Inflation driven by a vector field

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The possibility of inflationary models in which inflation is driven by a vector field rather than a scalar field is discussed. The vector field A^ρ is taken to be self-coupled through a "potential" $V(\xi)$, where $\xi = A^\rho A_\rho$. If V has a flat region, where $|\xi V'| \ll V$, then the Universe can undergo a period of isotropic inflation in which the space is approximately de Sitter. Because the vector field's stress tensor is not isotropic, the Universe will exit inflation into an anisotropic expansion. If the stable minimum of V occurs at $\xi = \xi_0 = 0$, then this anisotropy will damp away during the reheating period. If this minimum occurs at a nonzero value of ξ_0, then the anisotropy can be small at late times if collisionless particles, such as gravitons, are generated during reheating. In this case, the observed limits on anisotropy of the cosmic microwave background require that $\xi_0 \lesssim (10^{15}\text{ GeV})^2$. Finally, even if V does not have the flat region needed for de Sitter inflation, it is possible to have anisotropic inflation in which the Universe expands at different exponential rates in different directions. The conditions under which this can occur are discussed, and the stability of the resulting solutions is analyzed.

I. INTRODUCTION

Most versions of inflationary cosmology require a scalar field (the "inflaton") whose dynamics governs the duration and end of inflation.[1] The parameters of this scalar field must be rather finely tuned in order to allow adequate inflation and an acceptable magnitude for density perturbations. The need for this field is one of the less satisfactory features of inflationary models. Consequently, it is of interest to explore variations of inflation in which the role of the scalar field is played by some other field.[2] In this paper, models in which this role is played by a vector field are investigated.

II. INFLATING SOLUTIONS WITH VECTOR FIELDS

A. Coupled Einstein-vector field equations

We assume that the vector field A^μ is described by a Lagrangian of the form

$$\mathcal{L} = \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + V(\xi) , \tag{1}$$

where $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$, $\xi = A_\alpha A^\alpha$, and V is a "potential." If, for example, $V = \frac{1}{2} m^2 \xi$, then A^μ is a free, massive vector field.[3] More generally, this is a self-coupled (nongauge) field. For our purposes, V is an arbitrarily given function, but it may be regarded as an effective interaction arising from the coupling of A^μ to other matter fields analogous to the effective potential for a scalar field. This interaction might arise from a gauge field with spontaneously broken U(1) symmetry, much as the massive vector field arises in the Abelian Higgs model.

The equation of motion which follows from Eq. (1) is

$$\Box A_\mu - \nabla_\nu (\nabla_\mu A^\nu) - 2V'(\xi) A_\mu = 0 \tag{2}$$

and the energy-momentum tensor for the vector field is

$$T_{\mu\nu} = F_{\mu\beta} F_\nu{}^\beta - \tfrac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - g_{\mu\nu} V + 2V' A_\mu A_\nu . \tag{3}$$

Let A_z be the nonzero spatial component of A_μ. Because the stress tensor need not be isotropic, the spacetime will not be Robertson-Walker type. We may assume a Bianchi type-I metric for our purposes:

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2) + b^2(t) dz^2 . \tag{4}$$

The Einstein equations for this metric are

$$2\frac{\dot a \dot b}{ab} + \frac{\dot a^2}{a^2} + 8\pi\rho , \tag{5a}$$

$$2\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} = -8\pi p_z , \tag{5b}$$

and

$$\frac{\ddot a}{a} + \frac{\dot a \dot b}{ab} + \frac{\ddot b}{b} = -8\pi p_x , \tag{5c}$$

and the conservation law (with $p_x = p_y$) is

$$\dot\rho + \left[2\frac{\dot a}{a} + \frac{\dot b}{b} \right] \rho + 2\frac{\dot a}{a} p_x + \frac{\dot b}{b} p_z = 0 . \tag{6}$$

We may take Eqs. (5a) and (5b) to be the independent Einstein equations determining $a(t)$ and $b(t)$, as Eq. (5c) follows from these equations and the conservation law.

