Time variation of fundamental constants: Bounds from geophysical and astronomical data

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Consistent bounds for the simultaneous variations of fundamental constants in the standard model of fundamental interactions are obtained from astronomical, astrophysical, and geophysical data. These bounds exclude the Dirac large-number hypothesis and, in general, any theory demanding a large variation of the fundamental constants. They also impose severe constraints on Kaluza-Klein and superstring theories, and should be considered as strong tests of the equivalence principle.

I. INTRODUCTION

The standard model (SM) of fundamental interactions together with general relativity (GR) provides a consistent description of all known low-energy phenomena [i.e., low compared with the grand unified (GU) energy scale], in good agreement with experiment. This model depends on a set of parameters called the "fundamental constants." These are supposed to be universal parameters: i.e., time, position, and reference-frame invariant. Indeed, Einstein's equivalence principle, on which GR is based, implies such an invariance.

However, the time variation of fundamental constants has been an active subject of research since the introduction of the large-number hypothesis (LNH) by Dirac long ago.[1] This hypothesis was based on the existence of several large dimensionless numbers, such as the ratio of electrostatic and gravitational potentials in the hydrogen atom, whose value is near the ratio of the age of the Universe and a typical period of the hydrogen atom. Assuming that the former quantity is proportional to the latter, the existence of the unnatural large number is "explained." The simplicity of the LNH and its large predictive power leads to numerous theoretical and experimental researches on the time variation of fundamental parameters.

On the theoretical side, there have been many proposals, both phenomenological[2] and theoretical,[3] leading to a time variation of the fine-structure constant. Unifying schemes such as Kaluza-Klein[4] or superstring theories[5] provide a very general framework to study the time variation of fundamental constants. Indeed, it has been shown that Kaluza-Klein theories have cosmological solutions where the fundamental constants do vary,[6,7] and the same occurs in superstring theories.[8]

Partially inspired by these theoretical results, many attempts have been made to set observational or experimental bounds on the time variation of fundamental constants. Table I shows a summary of the most accurate bounds obtained from several sources, assuming the given constant is the only one which varies in time. This would give the right order of magnitude if there were no correlations between the variation of the constants to cancel its effect on any given physical observable. However, there are reasons to expect that several constants may vary simultaneously and that correlations are a consequence of deep theoretical results. For instance, the validity of Einstein's gravitational equations implies that the product of the gravitational constant and the mass of the body must be time independent.[17] Therefore, it is interesting to analyze the time variation of fundamental constants without the assumption of no conspiracy, and so we shall attempt to do so in this paper.

The time variation of fundamental constants will produce a host of different phenomena: changes in atomic and nuclear spectra,[9,10] variation of planetary radii and moments of inertia,[13] orbital evolution,[10] and anomalous luminosities of faint stars.[14] Nucleosynthesis, both cosmological[15] or stellar,[16] has also been used to set bounds on the variability of fundamental parameters. In this paper we shall analyze short-term local phenomena—astronomical and geophysical data based on time intervals much shorter than the age of the Universe—and so set bounds on the variability of the fundamental constants today in the solar system. Since other astrophysical and cosmological data refer to very different time scales, it seems reasonable to analyze these latter events in a separate way.

Our paper is organized as follows. In Sec. II we expose a simple phenomenological framework to study the time variation of fundamental constants and our choice of fundamental parameters is explained. In Sec. III we discuss the observational evidence available from astronomical

TABLE I. Sample bounds on the time variation of fundamental constants. These bounds assume that only a single constant varies.

Magnitude M	Bound on $\dot{M}/M$ (yr^{-1})	Reference
G_N	10^{-12}	10
α	4×10^{-12}	9
α	10^{-17}	11
$g_p m_e/m_p$	8×10^{-12}	9

and geophysical phenomena, and in Sec. IV we state our conclusions. Several appendixes discuss details of the analysis for each member of the data set.

II. A PHENOMENOLOGICAL MODEL

In this section we shall develop a very simple phenomenological model for the analysis of the consequences of the time variation of fundamental constants. It will be based on the adiabatic hypothesis, i.e., that the main changes in observable quantities are due to the time variation of the parameters, neglecting the necessary modifications of the SM. Although such a procedure will yield correct expressions for the change in observable quantities, one would not be able to relate the rate of change to interesting quantities, such as the Hubble constant or the contraction rate of extra dimensions, without a deeper analysis. This is because the Lagrangian obtained by simple substitution of time-varying parameters is generally inconsistent (Ref. 3 develops a simple consistent model for the time variation of α, the fine-structure constant).

To begin with, we must choose a definite system of units. In a world of time- and (space-) independent parameters, this choice is completely arbitrary; but this will not be so in a world with time-varying parameters.[17] Different systems of units can be chosen so that different parameters are time independent. In simple model theories with a time-varying gravitational constant, two such systems are the gravitational units, where G_N is time independent but atomic parameters are time dependent; and atomic units, where the opposite occurs. In a much more complex theory, such as the SM, very many different systems of units are possible.

In order to specify our system of units we shall first assume the constancy of c and $\hbar$, since this assumption simply fixes the length-to-time units ratio, while an $\hbar$-varying theory can be transformed into a G_N- (or α-) varying theory with a suitable conformal transformation. Moreover, we can use a finite, time-dependent renormalization-group transformation to select any dimensional quantity as a time-dependent energy unit.[18] With such a choice, which amounts to taking a time-varying renormalization point, one builds the desired system of units. There are several choices for the energy standard, defining several different systems of units having different physical meaning. Any of these systems will be related to any other through a finite renormalization-group transformation, although its explicit construction may be extremely difficult to carry out. In this paper we shall introduce the Salam-Weinberg system of units (SWU's), where the mass of the intermediary vector meson W,M_W, is taken as the time-independent energy unit. All our analysis will be carried out in SWU's.

There are several constraints between the fundamental parameters in the SM, and some difficulties related to the evaluation of the effect of their time variation on observable quantities. We shall list some of these problems and the corresponding choice of variable quantities.

(a) *Gravitational interactions.* In spite of many attempts of unification with other fundamental interactions, gravitation remains in isolation and its only parameter, Newton's gravitational constant G_N, is still unrelated to other fundamental constants. (See, however, Ref. 7.) We shall take G_N as one of our fundamental time-dependent parameters.

(b) *Electroweak interactions.* The Salam-Weinberg unification of electroweak interactions is well supported by experiment not only at the tree level, but also at the radiative correction level.[19] We shall assume the validity of the fundamental relations between the parameters of the theory: a consequence of the adiabatic hypothesis.

We choose the fine-structure constant α and Fermi weak-interaction constant G_F as our fundamental time-varying parameters. These are related to other fundamental parameters of the theory through the equations

$$\alpha=\alpha_1\sin^2\theta_W , \tag{2.1a}$$

$$G_F=\sqrt{2}\alpha_1/8M_W^2 . \tag{2.1b}$$

In these equations, α_1 and α_2 are the SU(2)⊗U(1) coupling constants and the Weinberg angle is defined as

$$\tan^2\theta_W=\alpha_1(M_W)/\alpha_2(M_W) . \tag{2.2}$$

The intermediate boson masses M_W and M_Z can be expressed using the vacuum expectation value (VEV) of the Higgs field, v:

$$M_W^2=\frac{v^2}{2}\alpha_1 , \tag{2.3a}$$

$$M_Z^2=M_W^2/\cos^2\theta_W . \tag{2.3b}$$

Thus, in SWU's the time variation of all fundamental parameters in the electroweak sector of the theory is fully determined by the time variation of α and of G_F. The latter quantity is, however, not directly observable: only the time variation of the product $G_F\cos^2\theta_C$, where θ_C is Cabibbo's angle, is directly measurable since there are no long-time high-precision measurements in the leptonic sector of the theory.

