Finite nonlocal gauge field theory

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A canonical quantization formalism for gauge fields is presented, based on massless nonlocal vector and second-rank tensor field Lagrangians. The Lagrangians describing quantum electrodynamics, electroweak theory, and gravitation within the context of the nonlocal formalism are shown to lead to finite, gauge-invariant, and unitary theories to all orders in perturbation theory. The generalized electroweak theory does not contain any gauge hierarchy problems, associated with the Higgs-meson perturbation theory, and it describes a nontrivial field theory.

I. INTRODUCTION

A new field theory based on nonlocal fields has been proposed.[1,2] A perturbation theory was developed,[2] which was shown to lead to an ultraviolet-finite unitary S matrix to all orders. The concept of microscopic causality, as formulated in strictly localizable perturbation theory, was extended for certain classes of entire functions that occur in the propagators. In the previous work, an attempt was made to associate the nonlocal fields with an internal, infinite-spin degree of freedom called "superspin." In the following, we shall devote our attention to the nonlocal aspects of a gauge theory including gravity.

The nonlocal scalar field $\Phi(x)$ was defined in terms of softening coefficients c_j, which determine the growth behavior of the scattering amplitudes as $p^2 \rightarrow \infty$. The behavior of these coefficients for large j was determined by an inversion formula in Euclidean space, combined with suitable physical boundary conditions at $x^2 \rightarrow 0$ and $x^2 \rightarrow \infty$. The c_j's for lower values of j are determined in perturbation theory, thus leading to a specific nonlocal field theory that can be used to predict cross sections.

In the following, we shall investigate the properties of nonlocal quantum electrodynamics (QED), electroweak theory, and gravitation. The S-matrix formalism in perturbation theory is developed within the standard canonical quantized scheme with a generalized Gupta-Bleuler prescription for dealing with negative-norm states. The Higgs sector in the nonlocal electroweak theory is shown to be nontrivial, in the sense of Landau,[3] for a chosen value of the nonlocal energy scale $M_W = 1/l_W$, where l_W is the length scale associated with weak interactions. In a separate work,[4] it has been demonstrated in a calculation of the vacuum-polarization processes in QED and quantum chromodynamics (QCD) that the running coupling constants tend to their "bare" charge values as $p^2 \rightarrow \infty$, leading to an ultraviolet fixed point in the β function. Thus, nonlocal QED becomes a nontrivial field theory, while nonlocal QCD is not an asymptotically free theory.

II. NONLOCAL QUANTUM ELECTRODYNAMICS

Let us begin our study of massless gauge fields by considering the spin-$\frac{1}{2}$ and the spin-1 nonlocal fields. We shall construct an extended version of QED. The free Lagrangian takes the form

$$\mathcal{L}_0 = -\tfrac{1}{2}{:}\partial_\mu A^\nu(x)\partial^\mu A_\nu(x){:} - {:}\bar{\psi}(x)(-i\gamma^\mu\partial_\mu + m)\psi(x){:} \;, \tag{2.1}$$

where A and ψ are the local point-particle electromagnetic potential and Dirac electron field operators, respectively. The interaction Lagrangian is given by

$$\mathcal{L}_I = e{:}\mathcal{A}^\mu(x)\bar{\Psi}(x)\gamma_\mu\Psi(x){:} + \tilde{\mathcal{L}}_I \;, \tag{2.2}$$

where $\tilde{\mathcal{L}}_I(\psi, A_\mu)$ contains higher-order interactions necessary to restore gauge invariance. The nonlocal fields $\mathcal{A}_\mu$ and Ψ are given in terms of the local point-particle fields A_μ and ψ by

$$\mathcal{A}_\mu(x) = \int d^4y\, B(x-y)A_\mu(y) = B(\Box_x)A_\mu(x) \;, \tag{2.3a}$$

$$\Psi(x) = \int d^4y\, B(x-y)\psi(y) = B(\Box_x)\psi(x) \;, \tag{2.3b}$$

where $B(\Box_x)$ is a Lorentz-invariant operator distribution.[2]

Our Lagrangian for QED is invariant under the gauge transformations

$$\delta A_\mu(x) = \partial_\mu\lambda(x) \;, \tag{2.4}$$

$$\delta\psi(x) = ie\int d^4y\, d^4z\, V(x,y,z)\lambda(y)\psi(z) + ie^2\int d^4y\, d^4z d^4w\, U^\mu(x,y,z,w) \times\lambda(y)A_\mu(z)\psi(w) + \cdots \;. \tag{2.5}$$

Here, $\lambda(x)$ is an arbitrary scalar field and $\Box\lambda(x)=0$. If we substitute the nonlocal field redefinitions (2.3a) and (2.3b) into (2.2), then the Lagrangian is invariant under the transformations (2.4) and (2.5), provided we include the higher-order interactions contained in $\tilde{\mathcal{L}}_I$. A detailed proof of gauge invariance will be published elsewhere. We refer to the representation of the Lagrangian involving the fields defined by (2.3a) and (2.3b) as the "B representation." The nonlocal nature of the Lagrangian becomes explicit in the B representation.

We can now set up a Gupta-Bleuler formalism with an indefinite metric[5] for the $\mathcal{A}_\mu(x)$ nonlocal gauge field. The vacuum state is postulated to satisfy

$$\partial_\mu \mathcal{A}^{\mu(+)}|0\rangle = B(\Box_x)\partial_\mu A^{\mu(+)}|0\rangle = 0 , \quad (2.6)$$

where we have used the fact that $[B(\Box_x),\partial_x]=0$. This is the Gupta-Bleuler condition that is used to remove the unphysical states in the interaction Lagrangian. The quantization of the nonlocal fields could, of course, be formulated using the modern methods of path integrals. This will be considered elsewhere.