We are interested in homogeneous solutions, so $A_\mu = A_\mu(t)$. In the metric of Eq. (4), Eq. (2) implies that $V' A_t = 0$ for such solutions and hence $A_t = 0$, if $V' = 0$. The equation for the only nonzero component A_z becomes

$$\ddot A_z + \left[2\frac{\dot a}{a} - \frac{\dot b}{b} \right] \dot A_z + 2V' A_z = 0 . \tag{7}$$

The energy density and pressures for the vector field are

$$\rho=\tfrac{1}{2}b^{-2}\dot{A}_z^2+V\ ,$$
$$p_x=p_y=\tfrac{1}{2}b^{-2}\dot{A}_z^2-V\ , \qquad (8)$$
$$p_z=-\rho+2\xi V'\ ,$$

with $\xi=b^{-2}A_z^2$.

B. "New inflation" models

We are interested in finding solutions to the coupled Einstein-vector-field equations which exhibit inflationary behavior. If $V \gg \max(|2\xi V'|, \tfrac{1}{2}b^{-2}\dot{A}_z^2)$, then $p_x=p_y \approx p_z \approx -\rho$, so the Universe rapidly approaches de Sitter space. Thus one possible class of models for vector-field-driven inflation are those in which V has the form illustrated in Fig. 1. Here V has a form similar to that in scalar new inflation, except that the slope at the origin is nonzero. Inflation occurs near $\xi=0$. If V is linear in ξ (quadratic in A_z) near the origin, then the behavior during inflation is represented by

$$V=V_0-\tfrac{1}{2}\mu^2\xi\ , \qquad (9)$$

an unstable field of mass square $-\mu^2$. The metric is approximately that of de Sitter space:

$$a(t)=b(t)=e^{Ht},\quad H=(8\pi V_0/3)^{1/2}\ , \qquad (10)$$

and the solutions of Eq. (7) are of the form $A_z(t)=Ae^{\gamma t}$, where $\gamma^2+H\gamma-\mu^2=0$. Hence the growing mode is that for which

$$\gamma=\tfrac{1}{2}(\sqrt{H^2+4\mu^2}-H)\ . \qquad (11)$$

The energy density of the vector field becomes

$$\rho=V_0-\frac{4\pi}{3}\gamma\mu A^2 e^{2(\gamma-H)t}\ . \qquad (12)$$

Note that although A_z grows for any nonzero μ, there is an instability of the spacetime only if $\gamma>H$, or equivalently, $\mu>2H$. This is due to the fact that V is a function of $\xi=A_z^2/b^2$ rather than A_z alone, and ξ grows only if $\gamma>H$. This is why one needs to require that $V'<0$ at $\xi=0$ in order to have a finite period of inflation. If this condition is not satisfied, then inflation either continues forever, or must end through quantum tunneling. This is analogous to the case of scalar inflation which occurs at a minimum of the potential. If there is an instability, then the duration of inflation is of order $(\gamma-H)^{-1}$, so with appropriate choices of μ and V_0, adequate inflation to solve the horizon and flatness problems would be obtained. In particular, to obtain inflation for a time of the order of 10^2H^{-1} these parameters must be tuned to a precision of 10^{-2}.

After inflation ends in this type of model, anisotropy will develop, as generally $p_z \neq p_x=p_y$. At first sight, this would seem to undo the beneficial effects of inflation. However, if the anisotropy ceases before the cosmic microwave background is formed, no conflicts with observation will arise. Reheating of the Universe can occur much as in scalar inflation; if the vector field is coupled to

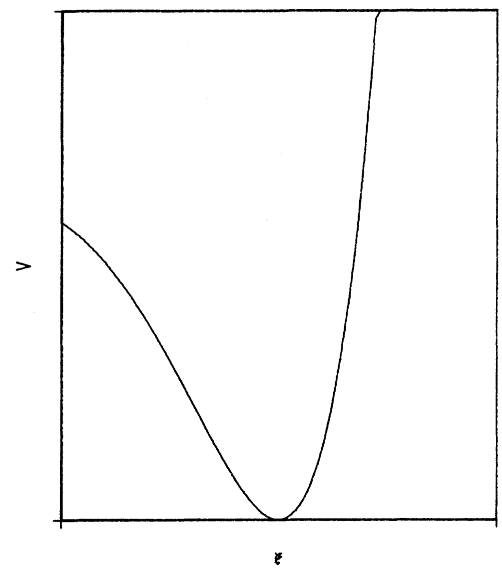

FIG. 1. A potential which leads to de Sitter inflation near $\xi=0$ and reheating at $\xi\neq 0$. Unlike scalar inflation, here we need $V'<0$ at the origin if inflation is to ever end.

other fields, quanta of these fields will be created during the rollover period. It is also possible for the time dependence of the spacetime metric to create particles of all types, regardless of their coupling to A^{μ}. This latter mechanism for reheating was discussed in the context of scalar inflation in Ref. 4. In the present model, this gravitational particle creation will be enhanced by the anisotropy[5] and hence become a more efficient reheating mechanism.

