Pion fluctuations around a moving and rotating Skyrmion

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Fluctuating pion fields around a moving and rotating Skyrmion are quantized by Dirac's method. Both the rotational and translational collective motion of the Skyrmion are described by collective coordinates and then all six zero-frequency modes of the pion field are eliminated. The method allows us to treat fluctuations around a soliton consistently with its collective rotational and translational motion. Pion-baryon linear couplings are studied and the $\Delta \rightarrow N\pi$ decay rate is calculated.

I. INTRODUCTION

For the past several years, the Skyrme model, in which the nucleon emerges as a soliton of a nonlinear pion field equation, has provided us with some important understanding of hadronic structure at low energy.[1] Its close tie with quantum chromodynamics is its main strength in comparison with most other baryon models.[2] The numerical predictions for many static properties of the nucleon, such as nucleon size and magnetic moments, are typically within 30% of the experimentally measured values.[3,4] The model is successful also in describing the qualitative behavior of pion-nucleon scattering amplitude in high partial waves, although it is not so satisfactory for the lower partial waves.[5]

The nucleon-nucleon interaction has also been studied in the Skryme model. The static potential between two solitons has been calculated.[6] A strong short-distance repulsion and a long-range one-pion-exchange potential are well reproduced, although the medium-range attraction, which is responsible for the nuclear binding, does not appear in the static potential. In the conventional picture of the nuclear force, an effective pion-nucleon field theory with Yukawa coupling is remarkably successful. The medium-range attraction is attributed to two or more pion exchanges between nucleons. In the Skyrme model the two-pion exchanges are not described at the classical level. Therefore the pion field must be quantized around the Skyrmion. We must also understand the correspondence between the quantized Skyrme model and the effective pion-nucleon theory. The relationship may not be straightforward, because one naively expects no linear (Yukawa-type) pion-Skyrmion coupling. Remember that the Skyrmion is a stable classical solution of the Euler-Lagrange equation for the pion field. Any linear quantum fluctuation must vanish around such a solution.

It has been pointed out, however, that two global symmetries of the Lagrangian, i.e., the isospin (or spin) rotational invariance and translational invariance, modify the naive quantization.[7] The classical soliton solution breaks those symmetries of the original Lagrangian. Thus in quantizing field fluctuations around the soliton, one encounters zero-frequency modes associated with the broken global symmetries. Schnitzer[8] was the first to develop a systematic chiral expansion of the pion-Skyrmion system. But in his early work, the zero modes were not treated carefully. More recent work shows that the introduction of collective coordinates and the elimination of the zero modes induce a new linear Yukawa-type pion-nucleon coupling. The method of the Dirac quantization[9,10] has been applied by Saito, Otofuji, and Yasuno[11] to the standard Skyrme model and by Zahed and his coworkers[12] to the vector-meson stabilized version of the Skyrme model. Work along this line also includes that of Holzwarth, Hayashi, and Schwesinger.[13] In most of the above-mentioned work, three zero-frequency modes associated with the translational motion have been ignored.

In this paper, we discuss the quantization of field fluctuations around the single Skyrmion with all six zero-frequency modes being taken into account by introducing both the rotational and translational collective coordinates. The formalism is analogous to Ref. 12, where the Dirac quantization method is applied to a gauge-field soliton. Our main aim is to study the effects of the translational motion of the Skyrmion in the pion-baryon coupling. In Sec. II, we present the Dirac quantization method for the Skyrme model. We introduce collective coordinates for the rotational and translational motions of the Skrymion as well as quantum fluctuations around the soliton. Then zero modes are eliminated by imposing constraints on the quantum variables. Under the constraints, the quantization leads us to a Hamiltonian which contains a Yukawa-type coupling of the pion and Skyrmion. In Sec. III, the pion-baryon coupling matrix elements are calculation. We use a plane-wave approximation for the pion field. Comparing the coupling matrix element with the effective pion-nucleon (Δ) field theory, the coupling constant and the form factor are calculated. The $\Delta \rightarrow N\pi$ decay rate is calculated as an example. Behavior of the form factors obtained is discussed. Conclusions and discussions are given in Sec. IV.

II. THE DIRAC QUANTIZATION

We choose the simplest Skyrme-model Lagrangian given by

$$\mathcal{L}(x)=\frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U(x)\partial^\mu U^\dagger(x)] + \frac{1}{32e^2}\mathrm{Tr}\{[\partial_\mu U(x)U^\dagger(x),\partial_\nu U(x)U^\dagger(x)]^2\} + \frac{m_\pi^2 f_\pi^2}{2}\mathrm{Tr}(U-1) \tag{1}$$

with

$$U(x)=\exp(i\boldsymbol{\tau}\cdot\boldsymbol{\phi}/f_\pi)\ ,$$

where the first term is the Lagrangian of pion fields $\boldsymbol{\phi}/f_\pi$ in the nonlinear sigma model and the second term is the Skyrme term introduced to stabilize the soliton solution. The third term introduces the pion mass. The Skyrmion is a chiral-soliton solution for the above Lagrangian with *hedgehog* symmetry:

$$U_0(x)=\exp(i\boldsymbol{\tau}\cdot\boldsymbol{\phi}_s/f_\pi)=\exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)]\ ,$$

where the chiral profile $F(r)$ satisfies the boundary conditions: $F(0)=\pi$ and $F(\infty)=0$. The Skyrme Lagrangian is invariant under an isospin SU(2) rotation: $U\to AUA^\dagger$ with an arbitrary $A\subset\mathrm{SU}(2)$ matrix. A rotating soliton is described by promoting A to a time-dependent collective coordinate. By quantizing the collective coordinate, one obtains the physical states with definite spin and isospin corresponding to N and Δ. The Skyrme Lagrangian is also invariant under spatial translation $U(\mathbf{x})\to U(\mathbf{x}-\mathbf{q})$. Again we can promote $\mathbf{q}$ as a time-dependent collective coordinate and quantize it. For a moving soliton, the quantization adds the kinetic energy term to the Hamiltonian to the lowest order in $\mathbf{P}^2$ (Ref. 14).

Next we introduce quantum fluctuation around the rotating and moving soliton:

$$\phi^i(\mathbf{x},t)=A_{i,j}(\boldsymbol{\alpha}(t))\phi_s^j(\mathbf{x}-\mathbf{q}(t))+\bar{\eta}^{\,i}(\mathbf{x},t)\ , \tag{2}$$

where $\bar{\eta}^{\,i}(\mathbf{x},t)$ is the time-dependent pion field and $\phi_s(\mathbf{x})$ is the hedgehog soliton in its rest frame, $U_0(\mathbf{x})=\exp[i\boldsymbol{\tau}\cdot\boldsymbol{\phi}_s(\mathbf{x})]$. $\boldsymbol{\alpha}(t)$ represents three angles specifying the isospin direction of the soliton. We find it convenient to recast the Skyrme Lagrangian into a different form before we substitute (2) into (1):

$$\mathcal{L}(x)=\tfrac{1}{2}\partial_\mu\phi^i K_{ij}\partial^\mu\phi^j-m_\pi^2 f_\pi^2(1-\cos\phi/f_\pi)\ , \tag{3}$$

where

$$K_{ij}=g_{ij}+\frac{1}{e^2}X_{iljm}\partial_n\phi^l\partial_n\phi^m\quad(i,j,\ldots=1,2,3) \tag{4}$$

and

$$g_{ij}(\phi)=\delta_{ij}^T\frac{\sin^2(\phi/f_\pi)}{(\phi/f_\pi)^2}+\hat{\phi}_i\hat{\phi}_j\ , \tag{5}$$

$$X_{iljm}(\phi)=(\delta_{ij}^T\delta_{lm}^T-\delta_{im}^T\delta_{lj}^T)\frac{\sin^4(\phi/f_\pi)}{(\phi/f_\pi)^4} + (\delta_{ij}^T\hat{\phi}_m\hat{\phi}_l+\delta_{ml}^T\hat{\phi}_i\hat{\phi}_j-\delta_{il}^T\hat{\phi}_m\hat{\phi}_j - \delta_{mj}^T\hat{\phi}_i\hat{\phi}_l)\frac{\sin^2(2\phi/f_\pi)}{4(\phi/f_\pi)^2} \tag{6}$$

with $\delta_{ij}^T=\delta_{ij}-\hat{\phi}_i\hat{\phi}_j$ and $\hat{\phi}_i=\phi_i(x)/\phi(x)$. The magnitude of ϕ is defined by $\phi=(\phi_i^2)^{1/2}$.

Substituting (2) into (3), we find that the generalized momenta defined by

$$\bar{I}_i=\frac{\partial L}{\partial\dot{\alpha}_i},\quad \bar{P}_{i'}=\frac{\partial L}{\partial\dot{q}_{i'}},\quad \bar{\pi}_i=\frac{\partial\mathcal{L}}{\partial\dot{\bar{\eta}}_i} \tag{7}$$

satisfy constraint equations (ψ constraints according to Dirac[9])

$$\bar{I}_i-\int\bar{\pi}_j(\mathbf{x},t)\frac{\partial A_{jl}}{\partial\alpha_i}\phi_s^l=0\ ,\qquad \bar{P}_{i'}-\int\bar{\pi}_j(\mathbf{x},t)A_{jl}\frac{\partial\phi_s^l}{\partial q_{i'}}=0\ , \tag{8}$$

where $i,i'=1,2,3$. Here, as well as in the rest of this text, the integration $\int$ is understood as integration over x, i.e., $\int=\int d^3\mathbf{x}$. By defining

$$Q_a=(\alpha_i(t),q_{i'}(t)),\quad \bar{\mathcal{P}}_a=(\bar{I}_i,\bar{P}_{i'})\ , \tag{9}$$

with $i,i'=1,2,3$ and $a=(i,i')$, Eq. (8) can be expressed in a more compact form:

$$\psi_a\equiv\bar{\mathcal{P}}_a-\int\bar{\pi}_i\frac{\partial}{\partial Q_a}(A_{il}\phi_s^l)=0\quad(a=1,\ldots,6)\ . \tag{10}$$

