SU(3)×SU(2)×U(1) model with Fritzsch mass matrices

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Discrete symmetries associated with the roots of unity are employed to construct Fritzsch-type mass matrices for the three generations of fermions in the standard model with SU(3)×SU(2)×U(1) gauge symmetry. The model requires four conventional Higgs doublets to account for the hierarchy of fermion masses and mixings.

In spite of the successes of the standard model in describing well the strong and electroweak interactions[1] there still remains the fundamental problem of understanding the pattern of masses and mixing angles of the quarks and leptons of the three families. In the absence of a dynamical theory, specific forms[2] of mass matrices have been put forward that relate masses to mixing angles and also give mass relations. In all these schemes the mass of the top quark remains a model-dependent parameter. Of all the mass matrices considered, the form suggested by Fritzsch[3] is the most popular. The elegance of a Fritzsch-type mass matrix lies in the fact that the elements of the matrix are easily related to the physical masses of the fermions and for the quarks contact with the Kobayashi-Maskawa[4] mixing matrix of weak interactions is also easily established. To see this, all fermion fields are initially taken as eigenstates of weak interactions. Left-handed fermions are weak-interaction doublets and right-handed fermions are weak-interaction singlets. Their transformations under the SU(3) ×SU(2)×U(1) gauge interactions are

$$\begin{pmatrix} u_i^0 \\ d_i^0 \end{pmatrix}_L, \begin{pmatrix} c_i^0 \\ s_i^0 \end{pmatrix}_L, \begin{pmatrix} t_i^0 \\ b_i^0 \end{pmatrix}_L \sim (3,2,\tfrac{1}{3}) \,, \quad u_R^0, c_R^0, t_R^0 \sim (3,1,\tfrac{4}{3}), \quad d_R^0, s_R^0, b_R^0 \sim (3,1,-\tfrac{2}{3}) \,; \tag{1}$$

$$\begin{pmatrix} \nu_e^0 \\ e^0 \end{pmatrix}_L, \begin{pmatrix} \nu_\mu^0 \\ \mu^0 \end{pmatrix}_L, \begin{pmatrix} \nu_\tau^0 \\ \tau^0 \end{pmatrix}_L \sim (1,2,-1) \,, \quad e_R^0, \mu_R^0, \tau_R^0 \sim (1,1,-2) \,, \tag{2}$$

where $i=1,2,3$ for color which will be suppressed henceforth. In what follows neutrinos will be taken to be massless. In order to discuss mass eigenstates the weak-interaction eigenstates are grouped according to their weak-isospin (I_3^w) quantum number. Since leptoquark transitions do not occur in the standard model, there are three fermion mass matrices to consider. These involve the $(I_3^w)=\frac{1}{2}$ quark fields $\Psi^{U^0}=(u^0,c^0,t^0)$, the $(I_3^w)=-\frac{1}{2}$ quark fields $\Psi^{D^0}=(d^0,s^0,b^0)$, and the $(I_3^w)=-\frac{1}{2}$ lepton fields $\Psi^{E^0}=(e^0,\mu^0,\tau^0)$. These mass matrices are referred to as M^U,M^D,M^E matrices. A feature common to M^U,M^D,M^E is that two of the three eigenvalues correspond to the masses of the fermions of the first two generations and are vanishingly small as compared with the mass of the third-generation fermion. Also from weak-interaction phenomenology the mixings between the fermions is inferred to be small. These considerations can be met in one possible way if the mass matrices M^U,M^D,M^E in the first approximation contain one driving mass term and there is weak mixing only among the fermions adjacent to each other (nearest-neighbor mixing). With the ansatz all Fritzsch mass matrices are of the form

$$\overline{M}=\begin{pmatrix} 0 & Ae^{i\alpha} & 0 \\ \overline{A}e^{i\overline{\alpha}} & 0 & Be^{i\beta} \\ 0 & \overline{B}e^{i\overline{\beta}} & Ce^{i\gamma} \end{pmatrix} \tag{3}$$