(c) *Strong interactions.* In the low-energy regime, strong interactions are effectively isolated but the single coupling constant α_3 is very big and nonperturbative effects are dominant. However, if u-d quarks are massless there is a single parameter in the theory, namely, the QCD scale parameter Λ, and we shall choose it as our fundamental parameter, since all static observables with dimension of mass must be proportional[20] to Λ. More precisely, any quantity σ with dimension D must satisfy an equation of the form

$$\sigma=\Lambda^D f\left[\frac{Q}{\Lambda}\right] , \tag{2.4}$$

where Q is a (set of) quantity specifying the energy scale while σ is measured and Λ is the scale parameter. However, for static quantities such as the proton mass, Eq. (2.4) takes the form

$$\sigma=\Lambda^D f\left[\frac{\sigma}{\Lambda^D}\right] \tag{2.5}$$

since the only scale parameter is σ itself. The solution of Eq. (2.5) has the form

$$\sigma = X\Lambda^D , \tag{2.6}$$

where X is a well-defined dimensionless numerical constant. As a consequence of (2.5) we see that dimensionless static observables are time independent in massless QCD with time-varying Λ. All low-energy static quantities, such as nuclear masses and radii, will satisfy an equation of the form (2.6). Even in the presence of massive u-d quarks, provided their masses are small enough, the dominant contribution to the ground state (or to a low-energy state) will be of the form (2.6). In this paper we shall assume massless quarks and so we shall apply Eq. (2.6) to compute the time variation of all nuclear masses and radii.

(d) *Higgs sector.* In the absence of a well-defined theory of the Higgs sector of the SM, a host of experiments would be necessary to analyze the time variation of the fundamental parameters in this sector. However, only a few of them, namely, the electron mass and the Cabibbo angle, are relevant in the low-energy regime and we may take them as fundamental parameters. That the time variation of the Cabibbo angle cannot be directly observed by our choice of massless u-d quarks implies that θ_C is time independent in our model.

(e) *Thermodynamical considerations.* The time variation of fundamental parameters will produce changes in the equation of state of macroscopic bodies that can be computed using simple thermodynamic considerations of a very general nature. Under an adiabatic change $\delta\lambda$ of a parameter λ, the free energy of the system will change in the form[21]

$$\delta F = \delta\lambda \left[\frac{\partial F}{\partial \lambda} \right]_{T,V} = \delta\lambda \left\langle \frac{\partial H}{\partial \lambda} \right\rangle_{T,V} . \tag{2.7}$$

The Hamiltonian H, however, can be written in the form

$$H = T + U , \tag{2.8}$$

where, for an electron gas, U accounts for the Coulomb interaction. In this case, the second term in the right-hand side is homogeneous of the first degree in α, while the first one is homogeneous of degree -1 in m_e. So we find

$$\dot F = -\frac{\dot m_e}{m_e}\langle T\rangle + \frac{\dot\alpha}{\alpha}\langle U\rangle . \tag{2.9}$$

In addition, both $\langle T\rangle$ and $\langle U\rangle$ can be written in terms of observable quantities using energy conservation and the virial theorem,

$$E = \langle T\rangle + \langle U\rangle , \tag{2.10a}$$

$$3pV = 2\langle T\rangle + \langle U\rangle , \tag{2.10b}$$

and in this way we can express the change in free energy in terms of observable quantities:

$$\langle T\rangle = 3pV - E , \tag{2.11a}$$

$$\langle U\rangle = 2E - 3pV . \tag{2.11b}$$

(f) *Time units transformation.* Several of the upper bounds observed are reported in a system of units different from SWU's we have in this work. For instance, astronomical observations are reported either in ephemeris time (where both G_N and planetary masses are assumed time independent) or in atomic time. Simple transformation rules can be derived observing that the observation time (atomic, ephemeris, or whatever you wish) t_1 will be related to SWU's time t in the form

$$t_1 = t + \tfrac{1}{2}\theta t^2 , \tag{2.12}$$

where θ is some linear combination of the time derivatives of the fundamental constants. The time derivatives in SWU's will be related to the reported ones through the equations

$$\frac{d\alpha}{dt} = \frac{dt_1}{dt}\frac{d\alpha}{dt_1} = (1+\theta t)\frac{d\alpha}{dt_1} , \tag{2.13a}$$

$$\frac{d^2\alpha}{dt} = \theta\frac{d\alpha}{dt_1} + (1+2\theta t)\frac{d^2\alpha}{dt_1^2} \tag{2.13b}$$

the last term being generally negligible. The application of these procedures to particular cases will be discussed in the appendixes.

(g) *Renormalization-group equations.* There are several parameters in our model that cannot be computed in a model-independent way from our fundamental parameters: namely, the time variation of the strong-interaction constant α_3 and of the renormalization point μ. We shall call these model-dependent parameters, since they can be computed within a larger model that contains the SM as a low-energy limit. In this paper, we shall limit ourselves to show how these parameters can be computed in a grand unified theory (GUT) which can be itself a low-energy limit of the Kaluza-Klein or superstring model.

We assume that at the grand unification scale Λ_u all the running coupling constants have a common value α_u. This is related to the Salam-Weinberg scale ($\mu \approx M_W$) values of the running constants α_i through the renormalization-group equations. Following Ref. (7) we write them in the form

$$\frac{\dot\alpha_i}{\alpha_i^2} = \frac{\dot\alpha_\mu}{\alpha_u^2} + \frac{1}{\pi}\sum_j C_{ij}\left[\frac{\dot\Lambda_u}{\Lambda_u} - \frac{\dot\mu}{\mu}\right] , \tag{2.14}$$

where we have neglected the fermion contributions, and the constants C_{ij} have well-defined values in the SM. Assuming no new physics, $\sum C_{ij}$ is equal to -2, $\frac{5}{3}$, and $\frac{7}{2}$ for $i=1,2,3$. In addition, α_3 and μ are related with Λ through the relation

$$\frac{\dot\Lambda}{\Lambda} = \frac{6\pi}{33-2N_F}\frac{\dot\alpha_3}{\alpha_3^2} + \frac{\dot\mu}{\mu} . \tag{2.15}$$

These four equations are enough to find the time variation of the model-dependent parameters α_3 and μ, and the GUT parameters α_u and Λ_u.

III. ANALYSIS OF OBSERVATIONS

In this section we shall analyze and discuss different observations of geophysical, astronomical, and geochemical nature in order to obtain bounds for the time variation of fundamental constants. Our discussion will be mainly qualitative, leaving most quantitative details for the appendixes.

A. Planetary radii

Planetary radii will change under a time variation of fundamental constants because of the variation in cohesion of matter and the pull of gravity. The theory of those variations has been modeled on those of Refs. 13 and 22, and is sketched in Appendix A.

The final result for the variation of planetary radii as observed from structural changes in the planetary surface is given by Eq. (A.12). The coefficients of these equations can be computed if density, pressure, and bulk modulus distributions of the planet are known. This is true for Earth, where seismological data yield accurate distributions of these quantities,[23] and for some smaller bodies of the solar system, such as Mercury and the Moon, whose chemical composition can be inferred from geophysical observations and where a linearized equation of state is a good approximation because of its small compression.

McElhinni, Taylor, and Stevenson report upper bounds for the change in the radius of several planets from a variety of geophysical observations. The mean rates of variation for the Moon and Mercury radii are shown in Table III, together with the coefficients of the conditional equations. The variation of Earth radii has not been included because in spite of the accuracy of the observations, its complicated geological history makes the determination of its paleoradius unreliable.

B. Earth's moment of inertia

The variation of Earth's moment of inertia can be computed from the change in angular velocity induced on the Earth-Moon system because of conservation of angular

TABLE II. Observational data. The columns show the data number (correlated with the conditional equation number in Table III), a simple data description, the observed value and the corresponding standard deviations (in units of 10^{-11} yr^{-1}), the system of units of the observation and the references.