With naive canonical commutation relations (Poisson brackets)

$$\{\bar{\mathcal{P}}_a,Q_b\}=\delta_{ab}\quad(a,b=1,\ldots,6)\ ,\qquad \{\bar{\pi}_i(\mathbf{x},t),\bar{\eta}_j(\mathbf{y},t)\}=\delta_{ij}\delta(\mathbf{x}-\mathbf{y})\quad(i,j=1,2,3)\ , \tag{11}$$

we see that Eqs. (10) are the first-class constraints of Dirac;[11] i.e., they satisfy

$$\{\psi_a,\psi_b\}=0,\quad\{H,\psi_a\}=0\ .$$

The Hamiltonian is determined only up to linear combinations of the ψ_a's:

$$H'=H+\lambda_a\psi_a\ ,$$

where the λ_a's are arbitrary functions of $\bar{\mathcal{P}}$ and Q. The canonical equations of motion derived from such a Hamiltonian will therefore contain some arbitrary functions. (The number of the arbitrary functions in the general solution of the equations of motion is equal to the number of independent first-class constraints.) To eliminate this arbitrariness, we impose the following "gauge" conditions (χ conditions):

$$\chi_a=\int\bar{\eta}_i K_{ij}\frac{\partial}{\partial Q_a}A_{jl}\phi_s^l=0\quad(a=1,\ldots,6)\ , \tag{12}$$

which satisfies

$$\det|\{\psi_a,\chi_b\}|\neq 0\ .$$

The physical meaning behind it is clear: the pion fluctuation should be orthogonal to the infinitesimal translation and rotation of the soliton. The set of constraints including both (10) and (12) is second class now. Therefore, according to Dirac, by appropriate modification of the canonical brackets (11), the system can be described by the ordinary Hamiltonian equation of motions. Canonical quantization can then be carried out in the usual way. The transformation to a new variable set, $\tilde{\pi}$ and $\tilde{\mathcal{P}}$,

$$\tilde{\pi}=\bar{\pi}-\bar{\mathcal{P}}\mu^{-1}K\frac{\partial A\phi_s}{\partial Q} ,$$
$$\tilde{\mathcal{P}}=\bar{\mathcal{P}}(1-\mu^{-1}\Xi-\mu^{-1}\Omega) , \quad (13)$$

is canonical and satisfies

$$\bar{\mathcal{P}}\dot{Q}+\int\bar{\pi}\dot{\bar{\eta}}=\bar{\mathcal{P}}\dot{Q}+\int\tilde{\pi}\dot{\bar{\eta}}+\bar{\mathcal{P}}\mu^{-1}\int K\frac{\partial A\phi}{\partial Q}\dot{\bar{\eta}}$$
$$=\tilde{\mathcal{P}}\dot{Q}+\int\tilde{\pi}\dot{\bar{\eta}} , \quad (14)$$

where

$$\mu_{ab}=\int\frac{\partial A\phi_s}{\partial Q_a}K\frac{\partial A\phi_s}{\partial Q_b}K ,$$

$$\Xi_{ab}=\int\bar{\eta}K\frac{\partial}{\partial Q_a}\frac{\partial A\phi_s}{\partial Q_b} \quad (a,b=1,\ldots,6) , \quad (15)$$

$$\Omega_b=\int\bar{\eta}\frac{\partial K}{\partial Q_a}\frac{\partial A\phi_s}{\partial Q_b} .$$

In terms of the new variables $\tilde{\pi}$ and $\tilde{\mathcal{P}}$ the Hamiltonian is

$$H=\tilde{P}\dot{Q}+\int\tilde{\pi}\dot{\bar{\eta}}-\int\mathcal{L}$$
$$=-\int\tfrac{1}{2}\tilde{\pi}K^{-1}\tilde{\pi}+\int\tfrac{1}{2}\nabla(A\phi_s+\bar{\eta})K\nabla(A\phi_s+\bar{\eta})+m_\pi^2f_\pi^2[1-\cos(A\phi_s+\bar{\eta})/f_\pi]$$
$$+\int\tfrac{1}{2}\tilde{\mathcal{P}}^T(1-\mu^{-1}\Xi-\mu^{-1}\Omega)^{-1}\mu^{-1}(1-\mu^{-1}\Xi-\mu^{-1}\Omega)^T\tilde{\mathcal{P}} \quad (16)$$

and the $\psi=0$ condition becomes

$$\psi_a\equiv\int\tilde{\pi}_i\frac{\partial}{\partial Q_a}(A_{il}\phi_s^l)=0 \quad (a=1,\ldots,6) . \quad (17)$$

The conditions (12) and (17) require that $\tilde{\pi}$ and $\bar{\eta}$ be orthogonal to the zero modes $(\partial/\partial Q_a)(A_{il}\phi_s^l)$.