with the hierarchy $C \gg B,\overline{B} \gg A,\overline{A}$. The matrices M^U,M^D,M^E are of this form. It is convenient to write $\overline{M}$ in the form $\overline{M}=P_1MP_2$ where P_j are diagonal phase matrices $P_j=\mathrm{diag}(e^{i\alpha_j},e^{i\beta_j},e^{i\gamma_j})$, $j=1,2$ and $\alpha=\beta_2+\alpha_1$, $\overline{\alpha}=\beta_1+\alpha_2$, $\beta=\gamma_2+\beta_1$, $\overline{\beta}=\gamma_1+\beta_2$, $\gamma=\gamma_1+\gamma_2$. Economy in the choice of parameters results if one takes $\alpha=\overline{\alpha}$, $\beta=\overline{\beta}$, $A=\overline{A}$, and $P_1=P_2$. With these simplifications the matrix M is of the form

$$M=\begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} . \tag{4}$$

In this form A,B,C can be related to the eigenvalues $(M_1,-M_2,M_3)$ of M; $\mathrm{Tr}M$ gives $C=M_1-M_2+M_3$, $\mathrm{Det}M$ gives $A^2C=M_1M_2M_3$, and $\mathrm{Tr}M^2$ gives $2(A^2+B^2)+C^2=M_1^2+M_2^2+M_3^2$. From these relations one gets $A=[M_1M_2M_3/(M_1-M_2+M_3)]^{1/2}$- and $B=[(M_3-M_2)(M_3+M_1)(M_2-M_1)/(M_1-M_2+M_3)]^{1/2}$. The orthogonal matrix J that diagonalizes M is given by

$$J=\begin{pmatrix} \left[\frac{M_3(M_3-M_2)M_2}{(M_3-M_1)(M_2+M_1)(M_3-M_2+M_1)}\right]^{1/2} & \left[\frac{M_1(M_3-M_2)}{(M_3-M_1)(M_2+M_1)}\right]^{1/2} & -\left[\frac{(M_2-M_1)M_1(M_1+M_3)}{(M_3-M_1)(M_1+M_2)(M_1-M_2+M_3)}\right]^{1/2} \\ \left[\frac{M_1(M_1+M_3)M_3}{(M_1+M_2)(M_2+M_3)(M_1-M_2+M_3)}\right]^{1/2} & -\left[\frac{M_2(M_3+M_1)}{(M_1+M_2)(M_2+M_3)}\right]^{1/2} & \left[\frac{(M_1-M_2)M_2(M_2-M_3)}{(M_1+M_2)(M_2+M_3)(M_1-M_2+M_3)}\right]^{1/2} \\ \left[\frac{M_2(M_2-M_1)M_1}{(M_2+M_3)(M_3-M_1)(M_1-M_2+M_3)}\right]^{1/2} & \left[\frac{M_3(M_2-M_1)}{(M_2+M_3)(M_3-M_1)}\right]^{1/2} & \left[\frac{(M_1+M_3)M_3(M_3-M_2)}{(M_3-(M_1)(M_2+M_3)(M_1-M_2+M_3)}\right]^{1/2} \end{pmatrix} . \quad (5)$$

Thus each mass matrix M^U, M^D, M^E has its own rotation matrix J^U, J^D, J^E for diagonalization. Let the eigenstates of the mass matrix for the quarks be denoted by $\Psi^U=(u,c,t)$, $\Psi^D(d,s,b)$. These are related to the quark eigenstates of weak interactions through J and P, $\Psi_L^U = J^U P^U \Psi_L^{U^0}$, $\Psi_L^D = J^D P^D \Psi_L^{D^0}$. Expressing the weak interactions $(g/2\sqrt{2})\Psi_L^{U^0} W \Psi_L^{U^0}$ in terms of the mass eigenstates gives $(g/2\sqrt{2})\Psi_L^U W (J^U)^T (P^U)^* P^D J^D \Psi_L^D$ from which the Kobayashi-Maskawa matrix V_{KM} in terms of the elements of the Fritzsch matrices is

