Anisotropy dissipation in the early Universe: Finite-temperature effects reexamined

Juan Pablo Paz*
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
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We reexamine the role that finite-temperature effects play in the dissipation of the initial anisotropy of the Universe. The issue was previously studied both in the zero- and finite-temperature cases. Our results show that the finite-temperature corrections do not introduce new qualitative behavior and that the anisotropy dissipation scenario is approximately the same as in the zero-temperature case. We use a formalism that is based on the extension to finite temperature of the closed-time-path method (it can be used to study systems out of thermal equilibrium). The effective equations that we obtain are real and causal. We compare our results with others obtained in the zero- and finite-temperature cases. We discuss some qualitative differences between the formalisms that are more frequently used to study real-time processes at finite temperature.

I. INTRODUCTION

In recent years there has been some interest in studying the modifications that quantum effects can produce on the classical predictions of general relativity. It was thought that these effects could provide a way out of the singularity theorems that had been proven in a classical context.[1] In the absence of a complete quantum theory that includes quantum-gravity effects, there are some heuristic approaches to these problems that find its justification mainly through the use of "common sense." The semiclassical approach[2] consists in assuming the existence of a background space-time with a metric that is treated classically. In this background, all the propagating fields are studied using the postulates of quantum mechanics. The classical background metric is coupled with the quantum fields through a set of "semiclassical Einstein's equations," the source of which is the expectation value of the energy-momentum-tensor operator of all the existing fields (eventually including the contribution of the metric fluctuations). Although it is still unclear how these semiclassical equations arise from an approximation to the full quantum gravity theory, "common sense" indicates that such a limit could exist (see Ref. 3 for an extended discussion on these issues). Many authors have worked on this topic studying the kind of solutions that the semiclassical equations possess. The series of papers by Anderson[4] can be used as a catalog of the properties that these solutions have in some particularly simple cases. When the fields that are present in the Universe are assumed to be free, massless, and conformally coupled to the space-time curvature, the semiclassical equations can be written explicitly as an ordinary local differential equation (if one also assumes that the Universe is a Robertson-Walker one). There are some free parameters in the equations that can be used to play with. They are related not only to the number of fields present in the model but also to the renormalized constant associated with the R^2 term that should be present in the gravitational classical action. The series of studies summarized in Anderson's papers conclude that among the solutions that exhibit an asymptotic classical behavior (in the future) there could exist families of solutions of the semiclassical equations that do not have an initial singularity. There exist also many families of asymptotically classical cosmological solutions without event horizons.

Thus, quantum effects can play an important role not only in softening the initial singularity but also in explaining some of the observable features of our present Universe. Many years ago, it was suggested that these effects could be responsible for the isotropy of our Universe (or, at least, they could be one of those responsible). Zel'dovich[5] proposed a mechanism that could have helped to dissipate the initial anisotropy of the Universe. The basic idea of the model was very simple: the anisotropic expansion of the Universe can produce a large amount of particles even for free, massless, and conformally coupled fields. These particles react back on the space-time metric via the semiclassical Einstein equations. The back reaction can introduce an effective dissipation and help to damp away the initial anisotropy. The mechanism was first studied by Zel'dovich and Starobinski[6] and later by Hu and Parker.[7] A systematic study of this same issue was done by Hartle and Hu.[8] In a series of papers these authors computed the effective action for the space-time metric (that incorporate the quantum effects of the matter fields) in an expansion in powers of the anisotropy. Writing the metric as

$$ds^2=a^2(\eta)(d\eta^2-e^{2\beta}{}_{ij}dx^i dx^j)\ , \tag{1.1}$$

where $\beta=\beta(\eta)$ is a 3×3 matrix, they obtained the effective action up to second order in β. Using this result, the effective equations that govern the dynamical evolution of the anisotropy were written and solved numerically (see also Ref. 9). The result was that the anisotropy can be, in fact, dissipated away by the quantum effects in an efficient manner.

After these pioneering works, some other results appeared. A conceptually interesting approach was followed by Calzetta and Hu in Ref. 10. In Ref. 8 the standard functional techniques of quantum field theory were used in order to compute the effective action. These techniques were developed in order to study scattering pro-

cesses and are not designed to deal with problems that are formulated in terms of initial-value-like boundary conditions. As a consequence, the "effective equations" that are derived by varying the usual effective action are not causal. Moreover, the effective action can be complex (its imaginary part is related with the probability of creating particles) and, thus, the effective metric found in Ref. 8 can be a complex quantity without a clear physical interpretation. The dissipative process would be much more understandable when studied using a method that allows the use of boundary conditions posed in terms of initial values. This method was first developed by Schwinger and Keldish.[11] It is known as the closed-time-path (CTP) formalism. Calzetta and Hu applied these formalisms to the anisotropy dissipation problem and found effective equations that govern the dynamical evolution of the (real) effective metric. These equations are real and causal and when written in a Fourier representation the dissipative effects can be analyzed in some simple cases. In fact, the resulting equations have a form that resembles that of a generalized damped oscillator (see Ref. 12). The energy dissipated away during all the process can be shown to be exactly the same as the energy carried by the created particles. A similar approach based on the use of the CTP formalism was followed by Jordan in Ref. 13. He also obtained the effective equation for the evolution of the anisotropy. In his paper, he showed that the effective equations have unstable modes. The existence of these modes makes the numerical resolution of the equations a rather difficult task. The instabilities do not show up when one works in the in-out formalism since from the beginning one imposes in-out boundary conditions that discard runaway solutions. The CTP formalism, thus, has the virtue of generating real and causal equations that allow a more intuitive interpretation but has also the major drawback of producing equations with runaway solutions. As emphasized by Jordan, the existence of these unstable modes is telling us something about the inadequacy of the approximation used to study the problem (one loop).

All the previous results were derived assuming that the quantum state of all the matter of the Universe was a pure one. However, as the early Universe seemed to have been a very hot "place," finite-temperature effects are worth studying. This was noticed by Amsterdamski[14] who analyzed the influence that finite-temperature effects can have on the anisotropy dissipation scenario. Surprisingly, the conclusion of his work was that the finite-temperature corrections can drastically change the behavior of the solution to the effective field equations. Although the anisotropy is still dissipated away, the role that the particle creation mechanism has in this process was obscured (Amsterdamski obtained some solutions that at finite temperature exhibited a less rapid anisotropy dissipation with a much bigger amount of created particles than in the zero-temperature case). The finite-temperature effects were incorporated into the formalism by using the thermo field dynamics (TFD) approach.[15] This well-known method (that implicitly imposes in-out-like boundary conditions) is believed to be suitable when one is interested in analyzing real-time processes in a system at finite temperature.

In this paper, we will restudy the effect of finite temperature on the anisotropy damping mechanism by using a different approach. The method that we are going to use was designed to study systems out of thermal equilibrium. One of the points that we want to stress here is that TFD (as presented in Ref. 16 and used in Ref. 14) is a method that is only applicable if thermal equilibrium is maintained.[17] This can be understood as follows. In TFD, the thermal averages are computed as vacuum expectation values by adding spurious degrees of freedom and tracing them out. The notion of a "thermal vacuum" plays a central role and the techniques of ordinary quantum field theory (QFT) at zero temperature (namely, perturbation theory, Feynman rules, etc.) are used in order to compute matrix elements of operator products between in and out thermal vacua. However, the results obtained in this way are physically meaningful only if the in thermal vacuum is the same state as the out thermal vacuum. Otherwise, the trick does not work. The thermal equilibrium needs to be preserved throughout the evolution of the system. We will give some other arguments on the nonapplicability of the usual functional version of TFD to study systems out of equilibrium later [all the conclusions that concern TFD are also applicable to other methods that in essence are equivalent to TFD such as the one presented by Niemi and Semenoff (NS) in Ref. 18].