As in standard QED, we write the S matrix in the form of Eq. (7.7), in Ref. 2. The "chronological" contraction of the nonlocal fields implies the use of the causal propagator. The causal propagator for the photon will have the form

$$\begin{aligned} D^c_{s\mu\nu}(x-y) &= \mathcal{A}^{\cdot}_\mu(x)\mathcal{A}^{\cdot}_\nu(y) \\ &= B(\Box_x)B(\Box_y)\langle 0|T[A_\mu(x)A_\nu(y)]|0\rangle \\ &= -\frac{\eta_{\mu\nu}}{(2\pi)^4 i}\int \frac{d^4k\,\Pi(k^2)}{-k^2-i\epsilon}e^{ik\cdot(x-y)} , \end{aligned} \quad (2.7)$$

where $A_\mu(x)$ is the usual free photon field:

$$A_\mu(x) = (2\pi)^{-3/2}\int \frac{d^3\mathbf{p}}{(2\omega)^{1/2}}[a_\mu(\mathbf{p})e^{-ip\cdot x} + a^*_\mu(\mathbf{p})e^{ip\cdot x}] . \quad (2.8)$$

The causal electron propagator is given by

$$\begin{aligned} S^c_s(x-y) &= \Psi^{\cdot}(x)\bar{\Psi}^{\cdot}(y) \\ &= B(\Box_x)B(\Box_y)\langle 0|T[\psi(x)\bar{\psi}(y)]|0\rangle \\ &= \frac{1}{(2\pi)^4 i}\int \frac{d^4p\,\Pi(p^2)}{m-p\cdot\gamma-i\epsilon}e^{ip\cdot(x-y)} . \end{aligned} \quad (2.9)$$

We now follow the same methods of constructing the perturbation theory as developed for the nonlocal scalar fields in Ref. 2. According to the results obtained, the function Π satisfies the following Efimov conditions:

$$\begin{aligned} &(1)\ \Pi(z) \text{ is an entire analytic function} \\ &\quad\ \text{of the order } \tfrac{1}{2}\le\gamma\le 1 , \\ &(2)\ [\Pi(z)]^* = \Pi(z^*) , \\ &(3)\ \Pi(x)>0 \ \text{ for real } x , \\ &(4)\ \int_o^\infty dv\,\Pi(v)<\infty . \end{aligned} \quad (2.10)$$

The S matrix is expanded in powers of the normal-ordered operators of the nonlocal photon field $\mathcal{A}_\mu(x)$ and the nonlocal electron field $\Psi(x)$ in the form

$$\begin{aligned} S = \sum_{n!m!r!} &\int d^4k_1 \cdots \int d^4k_n \int d^4p_1 \cdots \int d^4p_m \int d^4q_1 \cdots \int d^4q_r \\ &\times G^{\mu_1\cdots\mu_n}(k_1,\ldots,k_n;p_1,\ldots,p_m;q_1,\ldots,q_r) :\mathcal{A}_{\mu_1}(k_1)\cdots\mathcal{A}_{\mu_n}(k_n)\Psi(p_1)\cdots\Psi(p_m)\bar{\Psi}(q_1)\cdots\bar{\Psi}(q_r): . \end{aligned} \quad (2.11)$$

Gauge invariance is satisfied to all orders, if the coefficient functions $G_{\mu_1\cdots\mu_n}$ in the expansion obey the conditions

$$\begin{aligned} &k^{\mu_1}G_{\mu_1\cdots\mu_i\cdots\mu_n}(\cdots)=0 , \\ &k^{\mu_i}k^{\mu_j}G_{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_n}(\cdots)=0 . \end{aligned} \quad (2.12)$$

These conditions are satisfied, given the gauge invariance of our Lagrangian under the gauge transformation (2.4) and (2.5), provided that the other momenta in the function $G_{\mu_1\cdots\mu_n}$ are on the mass shell and that the higher-order interactions in $\tilde{\mathcal{L}}_I$ are chosen correctly.

The perturbation series will be ultraviolet finite to all orders and the proof of the unitarity of the S matrix follows from the Cutkosky rule, formulated in Euclidean momentum space, using our regularization techniques based on the Efimov regulating function R^δ. Moreover, the commutation relation for the nonlocal photon fields $\mathcal{A}_\mu(x)$ leads to a local microcausality condition, as in the case of the scalar superspin fields, provided certain restrictions are imposed on the entire functions $\Pi(p^2)$.

As in standard QED, the perturbative expansion will diverge because of the $n!$ number of n-loop Feynman diagrams. However, in contrast with string theories, the perturbation expansion is Borel summable. The same holds true of our scalar polynomial Lagrangian. We can see this by expanding the functional integral

$$Z = \int d[\phi]\exp\left[i\int d^4x[\mathcal{L}_0(\phi)+\mathcal{L}_I(\Phi)+\mathcal{J}\Phi]\right] \quad (2.13)$$

in powers of $\mathcal{L}_I$. $\mathcal{J}$ is an external source current. The divergence can be deduced from our interaction Lagrangian $\mathcal{L}_I = g{:}\Phi^p{:}$ to be of the form

$$\sum(-g)^n[(p-2)n/2]! , \quad (2.14)$$

which is Borel summable, and indicates the existence of a stable perturbative vacuum state.[6] This is to be contrasted with the bosonic string perturbation series that diverges like $\sum g^{2h}h!$, where h denotes the number of handles associated with the moduli space of Riemann surfaces.[7] Such a divergence is similar to that of a $g{:}\phi^3{:}$ theory, consistent with the form of certain string field theories, which leads to an unstable vacuum state.

Since our field theory is finite, there will not occur any infinite counterterms in the perturbation theory. A finite renormalization of our QED is performed by introducing the physical coupling constant e_r and the electron mass m_r, instead of the initial coupling constant e and the electron mass m.

We picture the photon and the electron as being extended objects. When $\Pi(p^2)\sim 1$, the nonlocal photon and electron become pointlike and we obtain the standard ultraviolet-divergent QED.

