Quantum cosmological approach to the cosmic no-hair conjecture in the Bianchi type-IX spacetime

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The propriety of the cosmic no-hair conjecture to the Bianchi-type-IX spacetime is discussed from a quantum cosmological point of view. It is shown that most, but not all, classical universes which are created quantum cosmologically are inflationary. The probability of inflation among such universes is also discussed.

I. INTRODUCTION

The observed large-scale isotropy, homogeneity, and flatness of our Universe implies that it can be described in terms of the Friedmann-Robertson-Walker (FRW) spacetime filled with energy nearly equal to the critical density. It is an important cosmological problem to explain why spacetime has such a highly symmetric and beautiful structure. The conventional big-bang cosmology has no answer. It merely resorts to a very unnatural fine-tuning of the initial condition.

On the other hand, inflationary cosmology may provide an answer to this problem.[1] Indeed if the Universe experiences a sufficiently long period of accelerated expansion, the horizon and flatness problems are solved. In studying inflation in the early Universe, one usually assumes the FRW spacetime at the outset. In order to see whether inflation provides a natural solution to the horizon problem, however, we should start from an anisotropic and inhomogeneous spacetime. If inflation is shown to occur from a wide range of initial conditions, we may conclude that inflation is a natural phenomenon which leads to the large-scale spatial property of the present Universe.

Concerning this problem, there is a conjecture called the cosmic no-hair conjecture, which asserts that any spacetime approaches de Sitter spacetime if there exists a positive (effective) cosmological constant Λ (Ref. 2). We know, however, that all spacetimes do not always realize de Sitter expansion. For example, closed FRW spacetime may recollapse without inflation even if there is a positive cosmological constant. Hence what is important to examine is the extent to which this conjecture is true.

For homogeneous but anisotropic spacetimes, which are classified into several Bianchi types, Wald showed that all the initially expanding Bianchi-type spacetimes except type IX approach de Sitter spacetime in one expansion time.[3] In the case of type-IX spacetime, some approach de Sitter spacetime while others recollapse without inflation. Wald also gave a criterion for inflation in the Bianchi type-IX spacetime.[3] That is, any type-IX spacetime approaches the de Sitter spacetime if the following inequality is satisfied at some time:

$$\Lambda > \tfrac{1}{2} R^{(3)}_{\max} , \tag{1}$$

where $R^{(3)}_{\max}$ is the maximum of the spatial scalar curvature with a fixed spatial proper volume which is realized in the the isotropic case.

Although Wald's above argument is simple and convincing, it is not satisfactory in discussing the generality of inflation in type-IX spacetime for the following two reasons. One is that the above inequality is a sufficient condition for inflation and there may be many inflationary solutions without satisfying the above criterion at the outset. The other is that, even if we content ourselves with the *sufficient* condition (1), the measure which satisfies inequality (1) is not yet certain in the possible initial phase space of classical universes.

In order to examine the naturalness of inflation in a specific spacetime, it is in general necessary to define a measure on the initial phase space and to assign the probability of their realization. It has been argued in the literature[4] how one should define a natural measure in the configuration space of the cosmological solutions and the probability of inflation has been discussed classically by simply estimating the area of inflationary trajectories in phase space in some simple spactimes.[5]

In the case of the Bianchi type-IX spacetime, however, it has been shown to exhibit a chaotic behavior near the singularity with or without the cosmological constant, like a particle moving in a triangle potential well.[6,7] Hence it is impossible to predict which trajectory will inflate if one sets the initial condition near the classical singularity.[7] This means that we cannot hope to discuss the probability of inflation in this spacetime in terms of the above method. But such a classical analysis may not be valid in the early Universe before the onset of inflation, especially near the singularity, because quantum-gravitational effect may play an important role.

In the present paper we would like to discuss the naturalness of inflation of the Bianchi type-IX spacetime with a positive cosmological constant making use of the essence of quantum cosmology advocated by Hartle and Hawking[8] or Vilenkin,[9] that is, the creation of the Universe from the Euclidean era and the avoidance of the classical singularity.

In the full quantum theory in which the quantum state of the Universe may be described by the wave function Ψ, it is rather fuzzy where classical universes emerge. As will be shown below, however, since it is hardly possible to calculate the wave function in the whole mini-superspace, we will employ a semiclassical analysis. In the present analysis based on the above spirit of quantum cosmology, we boldly presume that each classical universe starts its evolution on the boundary between the Euclidean and the Lorentzian region, the surface on which the superpotential vanishes, with a time-symmetric configuration or with vanishing momenta. We calculate the amplitude from the zero three-geometry to such a classical configuration in terms of the stationary-phase approximation of Klauder's coherent-state path integral[10] and thereby assign a probability to each initial configuration. Then we trace their classical evolution to see if they inflate, so that we can examine the generality of inflation among the universes thus created.