Returning to the anisotropy damping problem, we should say that in this case, as well as in many other problems involving quantum fields in curved spaces, thermal equilibrium is not maintained. The reason is that the particles produced by metric evolution are not distributed in general according to a thermal spectrum. These kinds of problems should be studied by using techniques that can deal with a system out of thermal equilibrium. This is true in general although in many cases it is possible to assume the existence of a quasiequilibrium situation and use approximated schemes such as those developed by Hu in Ref. 19. In our case, the equilibrium is maintained "up to second order in the anisotropy." Thus, the calculations that we are going to present below are not very much affected by the nonequilibrium situation since we will work up to this order assuming that the anisotropy is small. However, the use of the nonequilibrium method allows us to obtain real and causal equations (with the same benefits and disadvantages as in the zero-temperature case). The method that we are going to use is an extension of the zero-temperature CTP formalism. It can deal with nonequilibrium situations since the only input that it needs is information about the state of the system at a given instant. The method can be applied for general initial states and its main features were described by Calzetta and Hu in Ref. 20. If one studies a system whose density matrix describes an initial thermal equilibrium state, the method reduces to the one presented by Weiss and Semenoff in Ref. 21.

We think that there is some confusion in the literature about the applicability of the different finite-temperature techniques. One can often read that TFD and NS methods are equivalent to other closed-time-path formal-

isms such as one that we use here or the ones described by Weiss and Semenoff,[21] Zhao *et al.*,[11] etc. We hope that this paper will help to clarify the issue. The first class of formalisms can deal only with equilibrium systems. These methods are constructed by using the fact that the system is in thermal equilibrium both in the far past and the far future. The second class of methods can be used to deal with nonequilibrium situations. No assumption is made concerning the state of the system in the far future region. This is a conceptual difference between the two classes of methods. In fact even when equilibrium is maintained, the two classes of formalisms are different in the same sense that the zero-temperature in-out formalism is different from the in-in one. In fact, by examining the zero-temperature limit one can realize that the TFD-NS methods yield to the usual in-out formalism of QFT at zero temperature. The reason is that, as we mentioned, in these approaches it is necessary to assume the existence of an out vacuum state (that is also assumed to be equal to the in vacuum state). It is worth noting that the effective equations derived from these methods are noncausal (since one is putting in-out boundary conditions from the beginning). The second class of formalisms (CTP, Weiss and Semenoff, and Zhao *et al.*) yield to the in-in (Schwinger-Keldish) formalism of QFT in the zero-temperature limit. Thus, the effective equations derived in this case are real and causal.

The paper is organized as follows. In Sec. II we present our formalism. First, we describe the techniques that we are going to use. No original results are included here. The presentation is made trying to avoid the technical complications introduced by the matrix structure of the CTP method. We prove a simple lemma that allows us to easily compute the effective action from which the effective equations can be derived. The lemma states the following result: In the one-loop approximation, the effective field equations in the CTP formalism can be derived from an effective action Γ, that can be written

$$\Gamma = S - \frac{i}{2}\mathrm{tr}\ln G + O(\mathbf{h}) , \tag{1.2}$$

where

$$G(x,x') = \langle T[\phi(x)\phi(x')] \rangle . \tag{1.3}$$

This result is also true in the non-zero-temperature case. It allows us to forget about some technical complications introduced by the matrix structure of the CTP formalism and makes the calculations needed to compute the CTP effective action very similar to the ones done in the in-out approach (a similar result was obtained by Jordan[22] in the zero-temperature case). Finally, in this section we discuss some differences between the finite-temperature formalisms.

In Sec. III we are going to face the study of the anisotropy damping problem. Assuming that the Universe is filled only with a single scalar field (massless and conformally coupled) we compute the CTP effective action at finite temperature up to second order in the anisotropy and derive the effective cosmological equations. This equations are real and causal. The zero-temperature limit of them coincides with the system obtained by Calzetta and Hu in Ref. 10 and by Jordan in Ref. 13. We compare our results with the ones obtained by Amsterdamski.[14] There are conceptual and technical differences between these results and ours. The technical ones are less interesting but worth noting. The change in the behavior of the solutions of the effective equation found in Ref. 14 is not found here due to a difference in the sign in some terms of the equations (a technical problem that is present in Amsterdamski's paper). The conceptual differences are related to the difference between our approach and the one based on the use of thermo field dynamics. Our main conclusion concerning the anisotropy damping issue is that finite-temperature effects do not modify the qualitative features of the mechanism. Technically, we can say that there are local and nonlocal finite-temperature corrections to the effective equations. The local corrections do not incorporate new effects (this is where the difference in the signs of some terms is really important and the origin of Amsterdamski's wrong results). The nonlocal corrections can produce more dissipation. It is possible to relate the "amount of dissipation" in the zero-temperature case with the finite-temperature one. The easiest way to do it is by writing the evolution equation for the anisotropy in a Fourier representation. Doing this, we can relate the dissipative effects with the imaginary terms present in this equation. As was done by Calzetta and Hu in the zero-temperature case, one can define a "viscosity function" $\nu(\omega)$ by writing the imaginary term that multiplies $\beta_{ij}(\omega)$ as $i\omega\nu(\omega)\beta_{ij}(\omega)$. The viscosity function is the one that determines, for example, the linear response behavior of the system and can be readily identified as the source of the dissipative effects. We will show that the viscosity function at finite temperature can be related to the one at zero temperature as

$$\nu_T(\omega) = \nu_0(\omega)\coth\frac{|\omega|}{2\mathbf{T}} . \tag{1.4}$$

The finite-temperature equations have the same drawbacks as the ones obtained in the zero-temperature case in Refs. 10 and 13, namely, the existence of unstable modes that prevents a successful numerical integration. In Sec. III we comment more on this issue.

In Sec. IV we briefly summarize our results.

II. CLOSED-TIME-PATH FINITE-TEMPERATURE FORMALISM

In this section we are going to introduce the main features of our formalism. As we mentioned in the Introduction it is a generalization of the closed-time-path method. So, let us review the main features of the zero-temperature case. In doing this, we are also going to show how some useful results concerning the computation of the CTP effective action arise. We hope that these results will be useful in order to demystify the CTP formalism and to convince people that it is as easy (or as difficult) as the ordinary in-out one. The experienced reader who is not interested in learning simple techniques on the closed-time-path formalism can skip this part.

The generating functional for expectation values at

zero temperature is defined not as the vacuum-to-vacuum persistence amplitude under the action of a single source but as the overlap between the states obtained by evolving the initial vacuum under the action of two different sources. Thus

$$Z[J_1,J_2]={}_{J_2}\langle 0|0\rangle_{J_1} = \left\langle 0 \left| \bar{T}\left[\exp\left[-i\int {}^{t}J_2\phi\right]\right] \times T\left[\exp\left[i\int {}^{t}J_1\phi\right]\right] \right| 0 \right\rangle , \tag{2.1}$$

where t is an arbitrary time. This generating functional can be used to obtain all kinds of two-point functions (time ordered, antitime ordered, etc.) by deriving with respect to the currents and then replacing $J_i=0$. For example,

$$G_{11}(x,x')=(-i)\frac{\delta^2 Z[J_1,J_2]}{\delta J_1(x)\delta J_1(x')}\bigg|_{J_1=J_2=0} =i\langle T[\Phi(x)\Phi(x')]\rangle , \tag{2.2a}$$

$$G_{22}(x,x')=(-i)\frac{\delta^2 Z[J_1,J_2]}{\delta J_2(x)\delta J_2(x')}\bigg|_{J_1=J_2=0} =i\langle \bar{T}[\Phi(x)\Phi(x')]\rangle , \tag{2.2b}$$

$$G_{12}(x,x')=(+i)\frac{\delta^2 Z[J_1,J_2]}{\delta J_1(x)\delta J_2(x')}\bigg|_{J_1=J_2=0} =i\langle \Phi(x')\Phi(x)\rangle =G_{21}(x',x) . \tag{2.2c}$$

The generating functional has a standard functional representation that can be obtained from (2.1) by using the functional integral representation of the matrix element of a time-ordered product of Heisenberg operators. The equation for $Z[J_1,J_2]$ is

$$Z[J_1,J_2] = \int \mathbb{D}\Phi_1\mathbb{D}\Phi_2 \exp(\{S[\Phi_1]-S^*[\Phi_2] +(J_1,\Phi_1)-(J_2,\Phi_2)\}) =\exp(+iW[J_1,J_2]) . \tag{2.3}$$

Here, the functional integral should be made by summing over all the field histories Φ_1 and Φ_2 that coincide over some spacelike hypersurface (usually taken as $t=T=+\infty$). The name closed time path found its origin in this last formula. In fact, the double functional integral can be thought of as being a single functional integral over a field that takes values in a closed "temporal" axis. This field is equal to Φ_1 when $-\infty<t<T$ and to Φ_2 when t goes back from T to $-\infty$. This way of looking at the functional integral is not always the more convenient at the time of computing. There is another approach that is worth using for computational purposes. From Eq. (2.3) we can notice that $Z[J_a]$ could be interpreted as the generating functional for the theory of two fields with a classical action defined as

$$S[\Phi_1,\Phi_2]=S[\Phi_1]-S^*[\Phi_2] \tag{2.4}$$

(the notation S^* means that the sign of the $i\epsilon$ factor usually added to the m^2 term should be also reversed).