The question of whether the anisotropy eventually vanishes is nontrivial. The stable minimum of V at $\xi=\xi_0$ is a point where $V(\xi)=V'(\xi_0)=0$. If the system is to remain at this point, we must have

$$A_z(t)=\xi_0^{1/2}b(t)\ ; \qquad (13)$$

i.e., A_z must continue to grow in an expanding Universe. From Eq. (8) we find that the vector-field energy density and pressures are now

$$\rho=p_x=p_y=-p_z=\tfrac{1}{2}\xi_0(\dot{b}/b)^2 \qquad (14)$$

so the pressures are anisotropic. After reheating, there will be other forms of matter present. If this matter consists of particles which are strongly coupled to one another, its stress tensor will be isotropic and unable to compensate for the anisotropy of the vector field. However, if a collisionless fluid is present, it is capable of canceling the anisotropy in the vector-field stress tensor. This effect was discussed by Misner[6] in the context of models where the anisotropy arises from the initial conditions.

This collisionless fluid can in principle consist of any particles which are produced at the time of reheating, but which subsequently do not interact significantly apart from their gravitational effects. In Misner's original studies, it was assumed to consist of neutrinos. In the present context, it should be a particle whose characteristic energy scale below which it decouples is of the order of the scale at which inflation occurs. Thus for inflation close to the Planck scale, gravitons may be produced in sufficient numbers to serve this purpose. For inflation at lower scales, the physical candidates are less clear, but could include hypothetical particles such as supersymmetric partners of known particles. Neutrinos are also a possibility if either inflation occurs at a low enough energy scale, or if an extended period of anisotropy can be tolerated until neutrinos become collisionless.

Let $\tilde{p}_i$ be the pressures of such a collisionless, relativistic fluid

$$\tilde{p}_x = \tilde{p}_y = \frac{c_1}{a^3 b} \quad \text{and} \quad \tilde{p}_z = \frac{c_2}{a^2 b^2} \;, \tag{15}$$

where c_1 and c_2 are constants. A difference in the rates of expansion of a and b causes $\tilde{p}_x$ and $\tilde{p}_z$ to red-shift differently. This in turn reacts against the differential expansion and tends to drive the system toward isotropic expansion. Thus, if a sufficient density of gravitons or other collisionless particles is generated upon reheating, there is a possibility of isotropizing the Universe. We would have that the total stress tensor is isotropic because the anisotropy of the vector field's contribution is canceled by that of the collisionless fluids, and the Universe is a radiation-dominated Robertson-Walker universe. Because $a(t) \propto b(t) \propto t^{1/2}$, from Eqs. (14) and (15), both contributions redshift as t^{-2} and remain in balance. Once the Universe evolves to a matter-dominated stage, where $a(t) \propto b(t) \propto t^{2/3}$, then $\tilde{p}_i \propto t^{-8/3}$ but the vector-field pressures remain $p_i \propto t^{-2}$. At this point the vector-field stress tensor ceases to be canceled, and the expansion begins to become anisotropic again.

We can estimate the rate of growth of this anisotropy by considering the effect of a small anisotropic perturbation of the form of Eq. (14) to the stress tensor of a matter-dominated Robertson-Walker universe. Let

$$a = t^{2/3} + \delta_1 \quad \text{and} \quad b = t^{2/3} + \delta_2 \;. \tag{16}$$

Then the linearized equations for δ_1 and δ_2 are

$$9t^2\ddot{\delta}_1 + 6t\dot{\delta}_1 - 2\delta_1 = 8\pi\xi_0 t^{2/3} \tag{17a}$$

and

$$3t\dot{\delta}_2 + \delta_2 + 6t\dot{\delta}_1 + 2\delta_1 = 4\pi\xi_0 t^{2/3} \;. \tag{17b}$$