The rest of the paper is organized as follows. In Sec. II after introducing the Wheeler-DeWitt equation in the present mini-superspace, we discuss the applicability of the proposed prescriptions for giving the boundary condition to the present problem. Then in Sec. III we introduce our semiclassical approach. Finally Sec. IV is devoted to results and discussions.

II. BIANCHI TYPE-IX MINI-SUPERSPACE

A. Wheeler-DeWitt equation

The metric of the Bianchi type-IX spacetime is written as

$$ds^2=-dt^2+a^2e^{2\beta_{ij}}\omega^i\omega^j , \tag{2}$$

where a differential one-form ω^i satisfies $d\omega^i=\epsilon^i_{jk}\omega^j\wedge\omega^k$ and the traceless tensor β_{ij} is parametrized as

$$(\beta_{ij})=\mathrm{diag}(\beta_+ +\sqrt{3}\beta_-,\beta_+ -\sqrt{3}\beta_-,-2\beta_+) .$$

Then the Einstein action with a positive cosmological constant Λ reads

$$\begin{aligned}S&=\frac{1}{2\kappa^2}\int (R-2\Lambda)\sqrt{-g}\,d^4x+(\text{surface term})\\&=\frac{6\pi^2}{\kappa^2}\int dt\left[a\dot{a}^2-a^3\dot{\beta}_+^2-a^3\dot{\beta}_-^2\right.\\&\qquad\left.+a(U[\beta_+,\beta_-]-1)+a^3\frac{\Lambda}{3}\right],\\U[\beta_+,\beta_-]&=1+\tfrac{2}{3}e^{4\beta_+}[\cosh(4\sqrt{3}\beta_-)-1]\\&\qquad+\tfrac{1}{3}e^{-8\beta_+}-\tfrac{4}{3}e^{-2\beta_+}\cosh(2\sqrt{3}\beta_-) ,\end{aligned} \tag{3}$$

where κ^2 is equal to 8π times the Newtonian gravitational constant and an overdot denotes differentiation with respect to the Lorentzian time. The Lorentzian momenta conjugate to a, β_+, and β_- are

$$P_a=\frac{12\pi^2}{\kappa^2}a\dot{a},\quad P_{\beta_+}=-\frac{12\pi^2}{\kappa^2}a^3\dot{\beta}_+ ,$$

and (4)

$$P_{\beta_-}=-\frac{12\pi^2}{\kappa^2}a^3\dot{\beta}_- ,$$

respectively, by which the Hamiltonian constraint is expressed as

$$\frac{\kappa^2}{24\pi^2a}P_a^2-\frac{\kappa^2}{24\pi^2a^3}(P_{\beta_+}^2+P_{\beta_-}^2)-\frac{6\pi^2}{\kappa^2}\left[a(U[\beta_+,\beta_-]-1)+a^2\frac{\Lambda}{3}\right]=0 . \tag{5}$$

The standard procedure of canonical quantization yields the Wheeler-DeWitt equation to the wave function of the universe $\Psi[a,\beta_+,\beta_-]$,

$$\begin{aligned}&\left[\frac{\partial^2}{\partial a^2}-\frac{1}{a^2}\left[\frac{\partial^2}{\partial\beta_+^2}+\frac{\partial^2}{\partial\beta_-^2}\right]+\frac{144\pi^2}{\kappa^4}a^2\left[U[\beta_+,\beta_-]-1+a^2\frac{\Lambda}{3}\right]\right]\Psi[a,\beta_+,\beta_-]\\&\qquad\equiv(G^{AB}\partial_A\partial_B-W[a,\beta_+,\beta_-])\Psi[a,\beta_+,\beta_-]=0 ,\end{aligned} \tag{6}$$

where we have omitted terms with first-order derivatives which depend on the operator ordering.[11] In the last expression G^{AB} ($A,B=a$, β_+, and β_-) represents the supermetric of the minisuperspace and

$$W[a,\beta_+,\beta_-]=-\frac{144\pi^2}{\kappa^4}a^2\left[U[\beta_+,\beta_-]-1+a^2\frac{\Lambda}{3}\right] \tag{7}$$

is the superpotential.