We must stress that in this formalism we did not double the number of degrees of freedom. Although we can work as if we were dealing with the theory of two fields, we must remember that we have to impose boundary conditions on the functional integral. In fact, for a free field theory, i.e., for a theory with a quadratic classical action, we can show that taking into account the boundary conditions, the generating functional can be written as

$$Z[J_1,J_2]=\exp\left[\frac{i}{2}(J_a G_{ab} J_b)\right] , \tag{2.5}$$

where we are summing over the index a and b and the elements of the matrix propagator are the ones defined in (2.3).

The usual techniques derived in the context of the *in-out* formulation can be used almost directly in the CTP formalism by using this two-component notation. We can use many of the results obtained when working in the in-out formalism with a doublet field. The contact with the original single field theory is made when one relates the propagator matrix G_{ab} with the expectation values defined in (2.2). This matrix structure and the proliferation of many indices is something that cause some concern but, as we are going to show, the technical complications can be avoided in some cases. The CTP formalism can be used in order to derive effective equations for expectation values. They are obtained from an object that is analog to the usual effective action.

By doing a Legendre transform of $W[J_a]$ we can define an *in-in effective action* (or CTP effective action) as

$$\Gamma[\phi_a]=W[J_a]-(J_a\phi_a) , \tag{2.6}$$

where

$$\phi_a=\frac{\delta W[J_b]}{\delta J_a(x)} . \tag{2.7}$$

Note that when $J_a=0$, ϕ_a is nothing but the mean value of the field. The effective equation for the mean value is obtained as

$$\frac{\delta\Gamma[\phi_1,\phi_2]}{\delta\phi_1(x)}\bigg|_{\phi_1=\phi_2=\phi(x)}=0 . \tag{2.8}$$

So, in order to obtain the equation for ϕ, we must derive the effective action with respect to one of the fields treating the other as an independent variable and then replace $\phi_1=\phi_2=\phi$. Thus the effective equation is not derived directly from a variational principle formulated entirely in terms of $\phi(x)$.

The computation of the effective action is not an easy

task. The background-field method provides us with a recipe that is particularly useful if we want to compute the effective action in a loop expansion. The recipe states (see Ref. 23) that if the classical action is $S[\Phi]$, the effective action $\Gamma[\phi]$ can be written as

$$\Gamma[\phi]=S[\phi]-\frac{i}{2}\mathrm{tr}(\ln G)+O(\mathbf{h}) , \tag{2.9}$$

where G is the free propagator of the theory defined by the classical action

$$\bar{S}[\chi]=S[\phi+\chi]-S[\phi]-\int dx \frac{\delta S[\phi]}{\delta\phi(x)}\bigg|_{\phi(x)}\chi(x) . \tag{2.10}$$

It is worth noting that the propagator defined by (2.10) is a functional of the background field $\phi(x)$.

We can apply this recipe to compute the closed-time-path effective action. In that case our starting point will be the classical action (2.4). The propagator that appears in (2.9) will be a matrix that solves the system

$$\mathbf{A}_{ab}G_{bc}=-i\delta_{ac} , \tag{2.11a}$$

where

$$\mathbf{A}_{11}=\mathbf{A}_{11}(\phi_1)=\frac{\delta^2 S}{\delta\phi(x)\delta\phi(x')}\bigg|_{\phi=\phi_1} , \tag{2.11b}$$

$$\mathbf{A}_{22}(\phi_2)=-\mathbf{A}_{11}^{*}(\phi_2) , \tag{2.11c}$$

and

$$\mathbf{A}_{12}=\mathbf{A}_{21}=0 . \tag{2.11d}$$

A close observation of Eqs. (2.11) can yield the conclusion that we stated in the Introduction. If we are interested in deriving the effective equations we can use Eq. (2.9) but forget about the matrix structure of the propagator. In fact, the equations of motion for $\phi(x)$ are obtained by applying (2.8) to (2.9). The derivative of the first term will give the usual classical field equation. The second term can be derived as

$$\frac{\delta}{\delta\phi_1(x)}[\mathrm{tr}(\ln G)]=-\int dy\, dz\, G_{ab}(z,y)\frac{\delta}{\delta\phi_1(x)}G_{ba}^{-1}(y,z) \tag{2.12}$$

$$=-\int dy\, dz\, G_{11}(z,y)\frac{\delta}{\delta\phi_1(x)}G_{11}^{-1}(y,z)$$

$$=\frac{\delta}{\delta\phi_1(x)}[\mathrm{tr}(\ln G_{11})] . \tag{2.13}$$

As a consequence of this equation, we conclude that the CTP effective action can be approximated as

$$\Gamma[\phi_1,\Phi_2]=S[\phi_1]-\frac{i}{2}\mathrm{tr}(\ln G_{11})+F+O(\mathbf{h}) , \tag{2.14a}$$

where we can be assured that

$$\frac{\delta F}{\delta\phi_1(x)}\bigg|_{\phi_1=\phi_2=\phi(x)}=0 . \tag{2.14b}$$

Thus, we demonstrated (in the zero-temperature case) the lemma announced in the Introduction and obtained an equation that very much resembles the one that we get in the usual in-out approach: i.e.,

$$\Gamma[\phi]=S[\phi]-\frac{i}{2}\mathrm{tr}(\ln G_F)+O(\mathbf{h}) .$$

By remembering (2.2a) the similarity is clear but it is worth noting the difference. The only, but crucial, one is in the boundary condition that defines the propagator (2.3a) as an expectation value and not as an in-out matrix element. The difference between the in-out and the CTP result is much more clearly seen when one evaluates the propagator perturbatively. Unfortunately, this is an unavoidable task since we can solve Eq. (2.11) only in some extremely simple situations (free fields in flat space-time, etc.). When the propagator cannot be found exactly one should make use of some perturbative expansion that can be implemented in the following way. We can write the operator $\mathbf{A}$ appearing in Eq. (2.11) as

$$\mathbf{A}=A+V , \tag{2.15}$$

where A is an operator whose propagators are known and V contains all the perturbative terms. Let us define the zero-order propagators $\mathcal{G}_{ab}$ as

$$A_{ab}\mathcal{G}_{bc}=\delta_{ac} . \tag{2.16}$$

Taking into account Eqs. (2.11) we can be assured that in general the matrix V will be diagonal ($V_{ab}=V_a\delta_{ab}$) and that the matrix element V_a will depend on the background field ϕ_a. The perturbative expansion for the propagator reads as

$$iG_{11}=\mathcal{G}_{11}-\mathcal{G}_{1n}V_n\mathcal{G}_{n1}+\mathcal{G}_{1n}V_n\mathcal{G}_{nm}V_m\mathcal{G}_{m1}+\cdots . \tag{2.17}$$

Note that this expression is different from the one that one gets when computing the in-out expectation value. The terms with $n,m=2$ are new and are caused by imposing the boundary conditions (note that all the terms containing some 2-type index are solutions of the homogeneous equation). If we want to compute the effective action using (2.14) we need to take the log and the trace of (2.17). Making use of the Taylor expansion for the logarithm and noting that in general $\mathcal{G}_{11}^{-1}\mathcal{G}_{12}=0$, we get

$$\Gamma[\phi_1,\phi_2]=S[\phi_1]-\frac{i}{2}\mathrm{tr}\ln(-i)\mathcal{G}_{11}$$

$$+\frac{i}{2}\mathrm{tr}(V_1\mathcal{G}_{11}-\tfrac{1}{2}V_1\mathcal{G}_{11}V_1\mathcal{G}_{11}+V_1\mathcal{G}_{12}V_2\mathcal{G}_{21})$$

$$+F+O(\mathbf{h}) . \tag{2.18}$$

Equation (2.18) clearly shows which is the only difference between the perturbative evaluation of the CTP and the in-out effective action. The term containing the $\mathcal{G}_{12}$ propagators is the only new one that contributes to the ϕ_1 equation. Moreover, it is quite simple to see its effect. When we derive (2.18) with respect to ϕ_1 and put $\phi_1=\phi_2=\phi(x)$ it is easy to see that the contribution of the last two terms in the first line of (2.18) is simply

$$\frac{i}{2}V'(x)\int dy\,[\mathcal{G}_{11}^2(x,y)-\mathcal{G}_{12}^2(x,y)]V(y) , \tag{2.19}$$

where $V'(x)=[\delta V/\delta\phi(x)]$. The structure of (2.19) is clearly causal. In fact, if the point y lies in the future light cone of x, the time-ordered products are equal to the ordinary ones and the two terms in (2.19) cancel. So, the temporal integral in this equation goes until $y_0=x_0$.