The solution of these equations which vanishes when $\xi_0 = 0$ is

$$\delta_1 = \tfrac{8}{9}\pi\xi_0 t^{2/3}\ln(t/t_0), \quad \delta_2 = -2\delta_1 \;, \tag{18}$$

where the initial condition $\delta_1 = \delta_2 = 0$ at $t = t_0$ has been imposed. If $t_0 = t_{\rm eq}$, the time at which matter begins to dominate, then at the present time, $t/t_0 \approx 10^6$. The observational limits on the quadrupole anisotropy of the cosmic blackbody radiation tell us that[7]

$$\Delta T/T = a^{-1}|\delta_1 - \delta_2| < 8\times 10^{-5} \;. \tag{19}$$

This implies

$$\xi_0 \lesssim 10^{-6} = (10^{16}\ \text{GeV})^2 \;. \tag{20}$$

Thus if ξ_0 is characterized by an energy scale of the order of the grand-unified-theory (GUT) scale or lower, then the anisotropy will not violate observational limits. It is of interest to note that $\delta_1 > 0$ and $\delta_2 < 0$ $(t > t_0)$. Hence, the type of anisotropy produced by this scenario necessarily corresponds to a higher temperature along one axis than along the two orthogonal directions.

C. "Chaotic inflation" models

The models discussed above in which A_z is initially zero and later evolves to nonzero values is the vector analog of new inflation. An alternative possibility is that initially A_z was nonzero and that at late times it evolves toward zero. This would be analogous to the situation in chaotic inflation. In this case $V(\xi)$ must have a minimum at $\xi = 0$ and be rather flat for large ξ, as illustrated in Fig. 2. The need for flatness arises from the constraint that the period of inflation be sufficiently long. Because V is a function of $\xi = A_z^2/b^2$, the growth of b tends to drive the system toward $\xi = 0$. Thus the effect which tended to prolong inflation in the previous new inflation-type models now tends to shorten it.

Let us consider an explicit example of a suitable $V(\xi)$:

$$V(\xi) = V_0(1 - e^{-m^2\xi/2V_0}) \;. \tag{21}$$

Note that $V \approx \frac{1}{2}m^2\xi$ when $\xi \lesssim V_0 m^{-2}$, so the field is approximately a free massive vector field when ξ is small. During inflation the equation for $A_z(t)$ is

$$\ddot{A}_z + H\dot{A}_z + 2V' A_z = 0 \;. \tag{22}$$

However, for $\xi \gg V_0 m^{-2}$, V' is exponentially small and A_z is at most a linearly decreasing function of time. We may estimate the length of the period of inflation, τ, by

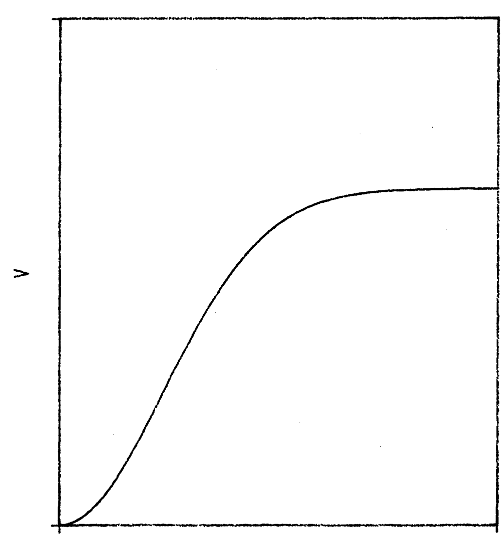

FIG. 2. A potential which leads to de Sitter inflation at $\xi \neq 0$ and reheating as $\xi \to 0$.

taking $A_z = A_z(0)$, its initial value, and setting

$$m^2\xi(\tau) = V_0 . \tag{23}$$

Then using $b(\tau) = e^{H\tau}$ and $H^2 = 8\pi V_0/3$ yields

$$e^{H\tau} = m A_z(0) V_0^{-1/2} \tag{24}$$

for the net expansion during inflation. Since we want to have $e^{H\tau} \gtrsim 10^{28}$, we can think of this relation as putting an upper bound on V_0 if m and $A_z(0)$ are specified. For example, if $m \simeq A_z(0) \simeq m_{\rm Pl}$, the Planck mass, then

$$V_0 \lesssim (10^5 \text{ GeV})^4 . \tag{25}$$

Equivalently, for arbitrary m and V_0 we could always obtain adequate inflation with a large enough $A_z(0)$. However if the required initial value of A_z is far above the Planck scale, it would be hard to see how such initial conditions could arise. Thus, in this type of model considerable fine-tuning is needed for adequate inflation.