B. Problems with boundary conditions

We can in principle determine the quantum state of the Universe Ψ by solving the Wheeler-DeWitt equation under an appropriate boundary condition. There have been two distinct proposals, in determining the wave function the Universe: namely, the Hartle-Hawking[8] and Vilenkin[9] proposals. Let us see their applicability to the present mini-superspace.

In the Hartle-Hawking proposal, the wave function Ψ is given in terms of the Euclidean path integral over possible compact Euclidean manifolds. However, since we cannot perform such an integration directly except for some simple mini-superspace models, some prescriptions have been given to obtain the wave function approximately.[8] In the case of simple systems for which we can explicitly write down the classical Euclidean action $I_{\rm cl}$, we may approximate the wave function as

$$\Psi\approx e^{-I_{\rm cl}} \tag{8}$$

in the Euclidean region where the superpotential is $W\gtrsim 0$. In the Lorentzian region, where $W\lesssim 0$, the wave function is given by the analytic continuation of (8).

Such an analysis of the present mini-superspace has been done by Amsterdamski,[12] where the condition that the anisotropy parameters $\beta_\pm$ should be small was inevitable to evaluate the semiclassical wave function analytically. In his approach the scale factor is assumed to evolve in the same way as in the de Sitter mini-superspace model, so that it is impossible to discuss the naturalness of inflation there. General cases with a large anisotropy are too complicated to allow analytic expression.

For those cases which cannot be treated analytically, it has been prescribed to utilize expression (8) only to give the Cauchy data for the Wheeler-DeWitt equation and to solve the equation numerically. In the present mini-superspace the supermetric reads

$$G_{AB}dX^A dX^B = -da^2 + a^2 d\beta_+^2 + a^2 d\beta_-^2$$
$$= -du_\mp dv_\mp + a^2 d\beta_\pm^2 \,, \tag{9}$$

where $u_\mp \equiv ae^{\beta_\mp}$ and $v_\mp \equiv -ae^{-\beta_\mp}$. We draw a conformal diagram of the mini-superspace. Figure 1 is an example which illustrates the (u_-,v_-) surface with β_+ being constant.[14] As seen there, the Cauchy null hypersurface lies in the Lorentzian region where we cannot specify the wave function as (8) because there are no Euclidean stationary paths which are regular at $a=0$.

The other proposal has been presented by Vilenkin, in which the initial singularity is avoided because the classical universe emerges as a result of quantum tunneling from "nothing." In this picture the wave function should possess only expanding mode at the boundary between Euclidean and Lorentzian regions and only outgoing modes at singular boundaries of the superspace.[9] As seen in Fig. 2, however, the Euclidean region of the present mini-superspace becomes thinner as anisotropy increases. Furthermore the classical singularity $a=0$ is exposed to the Lorentzian region with some large anisotropy. Thus the naive tunneling analogy breaks down there.

Recently Del Campo and Vilenkin reported that they have calculated the wave function in the present mini-superspace in some limiting cases of large and small anisotropy based on the tunneling boundary condition with the help of the finiteness condition of the wave function.[13] However, their analysis is not satisfactory enough to the present problem since they did not give the wave function with moderate anisotropy which may be important for discussing the naturalness of inflation.

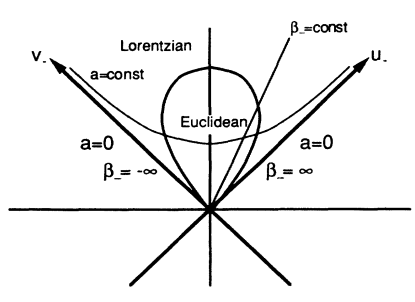

FIG. 1. A conformal diagram of the Bianchi type-IX mini-superspace with a positive cosmological constant. The (u_-,v_-) surface is shown.

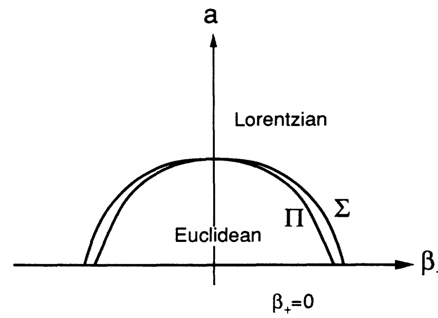

FIG. 2. The (a,β_-) plane illustrating the Euclidean region and the Lorentzian region in the present mini-superspace. The superpotential vanishes on the line II and the line Σ indicates the surface which corresponds to the maximum of a for the classical Euclidean four-geometries which are regular at $a=0$.