$$V_{KM} = (J^U)^T (P^U)^* P^D J^D . \quad (6)$$

The effective phase matrix $P^{\bar{U}D} = (P^U)^* P^D$ has only diagonal elements expressed in terms of two parameters σ and χ, $P^{\bar{U}D} = \mathrm{diag}(2\pi N + \sigma, \sigma+\chi, \chi)$ where $N = 0, \pm 1, \pm 2, \ldots$ denotes periodicity. It is interesting to note that in the absence of weak mixing terms A, B, the mass matrix with just one element C is equivalent to a matrix with all elements equal. Such rank-one matrices are suggestive of a universal mass-generating interaction[5] that mixes the three generations with equal masslike terms.

In the past, Fritzsch-type mass matrices for the quarks and leptons of the three generations have been written down only in models with extended gauge symmetry such as $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ and grand unified models such as SO(10). In this paper Fritzsch mass matrices are derived in the standard model with $SU(3) \times SU(2) \times U(1)$ gauge interactions. The additional ingredients required are discrete symmetries[6] and an extended Higgs structure. Let the elements of the discrete symmetries be the square roots $(1,-1)$ and the cube roots[7] $(1, e^{2i\pi/3}, e^{4i\pi/3})$ of unity. These are denoted by Σ^0, Σ, and $(\omega^0, \omega^1, \omega^2)$. Under the discrete symmetries, the left-handed and right-handed quarks and leptons are taken to transform in the following way:

$$\begin{bmatrix} u_i^0 \\ d_i^0 \end{bmatrix}_L \to \Sigma^0 \omega \begin{bmatrix} u_i^0 \\ d_i^0 \end{bmatrix}_L, \quad u_R^0 \to \Sigma^0 \omega^2 u_R^0, \quad d_R^0 \to \Sigma^0 \omega^2 d_R^0 , \quad (7)$$

$$\begin{bmatrix} c_i^0 \\ s_i^0 \end{bmatrix}_L \to \Sigma \omega^2 \begin{bmatrix} c_i^0 \\ s_i^0 \end{bmatrix}_L, \quad c_R^0 \to \Sigma \omega c_R^0, \quad s_R^0 \to \Sigma \omega s_R^0 , \quad (8)$$

$$\begin{bmatrix} t_i^0 \\ b_i^0 \end{bmatrix}_L \to \Sigma^0 \omega^0 \begin{bmatrix} t_i^0 \\ b_i^0 \end{bmatrix}_L, \quad t_R^0 \to \Sigma^0 \omega^0 t_R^0, \quad b_R^0 \to \Sigma^0 \omega^0 b_R^0 , \quad (9)$$

$$\begin{bmatrix} \nu_e^0 \\ e^0 \end{bmatrix}_L \to \Sigma^0 \omega \begin{bmatrix} \nu_e^0 \\ e^0 \end{bmatrix}_L, \quad e_R^0 \to \Sigma^0 \omega^2 e_R^0 , \quad (10)$$

$$\begin{bmatrix} \nu_\mu^0 \\ \mu^0 \end{bmatrix}_L \to \Sigma \omega^2 \begin{bmatrix} \nu_\mu^0 \\ \mu^0 \end{bmatrix}_L, \quad \mu_R^0 \to \Sigma \omega^2 \mu_R^0 , \quad (11)$$

$$\begin{bmatrix} \nu_\tau^0 \\ \tau^0 \end{bmatrix}_L \to \Sigma^0\omega^0 \begin{bmatrix} \nu_\tau^0 \\ \tau^0 \end{bmatrix}_L , \quad \tau_R^0 \to \Sigma^0\omega^0\tau_R^0 \ . \tag{12}$$