The problem of anisotropy at late times does not arise in this class of models because $A_z \rightarrow 0$ and hence the vector field's stress tensor vanishes. The vector field will in general oscillate around $A_z = 0$, but these oscillations will be damped both by the coupling of A^μ to any other fields and by the cosmological expansion, the effect of which is represented by the $\dot{A}_z$ term in Eq. (7). Once the anisotropy of the stress tensor disappears, the expansion does not immediately become isotropic, but it does so rapidly enough that it is soon negligible. This may be seen by examining the exact solutions of Einstein's equations for Bianchi type-I metrics with an isotropic fluid as source. For a pressureless fluid, the relevant solution was given by Heckmann and Schücking[8] and for radiation by Thorne.[9] If one examines the late-time behavior of these solutions, one finds that

$$\frac{a-b}{a} = O(t^{-\sigma}) ,$$

where $\sigma = 1$ for the pressureless case and $\sigma = \frac{1}{2}$ for the radiation case. The above quantity will be of order unity just after inflation, but will have decayed to order $(t_I/t_D)^\sigma$ by the time of decoupling t_D, where t_I is the time of inflation. New inflation-type models and the chaotic inflation-type models with vector fields may be compared by stating that for the latter, obtaining adequate inflation requires a rather special choice of parameters, but the disappearance of anisotropy at late times arises naturally. For the former, the reverse is true.

D. Anisotropic inflation

So far we have assumed that the inflation driven by a vector field is described by de Sitter space, i.e., there is a flat region of the potential where V' is sufficiently small to ensure that $p_x = p_y \approx p_z \approx -\rho$. However, the solution of the horizon and flatness problems does not require that the inflationary expansion itself be isotropic. All that is required is that all directions expand by factors of at least 10^{28}. This could conceivably occur anisotropically so long as this anisotropy later disappears. Let us examine the condition under which a vector field gives rise to a Universe which is exponentially expanding at different rates along different axes. We will look for solutions where $\xi = \xi_0 = \text{const}$, so

$$A_z(t) = \xi^{1/2} b(t) , \tag{26}$$

and let

$$a(t) = a_0 e^{H_1 t}, \quad b(t) = b_0 e^{H_2 t} , \qquad V_0 = V(\xi_0), \quad \text{and } V_0' = V'(\xi_0) . \tag{27}$$

The Einstein equations, Eqs. (5a) and (5b) and the vector-field equation of motion, Eq. (7) now yield three algebraic equations relating H_1, H_2, V_0, and V_0'. These equations may be expressed as

$$H_1 = -H_2^{-1} V_0' , \tag{28a}$$

$$H_2^2 = -(1+8\pi\xi)^{-1} V_0' , \tag{28b}$$

and

$$8\pi(1+8\pi\xi_0)V_0 + (3+16\pi\xi_0)(1+4\pi\xi_0)V_0' = 0 . \tag{29}$$

Equation (29) is the constraint which must be satisfied in order to have anisotropic inflation. If there is a point ξ_0 at which it is satisfied, then Eqs. (28a) and (28b) yield the values of H_1 and H_2 at that point. Clearly we can only have such solutions in regions where $V' < 0$. A given potential $V(\xi)$ can have several points at which Eq. (29) is satisfied, and these points can correspond to either stable or unstable solutions.