Thus the Bianchi type-IX mini-superspace seems to be too complicated for the usual prescriptions to be directly applicable. A similar problem arises even in the isotropic mini-superspace if a scalar field which is nonminimally coupled with spacetime curvature is present.[14] In the next section we describe our semiclassical approach to analyze the problem quantum cosmologically.

III. SEMICLASSICAL ANALYSIS

A. Quantum regime

As has been shown in some mini-superspace analyses,[8,9,15,16] the wave function Ψ is monotonic in the Euclidean region and exhibits an oscillatory behavior in the Lorentzian region where Ψ describes a set of classical universes. As stated in the Introduction, though there is no definite boundary between the Euclidean and the Lorentzian regions, the wave function becomes oscillatory near the surface Π on which the superpotential W changes it sign.

The present mini-superspace was first analyzed by Hawking and Luttrell.[11] They conjectured that the classical universes which are created quantum cosmologically start evolving near the surface II with small momenta. Their qualitative analysis, however, should be complemented by quantitative analysis in order to discuss the naturalness of inflation in this mini-superspace.

Wright and Moss[17] proceeded the analysis by numerically calculating the wave function with semiclassical approximation (8), $\Psi \approx e^{-I_{\rm cl}}$, where $I_{\rm cl}$ is the classical Euclidean action

$$I_{\rm cl} = \frac{6\pi^2}{\kappa^2}\int d\tau \left[-aa'^2 + a^3\beta_+'^2 + a^3\beta_-'^2 \right.$$
$$\left. + a(U[\beta_+,\beta_-]-1) + a^3\frac{\Lambda}{3} \right] , \tag{10}$$

evaluated with the regularity condition at $a=0$, where a prime denotes differentiation with respect to the Euclidean time $\tau \equiv -it$. They showed that the classical Euclidean paths extend beyond the surface II until a' vanishes except for the isotropic case when $a'=0$ on II. Though they did not evaluate the wave function in the oscillatory region, they concluded that the surface Σ, on which a' vanishes, not only corresponds the maximum of a for the Euclidean four-geometries but also minimum value of a for the bouncing Lorentzian four-geometries (see Fig. 2).

However, if we consider both Euclidean and Lorentzian Hamiltonian constraints on Σ, it turns out that we cannot set $P_a=0$ there except for the isotropic case. That is, since they read, respectively,

$$-\mathcal{P}_a^2+\frac{1}{a^2}(\mathcal{P}_{\beta_+}^2+\mathcal{P}_{\beta_-}^2)+W[a,\beta_+,\beta_-]=0 , \quad (11)$$

$$P_a^2-\frac{1}{a^2}(P_{\beta_+}^2+P_{\beta_-}^2)+W[a,\beta_+,\beta_-]=0 , \quad (12)$$

$P_a=\mathcal{P}_a=0$ inevitably implies that $W=0$ on Σ, where $\mathcal{P}$'s are the Euclidean momenta defined by

$$\mathcal{P}_a=-\frac{12\pi^2}{\kappa^2}aa', \quad \mathcal{P}_{\beta_+}=\frac{12\pi^2}{\kappa^2}a^3\beta'_+ ,$$

and (13)

$$\mathcal{P}_{\beta_-}=\frac{12\pi^2}{\kappa^2}a^3\beta'_- .$$

This suggests that the classical universes start on the $W=0$ surface II in the semiclassical approximation. Thus we assume that each universe starts its classical evolution on II with vanishing momenta or $P_a=P_{\beta_\pm}=0$.

In order to estimate the probability amplitude of the realization of the above initial state of classical universes, we should calculate the transition amplitude from the vanishing three-geometry or $a=0$ to each of the above configurations. However, we cannot estimate it in terms of the conventional path integral because we are specifying both the three-geometries $(a,\beta_\pm)$ and their momenta $(P_a,P_{\beta_\pm})$ of the initial state of the classical universes. Klauder has proposed the continuous-representation path integrals in order to calculate such amplitudes.[10] He showed that this integral picks up the main contribution in the transition amplitude between some specific states specifying the average values of both configurations and momenta. We may expect that this also true in our system. Since no proper quantum-cosmological method which is applicable to the present mini-superspace has been developed so far, we adopt it and estimate the probability of the realization of each classical trajectory by calculating the amplitude from $a=0$ to the initial state of each classical universe with his stationary phase approximation.