Eq. Description	Value (10^{-11} yr^{-1})	Unit	Ref.
Planetary paleoradius: $\dot{R}/R$			
(1) Mercury	0.0±0.012	SW	12
(2) Moon	0.0±0.015	SW	12
Lunar secular acceleration: $\dot{n}/n$			
(3) Mercury transits	−15.0±1.2	ET	29
(4) Ancient eclipses	−17.3±1.8	ET	25
(5) Growth rhythms	−14.2±2.4	AT	24,30
(6) LLR	−13.7±1.0	AT	28
(7) Tidal models	−15.2±3.0	SW	24
(8) Satellite data	−14.4±1.7	SW	31
Earth's secular acceleration: $\dot{\Omega}/\Omega$			
(9) Ancient eclipses	−24.3±2.0	ET	25
(10) Ancient equinoxes	−23.6±2.3	ET	25
(11) Growth rhythms	−22.5±1.0	AT	24
Viking ranging data			
(12) $\dot{G}_N/G_N$	0.0±1.2	AT	13
(13) $\dot{\beta}$	0.0±2.4	AT	13
Binary pulsar data			
(14) $\dot{n}/n$	1.0±1.2	AT	32
Laboratory data:			
(15) Clock rate diff.	−0.2±1.2	SW	36
Long-lived β decayers: $\dot{\lambda}/\lambda$			
(16) ^{187}Re	2.3±1.8	αU	39,41
(17) ^{40}K	0.0±0.29	αU	42
(18) ^{87}Rb	0.0±0.29	αU	42
Oklo phenomenon: $\dot{\sigma}/\sigma$			
(19) ^{149}Sm	0.0±69.0	SW	11,35

momentum (see Appendix B). The change in angular velocity of Earth can be directly measured, on the other hand, from the analysis of ancient astronomical observations[24,25] or from the analysis of paleontological data.[24] The observable quantity is an effective change in the moment of inertia, due to observation of the Sun from the rotary reference system of Earth[25] (see Appendix C). Neither of these observational methods is free of trouble: ancient astronomical observations cover a short period of the history of Earth, where small changes in the moment of inertia due to deglaciation effects are to be expected.[26] Paleontological data suffer from ambiguities in their interpretation.[27] Both methods are hampered by tidal friction. In order to obtain meaningful results from these data, a simultaneous analysis of the lunar acceleration and of Earth rotation results (following the pattern of Refs. 24–27) is necessary.

Paleontological data relies on the recording of tidal and climate phenomena on the shells of living animals and so records changes in the moment of inertia in atomic units where α and m_e are constants. Ancient astronomical observations measure the same change in ephemeris time where both G_N and planetary masses are assumed time independent. Both sets of data yield complementary information on the time variation of fundamental constants. The corresponding corrections have been introduced in the equations of Table III.

C. Orbital perturbations

The main perturbations induced on a Keplerian system by the time variation of fundamental constants will be an acceleration in longitude of the planet or satellite (Appendix C). This longitude acceleration cannot be observed in ephemeris time, since it is universal, but it can be observed in atomic time.

TABLE III. Coefficients of conditional equations. The columns show (1) the equation number (the same as that of Table II), (2) $\dot{\Lambda}/\Lambda$, (3) $\dot{\alpha}/\alpha$, (4) $\dot{m}_e/m_e$, (5) $\dot{G}_F/G_F$, (6) $\dot{G}_N/G_N$, (7) $\dot{n}/n$, (8) $\dot{\Omega}_N/\Omega_N$. All coefficients are in units of 10^{-11} yr^{-1}.

1	2	3	4	5	6	7	8
1	−0.039	1.30	1.32	−0.019	0.0	0.0	0.0
2	−0.008	0.9	0.9	−0.004	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	1.0	0.0
4	0.28	0.0	0.0	−0.11	0.0	1.0	0.0
5	−5.0	−2.0	−1.0	−2.0	0.0	1.0	0.0
6	−4.0	−4.0	−2.0	−2.0	0.0	1.0	0.0
7	−11.76	−1.61	−1.44	−3.94	0.0	1.85	1.0
8	−11.5	−1.61	−1.44	−3.83	0.0	1.85	1.0
9	−6.48	−2.0	−1.0	−1.83	0.0	1.85	1.0
10	0.0	0.0	0.0	0.0	0.0	1.0	0.0
11	0.0	0.0	0.0	0.0	0.0	1.0	0.0
12	−1.0	12.0	6.0	1.0	0.0	0.0	0.0
13	0.0	4.0	2.0	0.0	0.0	0.0	0.0
14	4.0	4.0	2.0	2.0	0.0	0.0	0.0
15	−1.0	3.0	1.0	0.0	0.0	0.0	0.0
16	5.62e2	2.16e4	−6.0e2	0.0	2.0	0.0	0.0
17	17.0	46.0	−1.0	0.0	2.0	0.0	0.0
18	2.71	1.07e3	−7.83	0.0	2.0	0.0	0.0
19	25.0	9.8e6	0.0	0.0	−707.0	0.0	0.0

Table II shows several determinations of the lunar tidal acceleration both in ephemeris[29] and atomic[30] time. Comparison of both data sets would yield the acceleration due to the variation of fundamental constants. However, because of the contamination of the data with tidal and deglaciation effects, we have analyzed it together with data on Earth's rotation, following the prescription of Ref. 25. The corresponding equations are shown in Table III.

The binary pulsar offers a second independent determination of orbital evolution due to the time variation of fundamental constants.[32] The astronomical system is very clean and the determination is very reliable.

Another determination of orbital evolution has been made on the motion of Mars as recorded from the Viking lander data.[13] The accuracy of these measurements is so high that it is possible to obtain a meaningful separation of the rate of variation of Newton's constant and of the mass. The corresponding equations are entries (5) and (6) in Table III. They include corrections for the conversion from atomic to SW units.

D. Long-lived β decayers

The half-life of long-lived β decayers, such as ^{187}Re or ^{40}K has been used by Dyson[33] to find upper bounds for the time variation of the fine-structure constant. These nuclei have a very long half-life that has been determined either in laboratory measurements or by comparison with the age of meteorites, as found from α-decay radioactivity analysis. Appendix D shows our analysis for the three β decayers ^{187}Re, ^{40}K, and ^{87}Rb.

In our phenomenological model, the abundance of any unstable nucleus will obey the following decay law:

$$N = N_0 e^{-(\lambda t + \dot{\lambda} t^2/2)} . \tag{3.1}$$

In standard physics, the effective decay constant can be found if the age of meteorites can be determined by means of any other present nucleus of known mean life. In our model, this nucleus will obey a decay law similar to Eq. (3.1), and the age of meteorites will have a different value for different nuclear species. For α and β decayers, the age in SWU's should be determined from the equations

$$M_\beta = \lambda_\beta^0 t + \tfrac{1}{2}\dot{\lambda}_\beta t^2 , \tag{3.2a}$$

$$M_\alpha = \lambda_\alpha^0 t + \tfrac{1}{2}\dot{\lambda}_\alpha t^2 , \tag{3.2b}$$

where M_α and M_β are the quantities measured from relative abundance data and the superscript 0 denotes the present-day value of the parameter. The α age of the sample is defined as

$$t_\alpha = \frac{M_\alpha}{\lambda_\alpha^0} \tag{3.3}$$

while the effective decay constant is obtained as

$$\lambda_\beta^{\text{eff}} = \frac{M_\beta}{t_\alpha} . \tag{3.4}$$

From these equations we obtain the time variation of the decay constants:

$$\frac{\dot{\lambda}_\beta}{\lambda_\beta}=\frac{2}{t_\alpha}\left[\frac{\lambda_\beta^{\text{eff}}}{\lambda_\alpha^0}-1\right]+\frac{\dot{\lambda}_\alpha}{\lambda_\alpha} . \tag{3.5}$$

The last term is similar to a conversion from some units to SWU. Using expressions for the logarithmic derivatives of the decay constants in Appendix D we find the coefficients for equations in Table III.