The subscript 0 on the fermion fields denotes weak-interaction eigenstates. To get the desired Yukawa couplings, four Higgs doublets H_1,H_2,H_3,H_4 are introduced. Under $SU(2)_L \times U(1)$ all four doublets transform as $(2,-1)$ and under the discrete symmetries the Higgs doublets transform in the following way:

$$\begin{aligned} &H_1 \to \Sigma\omega^0 H_1, \quad H_2 \to \Sigma\omega^2 H_2 \ , \\ &H_3 \to \Sigma^0\omega^0 H_3, \quad H_4 \to \Sigma\omega H_4 \ . \end{aligned} \tag{13}$$

The interaction Lagrangian for the fermion-fermion-Higgs-boson couplings that is invariant under SU(3) ×SU(2)×U(1) gauge interactions and the aforementioned discrete symmetries is

$$\begin{aligned} L_Y = {} & y_{uc}\bar{u}_L^0 H_1 c_R^0 + y_{cu}\bar{c}_L^0 H_1 u_R^0 + y_{ct}\bar{c}_L^0 H_2 t_R^0 + y_{tc}\bar{t}_L^0 H_2 c_R^0 + y_{tt}\bar{t}_L^0 H_3 t_R^0 + y_{ds}\bar{d}_L^0 \tilde{H}_1 s_R^0 \\ & + y_{sd}\bar{s}_L^0 \tilde{H}_1 d_R^0 + y_{sb}\bar{s}_L^0 \tilde{H}_4 b_R^0 + y_{bs}\bar{b}_L^0 \tilde{H}_4 b_R^0 + y_{bs}\bar{b}_L^0 \tilde{H}_4 b_R^0 + y_{bb}\bar{b}_L^0 \tilde{H}_3 b_R^0 + y_{e\mu}\bar{e}_L^0 \tilde{H}_1 \mu_R^0 + y_{\mu e}\bar{\mu}_L^0 \tilde{H}_1 e_R^0 \\ & + y_{\mu\tau}\bar{\mu}_L^0 \tilde{H}_4 \tau_R^0 + y_{\tau\mu}\bar{\tau}_L^0 \tilde{H}_4 \mu_R^0 + y_{\tau\tau}\bar{\tau}_L^0 \tilde{H}_3 \tau_R^0 + \text{H.c.} \ , \end{aligned} \tag{14}$$

where $\tilde{H} = i\sigma_2 H_a^*$ $(a=1,\ldots,4)$. The Fritzsch-type mass matrices for fermions are arrived at by substituting for the vacuum expectation values of H_1,H_2,H_3,H_4 and taking Yukawa couplings to be

$$\begin{aligned} &y_{uc}=y_1^U e^{i\alpha^U}, \quad y_{cu}=y_1^U e^{i\bar{\alpha}^U}, \quad y_{ct}=y_2^U e^{i\beta^U} \ , \\ &y_{tc}=y_1^U e^{i\bar{\beta}^U}, \quad y_{tt}=y_3^U e^{i\gamma^U}, \quad y_{ds}=y_1^D e^{i\alpha^D} \ , \\ &y_{sd}=y_1^D e^{i\bar{\alpha}^D}, \quad y_{sb}=y_2^D e^{i\beta^D}, \quad y_{bs}=y_2^D e^{i\bar{\beta}^D} \ , \\ &y_{bb}=y_3^D e^{i\gamma^D}, \quad y_{e\mu}=y_1^E e^{i\alpha^E}, \quad y_{\mu e}=y_1^E e^{i\bar{\alpha}^E} \ , \\ &y_{\mu\tau}=y_2^E e^{i\beta^E}, \quad y_{\tau\mu}=y_2^E e^{i\bar{\beta}^E}, \quad y_{\tau\tau}=y_3^E e^{i\gamma^E} \ . \end{aligned} \tag{15}$$