Further simplification can be made if we introduce soliton-fixed fields η and π via

$$\eta_i(x,t)=A_{ij}^{-1}(\alpha)\bar{\eta}_j(\mathbf{x}+\mathbf{q},t) ,$$
$$\pi_i(x,t)=A_{ij}^{-1}(\alpha)\bar{\pi}_j(\mathbf{x}+\mathbf{q},t) . \quad (18)$$

Equations (10), (12), and (17) will be made independent of α and q with the aid of (18).

We next expand the last term of our Hamiltonian (16) in terms of the pion field η. As expected, the η-independent terms include the rest mass M and the rotational and translational kinetic energies of the Skyrmion: $\bar{I}^2/2\Lambda+\bar{P}^2/2M$, where Λ, moment of inertia, and M, mass of the Skyrmion, are the diagonal matrix elements of the matrix μ.

The terms that are linear in η represent Yukawa-type couplings of the pion field to the soliton (N or Δ) of the form

$$H_Y=\int\tfrac{1}{2}(\tilde{I},\tilde{P})\mu_0^{-1}(-\mu_1+\Xi_1+\Xi_1^T+\Omega_1+\Omega_1^T)\mu_0^{-1}(\tilde{I},\tilde{P})^T$$
$$=\tfrac{1}{2}\tilde{I}_iM_{ij}\tilde{I}_j+\tfrac{1}{2}\tilde{I}_iM_{ii'}\tilde{P}_{i'}+\tfrac{1}{2}\tilde{P}_{i'}M_{i'j}\tilde{I}_j+\tfrac{1}{2}\tilde{P}_{i'}M_{i'j'}\tilde{P}_{j'} . \quad (19)$$

$i,j,i',j=1,2,3$ where we have expanded μ, Ξ, and Ω in the power of η, $\mu=\mu_0+\mu_1+\cdots$, etc., so that the matrix M is linear in η. The explicit form of M is given in the Appendix.

It is easily seen that we obtain pion-nucleon coupling of order $N_c^{-3/2}$, which is of higher order by $1/N_c^2$ than the π-N coupling expected in chiral symmetry.[15] In fact, the Goldberger-Treiman relation

$$\sqrt{4\pi}f_{\pi NN}=\frac{m_\pi g_A}{F_\pi}$$

leads the pion Yukawa coupling constant $f_{\pi NN}$ of order of $\sqrt{N_c}$. This is also consistent with the order of the one-pion-exchange energy between two nucleons. However, the Hamiltonian (16) for the Skyrmion-pion system does not have a $O(\sqrt{N_c})$ coupling, while the leading Yukawa coupling is given by (19). The $O(\sqrt{N_c})$ coupling vanishes in the Skyrme model due to the stability condition of the soliton solution. The soliton is a solution of the Euler-Lagrange equation and therefore is stable against a linear fluctuation. Thus no linear coupling of the pion (fluctuation) is allowed. The Yukawa term (19) emerges in imposing the constraint conditions, i.e., by the canonical transformation (13).

This apparent inconsistency can be solved by distinguishing the classical (or static) and the quantal (or nonstatic) couplings of pion and nucleon.[16] Because the Skyrmion is a solution of the classical equation, it is dressed by a static pion configuration (pion cloud) around a bare nucleon. This is analogous to the Coulomb field around a point charge. Indeed, if one places another Skyrmion at a distance R apart, then the total classical energy (order N_c) of the system contains a Yukawa potential, whose coupling is of the order of $\sqrt{N_c}$. When we consider the fluctuation around the (dressed) Skyrmion, we obtain the "leading" Yukawa coupling (19), which is the lowest-order nonstatic coupling. In the electromagnetism, the nonstatic coupling is the coupling of transverse photons via the spatial current, say $e\bar{\Psi}\gamma\Psi \sim e(\mathbf{P}/M)\bar{\Psi}\psi$, which is of higher order of $1/M \sim 1/N_c$.

III. PION EMISSION MATRIX ELEMENT

In dynamical processes such as $N \to N\pi$ and $\Delta \to N\pi$, where a nonstatic pion is emitted, we must evaluate the matrix element of the nonstatic Yukawa coupling H_Y of Eq. (19). Here we use the plane-wave approximation. Namely, instead of using the solution of the pion equation of motion derived from the Hamiltonian (16), we employ the plane wave

$$\bar{\eta}_c(\mathbf{x},t) = \int d^3k \frac{1}{\sqrt{2\omega_k V}} [a_{ck}\exp(i\mathbf{k}\cdot\mathbf{x} - i\omega_k t) + \text{H.c.}] , \tag{20}$$

where a_{ck} is an annihilation operator of pion field with isospin index c and momentum $\mathbf{k}$. This approximation is allowed only for large pion momentum $\mathbf{k}$, while for a small k the pion wave will be modified by the zero mode, because all the continuum solutions must be orthogonal to the zero modes. We neglect pion distortion in the present calculation, because the complete solutions are very complicated and we are interested here only in qualitative behavior.