E. The Oklo phenomenon

About 2×10^9 yr ago, a natural nuclear reactor operated for 6.5×10^6 yr in the uranium ore deposits in Oklo, Gabon. From an analysis of nuclear and geochemical data,[35] the operating conditions of the reactor could be reconstructed and the thermal neutron capture cross sections of several nuclear species measured.[35] In particular the ^{149}Sm capture cross section is strongly dependent on the position of a resonance level of the compound nucleus ^{150}Sm, being sensitive to small changes in its width and position.[11] Upper bounds for the variation of fundamental constants can be found if the functional dependence of the parameters can be derived. However, this is an extremely difficult task, since the capture level is an extremely complex state of a many-body system. In our analysis we treat it as a finite-temperature Fermi gas (Appendix E).

It is interesting to note that in our phenomenological model, the ^{149}Sm neutron capture resonance yields little information about the time variation of the strong-interaction parameter Λ. This is not difficult to understand: in a world of massless quarks, any nuclear energy must be exactly proportional to Λ and

$$\frac{\dot{\Lambda}}{\Lambda}=\frac{\Delta E}{E}\approx10^{-10}\ \text{yr}^{-1} \tag{3.6}$$

must be true. The much smaller bound found in Ref. 11 refers to the time variation of the depth of the nuclear well. As shown in Appendix E, this quantity seems unrelated to E_0 in our model.

The computation of the observable effects in the Oklo phenomenon is difficult and defiled with ambiguities. The tabulated coefficients may be wrong by an order of magnitude and we have multiplied by 3 the standard deviation of the measurement in order to take into account the theoretical uncertainties.

F. Laboratory experiments

There is a single laboratory experiment accurate enough to yield interesting bounds on the rate of variation of atomic constants.[36] In this experiment, a set of cesium atomic clocks were compared with a set of superconducting cavity stabilized oscillators. The frequency ratio between the hyperfine transition of cesium and the characteristic frequency of the cavity is given a simple function of the fundamental parameters such that

$$\frac{\dot{\nu}}{\nu}=\frac{\dot{m}_e}{m_e}-\frac{\dot{\Lambda}}{\Lambda}+3\frac{\dot{\alpha}}{\alpha} . \tag{3.7}$$

The measured value is shown in Table II and the corresponding equation in Table III.

G. Luminosity of faint stars

The luminosity of faint stars has been used to set strong constraints on scale-covariant theories of gravitation.[14] This anomalous luminosity is due to the radiation of internal energy of the star as it adjusts to the change in its structure due to the variation of G_N. Conservation of energy is essential for the derivation of this result.

However, in our model the time variation of fundamental constants induces no anomalous luminosity on such stars. Indeed, energy is not conserved in our model, but injected into the system by the variation of the constants. The correct energy balance equation will be (in the nonrelativistic limit)

$$\frac{dE}{dt}=\frac{\partial E}{\partial t}+\text{div}J , \tag{3.8}$$

where E is the local energy density and J is the energy current. The total time derivative is the energy production due to the time variation of fundamental constants, while the partial time derivative is the local change. However, our system is Hamiltonian in this approximation, and for any Hamiltonian theory the following identity holds,

$$\frac{dH}{dt}=\frac{\partial H}{\partial t} , \tag{3.9}$$

and taking the expectation value of the former equation we find that J vanishes and, as the luminosity is the flux of J, there is no anomalous luminosity in our model. A more direct proof of this result, based on the usual stellar structural equations is given in Appendix F.

IV. RESULTS AND CONCLUSIONS

The equations shown in Table III form an overdetermined set of constraints that the observational data must satisfy. A least-squares solution to the set of constraints is shown in Table IV, together with 95% confidence limits. These latter limits are much smaller than the Hubble rate (as can be seen from the last column of the table) and so we can exclude the Dirac large number hypothesis and, more generally, any theory showing a large variation of the fundamental constants. These theories should, however, satisfy the hypothesis of our phenomenological model. Scale-covariant theories do not satisfy them, and so they are not excluded by our present results.

Since our bounds form a consistent set, we can obtain from them bounds for the variation of other fundamental parameters of the SM, such as the mass of the intermediate vector boson Z or the vacuum expectation value (VEV) of the Higgs fields, v. The second part of Table IV shows these bounds, as computed from Eqs. (2.1a)–(2.3b) and the results of the first part of the table. These upper bounds are both consistent with experimental data and independent of any conspiration among the constants.

In the same way, we can find consistent upper bounds for the time variation of the fundamental constants at a GU scale. These results, together with the time variation rates of the model-dependent parameters, are shown in the last part of Table IV.

Both Kaluza-Klein and superstring theories predict time variation of fundamental constants depending on the cosmological model parameters. In these theories, the common value of the running coupling constants at the GU scale is related to the size of the extra dimensional space $R_I \approx \Lambda_u^{-1}$. In the case of Kaluza-Klein theories, $\alpha_u \propto R_I^{-2}$ and from this relation we find the result

$$|\dot{R}/R| \le 10^{-11}\ \mathrm{yr}^{-1} \qquad (4.1)$$

for the present contraction rate. Again, this result is independent of any conspiration between the different variation rates. Our bounds also impose very stringent constraints on the time variation of fundamental constants induced in superstring theories.

As we have mentioned before, Einstein's equivalence principle implies that all nongravitational constants of Nature must be time and position independent. The strong equivalence principle extends that statement to gravitational phenomena. Our results show that both forms of the principle of equivalence are very well satisfied, within a small fraction of the Hubble rate. Since the unrestricted validity of the principle of equivalence leads to general relativity as the only low-energy theory of gravitation, our results should be considered as an accurate verification of general relativity.

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TABLE IV. Results. The table shows the name of the parameter, the value, and the standard deviation for the fundamental parameters of our model, the 95% confidence limits as upper bounds and the same quantities in units of the Hubble constants. In order to get upper bounds a low value of $H_0 \ge 55$ km sec^{-1}Mpc^{-1} has been used.

(a) Values and bounds on fundamental parameters

$\dot{\Lambda}/\Lambda$	$(3.9\pm5.5)\times10^{-3}$	1.2×10^{-2}	2.2×10^{-3}
$\dot{\alpha}/\alpha$	$(-1.3\pm6.5)\times10^{-5}$	1.4×10^{-4}	2.6×10^{-5}
$\dot{m}_e/m_e$	$(0.0\pm5.1)\times10^{-3}$	1.1×10^{-2}	2.0×10^{-3}
$\dot{G}_F/G_F$	$(-1.8\times6.7)\times10^{-2}$	1.4×10^{-1}	2.6×10^{-2}
$\dot{G}_N/G_N$	$(0.3\pm2.2)\times10^{-1}$	4.8×10^{-1}	8.7×10^{-2}
$\dot{n}/n$	-14.62 ± 0.44		
$\dot{\Omega}_N/\Omega_N$	4.20 ± 0.90		

(b) Model-independent bounds on the SM parameters

$\dot{\alpha}_1/\alpha_1$	1.4×10^{-1}	2.6×10^{-2}
$\dot{\alpha}_2/\alpha_2$	4.2×10^{-2}	7.6×10^{-3}
$\dot{\theta}_W$	3.8×10^{-2}	6.9×10^{-3}
$\dot{M}_z/M_z$	2.1×10^{-2}	3.8×10^{-3}
$\dot{v}/v$	7.0×10^{-2}	1.3×10^{-2}
$\dot{f}_e/f_e$	2.8×10^{-1}	5.0×10^{-2}

(c) Bounds on GUT and model-dependent parameters

$\dot{\alpha}_U/\alpha_U$	6.1×10^{-2}	1.1×10^{-2}
$\dot{\alpha}_3/\alpha_3$	5.8×10^{-1}	1.0×10^{-1}
$\dot{\mu}/\mu$	5.2	0.95
$\dot{\Lambda}_U/\Lambda_U$	2.8	0.52

APPENDIX A: TIME VARIATION OF PLANETARY PARAMETERS

In order to compute the change in planetary parameters, we first approximate the planet as a nonrelativistic spherical distribution of ideal fluid, for which the following form of the equation of continuity holds:

$$\frac{1}{r}\frac{\partial[r^2\rho(r)v(r)]}{\partial r} = -\frac{\partial\rho}{\partial t} + \Theta , \qquad (A1)$$

where Θ represents a mass increase per unit volume rate.