To analyze the stability of our solutions, we need to consider small perturbations by writing

$$a(t) = a_0 e^{H_1 t}(1+\alpha) , \quad b(t) = b_0 e^{H_2 t}(1+\beta) , \quad A_z(t) = \xi_0^{1/2} b_0 e^{H_2 t}(1+\gamma) \tag{30}$$

and deriving linearized equations for α, β, and γ. The result is

$$\ddot{\gamma} + (2H_1 - H_2)\dot{\gamma} + H_2(2\dot{\alpha} - \dot{\beta}) + 4\xi_0 V_0''(\gamma - \beta) + 2(H_1 H_2 + V_0')\gamma = 0 , \tag{31a}$$

$$\ddot{\alpha} + 3H_1\dot{\alpha} - 4\pi\xi_0 H_2\dot{\gamma} + 4\pi\xi_0(4\xi_0 V_0'' + 2V_0' - H_2^2) \times (\gamma - \beta) = 0 , \tag{31b}$$

and

$$(H_1 + H_2)\dot{\alpha} + H_1\dot{\beta} - 4\pi\xi_0 H_2\dot{\gamma} - 4\pi\xi_0(H_2^2 + 2V_0') \times (\gamma - \beta) = 0 . \tag{31c}$$

These equations may be expressed as a set of five first-order equations. If we let $U^i = (\dot{\gamma}, \gamma, \dot{\alpha}, \alpha, \beta)$, then this set of equations is expressible as

$$\dot{U}^i = M^{ij} U^j . \tag{32}$$

The solution is unstable if the matrix M^{ij} has any eigenvalues with positive real parts. Writing out the eigenvalue equation for M^{ij} reveals two zero eigenvalues, and that the three remaining eigenvalues are solutions of a cubic equation:

$$\lambda^3+32H_2(4x_0+3)\lambda^2+4\pi(2x_0+1)^{-1}[4V_2x_0(2x_0^2+3x_0+1)-V_1(20x_0+9)]\lambda$$
$$-64\pi^2H_2^{-1}(2x_0+1)^{-3}x_0V_1(4x_0+3)[V_2(2x_0^2+3x_0+1)+V_1(4x_0+1)]=0\ , \quad (33)$$

where $x_0=4\pi\xi_0$, $V_1=V_0'/(4\pi)$, and $V_2=V_0''/(4\pi)^2$. If $\mathrm{Re}\lambda\le 0$ for all solutions of this equation, the solution of the Einstein-vector-field equations is stable. Otherwise it is unstable on a time scale of

$$\tau=(\mathrm{Re}\lambda_{\max})^{-1}\ , \quad (34)$$

where $\lambda_{\max}$ is the eigenvalue with the largest real part.

Note that if we rescale the potential by a constant factor A, i.e., $V\to AV$, then we have a new solution at the same time of ξ_0 with

$$H_1\to A^{1/2}H_1,\quad H_2\to A^{1/2}H_2\ ,$$
$$\lambda\to A^{1/2}\lambda,\quad \text{and}\quad \tau\to A^{-1/2}\tau\ . \quad (35)$$

The maximum expansion factors, $H_1\tau$ and $H_2\tau$, are unchanged by this rescaling. This scaling property allows us to adjust a solution to correspond to inflation at any given energy scale.

An example of a potential which exhibits anisotropic inflation is

$$V=x^2(x^2-Dx+C),\quad x=4\pi\xi\ . \quad (36)$$

If, for example, we set $C=1$ and take D in the range $2.0>D\gtrsim 1.9089$, then there is both a stable and an unstable solution. (Here we may take all quantities to be in Planck units. The above scaling property generates solutions at lower-energy scales.) As D approaches 1.9089, the instability time scale increases, and for $D\gtrsim 1.9089$, there are no solutions of Eq. (29). This behavior is illustrated in Figs. 3 and 4. With appropriate choices of the parameters of the potential, one can have anisotropic inflation at a marginally unstable point. This inflation will then last for a finite period of time and then cease. If the growing perturbation is in the direction to cause ξ to decrease (i.e., $\beta-\gamma>0$), then the system can evolve toward $\xi=0$. In this case, one has a reheating and isotropization scenario similar to that in chaotic-type de Sitter inflation based upon potentials such as that in Fig. 2. A question that needs to be studied further is how general the initial conditions may be for anisotropic inflation. That is, given a potential such as that of Eq. (36), which initial conditions cause the inflationary solution to be reached?