The matrix elements of H_Y defined in (19) are calculated for the $N' \to N + \pi$ process:

$$\langle N,\mathbf{p},\pi^l,\mathbf{k}|H_Y|N',\mathbf{p}\rangle = i\langle N|\tau^l(\sigma\mathbf{k})|N'\rangle \times \delta(\mathbf{p}-\mathbf{p}'-\mathbf{k})v^A_{\pi NN}(\mathbf{k}) \tag{21}$$

with the form factor

$$v^A_{\pi NN}(\mathbf{k}) = \frac{2m_N}{\Lambda^2}\frac{5}{2}[c_A(k)+c_B(k)] + \frac{2m_N}{M\Lambda}\left[-\frac{d(k)}{6}\right] ,$$

where

$$c_A(k) = -\tfrac{1}{15}\int R_1 \frac{3j_1(kr)}{k} d^3\mathbf{r} ,$$

$$c_B(k) = \tfrac{2}{15}\int R_2 \frac{3j_1(kr)}{k} d^3r , \tag{22}$$

$$d(k) = \tfrac{1}{3}\int (3N_1 = N_2 + N_3) j_0(kr) d^3\mathbf{r} .$$

The definitions of the functions R and N are given in the Appendix and $r = 2ef_\pi x$ is a dimensionless radial variable and k the momentum in units of $2ef_\pi$. Similarly, $\Delta \to \pi N$ and $\Delta \to \pi\Delta$ matrix elements are given by

$$v^A_{\pi N\Delta} = \frac{2m_N}{\Lambda^2}\frac{5}{8\sqrt{2}}(-4c_A - c_B) + \frac{2m_N}{M\Lambda}\frac{1}{2\sqrt{2}}\left[-\frac{5d}{2}\right] , \tag{23}$$

$$v^A_{\pi\Delta\Delta} = \frac{2m_N}{\Lambda^2}(-\tfrac{14}{30}c_A + \tfrac{34}{30}c_B) + \frac{2m_N}{M\Lambda}\frac{2d}{15} .$$

The form factors $c_A(k)$ and $c_B(k)$ are divergent at $k = 0$ if $m_\pi = 0$, because $R_1 \sim 2R_2$ fall off as r^{-2} for large r. This is an artifact of the chiral limit ($m_\pi = 0$). For the massless pion, the pionic cloud in the classical Skyrmion solution falls off as ρ^{-2} and thus the form factor diverges quadratically. This does not happen for a realistic pion mass $m_\pi = 138$ MeV, with which our numerical calculations have been done.

The Hamiltonian (19) is not Galilei invariant. The above calculation was done in the rest frame of the initial baryon and therefore we set $\mathbf{P}_{\text{tot}} = 0$. Then the last term of Eq. (19) does not contribute to the matrix element.

The quantum Hamiltonian H_Y is not uniquely defined. Instead of (19) we may choose

$$H_Y = \tfrac{1}{4}(\tilde{I}_i\tilde{I}_j M_{ij} + M_{ij}\tilde{I}_i\tilde{I}_j) + \cdots , \tag{24}$$

which has the same classical limit as that of (19) but will end up with different quantum theory. It is impossible to avoid this ambiguity in our derivation of H_y. In fact any linear combination of (19) and (24), $\alpha H_Y^A + \beta H_Y^B$, with $\alpha + \beta = 1$ is just as good as (19). Let us call H_Y in (19) as ordering A and H_Y in (24) ordering B and repeat our calculation for the ordering B to see how much our final results change. In place of (24) we now have

$$v^B_{\pi NN} = \frac{2m_N}{\Lambda^2}(-\tfrac{5}{12})c_A + \frac{2m_N}{M\Lambda}\frac{d}{6} + \frac{2m_N}{M^2}\frac{h}{18} ,$$

$$v^B_{\pi N\Delta} = \frac{2m_N}{\Lambda^2}\frac{1}{2\sqrt{2}}(-10c_A - \tfrac{15}{4}c_B) + \frac{2m_N}{M\Lambda}\frac{1}{2\sqrt{2}}\left[-\frac{d}{2}\right] + \frac{2m_N}{M^2}\frac{h}{12\sqrt{2}} , \tag{25}$$

$$v^B_{\pi\Delta\Delta} = \frac{2m_N}{\Lambda^2}(-\tfrac{34}{30}c_A + \tfrac{8}{5}c_B) + \frac{2m_N}{M\Lambda}\frac{2d}{15} + \frac{2m_N}{M^2}\frac{h}{90} ,$$

where c_A, c_B, and d are defined in Eq. (22) and h is defined by

$$h(k) = \tfrac{1}{3}\int \left[T_1 + 2T_2 + \frac{3T_3}{5}\right] 3j_1(kr)k\, d^3\mathbf{r} . \tag{26}$$

In numerical calculations we employ two sets of parameters: (1) $f_\pi = 54$ MeV and $e = 4.84$ (Ref. 3); (2) $f_\pi = 93$ MeV, and $e = 7.00$ (Ref. 4). The first two rows of Table I show the πNN, etc., "coupling constants," which are defined by $g_{\pi NN} = v_{\pi NN}$ ($k = 0$), etc. Note that this is not the static πNN coupling constant, but rather represents the strength of the nonstatic coupling. Corresponding form factors $F(k) = v(k)/v$ ($k = 0$) are plotted in Figs. 1–3. The form factors we obtained are very soft. The numerical results for ordering B are given in the third and fourth rows of Table I. Now the values of $g_{\pi NN}$ and $g_{\pi\Delta\Delta}$ are reduced by almost 50%. But the magnitude of $g_{\pi N\Delta}$ has increased by a factor of about 3. Also the sign (relative to $g_{\pi NN}$ and $g_{\pi\Delta\Delta}$) of $g_{\pi N\Delta}$ has been changed.