From (A1) we find

$$R^2\rho_0\dot{R} = \int_0^R r^2(-\dot{\rho}+\Theta)dr , \qquad (A2)$$

where ρ_0 is the planet surface density and $\dot{R} = v(R)$. In order to express $\dot{\rho}$ in terms of the temporal derivatives of fundamental constants, we shall use several thermodynamic results. First, we use the equation of state of the solid at zero temperature in the form

$$\dot{\rho} = \frac{\rho}{k}\dot{p} , \qquad (A3)$$

where k is the bulk modulus and p the pressure. However, the variation rate of the pressure can be computed from the thermodynamic relations derived in Sec. II and the identity

$$p = -\left[\frac{\partial F}{\partial V}\right]_T . \qquad (A4)$$

Taking the time derivative on both sides of this equation and using Eq. (2.9) we find

$$\dot{p} = \frac{\dot{\alpha}}{\alpha}\frac{\partial\langle U\rangle}{\partial V} + \frac{\dot{m}_e}{m_e}\frac{\partial\langle T\rangle}{\partial V} . \qquad (A5)$$

The mean values can be expressed in terms of the energy and the pressure from Eqs. (2.11), and we finally get

$$\dot{p} = \frac{\dot{\alpha}}{\alpha}(5p-3k) + \frac{\dot{m}_e}{m_e}(4p-3k) . \qquad (A6)$$

The above equation, once substituted in (A2) will give the effect of changes in the fine-structure constant and the electron mass on the planetary radii. On the other hand, the dominant contributions from the variation of G_N and Λ to $\dot{R}$ can be obtained from the integrated hydrostatic equilibrium equation

$$p(r) = \int_r^R \frac{G_N\rho(r')M(r')}{r'^2}dr' . \qquad (A7)$$

Taking the time derivative of Eq. (A7), and replacing $\dot{\rho}$ with Θ, the contribution due to mass increase, we can write

$$\dot{p} = \int_r^R \frac{G_N\rho(r')M(r')}{r'^2}dr' \left[\frac{\dot{G}_N}{G_N} + \frac{\Theta}{\rho} + \frac{\dot{M}(r')}{M(r')}\right] + G_N\rho_0\frac{M}{R^2}\dot{R} , \qquad (A8)$$

where

$$\dot{M}=4\pi\int_0^r \Theta r'^2 dr' \tag{A9}$$

and the mass production factor can be approximated exceedingly well by the variation of the nucleon mass:

$$\Theta=\rho\frac{\dot{\Lambda}}{\Lambda}+O\left[\frac{m_e}{\Lambda}\right] . \tag{A10}$$

So we obtain, for this contribution to the pressure variation,

$$\dot{p}=\left[\frac{\dot{G}_N}{G_N}+2\frac{\dot{\Lambda}}{\Lambda}\right]\rho+G_N\rho_0\frac{M}{R^2}\dot{R} . \tag{A11}$$

Collecting our previous results and substituting in Eq. (A2) we obtain the time variation of the planetary radius:

$$\frac{\dot{R}}{R}=\frac{\dot{\Lambda}}{\Lambda}\frac{M-2M_k}{3M_D}-\frac{\dot{G}_N}{G_N}\frac{M_k}{M_D}+\frac{\dot{\alpha}}{\alpha}\frac{M-\frac{5}{3}M_k}{M_D}+\frac{\dot{m}_e}{m_e}\frac{M-\frac{4}{3}M_k}{M_D} , \tag{A12}$$

where we have defined the quantities

$$M_k=4\pi\int_0^R dr\, r^2\rho(r)\frac{\rho(r)}{k(r)} , \tag{A13a}$$

$$M=\tfrac{4}{3}\pi R^3+\frac{G_N\rho_0 M}{3R}4\pi\int_0^R dr\, r^2\frac{\rho(r)}{k(r)} . \tag{A13b}$$

We can find an equation for the variation of planetary moment of inertia using the above results. For a spherical body,

$$I=4\pi\int_0^R dr\, r^4\rho(r) \tag{A14}$$

and taking the time derivative,

$$\dot{I}=4\pi\int_0^R dr\, r^4\dot{\rho}(r)+4\pi\rho_0 R^4 R . \tag{A15}$$

Recalling our previous results for $\dot{\rho}$ and $\dot{R}$ we find

$$\frac{\dot{I}}{I}=\left[2\frac{\dot{\Lambda}}{\Lambda}+\frac{\dot{G}_N}{G_N}+5\frac{\dot{\alpha}}{\alpha}+4\frac{\dot{m}_e}{m_e}\right]\frac{I_k}{I}+5\frac{I_D}{I}\frac{\dot{R}}{R} , \tag{A16}$$

where we have defined the quantities

$$I_k=4\pi\int_0^R dr\, r^4\rho(r)\frac{\rho(r)}{k(r)} , \tag{A17a}$$

$$I_l=G_N\rho_0\frac{M}{R}\frac{4}{5}\pi\int_0^R dr\, r^4\frac{\rho(r)}{k(r)} , \tag{A17b}$$

$$I_D=\tfrac{4}{5}\pi R^3\rho_0+I_l . \tag{A17c}$$

In order to compute the numerical values of the integrals, we have to know the density, pressure, and bulk modulus distribution of the planet. This is true for Earth, where seismological data yield accurate distributions of these quantities.[23] For small Earth-like bodies, such as Mercury and the Moon, even though not much is known about their interiors, their chemical composition has been inferred quite confidently.[23] In addition, a linearized equation of state is enough for them, because their small masses produce small compression. It is a good approximation to consider the Moon homogeneous and Mercury a nucleus plus mantle composite system. For Earth, the tables of Ref. 23 have been used.

APPENDIX B: ORBITAL PERTURBATIONS

The orbital perturbations induced by a time variation of fundamental constants have been discussed many times in the literature. In this appendix we shall briefly review the application of those results to our particular phenomenological model.

(a) Two-body systems (the binary pulsar). The particular case of a two-body system is very simple and the main perturbations can be derived from Kepler's third law and angular momentum conservation:

$$n^2\alpha^3=G_N(m_1+m_2) , \tag{B1}$$

$$M_r\alpha^2 n=L , \tag{B2}$$

where α is the semimajor axis, n the frequency or mean motion, and M_r the reduced mass of the system. From the time derivative of these equations we obtain

$$\frac{\dot{n}}{n}=2\frac{\dot{G}_N}{G_N}+5\frac{\dot{\Lambda}}{\Lambda} \tag{B3}$$

for the secular acceleration in longitude of the body. The last term comes from the planetary mass variation which, as we have seen, is mostly of nuclear nature.

The binary pulsar[32] is an exceedingly good example of a binary system. The observed acceleration in longitude is in good agreement with the prediction of general relativity and from the difference between theory and observation, the results quoted in Tables II and III are obtained.