In Klauder's formalism the dominant approximation for the amplitude $\langle p_f,q_f,t_f|p_i,q_i,t_i\rangle$ is given by $\exp(iS)$, where

$$S=\tfrac{1}{2}(p_f^{\rm cl}q_f-q_f^{\rm cl}p_f+q_i^{\rm cl}p_i-p_i^{\rm cl}q_i) + \int[\tfrac{1}{2}(p^{\rm cl}\dot{q}^{\rm cl}-\dot{p}^{\rm cl}q^{\rm cl})-\mathcal{H}(p^{\rm cl},q^{\rm cl})]dt . \quad (14)$$

Here the action functional is expressed by the complex solutions $q^{\rm cl}$ and $p^{\rm cl}$ of the classical Hamiltonian equations of motion,

$$\dot{q}=\frac{\partial\mathcal{H}}{\partial p} \quad\text{and}\quad \dot{p}=-\frac{\partial\mathcal{H}}{\partial q} , \quad (15)$$

subject to the boundary conditions $q_i^{\rm cl}+ip_i^{\rm cl}=q_i+ip_i$ and $q_f^{\rm cl}-ip_f^{\rm cl}=q_f-ip_f$. Such a solution may be found by appropriately choosing the complex variable w, where $q_i^{\rm cl}=q_i+w$ and $p_i^{\rm cl}=p_i+iw$, so that the time-evolved solution satisfies $q_f^{\rm cl}-ip_f^{\rm cl}=q_f-ip_f$.

We proceed the above procedure to estimate the amplitude

$$\langle (a,\beta_\pm) \text{ on II}, P_a=P_{\beta_\pm}=0|a=\beta_\pm=0,P_a=P_{\beta_\pm}=0\rangle ,$$

where the initial state is so taken that it satisfies the regularity condition. Indeed from the classical equations of motion, which may be derived from the Hamiltonian equations of motion (15),

$$\ddot{a}=\frac{-\dot{a}^2+U[\beta_+,\beta_-]-1}{2a}-\tfrac{3}{2}a(\dot{\beta}_+^2+\dot{\beta}_-^2)+\tfrac{1}{2}a\Lambda ,$$
$$\ddot{\beta}_\pm=-3\frac{\dot{a}}{a}\dot{\beta}_\pm-\frac{1}{2a^2}\frac{\partial U}{\partial\beta_\pm} , \quad (16)$$

it is evident that regularity at $a=0$ demands that $U=0$, $\dot{\beta}_\pm=0$, and $\dot{a}^2+1=0$. That is, $\beta_\pm=0$, $\dot{a}=\pm i$, and $P_a=P_{\beta_\pm}=0$. Since $\dot{a}$ is imaginary at the outset, our starting point of the path integral must be in the classically forbidden or the Euclidean region. Hence we perform the Wick rotation $t\rightarrow\pm i\tau$ and search for the complex stationary paths in Euclidean spacetime numerically.

Thus the transition amplitude is formally expressed as

$$\langle (a,\beta_\pm) \text{ on II}, P_a=P_{\beta_\pm}=0|a=\beta_\pm=0,P_a=P_{\beta_\pm}=0\rangle =\exp(\mp I) , \quad (17)$$

where the sign in the exponent depends on how we rotate the time axis. Since the Hamiltonian vanishes in the present case, I is given by

$$I=\tfrac{1}{2}\int(\mathcal{P}_a^{\rm cl}a'^{\rm cl}-\mathcal{P}_a'^{\rm cl}a^{\rm cl}+\mathcal{P}_{\beta_+}^{\rm cl}\beta_+'^{\rm cl}-\mathcal{P}_{\beta_+}'^{\rm cl}\beta_+^{\rm cl} +\mathcal{P}_{\beta_-}^{\rm cl}\beta_-'^{\rm cl}-\mathcal{P}_{\beta_-}'^{\rm cl}\beta_-^{\rm cl})d\tau , \quad (18)$$

where $a^{\rm cl}$, $\beta_\pm^{\rm cl}$, $\mathcal{P}_a^{\rm cl}$, and $\mathcal{P}_{\beta_\pm}^{\rm cl}$ are solutions of Euclidean Hamiltonian equations which satisfy the appropriate boundary conditions given by Klauder's prescription. Thus we may assign the realization probability of the initial state of each classical trajectory in terms of

$$|\langle (a,\beta_\pm) \text{ on II}, P_a=P_{\beta_\pm}=0|a=\beta_\pm=0,P_a=P_{\beta_\pm}=0\rangle|^2 .$$