By choosing this form for the Yukawa couplings, all mass matrices are of the form as in Eq. (1) with the Yukawa couplings y_{ij} not taken to be symmetric; only the magnitude of y_{ij} and y_{ji} are taken to be of the same order. As discussed previously, there exists the freedom to choose the phases $\alpha,\bar{\alpha},\beta,\bar{\beta},\gamma$ such that the resulting mass matrices are Hermitian with the appropriate choice for the phase matrices P; the matrix M can be related to the matrix of Eq. (2) as discussed in the preceding paragraphs. Thus the mass matrices take the forms

$$M^U = \begin{bmatrix} 0 & y_1^U\langle H_1\rangle & 0 \\ y_1^U\langle H_1\rangle & 0 & y_2^U\langle H_2\rangle \\ 0 & y_2^U\langle H_2\rangle & y_3^U\langle H_3\rangle \end{bmatrix} , \tag{16}$$

$$M^D = \begin{bmatrix} 0 & y_1^D\langle H_1\rangle & 0 \\ y_1^D\langle H_1\rangle & 0 & y_2^D\langle H_4\rangle \\ 0 & y_2^D\langle H_4\rangle & y_3^D\langle H_3\rangle \end{bmatrix} , \tag{17}$$

$$M^E = \begin{bmatrix} 0 & y_1^E\langle H_1\rangle & 0 \\ y_1^E\langle H_1\rangle & 0 & y_2^E\langle H_4\rangle \\ 0 & y_2^E\langle H_4\rangle & y_3^E\langle H_3\rangle \end{bmatrix} . \tag{18}$$

The vacuum expectation values $\langle H_a\rangle$, $(a=1,\ldots,4)$ determine the ground state of the scalar potential $V(H_1,H_2,H_3,H_4)$ where

$$\begin{aligned} V(H_1,H_2,H_3,H_4) = {} & \sum_{a,b=1}^{4} [-\mu_a^2(H_a^\dagger H_a) + (\tfrac{1}{2})\lambda_{ab}(H_a^\dagger H_a)(H_b^\dagger H_b)] \\ & + \sum_{a\neq b=1}^{4} [(\tfrac{1}{2})\lambda''_{ab}(\tilde{H}_a^\dagger H_b)(H_b^\dagger \tilde{H}_a) + (\tfrac{1}{2})\lambda'_{ab}(H_a^\dagger H_b)(H_b^\dagger H_a)] \\ & + \lambda_1(H_1^\dagger H_3)(H_1^\dagger H_3) + \lambda_2(H_1^\dagger H_2)(H_1^\dagger H_4) + \lambda_3(H_3^\dagger H_2)(H_3^\dagger H_4) \\ & + \lambda_4(H_2^\dagger H_4)(H_1^\dagger H_4) + \lambda_5(\tilde{H}_1^\dagger H_2)(\tilde{H}_2^\dagger H_4)^* + \lambda_6(\tilde{H}_2^\dagger H_3)(\tilde{H}_3^\dagger H_4)^* + \text{H.c.} \end{aligned} \tag{19}$$

The couplings $\lambda_{ab},\lambda'_{ab},\lambda''_{ab}$, are symmetric and all Yukawa couplings are taken to be real. The scales of $\langle H_a\rangle$, $a=1,\ldots,4$ are restricted by the electroweak scale

$$\sum_{a=1}^{4} \langle H_a\rangle^2 = (2\sqrt{2}G_H)^{-1} \ , \tag{20}$$

where G_F is the Fermi coupling constant of weak interactions. The desired hierarchies among the elements of the Fritzsch mass matrices are implemented by taking $\langle H_3\rangle \gg \langle H_2\rangle \gg \langle H_1\rangle$ and $\langle H_2\rangle \approx \langle H_4\rangle$ and the bound

$$\sum_{a=1}^{4} \langle H_a\rangle^2 = (174 \text{ GeV})^2 \ . \tag{21}$$

The doublets ensure that the tree-level boson mass relation $M_W = M_Z \cos\theta_W$ is still satisfied with the extended Higgs structure in the present model. The mixings of the flavors in the Yukawa couplings and the extended Higgs structure lead to flavor-changing neutral currents[8] mediated by neutral-Higgs-boson exchange. The currents contribute to the $K_L - K_S$ mass difference. There are seven neutral Higgs bosons that contribute to the $K_L - K_S$ mass difference. Their tree-level contribution is of the form