III. NONLOCAL ELECTROWEAK THEORY

Let us now consider weak interactions, restricting ourselves here to the leptonic interactions. The Lagrangian takes the form

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_I+\mathcal{L}_B \;, \tag{3.1}$$

where $\mathcal{L}_0$ is the free lepton Lagrangian density given by

$$\mathcal{L}_0=i[:\bar{\psi}^L(x)\gamma\cdot\partial\psi^L(x):+:\bar{\psi}_l^R(x)\gamma\cdot\partial\psi_l^R(x): \\ +:\bar{\psi}_{\nu_l}^R(x)\gamma\cdot\partial\psi_{\nu_l}^R(x):] \;. \tag{3.2}$$

Here, a summation over all different kinds of leptons is understood: $l=e,\mu,\ldots$, and the field $\psi^L(x)$ denotes a two-component left-handed lepton field:

$$\psi^L(x)=\begin{bmatrix}\psi_{\nu_l}^L(x)\\ \psi_l^L(x)\end{bmatrix} , \tag{3.3}$$

with $\psi^L(x)=\frac{1}{2}(1-\gamma_5)\psi(x)$ and $\psi^R(x)=\frac{1}{2}(1+\gamma_5)\psi(x)$ and, correspondingly,

$$\bar{\psi}^L(x)=(\bar{\psi}_{\nu_l}^L(x),\psi_l^L(x)) \;. \tag{3.4}$$

The interaction is described by

$$\mathcal{L}_I=-g\mathcal{J}_a^\mu(x)\mathcal{W}_{a\mu}(x)+g'\mathcal{J}_Y^\mu(x)\mathcal{B}_\mu(x)+\tilde{\mathcal{L}}_I \;. \tag{3.5}$$

The $\mathcal{J}_a^\mu$ and $\mathcal{J}_Y^\mu$ are the weak isospin and hypercharge nonlocal currents:

$$\mathcal{J}_a^\mu(x)=\tfrac{1}{2}\bar{\Psi}^L(x)\gamma^\mu\tau_a\Psi^L(x) \quad (a=1,2,3) \tag{3.6}$$

and

$$\mathcal{J}_Y^\mu(x)=-\tfrac{1}{2}\bar{\Psi}^L(x)\gamma^\mu\Psi^L(x)+\bar{\Psi}^R(x)\gamma^\mu\Psi^R(x) \;. \tag{3.7}$$

$\Psi^L(x)$ and $\Psi^R(x)$ are the nonlocal lepton field operators. The $\tilde{\mathcal{L}}_I$ term in (3.5) is required to restore the gauge symmetries. The τ_i are the 2×2 Hermitian Pauli matrices

$$\tau_1=\begin{bmatrix}0&1\\1&0\end{bmatrix},\quad \tau_2=\begin{bmatrix}0&-i\\i&0\end{bmatrix},\quad \tau_3=\begin{bmatrix}1&0\\0&-1\end{bmatrix} . \tag{3.8}$$

The quark doublets can be incorporated into the scheme by constructing a nonlocal quark field Ψ_q and including the nonlocal quarks in the $\mathcal{J}_a^\mu$ and $\mathcal{J}_Y^\mu$ currents.

The $\mathcal{W}_{a\mu}$ and $\mathcal{B}_\mu$ denote the intermediate charged-vector-boson field and the real boson field, respectively.

The $\mathcal{W}_{3\mu}(x)$ and $\mathcal{B}_\mu(x)$ are linear combinations of the two fields $\mathcal{A}_\mu(x)$ and $\mathcal{Z}_\mu(x)$:

$$\mathcal{W}_{3\mu}(x)=\cos\theta_W\mathcal{Z}_\mu(x)+\sin\theta_W\mathcal{A}_\mu(x) \;, \tag{3.9a}$$

$$\mathcal{B}_\mu(x)=-\sin\theta_W\mathcal{Z}_\mu(x)+\cos\theta_W\mathcal{A}_\mu(x) \;, \tag{3.9b}$$

where the angle θ_W denotes the Glashow-Weinberg angle. The electroweak coupling constants g and g' are related to the electric charge e by the standard equation

$$g\sin\theta_W=g'\cos\theta_W=e \tag{3.10}$$

and we use the normalization $\cos\theta_W=g/(g^2+g'^2)^{1/2}$. The boson Lagrangian $\mathcal{L}_B$ is given by

$$\mathcal{L}_B=-\tfrac{1}{4}\mathcal{G}_{a\mu\nu}(x)\mathcal{G}_a^{\mu\nu}(x) \;, \tag{3.11}$$

where

$$\mathcal{G}_a^{\mu\nu}=F_a^{\mu\nu}+g\epsilon_{abc}\mathcal{W}_b^\mu\mathcal{W}_c^\nu \;, \tag{3.12}$$

and

$$F_a^{\mu\nu}=\partial^\mu W_a^\nu-\partial^\nu W_a^\mu \;. \tag{3.13}$$

We can rewrite the interaction Lagrangian in the form

$$\mathcal{L}_I=-\mathcal{J}^{\mu\,\mathrm{em}}(x)\mathcal{A}_\mu(x) \\ -\frac{g}{2\sqrt{2}}[\mathcal{J}^{\mu\dagger}(x)\mathcal{W}_\mu(x)+\mathcal{J}^\mu(x)\mathcal{W}_\mu^\dagger(x)] \\ -\frac{g}{\cos\theta_W}[\mathcal{J}_3^\mu(x)-\sin^2\theta_W\mathcal{J}^{\mu\,\mathrm{em}}(x)/e]\mathcal{Z}_\mu(x)+\tilde{\mathcal{L}}_I \;, \tag{3.14}$$

where $\mathcal{J}^{\mu\,\mathrm{em}}$ is the nonlocal electromagnetic current:

$$\mathcal{J}^{\mu\,\mathrm{em}}(x)=-e\bar{\Psi}_l(x)\gamma^\mu\Psi_l(x) \;. \tag{3.15}$$