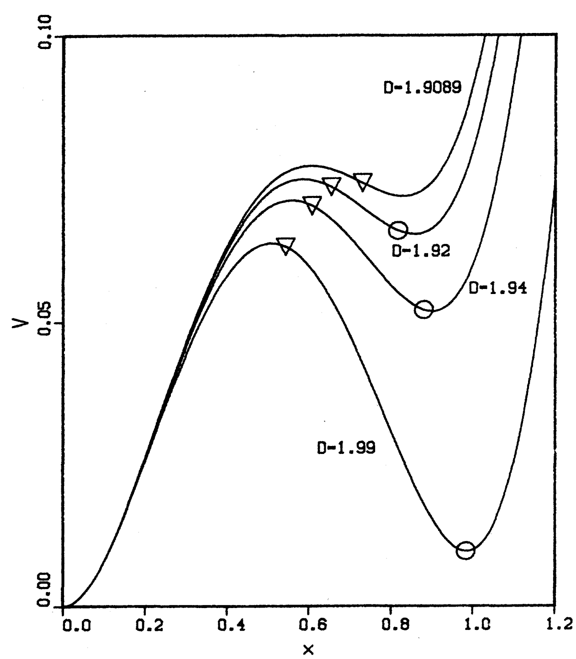

FIG. 3. The potential $V=x^2(x^2-Dx+1)$ is plotted for various values of D as a function of $x=4\pi\xi$. This potential leads to anisotropic inflation. The points marked with open circles represent stable solutions, whereas those marked with open triangles represent unstable solutions. For $D\gtrsim 1.9089$, there are no anisotropic inflation solutions for this potential.

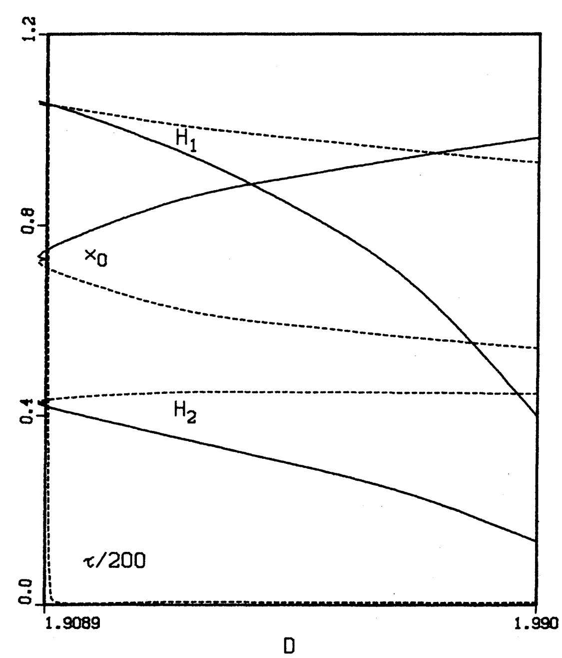

FIG. 4. The various parameters for the anisotropic inflation solutions of the potential of Eq. (36) are shown as functions of D with $C=1$. Here $x_0=4\pi\xi_0$ is the point at which the solution arises, H_1 and H_2 are the expansion rates, and τ is the instability time scale. Solid curves refer to stable solutions and dashed curves to unstable solutions.

III. QUANTUM FLUCTUATIONS AND DENSITY PERTURBATIONS

The peculiar features of vector-field inflation which we have encountered thus far are due to the anisotropy of the stress tensor. However, there is another distinction between scalar and vector fields which is relevant to inflation: the massless vector field is conformally invariant, whereas the massless, minimally coupled scalar field is not. This lack of conformal invariance causes such a scalar field to undergo large quantum fluctuations in de Sitter space, so that $\langle \phi^2 \rangle$ grows linearly in time;[10] $\langle \phi^2 \rangle = H^3 t/(2\pi)^2$. This growth of the rms value of ϕ has the effect of shortening the period of inflation. In the case of a vector field, this growth will not occur; in de Sitter space there is a de Sitter-invariant vacuum and $\langle A_\mu A^\mu \rangle = \text{const}$.