$$L^{\rm Higgs}_{\Delta S=2} = \sum_{i=1}^{7} G_F f_i \sqrt{m_d m_s} / M^2_{H_i^0} (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) \quad (22)$$

and is related to the $K_L - K_S$ mass difference by

$$2\langle K^0 \| L^{\rm Higgs}_{\Delta S=2} \| \bar{K}^0 \rangle \ll M_{K_L} - M_{K_S} \sim 3.5\times 10^{-15}\ {\rm GeV}\ , \quad (23)$$

where f_i are mixing angles in terms of the parameters of the Higgs potential. In evaluating the K^0-$\bar{K}^0$ matrix elements we use the vacuum-insertion method and also take the conservative approach of f_i and all Higgs-boson masses to be equal. This gives the following conservative lower bound on the Higgs-boson mass:

$$M_{H_i} \geq 8\sqrt{f}\ \ {\rm TeV}\ . \quad (24)$$

In what follows we will take all Higgs-boson masses of order 1 TeV (Ref. 9). This seems a reasonable value in view of the uncertainty in determining the mixing angles f_i and the approximate validity of the vacuum-insertion method.

CP violation in the present model is spontaneous rather than intrinsic in nature and comes about due to the possibility of having complex vacuum expectation values for H_a ($a = 1,\ldots,4$). These nonzero phases are denoted by θ_a ($a = 1,\ldots,4$). The constraints on the relative phases for spontaneous *CP* violation to occur are

$$\theta_2 - \theta_1 \neq n\pi/2,\quad \theta_3 - \theta_1 \neq l\pi/3,\quad \theta_4 - \theta_1 \neq m\pi/3\ , \quad (25)$$

where $n,l,m = 0, \pm 1, \pm 2, \ldots$. Next we establish the *CP*-violating character of the model in the presence of Higgs-boson masses of order 1 TeV. The ratio of the *CP*-violating to the *CP*-conserving matrix elements for the $\bar{K}^0$-K^0 system is related to the *CP*-violating parameter ϵ as follows:

$$\epsilon \approx \Delta(\theta){\rm Re}(f){\rm Im}(f)\frac{\pi m_d m_s M_W^2 \sin^2\theta_W}{\alpha M^2_{H^0} m_c^2}$$
$$\sim 10^{-3}\Delta(\theta){\rm Re}(f){\rm Im}(f)\ . \quad (26)$$

This quantity can easily be adjusted to the experimentally measured value of 2×10^{-3}. The strength of the *CP*-violating interaction is $10^{-6}G_F$ and is microweak in nature as opposed to the milliweak character encountered in multi-Higgs-boson models with natural flavor conservation.[10] The value of ϵ' is also readily accommodated around $10^{-3}\epsilon$. Finally, we consider the electric dipole moment of the neutron. There are two contributions to consider, one due to the charged Higgs boson and the other due to neutral Higgs boson. Once again all scalar-boson masses are taken to be approximately equal. The expression for the electric dipole moment of the neutron is determined to be

$$D_N \approx -\sum_q \frac{e G_F m_q^4 {\rm Im}(f_q)^2 \Delta_q}{24\pi^2 M^2_{H_0}} \ln\frac{m_q^2}{M^2_{H_0}}\ . \quad (27)$$

In view of our previous conservative estimates, the electric dipole moment of the neutron is found to be of order 10^{-27} *e*cm. It is to be noted that this value is 2 orders of magnitude smaller than the one predicted by models in which the Higgs interactions are flavor conserving. In working out the various *CP* effects the value of the top-quark mass used is 60 GeV. Finally we note that one immediate consequence of high-mass scalars of order 1 TeV in the Higgs potential is that some of the scalar couplings in the potential are greater than unity. This would seem to violate the perturbative unitary bound of Lee, Quigg, and Thacker.[11] The scalar sector of the theory becomes strongly interacting.[12]

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