Table II shows contributions of various terms of Eq. (19) to the coupling constants. Note that we would only have the first term of Eq. (19) if we had not considered the translational zero modes. We find that the first term gives the major contribution (about 80% to 90% depending on the particular coupling constant), while the second and third terms account for a 10–20% effect. The last term has no contribution at all to coupling constants be-

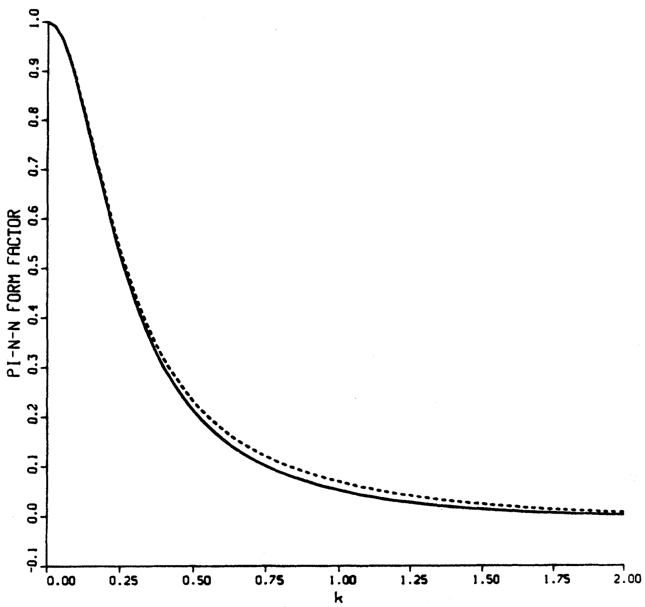

FIG. 1. Calculated πNN form factor $F_{\pi NN}(k)$ as a function of three-momentum transfer k in unit of $2ef_\pi$ ($=523$ MeV/c). The solid line is for the A ordering and the dashed line is for the B ordering.

cause $h(k=0)=0$.

The Δ decay width can be calculated using the formula

$$\Gamma(\Delta^{++}\to p+\pi^{+})=g^2_{\pi N\Delta}F^2_{\pi N\Delta}(\mathbf{k})\frac{|\mathbf{k}|^3}{12\pi}\frac{E_p}{m_p^2 m_\Delta} \tag{27}$$

with $|\mathbf{k}|=225.61$ MeV/c and $E_p=(\mathbf{k}^2+m_p^2)^{1/2}=966.7$ MeV, which gives $\Gamma=0.2705g^2_{\pi N\Delta}F^2_{\pi N\Delta}=17$ MeV for parameter set one and $=61$ MeV for parameter set two.

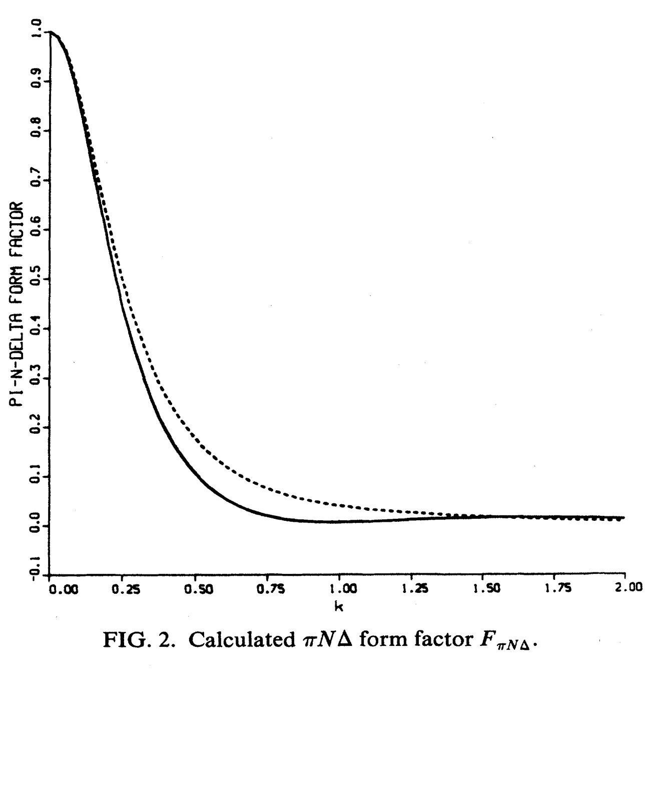

FIG. 2. Calculated $\pi N\Delta$ form factor $F_{\pi N\Delta}$.