Quantum fluctuations also play a key role in the formation of density perturbations in inflation.[11-14] The magnitude of the relative density perturbation generated in scalar inflation is[14]

$$\frac{\delta\rho}{\rho} \approx \frac{H\Delta\phi}{\dot{\phi}} . \tag{37}$$

Here $\Delta\phi$, the magnitude of the mean quantum fluctuations in ϕ, and $\dot{\phi}$ are evaluated at the time the scale in question leaves the horizon during the de Sitter phase, and $\delta\rho/\rho$ is evaluated at the time this scale reenters the horizon after reheating. This formula leads to a perturbation spectrum that is independent of the scale, apart from logarithmic factors. This arises because $\dot{\phi}$ is approximately constant for a long period and $\Delta\phi \approx H$, independent of the scale. Here $\Delta\phi$ may be defined as[11]

$$\Delta\phi = \left[(2\pi)^{-3} k^3 \int d^3x \, e^{i\mathbf{k}\cdot\mathbf{x}} \langle \phi(\mathbf{x},t)\phi(0,t) \rangle \right]^{1/2} . \tag{38}$$

It is of interest to consider what would happen if the scalar field were conformally invariant. (Note that scalar inflation only really works with a minimally coupled field; here we are using the conformal scalar field as a simplified analog of the vector field.) For such a field in de Sitter space, Eq. (38) yields

$$\Delta\phi = (16\pi^3)^{-1/2} H k |\eta| , \tag{39}$$

where $\eta = H^{-1} e^{-Ht}$ is the conformal time. At first sight, this appears to introduce a dependence on the scale k into $\Delta\phi$ and hence $\delta\rho/\rho$. However, at the time of horizon crossing, $|\eta| = k^{-1}$, so again $\Delta\phi$ and $\delta\rho/\rho$ are independent of k. Although scalar inflation with a conformal field is not equivalent to vector inflation, this result does suggest that vector inflation would also produce a scale-invariant perturbation spectrum, at least for de Sitter inflation. Further work is required to test this conjecture in detail. It is also not clear what effects anisotropic inflation will have upon the density perturbations.

IV. SUMMARY AND CONCLUSIONS

We have seen that vector fields can drive inflationary expansion. If V has a flat region, then a de Sitter expansion is possible. More generally, vector fields can give rise to anisotropically inflating universes. Such anisotropic inflation is still capable of solving the same cosmological puzzles as isotropic inflation. With suitable choices for the parameters of the model, adequate inflation is possible and excess anisotropy at late times may be avoided. A detailed study of density perturbations in these models remains to be done, but one expects isotropic vector-field inflation to yield a scale-free spectrum of perturbations.

ACKNOWLEDGMENTS

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[1] For a recent review, see S. K. Blau and A. H. Guth, in *Three Hundred Years of Gravitation,* edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1987), p. 524.

[2] This possibility was briefly discussed by T. Piran [Phys. Lett. B **181**, 238 (1987)], who noted that fields other than a scalar field may have difficulty producing a sufficiently long period. However, with a suitable choice of parameters a vector field can produce an arbitrary amount of inflation, as will be discussed in Sec. II.

[3] Vector fields in the presence of gravity have been discussed, for example, by G. Tauber, J. Math. Phys. **10**, 633 (1969).

[4] L. H. Ford, Phys. Rev. D **35**, 2955 (1987).

[5] See, e.g., N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982), Sec. 5.6.

[6] C. W. Misner, Astrophys. J. **151**, 431 (1968).

[7] D. J. Fixen, E. S. Cheng, and D. T. Wilkinson, Phys. Rev. Lett. **50**, 620 (1983); J. M. Uson and D. T. Wilkinson, in *Galactic and Extragalactic Radio Astronomy,* 2nd ed., edited by G. L. Verschuur and K. I. Kellermann (Springer, New York, 1988).

[8] O. Heckmann and E. Schücking, in *Gravitation: An Introduction to Current Research,* edited by L. Witten (Wiley, New York, 1962), p. 444.

[9] K. Thorne, Astrophys. J. **148**, 151 (1967).

[10] A. Vilenkin and L. H. Ford, Phys. Rev. D **26**, 1231 (1982); A. Linde, Phys. Lett. **116B**, 335 (1982).

[11] A. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982).

[12] S. Hawking, Phys. Lett. **115B**, 295 (1982).

[13] A. Starobinskii, Phys. Lett. **117B**, 175 (1982).

[14] J. Bardeen, P. Steinhardt, and M. Turner, Phys. Rev. D **28**, 679 (1983).