Thus, Eq. (2.18) is very useful when computing the effective action with the closed-time-path formalism and can be used in order to perform many computations straightforwardly.

Let us now turn our attention to the non-zero-temperature case. If the quantum state of the system is described with a density matrix, the generating functional for the propagators at finite temperature can be defined by analogy with Eq. (2.1) as

$$Z_\rho[J_1,J_2]=\mathrm{tr}\left\{\rho\bar{T}\left[\exp\left[-i\int {}^tJ_2\phi\right]\right]\times T\left[\exp\left[i\int {}^tJ_1\phi\right]\right]\right\} . \tag{2.20}$$

It is clear that this generating functional allows us to compute all kinds of real-time expectation values of operator products. It is worth noting that in this case we cannot interpret Eq. (2.20) as the "overlap" between the density matrices evolving under the action of two different sources (as we did with the generating functional in the zero-temperature case). The functional representation of (2.20) can be written as

$$Z_\rho[J_1,J_2]=\int \mathbb{D}\Phi_1\mathbb{D}\Phi_2\exp(i\{S[\Phi_1]-S^*[\Phi_2]+(J_1\Phi_1)-(J_2\Phi_2)\})\times\langle\phi_1(t_0)|\rho|\phi_2(t_0)\rangle , \tag{2.21}$$

where the functional integral should be made now by summing over all histories ϕ_1 and ϕ_2 that reduce to $\phi_1(t_0)$ and $\phi_2(t_0)$ at time t_0 and that satisfy $\phi_1=\phi_2$ at time t.

In order to make some computations we have to make an explicit choice of density matrix. An interesting approach was presented by Calzetta and Hu in Ref. 20. It consists in expanding the matrix element of ρ as a functional of the configurations $\phi_1(t_0)$ and $\phi_2(t_0)$. In this way one can study very general classes of states. The techniques that one needs to use turn out to be very similar to those developed first by Cornwall, Jackiw, and Tomboulis when they introduced the generating functional for n-particle irreducible diagrams at zero temperature.[24] Another possibility was exploited by Weiss and Semenoff.[21] It can be used if one considers a density matrix describing an initial thermal equilibrium state. In fact, if one assumes that

$$\rho\simeq\exp(-\mathcal{H}_0/\mathbf{T}) , \tag{2.22}$$

where $\mathcal{H}_0$ is the Hamiltonian of the system at time t_0, it is possible to write a functional integral representation for the matrix element that appears in (2.21):

$$\langle\phi_1(t_0)|\rho|\phi_2(t_0)\rangle=\int\mathbb{D}\phi_3\exp[-\mathcal{S}_E(\phi_3)] , \tag{2.23}$$

where $\mathcal{S}_E$ is the Euclidean action defined as

$$\mathcal{S}_E=\int_{-1/\mathbf{T}}^{0}d\tau\mathcal{H}_0 .$$

The functional integral in (2.23) is made by summing over all Euclidean histories that reduce to $\phi_1(t_0)$ when Euclidean time is zero and to $\phi_2(t_0)$ when Euclidean time is $(-1/\mathbf{T})$. By replacing (2.23) into (2.21) one can think of the generating functional as generated from an integration over a single field that takes values over a time path that lies in a complex plane. The path goes forward in time from t_0 to t parallel to the real axis, then backwards in time from t to t_0 and then goes parallel to the imaginary axis down to the point $t_0-i/\mathbf{T}$ that is identified with t_0.

If the Hamiltonian in (2.22) is not quadratic in the fields, it is necessary to introduce an extra current J_3 and define Feynman rules involving also the field Φ_3. If, on the contrary, $\mathcal{H}_0$ is a quadratic functional of the fields, the only influence that the presence of the density matrix will have in (2.21) will be to change the propagators from zero to finite temperature. Let us assume that $\mathcal{H}_0$ is indeed a quadratic functional (in Sec. III we will deal only with free fields) that can be written as

$$\rho=C\exp\left[-\int d\mathbf{k}a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}\Big/T_{\mathbf{k}}\right] , \tag{2.24}$$

where the operators $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^{\dagger}$ are annihilation and creation operators associated with the basis of Fock space in which the Hamiltonian is diagonal. It is simple to show that starting with the generating functional (2.20) we can define an "effective action" $\Gamma_\rho[\phi_1,\phi_2]$ and use the standard background-field method to establish the validity of Eq. (2.9). The only difference is that the propagators are now computed by using (2.2) but interpreting the expectation value as thermal [i.e., $\langle\cdots\rangle=\mathrm{tr}(\rho\cdots)$]. The same arguments used in the zero-temperature case can be used here to prove the finite-temperature analog to (2.14), that is the lemma we enunciated in the Introduction.

Finally, and for future convenience, let us write explicitly the finite-temperature propagators of a free scalar field in Minkowski space time. The action is

$$S=\tfrac{1}{2}\int dx\,[(\partial\phi)^2-m^2\phi^2] . \tag{2.25}$$

We assume that the density matrix is given by (2.24) with $\mathbf{T}_{\mathbf{k}}=\mathbf{T}$ (in this case, the creation and annihilation operators are the ones associated with the decomposition of the field in plane waves). Thus the generating functional is given by (2.5) with the propagators

$$G_{ab}(x,x')=-\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}[G_{ab}^{T=0}(k)+G_{ab}^{T}(k)] , \tag{2.26a}$$

where the zero-temperature part is given by

$$G_{11}^{T=0}(k)=(k^2-m^2+i\epsilon)^{-1}=G_{22}^{T=0}(k)^* , \tag{2.26b}$$

$$G_{12}^{T=0}(k)=(-2\pi i)\delta(k^2-m^2)\theta(k_0)=G_{21}^{T=0}(-k) \tag{2.26c}$$

and the finite-temperature part is

$$G^T(k)=(-2\pi i)\delta(k^2-m^2)n(k) \tag{2.26d}$$

with

$$n(k)=(e^{-|k_0|/\mathbf{T}}-1)^{-1} . \tag{2.26e}$$

Note that all the propagators G_{ab} have the same finite-temperature part.

With this we finish the presentation of our method. Now, let us discuss some aspects of the other class of real-time methods that are frequently used to incorporate finite-temperature effects in quantum field theory. Our aim is to give clear evidence that shows that TFD (as presented in Ref. 16 and used in Ref. 14) and other equivalent methods can only be used to study systems for which the thermal equilibrium is maintained. Among the many ways of introducing TFD or NS formalisms, we are going to use one that will allow us to understand the relation between the generating functional used in these methods and the one we use here. As we said, the definition of the generating functional (2.20) was inspired in the zero-temperature one (2.1). Let us try to naively generalize the usual in-out zero temperature generating functional in the following way. If we have a system that is described by a density matrix $\rho_{\rm in}$ (we work in the Heisenberg picture) and we act with an external source J, the density matrix will evolve according to

$$\rho_{\rm in}^J(t)=T\left[\exp\left[i\int {}^tJ\phi\right]\right]\rho_{\rm in}^{J=0}\bar{T}\left[\exp\left[-i\int {}^tJ\phi\right]\right] . \tag{2.27}$$