Moreover,

$$\mathcal{W}_\mu(x)=\frac{1}{\sqrt{2}}[\mathcal{W}_{1\mu}(x)-i\mathcal{W}_{2\mu}(x)] \;, \tag{3.16a}$$

$$-g\sum_{a=1}^{2}\mathcal{J}_a^\mu(x)\mathcal{W}_{a\mu}(x)=-\frac{g}{2\sqrt{2}}[\mathcal{J}^{\mu\dagger}(x)\mathcal{W}_\mu(x) \\ +\mathcal{J}^\mu(x)\mathcal{W}_\mu^\dagger(x)] \;, \tag{3.16b}$$

and

$$\mathcal{J}^\mu(x)=2[\mathcal{J}_1^\mu(x)-i\mathcal{J}_2^\mu(x)] \;. \tag{3.16c}$$

Equation (3.14) is the $SU(2)\times U(1)$-invariant Lagrangian in the Glashow-Salam-Weinberg (GSW) theory,[8] which is written in terms of the nonlocal fields. The second term in (3.14) is the intermediate-vector-boson Lagrangian with the coupling constant $g=2\sqrt{2}g_W$ where $(g_W/m_W)^2=G_F/\sqrt{2}$ and $G_F=1.027\times10^{-5}/m_P^2$ is the Fermi weak coupling constant. The last term describes the coupling of the weak neutral current to the $\mathcal{Z}^0$ boson and $\sin^2\theta_W=0.227\pm0.014$.

To make the model realistic, we must add to the Lagrangian (3.1) a Higgs-boson sector.

The Higgs Lagrangian has the standard form

$$\mathcal{L}_h=[D^\mu\phi(x)]^\dagger[D_\mu\phi(x)]-\mu^2\phi^\dagger(x)\phi(x)-\lambda[\Phi^\dagger(x)\Phi(x)]^2 \\ -g_l[\bar{\Psi}_l^L(x)\Psi_l^R(x)\Phi(x)+\Phi^\dagger(x)\bar{\Psi}_l^R(x)\Psi_l^L(x)]-g_{\nu_l}[\bar{\Psi}_l^L(x)\Psi_{\nu_l}^R(x)\Phi(x)+\Phi^\dagger(x)\bar{\Psi}_{\nu_l}^R\Psi_l^L(x)] \;, \tag{3.17}$$

where $\Phi(x)$ is the nonlocal Higgs field defined in the B representation by $\Phi(x)=B(\Box_x)\phi(x)$. The covariant derivatives are given by

$$D^\mu\psi_l^L(x)=[\partial^\mu+\tfrac{1}{2}ig\tau_a\mathcal{W}_a^\mu(x)B(\Box_x)-\tfrac{1}{2}ig'\mathcal{B}^\mu(x)B(\Box_x)]\psi_l^L(x)\ , \tag{3.18a}$$

$$D^\mu\psi_l^R(x)=[\partial^\mu-ig'\mathcal{B}^\mu(x)B(\Box_x)]\psi_l^R(x)\ , \tag{3.18b}$$

$$D^\mu\psi_{\nu_l}^R(x)=\partial^\mu\psi_{\nu_l}^R(x)\ , \tag{3.18c}$$

$$D^\mu\phi(x)=[\partial^\mu+\tfrac{1}{2}ig\tau_a\mathcal{W}_a^\mu(x)B(\Box_x)+\tfrac{1}{2}ig'\mathcal{B}^\mu(x)B(\Box_x)]\phi(x)\ . \tag{3.18d}$$

In the quantized theory, SU(2)×U(1) will be spontaneously broken by the vacuum expectation value of the Higgs field

$$\langle 0|\Phi(x)|0\rangle=\Phi_0=\begin{bmatrix}0\\ v/\sqrt{2}\end{bmatrix}\ , \tag{3.19}$$

where $v=(-\mu^2/\lambda)^{1/2}$ is not invariant under SU(2) ×U(1) gauge transformations, but is invariant under the U(1) gauge transformations of electromagnetism, thereby preserving a massless photon. The photon field and the Z^0 meson field are given by the standard combinations of $\mathcal{W}^{3\mu}$ and $\mathcal{B}^\mu$ fields[9]

$$\mathcal{A}^\mu=\sin\theta_W\mathcal{W}^{3\mu}+\cos\theta_W\mathcal{B}^\mu\ , \tag{3.20a}$$

and

$$Z^\mu=\cos\theta_w\mathcal{W}^{3\mu}-\sin\theta_w\mathcal{B}^\mu\ . \tag{3.20b}$$

The whole nonlocal formalism can be generalized to the SU(3)×SU(2)×U(1) standard model including quantum chromodynamics.

In our nonlocal field theory formalism, the chronological contraction rules for the $\mathcal{W}$ and Z fields are

$$D_{s\mu\nu}^c(x-y,m_W)=\mathcal{W}_\mu^{\cdot}(x)\mathcal{W}_\nu^{\dagger\cdot}(y)=B(\Box_x)B(\Box_y)\langle 0|T[W_\mu(x)W_\nu^\dagger(y)]|0\rangle=\frac{1}{(2\pi)^4i}\int\frac{d^4k\ \Pi(k^2)(-\eta_{\mu\nu}+k_\mu k_\nu/m_W^2)}{-k^2+m_W^2-i\epsilon}e^{ik\cdot(x-y)}\ , \tag{3.21}$$

and

$$D_{s\mu\nu}^c(x-y,m_Z)=Z_\mu^{\cdot}(x)Z_\nu^{\cdot}(y)=B(\Box_x)B(\Box_y)\langle 0|T[Z_\mu(x)Z_\nu(y)]|0\rangle=\frac{1}{(2\pi)^4i}\int\frac{d^4k\ \Pi(k^2)(-\eta_{\mu\nu}+k_\mu k_\nu/m_Z^2)}{-k^2+m_Z^2-i\epsilon}e^{ik\cdot(x-y)}\ . \tag{3.22}$$

