not have to be large compared to, say, the physical distance scales m^{-1} or g^{-2}. The problem is rather due to the masslessness of the photon which forces one to eventually take the limit $\mu\to0$.

We must, therefore, discuss the question of the photon mass now. The IR divergences discussed above occur only if the photon remains massless even after radiative corrections are taken into account. In $d=3$, a photon mass of the form $\frac{1}{4}m_\gamma\epsilon_{\mu\nu\lambda}A^\mu F^{\nu\lambda}$ can be dynamically generated by the fermions. This parity-violating mass term[6,7] is, at one loop, equal to[8,9] $m_\gamma=\sum_i(g_i^2/4\pi\times m_i/|m_i|$ where (m_i,g_i) denote the charge and mass of fermion i in Eq. (1). At higher loops, there is no further contribution to m_γ (Ref. 10). If such a mass is generated, then confinement is lost because there are no longer any IR problems. The Coulomb potential develops no divergences if m_γ is nonzero and $\mu\to0$. (The photon propagator in the presence of both a parity-violating and a parity-conserving mass term is available in Ref. 17.) The mass shift δm is also infrared finite and gauge invariant if one lets $\mu\to0$ keeping m and m_γ fixed. [Note that it is necessary to take μ nonzero when doing this computation and let $\mu\to0$ only at the end. If one puts $\mu=0$ to begin with, there are gauge-dependent singularities in the photon propagator (even though $m_\gamma \neq 0$) which make δm ξ dependent and Z_2 ill defined[6].] Thus there are no IR problems in defining the mass of any state, neutral or charged.

To obtain confinement, we must therefore assume that such a photon mass is not generated, assuming that the bare mass is zero. The simplest model in which this is true is one with two fermions whose masses are equal but have opposite signs, and whose gauge couplings are equal. The fermionic Lagrangian

$$\mathcal{L} = \bar{\psi}_1(i\partial\!\!\!/ - gA\!\!\!/ - m)\psi_1 + \bar{\psi}_2(i\partial\!\!\!/ - gA\!\!\!/ + m)\psi_2 \qquad (11)$$

is parity conserving if the parity transformation is defined to be $(t,x,y)\rightarrow(t,-x,y)$, $(A^0,A^1,A^2)\rightarrow(A^0,-A^1,A^2)$, $\psi_1\rightarrow\gamma^1\psi_2$, and $\psi_2\rightarrow\gamma^1\psi_1$. Clearly, the parity-violating mass term cannot be generated in such a model.

This completes our discussion of IR divergences and confinement in massive QED_3 The arguments put forward in favor of confinement are plausible but certainly not sufficient. For a comprehensive understanding of confinement in this theory, one must examine other aspects of the theory, for example, the behavior of large Wilson loops.[18]

The arguments in this paper also apply to QED_2. Since the massive Schwinger model has been extensively studied in many different ways,[2] we will be brief. The IR divergence in the one-loop mass renormalization of an elementary particle (fermionic or scalar) of charge g_i is given by $g_i^2/4\mu$. The potential between two particles is, to the same order in the charges, equal to $(g_i g_j/2\mu)\exp(-\mu r)$ whose IR-divergent part is $(g_i g_j/2\mu)$. So the divergent piece in the mass of a composite state equals $(1/4\mu)(\sum_i g_i)^2$ which only vanishes for neutral states.

I thank R. Kaul and R. Rajaraman for some enlightening discussions.

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