B. Fate of the classical universe

Having specified the initial state of the classical universes and their creation probability, we are now in a position to investigate the fate of each classical universe. We solve the Lorentzian equations of motion (16) starting from the surface II with the initial condition that all the momenta vanish. The calculation is done until either of the following sufficient conditions is satisfied:[3]

$$\dot{a} > 0 \text{ and } \Lambda > \frac{3}{a^2} \Longrightarrow \text{inflate} , \tag{19}$$

$$\dot{a} < 0 \text{ and } \Lambda < 3\left[\frac{\dot{a}}{a}\right]^2 \Longrightarrow \text{recollapse} . \tag{20}$$

Condition (19) is nothing but Wald's sufficient condition (1) and once it is satisfied, the spacetime always inflates. On the other hand, the criterion (20) is derived from the Raychaudhuri equation. From it we have

$$\dot{K} \leq \Lambda - \tfrac{1}{3}K^2 , \tag{21}$$

where $K = 3\dot{a}/a$ is the trace of the extrinsic curvature tensor. Thus if $\dot{a} < 0$ and $\Lambda - K^2/3 < 0$ or $\Lambda < 3(\dot{a}/a)^2$, $\dot{a}$ will never become positive, so that the spacetime will inevitably collapses to a singularity at least classically.

IV. RESULTS AND DISCUSSION

We have calculated according to the above algorithm and the main result is illustrated in Fig. 3. In this figure the creation surface of classical universes, $W[a,\beta_+,\beta_-]=0$, is projected on the (β_+,β_-) plane on which the scale factor is given by

$$a = a_*(\beta_+,\beta_-) \equiv \left[\frac{3(1-U[\beta_+,\beta_-])}{\Lambda}\right]^{1/2} . \tag{22}$$

The three curves in the figure stand for $a_*(\beta_+,\beta_-)=0$ lines, beyond which the singularity $a=0$ is exposed to the Lorentzian region and quantum-cosmological creation of the universe does not take place in the present scheme. In this figure, the initial configuration of each classical universe is depicted with a mark indicating its fate. That is, the circle stands for an inflating universe, while the cross indicates a universe which recollapses without inflation. This figure is independent of the values of the cosmological constant, because systems with different values of $\Lambda > 0$ are shown to be dynamically equivalent by rescaling the scale factor a and the time variable. In fact, dynamical behavior of the system $(a,\beta_\mp;\Lambda)$ is related with that of $(\tilde{a},\tilde{\beta}_\pm;\tilde{\Lambda})$ by the transformation rule