For the nonlocal lepton fields, we have

$$S_s^c(x-y,m_l)=\Psi_l^{\cdot}(x)\overline{\Psi}_l^{\cdot}(y)=B(\Box_x)B(\Box_y)\langle 0|T[\psi_l(x)\psi_l^\dagger(y)]|0\rangle=\frac{1}{(2\pi)^4i}\int\frac{d^4p\ \Pi(p^2)e^{ip\cdot(x-y)}}{m_l-p\cdot\gamma-i\epsilon}\ . \tag{3.23}$$

Here, the $W_\mu(x)$, $Z_\mu(x)$, and $\psi(x)$ are the point-particle free fields. The W and Z fields are given by

$$W^\mu(x)=(2\pi)^{-3/2}\int\frac{d^3\mathbf{k}}{(2\omega)^{1/2}}\sum_r[e_r^\mu(\mathbf{k})a_r(\mathbf{k})e^{-ik\cdot x}+e_r^\mu(\mathbf{k})b_r^*(\mathbf{k})e^{ik\cdot x}]\ , \tag{3.24a}$$

$$Z^\mu(x)=(2\pi)^{-3/2}\int\frac{d^3\mathbf{k}}{(2\omega)^{1/2}}\sum_r[e_r^\mu(\mathbf{k})c_r(\mathbf{k})e^{-ik\cdot x}+e_r^\mu(\mathbf{k})c_r^*(\mathbf{k})e^{ik\cdot x}]\ , \tag{3.24b}$$

and $\psi_l(x)$ has the same form as Eq. (4.12), in Ref. 2. The vectors $e_r^\mu(\mathbf{k})$, $r=1,2,3$, are a complete set of orthonormal polarization vectors

$$e_r(\mathbf{k})e_s(\mathbf{k})=\delta_{rs}\ . \tag{3.25}$$

These vectors satisfy the condition $k_\alpha e_r^\alpha(\mathbf{k})=0$ and the completeness relation gives

$$\sum_{r=1}^{3}e_r^\mu(\mathbf{k})e_r^\nu(\mathbf{k})=-\eta^{\mu\nu}+k^\mu k^\nu/m_V^2\ , \tag{3.26}$$

where m_V denotes either the $\mathcal{W}$ or Z particle mass. We can define the Wick theorem for our nonlocal field operators and solve the S matrix for the leptonic interactions in the same way as was done for the nonlocal scalar fields. The perturbation series will be convergent to all orders and the S matrix will be unitary, since the function $\Pi(k^2)$ satisfies the conditions (1)–(4) in (2.10). Thus, we can calculate all the *finite* radiative corrections in our nonlocal electroweak theory, using the generalized Feynman rules.

We shall see in the next section, that our formalism can lead to a finite quantum gravity theory. But this necessitates picturing the graviton as an extended particle. It is logical that we should then describe all the fundamental particles of nature as extended particles, whereby we arrive at our formulation of electroweak theory. Although there is, as yet, no experimental confirmation of the Higgs radiative corrections in the GSW theory, the predicted W and Z masses $m_W=\frac{1}{2}vg$ and $m_Z=m_W/\cos\theta_W$ agree well with the experiment.

In the standard renormalizable GSW theory, the renormalized Higgs-boson coupling constant $\lambda_{\rm ren}$ is related to the scale M_W by $1/\lambda_{\rm ren}=1/\lambda_{\rm bare}+(3/16\pi^2)\ln(M_W/\mu_0)$, so that for a fixed value of λ_{bare}, it follows that $\lambda_{\text{ren}} \to 0$ as $M_W \to \infty$ and the theory becomes trivial. Moreover, since $\lambda_{\text{ren}} = m_H^2/4v^2$, then $m_H \to 0$ as $M_W \to \infty$. The Higgs-boson mass produced by the Higgs self-energy graph is quadratically divergent in the standard GSW theory. The electroweak perturbation theory is unstable and we have to fine-tune the theory in an unnatural way. This is known as the hierarchy problem. If, in our generalized electroweak theory, we set the *physical* electroweak scale $M_W \sim 300$ GeV, and since the loop integrals in the region $M_W < |p| < \infty$ damp off exponentially fast, we shall not encounter a gauge hierarchy problem. Let us choose, in Euclidean momentum space, $\Pi(p^2) = \exp(-p^2/M_W^2)$, a choice that is compatible with the physical boundary conditions imposed on the propagators, as described in Ref. 2. A calculation gives $1/\lambda_{\text{ren}} = 1/\lambda_{\text{bare}} + (3/16\pi^2)\ln(M_W/\mu_0)$. For $M_W \gg \mu_0$, the β function for the $\lambda{:}\Phi^4$: Higgs sector has the standard form: namely, $\beta \simeq 3\lambda^2/16\pi^2$, and the nonlocal electroweak theory is nontrivial at all energies. In the standard GSW theory, if $M_W \to \infty$, then the renormalized Higgs interactions are zero. If, on the other hand, M_W is finite, then the theory can only be considered as an *effective* theory, i.e., as the limit of an, as yet, unknown more fundamental theory.[3] In the generalized electroweak theory, formulated using nonlocal fields, the finite scale M_W is a physical quantity associated with the length $l_W = 1/M_W$, related to the size of the extended particles, and the theory is fundamental at all energies.

Hopefully, future high-energy accelerators will be able to check the radiative Higgs corrections predicted by the nonlocal electroweak field theory, so that we can compare them with those predicted by the GSW theory. The details of the predicted radiative corrections in the nonlocal electroweak theory will be presented elsewhere.

IV. GRAVITATION

We shall now turn to a study of the gravitational field. In the present work, we apply the nonlocal field-theory formalism to Einstein's theory of gravitation. In a subsequent article, we will extend this work to the more general nonsymmetric gravitation theory[10] (NGT).