(b) *Viking ranging data.* Let us compare the equations of motion used in Ref. 13 with the corresponding equations in our phenomenological model. The perturbation on the acceleration of Mars will be, in the latter case,

$$\delta\mathbf{a}=-\left[\frac{\dot{G}_N}{G_N}+\frac{\dot{M}}{M}\right](t-t_0)\mathbf{a}-\frac{\dot{M}}{M}\mathbf{v} , \tag{B4}$$

where $\mathbf{a}$ is Newtonian acceleration,

$$\mathbf{a}=-G_N M\frac{r}{r^3} , \tag{B5}$$

and M is the mass of Mars.

The equations of motion of Ref. 13 are written for the case of pure G_N variation and for a scale-covariant theory with a scale variation parameter β:

$$\delta\mathbf{a}=-\frac{\dot{G}_N^*}{G_N}(t-t_0)\mathbf{a} , \tag{B6a}$$

$$\delta\mathbf{a}=-\beta[(t-t_0)\mathbf{a}-\mathbf{v}] . \tag{B6b}$$

Comparison of both sets of equations yield the results

$$\frac{\dot{G}_N}{G_N}+2\frac{\dot{M}}{M}=\frac{\dot{G}_N^*}{G_N}=(0.0\pm 2.1)\times 10^{-11}\ \mathrm{yr}^{-1} , \tag{B7a}$$

$$\frac{\dot{M}}{M}=\beta=(0.0\pm 4.2)\times 10^{-11}\ \mathrm{yr}^{-1} \tag{B7b}$$

and we shall interpret the uncertainties as two σ values. These data are measured in atomic time and a correction of the form

$$\frac{\dot{G}_N^{**}}{G_N}=\frac{\dot{G}_N^{*}}{G_N}-2\theta\ , \tag{B8a}$$

$$\dot{\beta}^{*}=\dot{\beta}-\theta \tag{B8b}$$

with

$$\theta=2\frac{\dot{m}_e}{m_e}-\frac{\dot{\Lambda}}{\Lambda}+4\frac{\dot{\alpha}}{\alpha} \tag{B9}$$

being the conversion between atomic time and SWU's.

(c) *Lunar acceleration and rotation of Earth.* These two phenomena are deeply related through tidal friction and the conservation of angular momentum. Their complex interaction will force us to solve at the same time for the rate of change of the fundamental constants, the Moon secular acceleration and Earth's geophysical changes in the moment inertia (as explained in Ref. 25). We shall very briefly review the elements of the theory necessary for our purpose.

The tides raised by the Sun and the Moon on Earth slowly brake its rotation and angular momentum is transferred between Earth's spin and orbital motion. In order to obtain bounds on the time variation of fundamental constants, these tidal effects must be taken into account. The angular momentum balance on the Earth-Moon system is given by

$$\frac{d}{dt}(M_M\alpha_M^2 n_M)=\tau_M\ , \tag{B10a}$$

$$\frac{d}{dt}(M_\oplus\alpha_\oplus^2 n_\oplus)=\tau_\odot\ , \tag{B10b}$$

$$\frac{d}{dt}(I\Omega)=-\tau_M-\tau_\odot\ , \tag{B10c}$$

and Kepler's third law yields the additional equations

$$n_M^2\alpha_M^3=G_N(M_\oplus+M_M)\ , \tag{B11a}$$

$$n_\oplus^2\alpha_\oplus^3=G_N(M_\oplus+M_\odot)\ . \tag{B11b}$$

In these equations Ω is the Earth's angular velocity, τ is minus the tidal torques produced by the Sun and Moon on Earth. From these equations we obtain equations analogous to (B3),

$$\frac{\dot{n}}{n}=2\frac{\dot{G}_N}{G_N}+5\frac{\dot{\Lambda}}{\Lambda}-3\frac{\tau_M}{L_M}\ , \tag{B12}$$

and an equation for the angular acceleration of the Earth:

$$\frac{\dot{I}}{I}+\frac{\dot{\Omega}}{\Omega}=-\frac{\tau_M}{L_\oplus}\left[1+\frac{\tau_0}{\tau_M}\right]\ . \tag{B13}$$

The last term in (B12) is the tidal acceleration of the Moon, which has been computed quite accurately from tidal models and satellite motion analysis[24,31]:

$$\frac{\dot{n}_r}{n}=-3\frac{\tau_M}{L_M}\ . \tag{B14}$$

Equation (B13) can be written in terms of the lunar tidal acceleration in the form

$$\frac{\dot{\Omega}}{\Omega}=1.84\frac{\dot{n}_T}{n}-\frac{\dot{I}}{I}\ , \tag{B15}$$

where numerical values for Earth's angular momentum and torque ratio have been introduced.[26] From these equations we see that the angular acceleration decomposes in various terms, namely, the tidal friction and changes in the moment of inertia due to geophysical or cosmological contributions:

$$\frac{\dot{\Omega}}{\Omega}=\frac{\dot{\Omega}_T}{\Omega}+\frac{\dot{\Omega}_N}{\Omega}+\frac{\dot{\Omega}_C}{\Omega}\ . \tag{B16}$$

The geophysical contribution $\dot{\Omega}_N$ is due probably to glacial rebound.[26] It is important for astronomical determinations of $\dot{\Omega}$ (covering the last 3000 years) but it is irrelevant for determinations from paleontological data.

Let us discuss very briefly the data available.

(a) The secular acceleration of the Moon has been determined from ancient eclipse data,[15] transits of Mercury,[29] paleontological data,[24,27,30] (these are ephemeris time determinations of $\dot{n}$) and from lunar laser ranging[28] (this being an atomic time determination). All these data are tabulated in Table II, and the corresponding constraint equations are displayed in Table III. Conversion to SWU terms are included.

(b) Earth's angular acceleration has been determined from ancient eclipses and equinoxes[25] and from paleontological data,[24,27,30] the former being ephemeris time and the latter atomic time determinations. However, both methods measure it with respect to the Sun position, and so an effective acceleration is determined[25]:

$$\frac{\dot{\Omega}_{\text{eff}}}{\Omega}=\frac{\dot{\Omega}}{\Omega}-\frac{\dot{n}_C}{n}\ . \tag{B17}$$

Tables II tabulates the observational values of Earth's angular acceleration together with computed values of the tidal friction acceleration. The corresponding constraint equations are tabulated in Table III.

APPENDIX C: LONG-LIVED β DECAYERS

In this appendix we shall compute the time variation of decay rates produced within our model by the time variation of fundamental constants.

(a) ^{187}*Re*. This nucleus decays to ^{187}Os through a first forbidden unique β transition. From the well-known theory of β decay we have the formal expression for the decay constant:

$$\lambda=\frac{G_F}{2\pi^3}f(W_0,Z)\langle S(W,Z)\rangle\ , \tag{C1}$$

where

$$f(W_0,Z)=\frac{1}{m_e^5}\int_{m_e}^{W_0}dW\,\rho W(W_0-W)^2F(Z,W) \tag{C2}$$

is the integrated statistical spectrum, ρ, W, m_e, are the electron momentum, energy, and mass, respectively, and

W_0 is the released energy. S is the shape factor and its mean value is defined as

$$\langle S \rangle = f^{-1}\int_0^{p_0} dp\, p^2 q^4 F(Z,W) S(Z,W) \ . \quad \text{(C3)}$$

The Fermi function F can be approximated for the very nonrelativistic electrons of ^{187}Re:

$$F(Z,W) \simeq \frac{n m_e Z\alpha}{p} \quad \text{(C4)}$$

and the shape factor, in the normal approximation, reads

$$S \simeq R^2 \left[q^2 + p^2 \frac{1+\nu^2}{F(Z,W)} \frac{2\pi\nu}{1-e^{-2\pi\nu}} \right] . \quad \text{(C5)}$$