We can compute the "overlap" between this matrix and the one that describes the out equilibrium state (of course, we are assuming that the equilibrium is maintained) and define

$$\begin{aligned}\zeta(J)&=\mathrm{Tr}(\rho_{\rm in}^J\rho_{\rm out})\\&=\mathrm{Tr}\left\{T\left[\exp\left[i\int {}^tJ\phi\right]\right]\rho_{\rm in}^{J=0}\right.\\&\quad\left.\times\bar{T}\left[\exp\left[-i\int {}^tJ\phi\right]\right]\rho_{\rm out}\right\} .\end{aligned} \tag{2.28}$$

This functional can be thought to be the generalization of the vacuum persistence amplitude. However, it is evident that $\zeta(J)$ is useless since its derivatives cannot be interpreted as expectation values of operator products (at least in a general case). However, we can invent a new functional by slightly generalizing $\zeta(J)$ as

$$\begin{aligned}Z(J_1,J_2)=\mathrm{Tr}\left\{\left[\exp\left[i\int {}^tJ_2\phi\right]\right]\rho_{\rm in}^{J=0}\right.\\\left.\times\bar{T}\left[\exp\left[i\int {}^tJ_1\phi\right]\right]\rho_{\rm out}\right\} .\end{aligned} \tag{2.29}$$

Now, we note that by deriving Z with respect to one of the currents and by evaluating the result in $J_1=J_2=0$, we can generate expectation values of (time-ordered or anti-time-ordered) operator products in the state defined by the density matrix

$$\rho_r=\rho_{\rm in}\rho_{\rm out} . \tag{2.30}$$

We can notice that if our aim is to compute the Green's functions of a system that is described by a density matrix ρ_r we can use the generating functional defined in (2.29). Moreover, we can freely choose the in and out density matrices (that are artifacts of the formalism). If the original system is in thermal equilibrium at temperature $\mathbf{T}$, so that

$$\rho_r=e^{-\mathcal{H}/\mathbf{T}} \tag{2.31}$$

then a particularly symmetric choice for the in and out densities is

$$\rho_{\rm in}=\rho_{\rm out}=\rho_r^{1/2} . \tag{2.32}$$

Replacing (2.32) in the original generating functional (2.29) it is easy to show that the result is precisely the same generating functional defined by Niemi and Semenoff in Ref. 18. In fact, in this case, it is possible to find a functional integral representation of $Z(J_1,J_2)$. The functional integral should be made by summing over fields that take values on a complex time path that goes from the point t_0 to $t_0-1/\mathbf{T}$ after passing through the points t_1, t_2, and t_3 that are defined as $t_1=t$, $t_2=t-1/2\mathbf{T}$, $t_3=t_0-1/2\mathbf{T}$. The method based on the use of this generating functional was proved to be equivalent to thermo field dynamics (see, for example, Ref. 16). Obviously, the choice (2.32) is not the only possible one. The freedom in choosing different in-out density matrices was also exploited in the literature (see Ref. 25) and the results obtained by using other possible "splittings" of ρ_r were proven to be equivalent [as it is obvious from (2.29)]. From the presentation we made here it is evident that the NS method (and, as a consequence, the TFD also) are only applicable if the system is such that the equilibrium is maintained. Moreover, the fact that in these methods one is imposing in-out boundary conditions from the beginning produces the zero-temperature limit to coincide with the usual in-out formalism. This can be seen by replacing $\rho_{\rm in}$ by $|0,\mathrm{in}\rangle\langle 0,\mathrm{in}|$ and $\rho_{\rm out}$ by $|0,\mathrm{out}\rangle\langle 0,\mathrm{out}|$ in (2.29). The result is simply

$$Z(J_1,J_2)=Z(J_1)Z^*(J_2) , \tag{2.33}$$

where $Z(J)$ is the usual vacuum persistence amplitude under the action of J.

It is quite frequent to find in the literature that the zero-temperature limit of TFD or NS formalism is the same as the zero-temperature limit of the CTP formalism we presented before. As we showed here, this assertion should be interpreted carefully.

III. THE ANISOTROPY DISSIPATION AT FINITE TEMPERATURE

Let us consider now the anisotropy dissipation problem. We will suppose that the metric of space-time is given by (1.1) and that the content of the Universe can be modeled by a free massless scalar field conformally coupled to the space-time curvature (we confess that this is a rather unrealistic picture). The action is written as

$$\mathbf{S}=S_g+S_m \ . \tag{3.1}$$

The matter action is

$$S_m=\tfrac{1}{2}\int d^4x\,(-g)^{1/2}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{R}{6}\phi^2\right] . \tag{3.2}$$

This action can be expanded in powers of the anisotropy β. In fact, it is quite easy to arrive at the following equations:

$$S_m=S_0+S_1+S_2 \ , \tag{3.3}$$

where

$$S_0=\tfrac{1}{2}\int d^4x(a^2\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-a''a\phi^2) \ , \tag{3.4a}$$

$$S=\int d^4x\ a^2\beta^{ij}\partial_i\phi\partial_j\phi \ , \tag{3.4b}$$

$$S_2=-\int d^4x\left[a^2\beta^{ik}\beta^j_k\partial_i\phi\partial_j\phi+\frac{a^2\phi^2}{12}\beta'_{ik}\beta'^{ik}\right] . \tag{3.4c}$$

We are using the following notation:

$$\beta_{ij}=\beta^{ij}=\beta^j_i$$

and a prime denotes the derivative with respect to the conformal time η. The gravitational action in (3.1) is

$$S_g=\int d^4x\,(-g)^{1/2}\left[-\frac{R}{K}+\epsilon_1R^2+\epsilon_2R_{\mu\nu}R^{\mu\nu}+\epsilon_3R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right] . \tag{3.5}$$

The constants that appear in (3.5) are the bare ones and can be written as the renormalized constants plus counterterms. The counterterms are well known (see, for example, Ref. 2) and for simplicity we will assume that the renormalized ϵ_i's constants are zero (this assumption is not relevant in what concerns the anisotropy damping mechanism). The expansion of the gravitational action up to second order in the anisotropy and the use of the standard counterterms yield

$$S_g=\int d^4x\ \mathrm{Tr}(K^{-1}a^2\beta'^2+\lambda\{3\beta''^2\ln\mu a-\beta'^2[(a'/a)^2+(a''/a)]\})+\frac{3\lambda}{(n-4)}\int d^4x\ \mathrm{Tr}\beta''^2 \ , \tag{3.6}$$

where the trace is an ordinary matricial one and $\lambda=(2880\pi^2)^{-1}$.

The state of the system will be described by a density matrix. The choice of the density matrix in our case is not a trivial issue. In fact, it is a well-known fact that the notion of the vacuum state (and thus the notion of particle) is not unambiguously defined in curved space-time and then does not have special symmetries (such as a timelike Killing vector) or asymptotic regions. This ambiguity is directly translated to the density matrix choice. If we decide to study a state described by a density matrix written as (2.22) we still have to define what we mean by the canonical Hamiltonian (i.e., we must choose a particular observer, that defines a global notion of time). In our case, we will choose a density matrix such as (2.22) where the Hamiltonian $\mathcal{H}_0$ will be the one associated with the action (3.4a) and the comoving observer in the isotropic RW space-time. That is to say, we are choosing a state that in the zero-temperature limit reduces to the conformal vacuum (this was the state used by the authors that studied the $\mathbf{T}=0$ case and also the one used by Amsterdamski in Ref. 14).