The gravitational field has special problems associated with it that are not encountered when considering other forces of nature. Einstein's gravitational field is expressed in geometrical language using a Riemannian spacetime, and the field equations are highly nonlinear. Much has been said about the difficulties of quantizing the metric of four-dimensional spacetime, since it is difficult to associate quantum-mechanical concepts such as the Heisenberg uncertainty principle with spacetime quantities. In the following, we shall not attempt to solve these fundamental problems, choosing instead to construct a finite perturbation theory of quantum gravity along the lines of the scalar and QED nonlocal field theory. We shall regard the Minkowskian spacetime as the zeroth-order approximation to the Riemannian spacetime. Such a solution should be physically valid for all gravitational fields, save those at the Planck energy $\sim 10^{19}$ GeV and beyond. Weak gravitational field solutions will be the only ones of experimental interest, since it is inconceivable that we will ever obtain direct experimental information about Planck-energy gravitational phenomena. The whole method can be based on an expansion about an arbitrary classical background and a curved spacetime.[11]

The resolution of the gravitational problem, even for weak fields, is central to the basic issues of the finiteness of field theory, because no really attractive solution to the infinities of quantum gravity has yet been found using pointlike gravitons. Thus, a solution to finite quantum gravity will affect the rest of field theory in a fundamental way. The basic solution of quantum gravity, using the extended graviton, leads to a nonlocal modification of the field theories of the other forces of nature. Hopefully, this fundamental modification will produce testable experimental predictions.

The Lagrangian density for Einstein's theory can be taken as[12,13]

$$\mathcal{L}_E = \kappa^{-2}\mathbf{g}^{\mu\nu}\left[\begin{Bmatrix}\alpha\\ \mu\beta\end{Bmatrix}\begin{Bmatrix}\beta\\ \nu\alpha\end{Bmatrix} - \begin{Bmatrix}\alpha\\ \mu\nu\end{Bmatrix}\begin{Bmatrix}\beta\\ \alpha\beta\end{Bmatrix}\right] . \tag{4.1}$$

Here, $\kappa = (16\pi G)^{1/2}$, where G is the Newtonian gravitational constant, $g = |\mathbf{g}^{\mu\nu}|$, $g^{\mu\nu} = (-g)^{-1/2}\mathbf{g}^{\mu\nu}$ and

$$\begin{Bmatrix}\alpha\\ \mu\nu\end{Bmatrix} = \tfrac{1}{2}g^{\alpha\lambda}(\partial_\nu g_{\mu\lambda} + \partial_\mu g_{\nu\lambda} - \partial_\lambda g_{\mu\nu}) . \tag{4.2}$$

Consider now an infinitesimal linear transformation

$$\delta x^\mu = -\eta^{\mu\lambda}\delta\omega_{\lambda\nu}x^\nu , \tag{4.3}$$

where the coefficients $\omega_{\mu\nu}$ satisfy $\omega_{\mu\nu} = -\omega_{\nu\mu}$. We define

$$\begin{aligned} \mathbf{t}^{\mu\nu,\rho} &= -\frac{\delta\mathbf{g}^{\alpha\beta}}{\delta\omega_{\mu\nu}}\frac{\partial\mathcal{L}_E}{\partial\mathbf{g}^{\alpha\beta}/\partial x^\rho} \\ &= 2\frac{\partial\mathcal{L}_E}{\partial\mathbf{g}^{\alpha\beta}/\partial x^\rho}(\eta^{\alpha\mu}\mathbf{g}^{\beta\nu} - \eta^{\alpha\nu}\mathbf{g}^{\beta\mu}) . \end{aligned} \tag{4.4}$$

The symmetrical gravitational pseudotensor density has the form

$$\begin{aligned} \mathbf{t}^{\mu\nu} &= \eta^{\mu\lambda}\mathbf{t}_\lambda^\nu \\ &= \eta^{\mu\lambda}\theta_\lambda^\nu + \tfrac{1}{2}\partial_\rho(\mathbf{t}^{\mu\nu,\rho} + \mathbf{t}^{\rho\mu,\nu} + \mathbf{t}^{\rho\nu,\mu}) , \end{aligned} \tag{4.5}$$

where

$$\theta_\nu^\mu = \frac{\partial\mathcal{L}_E}{\partial\mathbf{g}^{\alpha\beta}/\partial x^\mu}\partial\mathbf{g}^{\alpha\beta}/\partial x^\nu - \delta_\nu^\mu\mathcal{L}_E . \tag{4.6}$$

The total energy-momentum tensor is obtained by adding the matter-field energy-momentum-tensor density $\mathbf{T}^{\mu\nu}$ to (4.5). This gives

$$\Theta^{\mu\nu} = \eta^{\mu\lambda}(\mathbf{T}_\lambda^\nu + \mathbf{t}_\lambda^\nu) . \tag{4.7}$$

We shall now use the relation

$$\partial_\lambda\mathbf{R}_\nu^{\lambda\mu} = \tfrac{1}{2}(\mathbf{T}_\nu^\mu + \theta_\nu^\mu) , \tag{4.8}$$

where

$$\mathbf{R}_\nu^{\lambda\mu} = \frac{\partial\mathcal{L}_E}{\partial\mathbf{g}^{\nu\rho}/\partial x^\lambda}\mathbf{g}^{\mu\rho} - \tfrac{1}{2}\delta_\nu^\mu\frac{\partial\mathcal{L}_E}{\partial g^{\rho\sigma}/\partial x^\lambda}\mathbf{g}^{\rho\sigma} . \tag{4.9}$$

We also have

$$t^{\mu\nu,\rho}=-2(\mathbf{R}^{\rho\mu}_{\lambda}\eta^{\nu\lambda}-\mathbf{R}^{\rho\nu}_{\lambda}\eta^{\mu\lambda})\ . \tag{4.10}$$

From these results, we obtain

$$\partial_\alpha\partial_\beta(\eta^{\alpha\beta}\mathbf{g}^{\mu\nu}-\eta^{\nu\beta}\mathbf{g}^{\mu\alpha}+\eta^{\mu\nu}\mathbf{g}^{\alpha\beta}-\eta^{\mu\beta}\mathbf{g}^{\alpha\nu})=\kappa^2\Theta^{\mu\nu}\ . \tag{4.11}$$

Let us assume the de Donder coordinate conditions[14]

$$\partial_\nu\mathbf{g}^{\mu\nu}=0\ . \tag{4.12}$$

Then the field equations of gravitation become

$$\Box\mathbf{g}^{\mu\nu}=\kappa^2\Theta^{\mu\nu}\ . \tag{4.13}$$

On account of the conservation equations

$$\partial_\nu\Theta^{\mu\nu}=0\ , \tag{4.14}$$

the coordinate conditions (4.12) and the field equations (4.13) are compatible.