The second term is much larger than the first because of the small electron energy and we get

$$S \approx (\pi m_e Z\alpha)^2 R^2 = \langle S \rangle \ . \quad \text{(C6)}$$

Substitution of these results in (C1) yields the final result for the decay constant,

$$\lambda \simeq \frac{G_F^2}{2} \frac{R^2}{\pi^3} (Z\alpha)^3 E_0^3 m_e^4 \ , \quad \text{(C7)}$$

where E_0 is the limiting kinetic energy of the electron:

$$E_0 = W_0 - m_e \ . \quad \text{(C8)}$$

Now, the time variation rate of the released energy can be written in the form

$$\frac{\dot{W}_0}{W_0} = \frac{\dot{\Lambda}}{\Lambda} + \frac{\Delta E_C}{W_0} \frac{\dot{\alpha}}{\alpha} \quad \text{(C9)}$$

with ΔE_C the electrostatic energy difference between ^{187}Re and ^{187}Os, which can be estimated from the semiempirical mass formula. Also,

$$\frac{\dot{E}_0}{E_0} = \frac{W_0}{E_0}\frac{\dot{\Lambda}}{\Lambda} + \frac{\Delta E_C}{E_0}\frac{\dot{\alpha}}{\alpha} - \frac{m_e}{E_0}\frac{\dot{m}_e}{m_e} \ . \quad \text{(C10)}$$

So we get

$$\frac{\dot{\lambda}}{\lambda} = 2\frac{\dot{G}_F}{G_F} + 3\left[1 + \frac{\Delta E_C}{E_0}\right]\frac{\dot{\alpha}}{\alpha} + \left[3\frac{W_0}{E_0} - 2\right]\frac{\dot{\Lambda}}{\Lambda} + \left[4 - 3\frac{m_e}{E_0}\right]\frac{\dot{m}_e}{m_e} \ . \quad \text{(C11)}$$

The large numerical values of the coefficients of this formula are tabulated in Table III.

(b) ^{40}K. This nucleus decays to ^{40}Ca and ^{40}Ar through a third, forbidden unique transition. Because of its small charge we can make the approximation

$$F(Z,W) \approx 1 \quad \text{(C12)}$$

and the shape factor is[37]

$$S \approx R^6 [q^6 + p^6 + 7q^2p^2(q^2+p^2)] \ . \quad \text{(C13)}$$

Its mean value can be obtained with the approximation[38]

$$\langle S(p,q) \rangle \approx S \left[\frac{p}{2}, \frac{q}{2} \right] . \quad \text{(C14)}$$

After substitution in Eq. (C1) and making the integration we obtain the following result:

$$\frac{\dot{\lambda}}{\lambda} = 2\frac{\dot{G}_F}{G_F}\frac{\dot{\alpha}}{\alpha} - 6\frac{\dot{\Lambda}}{\Lambda} + 4.28\frac{\dot{m}_e}{m_e} + 6.93\frac{\dot{E}_0}{E_0} \ . \quad \text{(C15)}$$

The numerical evaluation for the coefficients proceeds as before, using Eqs. (C9) and (C10), and we obtain the coefficients tabulated in Table III.

(c) ^{87}Rb. This nucleus decays to ^{87}Sr through a third forbidden nonunique transition. Its shape factor, however, has been shown to have the same momentum dependence as a pure unique second forbidden transition.[41] Our treatment is similar to the one for ^{40}K and we obtain as a result the coefficients tabulated in Table III.

(d) *α decayers.* The decay constant for α decay can be written, with accuracy enough for our purposes, in the form

$$\lambda_\alpha = \lambda_0 e^{-Z\alpha(2M/E)^{1/2}} \ , \quad \text{(C16)}$$

where λ_0 is the strong contribution to λ (and so scales with Λ), M, and E are the mass and energy of the α particle. Taking the time derivative of (C16) we obtain

$$\frac{\dot{\lambda}_\alpha}{\lambda_\alpha} = \frac{\dot{\Lambda}}{\Lambda} - 2Z\alpha\frac{M}{p}\frac{\dot{\alpha}}{\alpha} \ . \quad \text{(C17)}$$

This equation gives the corrections discussed in Sec. III. The equations listed in Table III have already been corrected with (C16).

APPENDIX D: NEUTRON-CAPTURE RESONANCES

About 2×10^9 yr ago, a natural reactor operated for some time in the uranium ore deposits of Oklo, Gabon.[35] From an analysis of geological, chemical, and nuclear data, the operating conditions have been reconstructed and the values of some thermal capture cross sections estimated. In particular, it is known that the reactor started 1.8×10^9 yr ago and that it was operational for $(2.3\pm0.7)\times10^5$ yr. The neutron fluence was estimated as $\phi \approx 10^{21}\ n/\text{cm}^2$ and then temperature was in the range 300–1000 K. An excess in ^{150}Sm isotropic abundance, together with above-mentioned fluence, allowed us to infer that the thermally averaged neutron capture cross section (TACS) was

$$\langle \sigma_{\text{Oklo}} \rangle = (55\pm8)\times10^3 b \ , \quad \text{(D1)}$$

while the modern value is

$$\langle \sigma_{\text{now}} \rangle = (50\pm5)\times10^3 b \ . \quad \text{(D2)}$$

Both values differ by less than 10%, and the mean rate of variation of the cross section is

$$\frac{\dot{\sigma}}{\sigma} = (5\pm5)\times10^{-11}\ \text{yr}^{-1} \ . \quad \text{(D3)}$$

In this appendix we shall try to relate this change in the TACS to the time variation of fundamental constants within the model exposed in Sec. II. Other bounds have been obtained from a different set of hypotheses in Ref. 11.

The TACS can be expressed in the form

$$\langle \sigma v \rangle \propto W_0^{-1/2}\Gamma_{0M}\int_0^\infty dE\, E^{1/2}e^{-E/t}\frac{\Gamma_\gamma}{(E-E_0)^2+\Gamma_\gamma^2}$$
$$\propto \left[\frac{T}{W_0}\right]^{1/2}\Gamma_{0M}\mathrm{Re}[z\omega(z)] , \qquad \text{(D4)}$$

where T is the reactor working temperature, Γ_{0M} is the energy-independent neutron decay width, Γ_γ is the γ width, W_0 is the neutron separation energy, and E_0 is the resonance energy. $\omega(z)$ is the complex error function and

$$z^2=\frac{E_0+i\Gamma_\gamma}{T}, \quad \mathrm{Im}(z)>0 . \qquad \text{(D5)}$$

From Eq. (D4) we can compute the change of the cross section under the variation of the parameters. Using the numerical values

$$E_0\simeq 98 \text{ meV}, \quad T\approx 86 \text{ MeV} , \quad \Gamma_\gamma\simeq 63 \text{ MeV} \qquad \text{(D6)}$$

we get the result

$$\frac{\delta\langle\sigma v\rangle}{\langle\sigma v\rangle}=-\frac{1}{2}\frac{\delta W_0}{W_0}+\frac{\delta\Gamma_{0M}}{\Gamma_{0M}}+0.736\frac{\delta T}{T}$$
$$-0.441\frac{\delta E_0}{E_0}+0.206\frac{\delta\Gamma_\gamma}{\Gamma_\gamma} . \qquad \text{(D7)}$$

The changes in the widths can be related easily to the time variation of α and Λ. They will be very small compared with other contributions. The change in the reactor working temperature T can be related to the change in the uranium fission cross sections and energy. However, the large uncertainty in the temperature is much larger than any induced time variation and forces us to add this estimate to the total error.