We want to compute the CTP effective action in the semiclassical approximation. We are going to neglect completely the metric fluctuations and quantize just the scalar field (an assumption that can make sense if the number of scalar fields that are present in the Universe is large). So, we have to proceed exactly as explained in Sec. II. We want to compute the effective action $\Gamma[\Phi_1,\Phi_2]$ where the symbol Φ denotes now the set of fields $\{a,\beta,\phi\}$. As we are interested in computing the effective action for the metric, we will fix the background $\phi(x)=0$ (that is a solution of the effective equations). We can use Eq. (2.14a) and write

$$\Gamma[a_1,\beta_1;a_2,\beta_2]=S_g[a_1,\beta_1]-\frac{i}{2}\mathrm{tr}\ln\mathbf{G}_{11}+F+O(\mathbf{h}) \ , \tag{3.7}$$

where F makes a vanishing contribution to the equation for a and β while $\mathbf{G}_{11}$ is simply the thermal expectation value of the time-ordered product of the quantum field ϕ. Thus, the only thing that we need to do is to compute $\mathbf{G}_{11}$ perturbatively in β. For that purpose, it is worth using the conformally transformed propagator defined as

$$\mathbf{G}_{ab}(x,x')=a^{-1}(\eta)G_{ab}(x,x')a^{-1}(\eta') \ . \tag{3.8}$$

It is simple to show that the field equation for the propagators G_{ab} can be written as

$$\mathbf{A}_{ab}G_{bc}=-i\delta_{ac} \ , \tag{3.9a}$$

where

$$\mathbf{A}=A+V+W \ . \tag{3.9b}$$

The matrices are all diagonal and have the following nonvanishing elements:

$$A_{11}=-A_{22}=\eta^{\mu\nu}\partial_\mu\partial_\nu \ , \tag{3.10a}$$

$$V_{11}=V_{11}(a_{(1)},\beta_{(1)})=\frac{2}{a^2_{(1)}}\beta^{ij}_{(1)}\partial_i\partial_j \ , \tag{3.10b}$$

$$W_{11}=W_{11}(a_{(1)},\beta_{(1)})=-\frac{2}{a^2_{(1)}}\beta^{ik}_{(1)}\beta^j_{(1)k}\partial_i\partial_j+\frac{\beta^2_{(1)}}{6a^2_{(1)}} \ . \tag{3.10c}$$

V_{22} and W_{22} are obtained from (3.10b) and (3.10c) by changing the sign and replacing $a_{(1)}$ and $\beta_{(1)}$ by $a_{(2)}$ and $\beta_{(2)}$.

After defining the zero-order propagator as in (2.16),

we can use (2.18a) to compute the effective action (the only difference is that we are separating the first- and second-order contributions to the perturbation and denoting them as V and W). The explicit form of the zero-order thermal propagators is given in (2.26). It is easy to show that we can split the effective action as

$$\Gamma[a_{(1)},\beta_{(1)};a_{(2)},\beta_{(2)}]=\Gamma_{\rm RW}[a_{(1)}]+\Gamma_\beta[a,\beta_{(1)},\beta_{(2)}]+F+O(\hbar)\ . \tag{3.11}$$

Here $\Gamma_{\rm RW}$ is the finite-temperature effective action for a massless conformally coupled scalar field in a RW universe (it does not depend on the anisotropy) and

$$\Gamma_\beta[a,\beta_{(1)},\beta_{(2)}]=S_g(\beta_{(1)})+\frac{i}{2}{\rm tr}(V_1\mathcal{G}_{11}+W_1\mathcal{G}_{11}-\tfrac{1}{2}V_1\mathcal{G}_{11}V_1\mathcal{G}_{11}-V_1\mathcal{G}_{12}V_2\mathcal{G}_{21}) \tag{3.12}$$

is the anisotropy-dependent part of the effective action. In (3.12), S_g is the β-dependent part of the classical gravitational action and the trace means the integration $\int d^4x(-g)^{1/2}$ as well as the trace over the matrix indices.

The effective action Γ_β can be divided into a sum of a zero- and a finite-temperature part. The zero-temperature part reads as (3.12) where we have to replace the propagator by the zero-temperature one [see (2.26)]. It is not difficult to show that the result is exactly the same one obtained by Calzetta and Hu in Ref. 10 and by Jordan in Ref. 13. It reads

$$\Gamma_\beta^{T=0}[a,\beta_{(1)},\beta_{(2)}]=S_g(\beta_{(1)})-{\rm Tr}\int d\eta\,d\eta'\frac{d\omega}{2\pi}e^{i\omega(\eta-\eta')}\beta_{(1)}(\eta)[\beta_{(1)}(\eta'){\bf I}^{T=0}(\omega)-4i\theta(\omega){\rm Im}{\bf I}^{T=0}(\omega)\beta_{(2)}(\eta')]-\frac{3\lambda}{(n-4)}{\rm Tr}\int d^4x\,\beta_{(1)}''^2\ , \tag{3.13a}$$

where

$${\bf I}(\omega)=\frac{\omega^4}{1920\pi^2}(\ln|\omega^2/\mu^2|-i\pi)\ . \tag{3.13b}$$

The term proportional to $(n-4)^{-1}$ cancels with the counterterms contained in $S_g(\beta)$ [see (3.6)].

The finite-temperature part of Γ_β can be written easily from (3.12). It is

$$\Gamma_\beta^T[a,\beta_{(1)},\beta_{(2)}]=\frac{i}{2}{\rm tr}(V_1\mathcal{G}^T+W_1\mathcal{G}^T-V_1\mathcal{G}_{11}^{T=0}V_1\mathcal{G}^T-2V_1\mathcal{G}_{12}^{T=0}V_2\mathcal{G}^T)-\frac{i}{4}{\rm tr}[V_1\mathcal{G}^T(V_1+2V_2)\mathcal{G}^T]\ . \tag{3.14}$$

It is simple to show that the last term in (3.14) involving two thermal propagators does not contribute to the effective equation and, thus, need not be computed. It is also evident that the contribution to the effective equations coming from the first trace in (3.14) is of causal nature. This can be proved by using arguments similar to those used in Sec. II.

Now we will compute the relevant terms in (3.14). The term proportional to V_1 is the easiest one since the result is proportional to the trace of the matrix β that is zero. The one proportional to W_1 is not vanishing. It is

$$\frac{i}{2}{\rm tr}(W_1\mathcal{G}^T)=-\pi\int d^4x\frac{d^4k}{(2\pi)^4}\delta(k^2)n(k)\times\left[2\beta_{(1)}^{2ij}k_ik_j+{\rm Tr}\frac{\beta'^2}{6}\right]=-{\rm Tr}\int d\eta\left[\beta_{(1)}'^2\frac{\mathbf{T}^2}{144}+\beta_{(1)}^2\frac{\pi^2\mathbf{T}^4}{45}\right]\ , \tag{3.15}$$

where we normalized the space volume to one.

Amsterdamski got a similar result[14] but with a relevant difference in the sign of this term. This sign difference caused drastic changes in the behavior of the anisotropy.

The last two terms (the ones proportional to V^2) are not difficult to compute. By using the explicit form of the propagators we get

$$-\frac{i}{2}{\rm tr}(V_1\mathcal{G}_{11}^{T=0}V_1\mathcal{G}^T)=-{\rm Tr}\int d\eta\,d\eta'\frac{d\omega}{2\pi}e^{i\omega(\eta-\eta')}\times\beta_{(1)}(\eta)\beta_{(1)}(\eta'){\bf I}^T(\omega)\ , \tag{3.16a}$$

where

$${\bf I}^T(\omega)=\frac{2}{15\pi^2}\int dq\,q^5n(q)(\omega^2-4q^2+i\epsilon)^{-1}\ . \tag{3.16b}$$

The identity

$$\mathcal{G}_{12}^{T=0}(k)=-2i\theta(k_0){\rm Im}\mathcal{G}_{11}^{T=0}(k)$$

allows us to write

$$-\frac{i}{2}{\rm tr}(V_1\mathcal{G}_{12}^{T=0}V_2\mathcal{G}^T)=4i\,{\rm Tr}\int d\eta\,d\eta'\frac{d\omega}{2\pi}e^{i\omega(\eta-\eta')}\times\beta_{(1)}(\eta)\beta_{(2)}(\eta')\theta(\omega){\rm Im}[{\bf I}^T(\omega)]\ . \tag{3.16c}$$

Thus, the final result for the anisotropy's CTP effective action can be written as

$$\Gamma_\beta[a,\beta_{(1)},\beta_{(2)}]=S_g(\beta_{(1)})-\mathrm{Tr}\int d\eta\left[\beta'^2_{(1)}\frac{\mathbf{T}^2}{144}+\beta^2_{(1)}\frac{\pi^2\mathbf{T}^4}{45}\right]$$
$$-\mathrm{Tr}\int d\eta\,d\eta'\frac{d\omega}{2\pi}e^{i\omega(\eta-\eta')}\beta_{(1)}(\eta)[\beta_{(1)}(\eta)\mathbf{I}(\omega)-4i\theta(\omega)\mathrm{Im}\mathbf{I}(\omega)\beta_{(2)}(\eta')]\ , \tag{3.17a}$$

where

$$\mathbf{I}(\omega)=\mathbf{I}^{T=0}(\omega)+\mathbf{I}^{T}(\omega)\ . \tag{3.17b}$$