After some reduction, the Lagrangian becomes

$$\mathcal{L}_E=\frac{1}{4\kappa^2}\mathbf{g}^{\mu\nu}g^{\alpha\lambda}g^{\beta\rho}(2\partial_\beta g_{\mu\lambda}\partial_\alpha g_{\nu\rho}-2\partial_\alpha g_{\mu\nu}g_{\rho\lambda}-\partial_\mu g_{\beta\lambda}\partial_\nu g_{\alpha\rho}+\partial_\mu g_{\beta\rho}\partial_\nu g_{\alpha\lambda})\ . \tag{4.15}$$

To implement the de Donder harmonic gauge condition (4.12), we add to $\mathcal{L}_E$ the "noncovariant" term

$$\mathcal{L}'=\frac{1}{2\kappa^2}\eta_{\mu\nu}\partial_\alpha\mathbf{g}^{\mu\alpha}\partial_\beta\mathbf{g}^{\nu\beta}\ . \tag{4.16}$$

The total gravitational Lagrangian now becomes

$$\mathcal{L}_G=\mathcal{L}_E+\mathcal{L}'\ . \tag{4.17}$$

We expand $\mathbf{g}^{\mu\nu}$ about Minkowski flat space:

$$\mathbf{g}^{\mu\nu}=\eta^{\mu\nu}+\kappa\gamma^{\mu\nu}\ . \tag{4.18}$$

Here, $\gamma_{\mu\nu}$ is given by

$$\gamma_{\mu\nu}(x)=(2\pi)^{-3/2}\int\frac{d^3\mathbf{p}}{(2\omega)^{1/2}}[a_{\mu\nu}(\mathbf{p})e^{-ip\cdot x}+a^*_{\mu\nu}(\mathbf{p})e^{ip\cdot x}]\ . \tag{4.19}$$

$s_{\mu\nu}(x)$ is the nonlocal graviton field operator, given in terms of the local point graviton field $\gamma_{\mu\nu}(x)$ by the equation in the B representation:

$$s_{\mu\nu}(x)=\int d^4y\,B(x-y)\gamma_{\mu\nu}(y)=B(\Box_x)\gamma_{\mu\nu}(x)\ . \tag{4.20}$$

Since we have chosen $\mathbf{g}^{\mu\nu}$ as the interpolating field, the additional $\mathcal{L}'$ only modifies the free part of the Lagrangian, which takes the form

$$\mathcal{L}_{G0}=\tfrac{1}{4}[2\eta_{\alpha\beta}\partial_\nu\gamma^{\beta\mu}(x)\partial_\mu\gamma^{\alpha\nu}(x)-\eta^{\mu\nu}\eta_{\rho\alpha}\eta_{\lambda\beta}\partial_\nu\gamma^{\lambda\alpha}(x)\partial_\mu\gamma^{\rho\beta}(x)+\tfrac{1}{2}\eta^{\mu\nu}\eta_{\lambda\rho}\eta_{\alpha\beta}\partial_\mu\gamma^{\lambda\rho}(x)\partial_\nu\gamma^{\alpha\beta}(x)+\tfrac{1}{2}\eta_{\mu\nu}\partial_\alpha\gamma^{\mu\alpha}(x)\partial_\beta\gamma^{\nu\beta}(x)]\ . \tag{4.21}$$

The point-particle graviton field $\gamma_{\mu\nu}(x)$ satisfies the free field equation of motion

$$\Box\gamma_{\mu\nu}=0 \tag{4.22}$$

and the coordinate condition

$$\partial_\nu\gamma^{\mu\nu}=0\ . \tag{4.23}$$

The interaction Lagrangian takes the form

$$\mathcal{L}_I=\frac{\kappa}{4}[\eta_{\rho\alpha}\eta_{\lambda\beta}s^{\mu\nu}(x)\partial_\nu s^{\lambda\alpha}(x)\partial_\mu s^{\rho\beta}(x)-\tfrac{1}{2}\eta_{\lambda\rho}\eta_{\alpha\beta}s^{\mu\nu}(x)\partial_\mu s^{\lambda\rho}(x)\partial_\nu s^{\alpha\beta}(x)+2\eta_{\alpha\lambda}\eta_{\beta\rho}s^{\alpha\beta}(x)\partial_\nu s^{\lambda\mu}(x)\partial_\mu s^{\rho\nu}(x)+\eta^{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\lambda}\eta_{\eta\rho}s^{\lambda\rho}(x)\partial_\mu s^{\alpha\beta}(x)\partial_\nu s^{\sigma\eta}(x)-2\eta^{\mu\nu}\eta_{\rho\eta}\eta_{\alpha\lambda}\eta_{\beta\sigma}s^{\alpha\beta}(x)\partial_\nu s^{\lambda\eta}(x)\partial_\mu s^{\sigma\rho}(x)]+\tilde{\mathcal{L}}_I(\kappa)+O(\kappa^2)\ , \tag{4.24}$$

where $\tilde{\mathcal{L}}_I(\kappa)$ contains graviton diagrams that restore the gauge invariance to order κ.

The free Lagrangian that arises from $\mathcal{L}_E$ is invariant under the infinitesimal gauge transformation

$$\gamma_{\mu\nu}(x)\to\gamma_{\mu\nu}(x)+\partial_\mu\lambda_\nu(x)+\partial_\nu\lambda_\mu(x)\ , \tag{4.25}$$

where $\lambda_\mu(x)$ is an arbitrary vector field. By adding the noncovariant (noncovariant with respect to general coordinate transformations) piece $\mathcal{L}'$ to the Lagrangian, we break the gauge symmetry in the usual way.