In order to find the change in W_0 and E_0 we must use some nuclear model to estimate the different contributions of strong, electromagnetic, and weak interactions to the energy level. We shall treat the ground state and the neutron capture resonance level as a zero-temperature and a finite-temperature state of a Fermi gas. The corresponding energy differences will be given by

$$W_0=E_g(^{149}\mathrm{Sm})-E_g(^{150}\mathrm{Sm}) , \qquad \text{(D8a)}$$

$$E_0=E^*(^{149}\mathrm{Sm})-E_g(^{149}\mathrm{Sm})-W_0 , \qquad \text{(D8b)}$$

where E_g and E^* are the ground- and excited-state energies. The effective temperature T^* corresponding to the NCR level will be given by the well-known formula[21]

$$E^*=E_g\left[1+\tfrac{35}{12}\pi^2\frac{T^{*2}}{\mu^2}\right] \qquad \text{(D9)}$$

from which we estimate

$$\frac{T^*}{\mu}\approx\left[\frac{W_0}{E_g}\frac{12}{35\pi^2}\right] . \qquad \text{(D10)}$$

In order to estimate the contribution of electromagnetic and weak interactions to the energy, we shall use the approximation

$$\Delta E_i=\tfrac{1}{2}\int d^3x\, d^3x\, n(\mathbf{x})V_i(\mathbf{x}-\mathbf{x}')n(\mathbf{x}') , \qquad \text{(D11)}$$

where V_i is a local potential between two nucleons and n is the local nucleon number density. In momentum space,

$$\Delta E_i=\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}n(\mathbf{k})V_i(k)n(-\mathbf{k}) \qquad \text{(D12)}$$

or, using the Fermi-Dirac distribution for $n(\mathbf{k})$,

$$\Delta E_i=\frac{1}{(2\pi)^2}\int_0^\infty k^2dk\, V_i(k)$$
$$\times\left\{\exp\left[\left[\frac{k^2}{2m}-\mu\right]\Big/T\right]+1\right\}^{-2} . \qquad \text{(D13)}$$

We can evaluate this integral for small enough (T/μ) with well-known techniques[43] and get the first temperature-dependent corrections. For the electromagnetic and weak interaction we get

$$E_C(T)=E_C(0)\left[1-\frac{1}{2}\frac{T}{\mu}\right] , \qquad \text{(D14)}$$

$$E_W(T)=E_W(0)\left[1-\frac{3}{2}\frac{T}{\mu}\right] , \qquad \text{(D15)}$$

where $E_C(0)$ is the contribution to the ground-state energy of the Coulomb interaction, which can be computed from the semiempirical mass formula. The weak contribution to the ground state has been computed in Ref. 44 and in the SM it is equal to

$$E(0)=G_W2^{-3/2}V^{-1}/Z_S\{NZ[(3\alpha^2-1)+4A_nA_p]$$
$$+\tfrac{1}{2}A_n^2N^2(1+3\alpha_n^2)$$
$$+\tfrac{1}{2}A_n^2Z^2(1+3\alpha_p^2)\} , \qquad \text{(D16)}$$

where, for the SM,

$$A_n=\tfrac{1}{2}, \quad A_p=-2\sin^2\theta_W+\tfrac{1}{2} , \qquad \text{(D17a)}$$

$$\alpha_n=1, \quad \alpha_p=(1-4\sin^2\theta_W) . \qquad \text{(D17b)}$$

The Coulomb energy difference between the ground states of both samarium isotopes cannot be computed accurately from the semiempirical mass formula since both nuclei have the same charge. Instead, the main contribution can be computed from the interaction of the quadrupolar moment of the nucleus with the gradient of the nuclear electric field:

$$W_{0C}=-\tfrac{1}{2}Q_{33}\frac{Z\alpha}{R^3} . \qquad \text{(D18)}$$

The final results for the electrostatic and weak contributions to E_0 and W_0 are

$$W_{0W}=-6\times10^{-6}\text{ MeV} , \qquad \text{(D19a)}$$

$$E_{0W}=-3\times10^{-6}\ \text{MeV}\ , \tag{D19b}$$

$$W_{0C}=0.53\ \text{MeV}\ , \tag{D19c}$$

$$E_{0C}=-0.22\ \text{MeV}\ . \tag{D19d}$$

The strong-interaction contribution to E_0 and W_0 will be given by

$$\frac{\dot E_{0S}}{E_{0S}}=\frac{\dot E_{0W}}{E_{0W}}=\frac{\dot\Lambda}{\Lambda}\ . \tag{D19e}$$

We can compute the time variation of the capture resonance energy from the identity

$$\frac{\dot E_0}{E_0}=\frac{E_{0C}}{E_0}\frac{\dot E_{0C}}{E_{0C}}+\frac{E_{0W}}{E_0}\frac{\dot E_{0W}}{E_{0W}}+\frac{E_{0S}}{E_0}\frac{\dot E_{0S}}{E_{0S}} \tag{D20}$$

and our results (D19). We find

$$\frac{\dot\sigma}{\sigma}=25\frac{\dot\Lambda}{\Lambda}+9.8\times10^{9}\frac{\dot\alpha}{\alpha}-685\frac{\dot G_F}{G_F}\ . \tag{D21}$$

The standard deviation of this result, as computed from (D3) and the 50% error in temperature is

$$\sigma(\dot\sigma/\sigma)=2.2\times10^{-10}\ \text{yr}^{-1}\ , \tag{D22}$$

but this does not include the errors introduced by the nuclear model hypothesis. These are certainly large, and there may be uncertainties of an order of magnitude in the coefficients of Eq. (D21). We shall multiply the above standard deviation by a factor of 3 [i.e., we underweight (D21) by a factor of 10] in order to take model errors into account. The final equation is given in Table III.

APPENDIX E: LUMINOSITY OF WHITE DWARFS

In the first approximation we shall consider a white dwarf in the nonrelativistic limit, where simple analytical expressions are available for the thermodynamic variables. Because of the variation of the fundamental constants, the luminosity equation takes the form

$$\frac{dL}{dt}=4\pi r^2\rho\epsilon-\frac{\partial\mathcal{M}(r)}{\partial t}\ , \tag{E1}$$

where the energy production per unit mass ϵ is related to the energy density in the form

$$\rho\epsilon=\dot u \tag{E2}$$

and $\mathcal{M}$ is the relativistic energy density, with contributions from internal and gravitational energy. But the energy density can be written

$$u=\rho c^2+e(r)-\frac{G_N M(r)\rho(r)}{r}\ , \tag{E3}$$

where the local value of the internal energy is that of a nonrelativistic Fermi gas:

$$e=\frac{\hbar^2}{15m_e\pi^2}\left[\frac{3\pi^2\rho}{m_N\mu}\right]^{5/3}\ . \tag{E4}$$

From these equations we find

$$\rho\epsilon=\dot u=\frac{\dot\Lambda}{\Lambda}\rho c^2-\frac{\dot m_e}{m_e}e-\left[\frac{\dot G_N}{G_N}+2\frac{\dot\Lambda}{\Lambda}\right]\frac{G_N M(r)\rho(r)}{r}\ . \tag{E5}$$

Substituting in (E1) and integrating over the volume of the star gives

$$L=\frac{\dot\Lambda}{\Lambda}\left[Mc^2-\frac{12}{7}\frac{G_N M^2}{R}\right]-\frac{3}{7}\left[\frac{\dot m_e}{m_e}+2\frac{\dot G_N}{G_N}\right]\frac{G_N M^2}{R}-\frac{\partial\mathcal{M}}{\partial t}\ . \tag{E6}$$

The last term is equal to

$$\dot{\mathcal{M}}=\frac{\dot\Lambda}{\Lambda}Mc^2-\frac{3}{7}\left[\frac{\dot G_N}{G_N}+2\frac{\dot\Lambda}{\Lambda}-\frac{\dot R}{R}\right]\frac{G_N M^2}{R} \tag{E7}$$

and the time variation rate of the radius can be obtained from the mass-radius relation for white dwarfs:

$$\frac{\dot R}{R}=-\left[\frac{\dot G_N}{G_N}+2\frac{\dot\Lambda}{\Lambda}+\frac{\dot m_e}{m_e}\right]\ . \tag{E8}$$

Substituting (E7) and (E8) into (E6) we get the announced result.

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