From (3.17) it is easy to calculate the effective equation for the anisotropy β. We just have to use (2.8). The resulting equation will be nonlocal due to the existence of the last term in (3.17a). Explicitly we can derive (3.17) and obtain

$$(3\lambda\beta''\mu a)''+\left\{\lambda\beta'\left[\frac{a''}{a}+\left[\frac{a'}{a}\right]^2+\frac{\mathbf{T}^2}{144\lambda}-K^{-1}a^2\lambda^{-1}\right]\right\}'-\beta\frac{\pi^2\mathbf{T}^4}{45}-\int d\eta'\mathbf{K}(\eta-\eta')\beta(\eta')=0\ , \tag{3.18a}$$

where

$$\mathbf{K}(x)=\int\frac{d\omega}{2\pi}e^{i\omega x}\mathbf{K}(\omega) \tag{3.18b}$$

and

$$\mathbf{K}(\omega)=\mathbf{I}(\omega)-2i\theta(\omega)\mathrm{Im}\mathbf{I}(\omega)=\frac{\omega^4}{960\pi^2}\ln|\omega/\mu|+\frac{2}{15\pi^2}\mathbf{P}\int dq\,q^5n(q)(\omega^2-4q^2)^{-1}+i\mathrm{sgn}(\omega)\frac{\omega^4}{1920\pi}\coth\frac{|\omega|}{2\mathbf{T}}\ . \tag{3.18c}$$

For future convenience it is worth defining the variables

$$\mathbf{M}(x)=3\lambda\ln\mu a\ , \tag{3.19a}$$

$$k(x)=\lambda\left[\frac{a''}{a}+\left[\frac{a'}{a}\right]^2+\frac{\mathbf{T}^2}{144\lambda}-K^{-1}a^2\lambda^{-1}\right]\ . \tag{3.19b}$$

In terms of these variables (that are η-dependent through the dependence on the scale factor a) Eq. (3.18a) is more compact and reads

$$(\mathbf{M}\beta'')''+(k\beta')'-\beta\frac{\pi^2\mathbf{T}^4}{45}-\int d\eta'\mathbf{K}(\eta-\eta')\beta(\eta')=0\ . \tag{3.20}$$

The equation is real and causal (as expected). In fact, the kernel $\mathbf{K}(x)$ is real since the real (imaginary) part of its Fourier transform is an even (odd) function of ω. The causal structure of the kernel can be shown by noticing that all the singularities of $\mathbf{K}(\omega)$ are on the upper-half complex plane. These two properties were proven in the zero-temperature case in Refs. 10 and 13. By using some simple tricks we can find a closed expression for the kernel that makes more explicit its causal properties. In fact, we can show that

$$\int d\eta'\mathbf{K}(\eta-\eta')\beta(\eta')=\frac{1}{960\pi^2}\left[\int^\eta d\eta'\beta''''(\eta')\ln(\eta-\eta')\right]'-\frac{\Psi_{(2)}}{960\pi^2}\beta''''(\eta)$$
$$-\tfrac{1}{5}\beta''(\eta)\frac{\mathbf{T}^2}{144}+\beta\frac{\pi^2\mathbf{T}^4}{450}+\frac{1}{240\pi^2}\left[\int^\eta d\eta'\beta'''(\eta')g(\eta-\eta')\right]'\ , \tag{3.21a}$$

where

$$g(x)=2x\mathbf{T}^2\sum_{n\geq1}\frac{1}{n^2+4x^2\mathbf{T}^2}\ . \tag{3.21b}$$

It is worth noting that the first term in (3.21a) that carries the zero-temperature contribution can be thought of as being the $n=0$ term of the sum defining $g(x)$. This shows that our finite-temperature result matches very well with the zero-temperature one. This can also be noticed from (3.18c) where we see that the finite temperature increases the imaginary part of $\mathbf{K}(\omega)$. As we will see, this term can be related (in some cases) to the dissipative behavior of the system.

Equation (3.20) describes the evolution of β and is coupled to the other cosmological equation that can be obtained by deriving the effective action with respect to the scale factor $a(\eta)$. The resulting system is extremely coupled and rather intractable. A further simplification can be introduced if we solve the equation for the scale factor by assuming a perfect isotropic space-time and then plug the solution into the equation for the anisotropy. The procedure is correct since we are assuming that the anisotropy is small (the solution should be consistent with this assumption). The equation for $a(\eta)$ is thus obtained from

$$\frac{\delta\Gamma_{\rm RW}}{\delta a}=0 \ . \tag{3.22}$$

The solutions of this equation are catalogued in Anderson's papers.[4] Their late-time behavior is dominated by the finite-temperature contribution to the effective action and the Universe expands as an ordinary radiation-dominated RW one. The early time behavior is in turn dominated by the quantum effects that are incorporated by the anomalous terms in the effective action $\Gamma_{\rm RW}$. There are families of solutions that have an asymptotically classical behavior (in the far future) and do not possess event horizons (the singularity occurs for $\eta\rightarrow\infty$). Among them, the so-called "conformally complete" solution have an initial behavior like

$$a(\eta)=\exp(a\eta) \ , \tag{3.23a}$$

where

$$a=2\pi\mathbf{T} \ . \tag{3.23b}$$

For other families, the singularity takes place for a finite conformal time η_0 and the initial behavior is like

$$a(\eta)=(\eta-\eta_0)^{\gamma}, \quad \gamma>0 \ . \tag{3.24}$$

The solution that begins as (3.23) was used as the prototype to study the anisotropy dissipation[8,14] by people that used the in-out approach (the singular solutions (3.24) cannot be studied using the in-out method due to the noncausal structure of the equations).

In what follows we will show that our Eq. (3.20) has the same virtues and the same pathologies as the equations found in the zero-temperature case. First, we should notice that if we want to solve the equation by fixing some initial values on a given instant $\eta=\eta_0$, as the equation is nonlocal, we must provide the information on the values of the function for all $\eta<\eta_0$. As a consequence, the matter of choosing initial values turns out to be quite complicated. A possible way to proceed is to assume that some of the terms in (3.20) dominate the early-time behavior of the solution. In this way, one can find an approximate solution to simulate the initial behavior and use it for the nonlocal initial condition. Some authors[8,9] argued that the contribution of the nonlocal term to the early-time behavior of the solution could be negligible. Although the assumption is not very well justified, we can just do it and see if the results that arise are consistent. This approximation is known as the "local truncation." The local truncation of the zero-temperature in-in effective equations is nothing but the real part of the in-out analog. If one uses Eq. (3.23) in order to approximate the behavior of the scale factor, then the local truncation of (3.20) can be written as follows (after integrating once and neglecting the term proportional to $\mathbf{T}^4$, i.e., in the low-temperature limit):

$$\frac{d}{dx}\left[x\frac{d}{dx}q\right]+\sigma q=C \ , \tag{3.25a}$$

where

$$\beta'=q \ . \tag{3.25b}$$

C is a constant matrix that fixes the initial orientation of the rate of change of the anisotropy and x is the rescaled time defined as $x=\mathbf{T}\eta$. The constant σ comes from the contribution of the terms containing derivatives of $a(\eta)$ in (3.18) as well as from the one proportional to $\mathbf{T}^2$. It takes the value

$$\sigma=14\pi/3 \ . \tag{3.25c}$$

The zero-temperature result is reobtained by putting $\sigma=4\pi/3$. Thus, it is evident that the finite temperature does not affect the qualitative behavior of the solutions in the local truncation approximation. In fact our results are similar to the ones obtained by Hartle and Hu in the zero $\mathbf{T}$ case[8] using the in-out approach. So, it is possible to construct a solution of (3.25) that gives rise to a finite probability for the particle production and to a finite local term. The solution to the homogeneous equation (3.25) is simply modified Bessel functions $\mathbf{H}_0(\sigma x^{1/2})$ that behave as plane waves times $x^{1/4}$ when $|x|\rightarrow\infty$. For this kind of solutions the probability of particle creation remains finite (we recall that the probability of particle creation is related with the imaginary part of the in-out effective action that, as can be easily proven, is proportional to the integral of q'^2). In Ref. 14, due to the problem with the sign in (3.15) that we discussed above, the author used the value $\sigma=-2\pi$ in (3.25). In that case the solutions of that equation do not generate a finite particle production and the local truncation is useless. Here, using the local truncation it is possible to find solutions that generate finite particle production probability and finite nonlocal term both in the zero- and finite-temperature cases.