We can construct Feynman rules for the gravitons and use (4.24) to solve for the S matrix, in the same way as for the scalar nonlocal field $\Phi(x)$. A chronological contraction rule for the gravitons is defined by

$$D^c_{s\mu\nu\lambda\rho}=s^{\cdot}_{\mu\nu}(x)s^{\cdot}_{\lambda\rho}(y)=B(\Box_x)B(\Box_y)\langle 0|T[\gamma_{\mu\nu}(x)\gamma_{\lambda\rho}(y)]|0\rangle=(\eta_{\mu\lambda}\eta_{\nu\rho}+\eta_{\mu\rho}\eta_{\nu\lambda}-\eta_{\mu\nu}\eta_{\lambda\rho})\frac{1}{(2\pi)^4 i}\times\int\frac{d^4k\,\Pi(k^2)}{-k^2-i\epsilon}e^{ik\cdot(x-y)}\ . \tag{4.26}$$

To avoid any graviton states with negative probabilities occurring, we must introduce an indefinite metric in the Hilbert space for the components γ_{i0} $(i=1,2,3)$ and γ where $\gamma=\eta^{\mu\nu}\gamma_{\mu\nu}$. This involves imposing supplementary Gupta conditions of the form[13]

$$\partial_\nu\gamma^{(+)\mu\nu}|\alpha\rangle=0 \tag{4.27}$$

and

$$\gamma^{(+)}|\alpha\rangle=0\ , \qquad (4.28)$$

for an arbitrary state vector $|\alpha\rangle$. These conditions guarantee that only nine types of gravitons exist. In a pure gravitational field only γ_{12} and γ_{11} gravitons exist, because the other types of gravitons will be excluded by the supplementary Gupta conditions. The vacuum state must satisfy the conditions

$$\gamma_{\mu\nu}^{(+)}|0\rangle=0,\ \ \gamma^{(+)}|0\rangle=0 \qquad (4.29)$$

and only two types of gravitons can physically exist in a gravitational field.

The function $\Pi(k^2)$ satisfies conditions (1)–(4), in (2.10), and an analysis of the Feynman graviton diagrams shows that they are all finite and that the S matrix satisfies the unitarity condition to all orders in perturbation theory. However, as Feynman[13] has pointed out, in order to satisfy unitarity for the closed graviton loops, we must include ("fictitious") Faddeev-Popov ghost particles.[9] A calculation of the commutator of the nonlocal graviton fields gives

$$[s_{\mu\nu}(x),s_{\lambda\rho}(y)]_-=(\eta_{\mu\lambda}\eta_{\nu\rho}+\eta_{\mu\rho}\eta_{\nu\lambda}-\eta_{\mu\nu}\eta_{\lambda\rho})[\Pi(0)]^2\Delta(x-y)\ . \qquad (4.30)$$

Thus, the nonlocal graviton field satisfies a condition of microcausality for suitably chosen entire functions in the propagators for spacelike separations greater in magnitude than the Planck length $\sqrt{G}=1.6\times10^{-33}$ cm.

It would be interesting to repeat Duff's[15] loop-graph calculation using nonlocal perturbation theory, to see what quantum loop corrections are obtained for the tree-graph "extended particle" Schwarzschild solution. It is interesting that divergences occur in Duff's calculations already at the tree-graph level, unless the material particles are assumed to have a finite structure. In our theory, this finite particle structure appears as a basic feature and, consequently, we do not have such divergences occurring at the classical, tree-graph level of the calculations. Moreover, the nonlocal formalism provides a meaningful way to perform the finite loop calculations to all orders in quantum gravity.

V. CONCLUSIONS

By assigning a finite size to every particle, we have succeeded in developing a field-theory formalism that leads to a finite perturbation theory and a unitary S matrix for the basic massive spin-0 scalar fields, and for the massless gauge spin-1 and spin-2 fields of nature. The causality properties of a strictly localizable field were extended so that a condition of microcausality for the fields was satisfied. The other requirements of axiomatic field theory, such as the existence of a scattering theory, can also be included in the extended nonlocal field theory. A breakdown of causality must be expected at the scale of the size of the particle; e.g., for gravity this would occur at the Planck length. If these noncausal effects are small enough, then such a phenomenon is not unacceptable.

The nonlocal version of quantum gravity, based on Einstein's gravitational theory can be extended to the more general theory of spacetime described by the NGT (Ref. 10).

Now that we have a consistent, finite theory of gravitation that is solved using perturbation theory, how can we check that nonlocality is the mechanism that produces a finite field theory? An interesting possibility is to investigate the singularity structure of gravitational theory and the effects of nonlocal field theory on the formation of black-hole event horizons. However, such investigations do not lead to any direct experimental tests and they may suffer from the limitations of using perturbation theory. A calculation of the radiative corrections in the nonlocal electroweak theory could produce results that could be tested in high-energy accelerators. There is at present no experimental check of the radiative corrections in GSW theory involving Higgs mesons. The latter could differ in their quantitative behavior from the radiative corrections in the nonlocal electroweak theory. The problem of the origin of the fermion masses has to be investigated.

An important success of the nonlocal theory is that it resolves the field-theory triviality problem, first raised by Landau,[3] and the gauge hierarchy problem in a natural and fundamental way without introducing new observable particles as in the case of technicolor models, which introduce a new regime of strongly interacting particles at $E\sim1$ TeV, and supersymmetric models with their predicted new supersymmetric partners.[16]

With the failure of point-particle field theory to resolve the infinities in standard quantum gravity, we seem to be forced into a theoretical picture in which particles are topologically extended objects and field theory is intrinsically slightly nonlocal. The field theory developed here is an example of a self-consistent field theory, based on nonlocal fields, that can remove the unsatisfactory features of standard strictly local field theory. More work remains to be done to investigate many of the fundamental ramifications of such a theory and its implications for future particle physics.

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