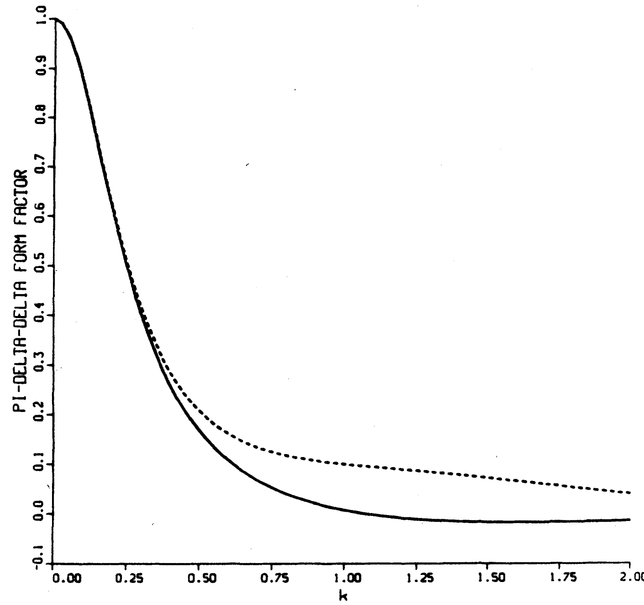

FIG. 3. Calculated $\pi\Delta\Delta$ form factor $F_{\pi\Delta\Delta}$.

With the B ordering we get for the Δ width $\Gamma=350.94$ MeV for parameter set one and 620 MeV for parameter set two. The experimental value is $\Gamma_{\text{expt}}=115$ MeV. Similar calculations on the Δ width without including the quantization of the translational motion have been performed by Saito,[11] Hayashi, and Schwesinger[13] using the standard Skyrmion model and Adami and Zahed[12] with a vector-meson stabilized Skyrmion model.

IV. CONCLUSION AND DISCUSSION

We have applied the Dirac quantization method to the fluctuating pion field around a moving and rotating Skyrmion with all six zero-frequency modes eliminated. The pion-nucleon and pion-Δ coupling matrix elements have been calculated and the $\Delta\to N\pi$ decay width evaluated. Though the coupling strength obtained agrees with that of phenomenological pion-nucleon (Δ) effective field theory within an order of magnitude, the ordering ambiguity in the quantization has prevented us from giving an unambiguous prediction. We found that the contribution of the translational motion of the Skyrmion to the πN coupling is less than 20%. This mainly because the inertia M for the translation is much larger than Λ for the rotation.

The form factors we obtain are very soft compared with the known $\Delta\to\pi N$ form factor. It has been suggested that the pion-nucleon vertex form factor becomes harder when vector mesons are introduced explicitly into the model Lagrangian.[17,18]

The static pion-baryon coupling constant can be obtained from the asymptotic (large-r) behavior of the soliton profile. It is known that the pion-nucleon coupling is consistent with the axial-vector coupling constant g_A via the Goldberger-Treiman relation. The semiclassical evaluation of the axial-vector current of the nucleon satisfies

TABLE I. Calculated pion-nucleon "coupling constants," $g_{\pi NN} \equiv v_{\pi NN}$ ($k=0$), etc. The first and second rows are those predicted by the ordering-A quantum Hamiltonian (19) with $f_\pi = 54$ MeV, $e = 4.84$ [those introduced by Adkins and Nappi (Ref. 13)] and $f_\pi = 93$ MeV, $e = 7.0$ [those introduced by Jackson and Rho (Ref. 4)], respectively. The third and fourth rows are obtained using the ordering-B Hamiltonian (24) again for the two sets of f_π and e.

	$g_{\pi NN}$	$g_{\pi N\Delta}$	$g_{\pi\Delta\Delta}$	$g_{\pi N\Delta}/g_{\pi NN}$	$g_{\pi\Delta\Delta}/g_{\pi NN}$
AN^A	24.54	−48.23	17.26	−1.96	0.70
JR^A	27.05	−63.44	20.43	−2.33	0.73
AN^B	−13.06	−151.4	−9.58	11.59	0.73
JR^B	−13.92	−177.8	−10.58	12.76	0.77

the PCAC (partial conservation of axial-vector-current) relation, too. Such classical pion-baryon coupling, however, does not describe dynamical pion-nucleon processes, such as $\Delta \to N\pi$ decay, because the pion is not a quantum particle but is treated as a classical field. Although the classical approach gives the static one-pion-exchange potential between two nucleons, the two-pion-exchange force does not appear unless the pion field is quantized. The present quantization of the pion fluctuation around the soliton will, in principle, give an effective pion-nucleon field theory with nonstatic couplings.[1] (In another paper, we have shown that the covariant one-meson-exchange amplitude can be obtained by including such nonstatic meson-soliton coupling in $1+1$ dimensions.[16])