Within the in-out approach ($\mathbf{T}=0$), Hartle and Hu[8] showed that, if it exists, there is a unique solution to the full nonlocal equation with an asymptotically classical behavior (in the far future) and a finite particle production. The solution that one finds with the local truncation is a candidate to approximate the initial behavior of such "unique" solution.

Let us now turn to some of the pathologies. It is easy to show that Eq. (3.20) can be rewritten in general in terms of q (if we neglect the term proportional to $\mathbf{T}^4$). In fact, after integrating once, we can write

$$\frac{d}{dx}\left[\mathbf{M}\frac{d}{dx}q\right]+kq-\mathcal{F}(q)=C \ , \tag{3.26a}$$

where

$$\begin{aligned}\mathcal{F}[q]&=\frac{1}{960\pi^2}\int^x dx'q'''(x')\ln(x-x')\\&\quad+\frac{1}{240\pi^2}\int^x dx'q''(x')g(x-x')\\&=\int\frac{d\omega}{2\pi}e^{i\omega x}\frac{\mathbf{K}(\omega)}{\omega^2}q(\omega)\end{aligned} \tag{3.26b}$$

and $\mathbf{K}(\omega)$ is defined in (3.18c).

This equation is similar to the one of a generalized oscillator with time-dependent parameters (mass and spring constant) and a generalized "friction" force. The friction force is velocity dependent and history dependent. We

see that the finite-temperature effects enhance the value of the spring constant (increasing the initial natural frequency of the oscillator) and add a new frictionlike term [by changing the real and the imaginary parts of $\mathbf{K}(\omega)$]. The study of Eq. (3.25) is rather complicated because of the combined effects of the time dependent "parameters" and the friction terms. The most important problem is that, as can be seen from the definitions (3.19), both the "mass" and the spring "constant" can be negative in some region. This can cause instabilities to occur. In the zero-temperature case, this fact was addressed by Jordan in Ref. 22. In that paper, Jordan showed that the instabilities are present in the classical regime. In fact, in that case the scale factor grows as $a^2 \simeq \eta^2$ (since the Universe expands as if it were radiation dominated). So, the mass is positive while k is negative. It can be shown [using the Fourier-transformed version of Eq. (3.26a) obtained under the assumption that $\mathbf{M}$ and k are constants] that the instabilities are present for all values of μ. In the quantum regime, the situation is different but the instability is still there. In that case, if we use the form of the scale factor given by (3.23a), we see that $k \simeq a^2 > 0$ while the mass is proportional to $\ln(\mu a)$. So the instability is present since the mass can be negative. However, if we choose the mass scale to be the Planck one (something that is not obligatory at all) the instability takes place at the Planck scale. These instabilities make the solution of the system by numerical methods a rather difficult task.

In the case where both $\mathbf{M}$ and k are positive, the role of the nonlocal term can be understood as generating dissipation. The way in which this can be studied is by looking at the Fourier-transformed version of Eq. (3.26) (again, in order to use Fourier techniques we should assume that the parameters are almost constants). In that equation, we can see that there is an imaginary term that plays a role that is similar to the one played by the term generated by a force proportional to the velocity in the equation for a damped oscillator. In fact, we can define a "viscosity function" by writing

$$i\nu(\omega)\omega q(\omega) = i\,\mathrm{Im}\mathbf{K}(\omega) q(\omega) \ . \tag{3.27}$$

The viscosity function at zero temperature is simply

$$\nu_0(\omega) = \frac{|\omega|^3}{1920\pi} \ . \tag{3.28a}$$

The result for the finite-temperature case can be read from (3.18c) and is

$$\nu_T(\omega) = \nu_0(\omega)\coth\frac{|\omega|}{2\mathbf{T}} \ . \tag{3.28b}$$

In order to relate this viscosity function with dissipation we can proceed to do two different calculations. The same kind of analysis was done in Refs. 26 and 27 when examining dissipative effects in quantum field theory in a different context. The first calculation consists in examining the linear response of the system to an impulsive force. We can perturb the system with an impulsive δ-function source and see how it evolves. The dissipative behavior should produce the damping of the initial perturbation. The short-time behavior of the system is dominated by the large-ω sector of the spectrum. An exponential damping arises because of the presence of the imaginary term and the initial behavior can be roughly approximated by

$$q(\eta) \simeq e^{-\eta/\tau} \ , \tag{3.29a}$$

where

$$\tau = 3240\pi\Omega^{-1/2}\tanh\Omega/2\mathbf{T} \tag{3.29b}$$

with

$$\Omega^2 = k/\mathbf{M} \ .$$

The finite-temperature effects increase both the viscosity function and the frequency Ω generating thus a decrease of the characteristic time τ (we stress again that this is valid only if $\mathbf{M}$ and k are positive).

The second calculation that could be done in order to understand the role of particle creation in the dissipative process is to compute the total energy dissipated by the frictionlike terms in Eq. (3.25) and to compare it with the energy carried by the particles produced during the process. The zero-temperature calculation was carried out in Ref. 20 (see also Ref. 27) and the finite-temperature one can be done straightforwardly. This ends our discussion on the anisotropy damping issue.

IV. CONCLUSIONS

Let us summarize what are the main results contained in this paper. We restudied the cosmological anisotropy damping mechanism proposed originally by Zel'dovich assuming that the state of the matter in the Universe is not the vacuum but describes a system that would be in thermal equilibrium if the Universe would have been isotropic (in the zero-temperature limit, this state reduces to the conformal vacuum). We used the closed-time-path formalism extended to include general quantum states. The effective equations that we derived are real and causal. In fact, the formalism deals with real mean values and not with matrix elements such as the usual functional in-out approach.

Our results show that the finite-temperature effects do not qualitatively modify the features of the zero-temperature anisotropy dissipation scenario. Some insight on the initial behavior of the solution can be gained by neglecting the nonlocal terms. In this way one obtains a solution that generates finite particle production probability (and a finite nonlocal term). The cosmological effective equations have instabilities that are related to the presence of higher derivative terms in the effective theory. The occurrence of such instabilities can be seen as evidence against the approximated theory in which this calculation is based (one-loop approximation to quantum gravity). Finite-temperature corrections, as expected, add nothing to the solution of this problem. The temperature can enhance the dissipative effects by increasing the value of the imaginary term (or viscosity function) in the effective equation for the Fourier transform of the anisotropy matrix (and can also make more

rapid the oscillations of the anisotropy rate of change).

Our result differs from the one obtained previously by Amsterdamski. The main difference is caused by a difference in the sign of some terms in our equations that cause qualitatively different behavior of the solutions of the effective equations.

In this paper we also discussed the differences between the most common methods used to incorporate finite temperature and to deal with real-time processes. In this particular aspect our results are not new but can serve in order to make clearer a subtle issue. We stressed that there are some methods that can be used to deal with systems out of thermal equilibrium. These methods are all generalizations of the closed-time-path approach to zero-temperature quantum field theory. As, in general, when one studies quantum fields in a dynamical spacetime equilibrium is not maintained, we are obligated to use these kind of techniques to attack cosmological problems. The other class of methods (that includes thermo field dynamics, Niemi and Semenoff, etc.) can only be used if the equilibrium is maintained throughout the evolution of the system. As in these approaches one is obligated to impose the equilibrium condition both in the in and the out regions (far past and future, respectively) this class of methods are conceptually equivalent to the usual in-out approach to quantum field theory at zero temperature (which is the zero-temperature limit of TFD, NS, etc.). Even if the system is such that the equilibrium is maintained, the two classes of methods are different in the same extent that the in-out and in-in formalism of zero-temperature QFT differ (they are different techniques designed to study different aspects of the same physical phenomenon). We will continue with the discussion on the different approaches to equilibrium and nonequilibrium quantum field theory elsewhere.[28]

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*On leave from Instituto de Astronomia y Fisica del Espacio, CC 67 Suc 28, Buenos Aires, Argentina. Electronic mail address: PAZ@UMDHEP.bitnet.

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