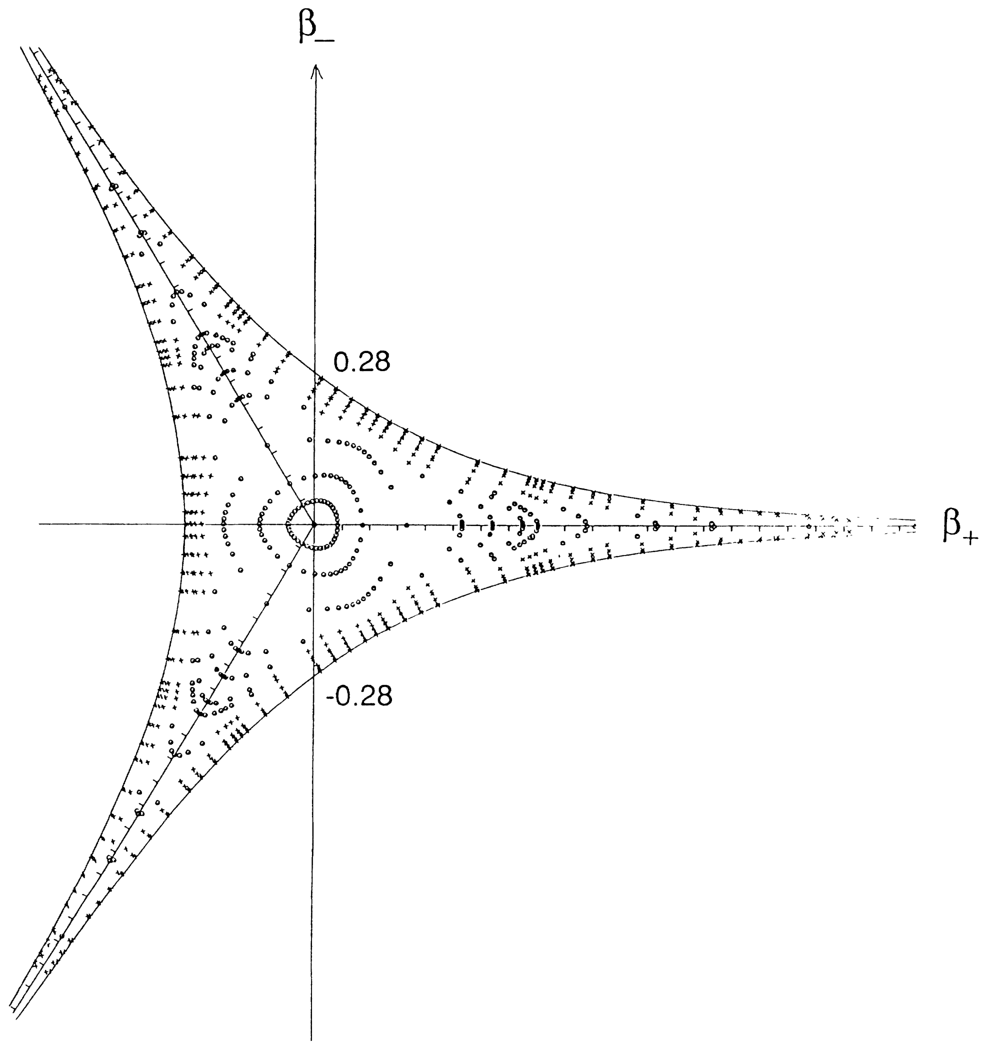

FIG. 3. Fate of the classical universes created on the $W[a,\beta_+,\beta_-]=0$ surface. Circles denote initial state of the inflationary universes, while crosses are those of noninflationary universes.

$$\sqrt{\Lambda}a=\sqrt{\Lambda}\tilde{a},\quad \beta_{\pm}=\tilde{\beta}_{\pm},\quad \text{and}\quad \sqrt{\Lambda}t=\sqrt{\Lambda}\tilde{t}\ .$$

What is remarkable in Fig. 3 is that the initial configuration of classical universes is clearly divided into inflationary trajectories and recollapsing trajectories. It is quite different from a result of classical analysis in which the system exhibits chaotic behavior near the classical singularity so that it is not predictable which trajectory inflates.[7]

As is seen in Fig. 3, in the present case most of the classical trajectories are inflationary except for those starting with a relatively large anisotropy and small spatial volume near the $a_*(\beta_+,\beta_-)=0$ lines. One should note, however, that there are classical universes which may inflate even if they start with a very large anisotropy and small spatial volume. For example, on the $\beta_-=0$ line there is an inflationary solution even if β_+ tends to infinity, in which case the equation of motion for a reads

$$\ddot{a}=\frac{-\dot{a}^2+1}{2a}-\tfrac{1}{2}a\Lambda\ , \tag{23}$$

which is the same as the equation of motion for the scale factor in the spatially flat FRW spacetime with the cosmological constant Λ.

It is also interesting to compare our result with Wald's criterion (1). On the creation surface of classical universes the criterion is not satisfied at any point except that $\Lambda=\frac{1}{2}R^{(3)}_{\max}$ at the isotropic point $\beta_{\pm}=0$. Hence a quantum-cosmological consideration picks up a set of inflationary universes without satisfying Wald's sufficient condition at the outset.

Next let us consider the generality of inflation in the present model. Though it seems most of the universes which are created quantum cosmologically are inflationary in Fig. 3, we should consider the probability distribution, which could be given by the absolute square of the transition amplitude, in order to see how general inflation is. Figure 4, which depicts the magnitude of the real part I_r of the stationary Euclidean action I Eq. (18) evaluated on the creation surface along the $\beta_-=0$ line, is helpful in the probability interpretation. This figure shows a generic feature of I_r on the whole $W[a,\beta_+,\beta_-]=0$ surface. That is, it is large for symmetric configurations and small for recollapsing universes.

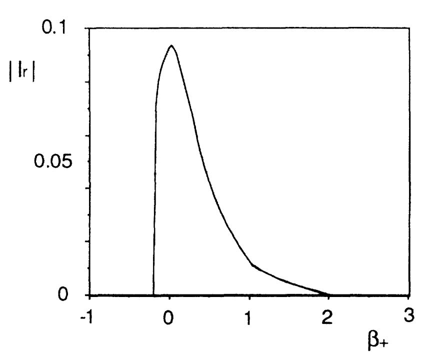

FIG. 4. Magnitude of I_r evaluated on the $W[a,\beta_+,\beta_-]=0$ surface along the β_- line. The vacuum energy density is taken to be $V=M_{\rm Pl}^4$.