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APPENDIX

The expressions of the matrix M in Eq. (19) are given below:

$$M_{ij} = \frac{f_\pi}{\Lambda^2} \int \eta_k [\delta_{ij}^T \hat{r}_k R_1 + (\delta_{ik}^T \hat{r}_j + \delta_{jk}^T \hat{r}_i) R_2]$$

with

$$R_1 = -2sc + \frac{1}{f_\pi^2 e^2} \left[2s^2 F'' + \frac{4s^2}{r} F' + 2sc(F')^2 - \frac{4s^3 c}{r^2} \right], \tag{A1}$$

$$R_2 = \frac{s^2}{F} + \frac{1}{f_\pi^2 e^2} \left[\frac{s^2}{F}(F')^2 - \frac{2s^4}{Fr^2} \right],$$

where $F = F(r)$ is the Skyrmion profile and $s = \sin(F)$, $c = \cos(F)$, and $F' = dF'/dr$. $\eta_k = \eta_k(\mathbf{x}, t)$ is the soliton-fixed pion field:

$$M_{ii'} = \frac{f_\pi}{\Lambda M} \int \eta_k (\epsilon_{k,ii'} N_1 + \epsilon_{ii'j} \hat{r}_j \hat{r}_k N_2 + \epsilon_{iji'} \hat{r}_{i'} \hat{r}_j N_3) \tag{A2}$$

with

$$N_1 = \frac{2s^2}{rF} + \frac{1}{f_\pi^2 e^2} \left[\frac{2s^2 (F')^2}{rF} - \frac{s^4}{r^3 F} \right],$$

$$N_2 = \frac{2sc}{r} - \frac{2s^2}{rF} + \frac{1}{f_\pi^2 e^2} \left[-\frac{2s^2 F''}{r} - \frac{s^2 F'}{r^2} - \frac{2sc(F')^2}{r} - \frac{2s^2 (F')^2}{r} + \frac{4s^3 c}{r^3} + \frac{s^4}{r^3 F} \right], \tag{A3}$$

$$N_3 = \frac{2scF'}{F} - \frac{2s^2}{rF} + \frac{1}{f_\pi^2 e^2} \left[\frac{s^2 (F')^3}{F^2} - \frac{4s^2 (F')^2}{Fr} + \frac{4s^3 c F'}{Fr^2} - \frac{2s^4}{Fr^3} \right],$$

$$M_{i'j'} = \frac{f_\pi}{M^2} \int \eta_k [\delta_{i'j'}^T \hat{r}_k T_1 + (\delta_{i'k}^T \hat{r}_{j'} + \delta_{j'k}^T \hat{r}_{i'}) T_2 + \hat{r}_{i'} \hat{r}_{j'} \hat{r}_k T_3], \tag{A4}$$

with

TABLE II. Different contributions to the "coupling constants."

	$g_{\pi NN}^A$	$g_{\pi NN}^B$	$g_{\pi N\Delta}^A$	$g_{\pi N\Delta}^B$	$g_{\pi\Delta\Delta}^A$	$g_{\pi\Delta\Delta}^B$
Rot. only	22.51	−11.03	−58.98	−153.5	18.88	−7.96
Total	24.54	−13.06	−48.23	−151.4	17.26	−9.58

$$
\begin{aligned}
T_1 = {} & \frac{2F'}{r} - \frac{2sc}{r^2} + \frac{1}{f_\pi^2 e^2}\left[\frac{4s^2F'}{r^3} - \frac{2s^2F}{r^4} - \frac{2s^4}{r^4F^2}\right] \\
& + \frac{1}{f_\pi^2 e^2}\left[-\frac{2s}{r^2}\left[c - \frac{s}{F}\right]\left[\frac{2s^2}{r^2} + (F')^2\right] + \frac{2s^4}{Fr^4} - \frac{2s^2FF'}{r^3} + \frac{2s^2F''}{r^2} + \frac{4sc(F')^2}{r^2}\right], \\
T_2 = {} & \frac{2s^2F'}{F^2r} - \frac{2s^2}{Fr^2} + \frac{2scF'}{Fr} - \frac{2s^2F'}{F^2r} \\
& + \frac{1}{f_\pi^2 e^2}\left[\frac{2s^2}{F^2r}\left[F' - \frac{F}{r}\right]\left[\frac{s^2}{r^2} + (F')^2\right] + \frac{s^2F'}{r^3}\left[1 - \frac{s^2}{F^2}\right] + \frac{2sF'}{r}\left[c - \frac{s}{F}\right]\left[(F')^2 + \frac{2s^2}{r^2}\right]\right] \\
& + \frac{1}{f_\pi^2 e^2}\left[\frac{s^2F'}{r^3}\left[1 - \frac{s^2}{F^2}\right] - \frac{s^4}{Fr^4} + \frac{2s^4(F')^2}{F^3r^2} - \frac{s^2FF'}{r^3} - \frac{s^2(F')^2}{r^2} - \frac{2sc(F')^3}{r} - \frac{2s^2F'F''}{r}\right], \\
T_3 = {} & 2F'' + \frac{1}{f_\pi^2 e^2}\left[\frac{6s^2F''}{r^2} + \frac{4sc(F')^2}{r^2}\left[c - \frac{s}{F}\right] + \frac{2s^2(F')^2}{r^2} + \frac{4sc(F')^3}{r} + \frac{4s^2F'F''}{r} - \frac{4sc(F')^2}{r^2}\right].
\end{aligned}
\tag{A5}
$$

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