In terms of I_r the probability amplitude that a classical universe is created with anisotropy $\beta_+\sim\beta_++d\beta_+$ and $\beta_-\sim\beta_-+d\beta_-$ is given by either

$$P_+[a,\beta_+,\beta_-]d\beta_+d\beta_-=\exp(2|I_r[a,\beta_+,\beta_-]|)d\beta_+d\beta_-\ , \tag{24}$$

or

$$P_-[a,\beta_+,\beta_-]d\beta_+d\beta_- =\exp(-2|I_r[a,\beta_+,\beta_-]|)d\beta_+d\beta_-\ , \tag{25}$$

depending on how we perform the Wick rotation, or in other words, how we take the integration path. In simple models the former corresponds to the Hartle-Hawking prescription[8] and the latter to the Vilenkin-Linde prescription.[9,18]

The former has an exponential peak on the isotropic state and this peak becomes sharper as Λ decreases. Hence this prescription predicts a symmetric universe which inflates. On the other hand, in the latter probability distribution it is likely that a classical universe emerges with large anisotropy and small spatial volume, which may or may not inflate. As the cosmological constant increases, inflation becomes more probable because the weight P_- becomes flatter.

Finally let us consider the probability of inflationary versus noninflationary universes. First, for the sake of comparison, we calculate the ratio of the area of the inflationary and noninflationary regions in Fig. 3, which turns out to be,

$$\int_{\rm inflationary}d\beta_+d\beta_-:\int_{\rm noninflationary}d\beta_+d\beta_- =63\%:37\%\ . \tag{26}$$

Next we evaluate the probability of inflation based on the probability amplitudes (24) and (25), keeping in mind that the maximum of $|I_r|$ is realized when $\beta_{\pm}=0$ and it is given approximately by

$$|I_r|_{\max}=\frac{60.4}{\kappa^2\Lambda}=0.096\frac{M_{\rm Pl}^4}{V}\ , \tag{27}$$

where we have defined the vacuum energy density $V=\kappa^{-2}\Lambda$. As two typical values of V, let us consider $V=M_{\rm Pl}^4$ and $V=10^{-11}M_{\rm Pl}^4$, the former suggested by primordial inflation[19] and the latter by the density fluctuation constraint.[20]

For $V=M_{\rm Pl}^4$, $2|I_r|_{\max}=0.19$ and $e^{\pm 2|I_r|}\sim 1$, so that there is no considerable difference between the two prescriptions. Indeed we yield

$$\int_{\text{inflationary}} P_+[a,\beta_+,\beta_-]d\beta_+d\beta_- : \int_{\text{noninflationary}} P_+[a,\beta_+,\beta_-]d\beta_+d\beta_- = 65\%:35\% \tag{28}$$

and

$$\int_{\text{inflationay}} P_-[a,\beta_+,\beta_-]d\beta_+d\beta_- : \int_{\text{noninflationary}} P_-[a,\beta_+,\beta_-]d\beta_+d\beta_- = 60\%:40\% \ , \tag{29}$$

On the other hand, for $V=10^{-11}M_{\rm Pl}^4$, we yield $2|I_r|_{\max}\sim 10^{10}$. Hence P_+ predicts de Sitter spacetime with a very high probability, while P_- predicts very anisotropic universes, most, but not all, of which recollapses without inflation. One may wonder that the above result is contrary to that in the mini-superspace model which contains the FRW scale factor a and homogeneous scalar field ϕ with a potential $V[\phi]=m^2\phi^2/2$ as degrees of freedom. This is because the potentials $V[\phi]$ and $U[\beta_+,\beta_-]$ are involved in the superpotentials of the Wheeler-DeWitt equations in a different manner. If we replaced Λ with the potential energy $V[\phi]$ in the present model, P_+ or the Hartle-Hawking prescription would predict isotropic universes with small $V[\phi]$ but no inflation, while P_- or the Vilenkin-Linde prescription would predict anisotropic universes with large $V[\phi]$, which may inflate.

In conclusion, we have considered the naturalness of inflation in Bianchi type-IX spacetime with a positive cosmological constant. First we argued about difficulties in both classical and quantum analyses of this spacetime. Then we have developed a semiclassical analysis scheme to see the fate of classical universes which presumably have typical trajectories of those created quantum cosmologically. Though our approach may not take full account of quantum-gravitational effects, we have obtained a definite result thanks to the semiclassical treatment. That is, we have found that most of the classical trajectories thus created experience inflation, even though they do not satisfy Wald's criterion for inflation at the start of classical evolution. We have also discussed the probabiities of inflation in this mini-superspace in terms of the transition amplitude given by the coherent-state path integral.

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