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Origin of proton spin: Rotating constituents?

Meng Ta-chung, Pan Ji-cai,* Xie Qu-bing,† and Zhu Wei‡
Fachbereich Physik der Freien Universität Berlin, Berlin, West Germany
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The question of whether the constituents of polarized protons are rotating about their polarization axes is discussed. Two collision experiments are proposed in which the effects due to rotating constituents should manifest themselves if such orbital motion exists.

The recent spin-asymmetry measurement of the European Muon Collaboration[1] (EMC) has attracted much attention.[2,3] The measurement suggests that only a rather small fraction of the proton spin is due to the spin of the quarks, in disagreement with the popular picture for nucleon structure. Several analyses[2,3] have been made and the obtained results had led some authors[2] to argue that the orbital angular momentum of the constituents may significantly contribute to proton spin, while other authors[3] argue that the data give no clear conclusion on the spin content of the proton and that errors, specifically those arising from the uncertainty of extrapolation, have been underestimated. That is, the above-mentioned experimental and the theoretical activities[1-3] have not only revived the discussion of an old question "Can the spin of a hadron be attributed to the orbital motion of its constituents?" but also have manifested the urgent need of more direct methods to answer this question.

The concept of velocity distribution for constituents inside a nucleon, together with its relation with the spin of the proton, was proposed by Yang and Kantor[4,5] in the later 1960s, and has been discussed in connection with the geometrical picture for elastic hadron-hadron collisions by Chou and Yang[5] in the early 1970s. The importance of orbital motion of the constituents inside the proton in polarized electron-nucleon scattering was pointed out by Sehgal[6] in 1974, in connection with the quark-parton model and related sum rules.[7] Since then, several authors[8] have discussed the role of orbital angular momentum in their calculations of sum rules for various structure functions. But, unfortunately, up to the present time, these discussions[4-6,8] did not seem to have attracted much attention, although the answer to the above-mentioned question may force us to reexamine the present understanding of nucleon structure.

The result of the recent EMC experiment[1] taken together with the subsequent analyses[2,3] is obviously a challenge to experimentalists as well as theorists working in this field. One of the problems all of us are now facing is the following. Can the question "Are there rotating constituents inside a polarized nucleon?" be answered experimentally in a simple, direct way?

In this paper, we show that it is possible. We propose two polarization experiments in which the effects of rotating constituents should clearly manifest themselves, if such rotation exists. These experiments are independent of the usual analyzing procedure. Hence, the results will be free from the problems associated with the extrapolations and integrations of spin-dependent structure functions, etc., mentioned above.

Experiment A. Multihadron production in deep-inelastic lepton-proton scattering *with polarized target.* Measurement of the azimuthal distributions of the produced charged hadrons, or that of the flow of hadronic energy.

We recall the following. Every lepton-proton scattering event is characterized by the energy transfer ν and the momentum transfer $\mathbf{q}$ by measuring the energy and the scattering angle of the outgoing lepton. Events associated with large invariant momentum transfer Q^2 $(=-q^2)$ values are those in which pointlike interaction between the virtual photon/vector boson and one of the constituents of the target proton take place. We also recall that many experimental facts suggest that the target proton can be envisaged as a system of infinitely many constituents. Such constituents are known in the literature either as "stuff" (in the geometrical model[9]), as "partons" (in the parton model[10]), or as "a sea of uncorrelated quark-antiquark pairs" (in quark-parton and/or QCD type of models[11]).

Under the assumption that these constituents can be considered as pointlike objects, the interaction between which can be neglected, jet-production processes have been successfully described. Hence this assumption can at least be considered as a useful phenomenological ansatz in understanding the existence of hadronic shower due to the large momentum transfer in deep-inelastic lepton-proton scattering processes. Here, the direction of the "current jet" coincides with that of the momentum-transfer $\mathbf{q}$ via a virtual photon/vector boson, provided that the struck constituent does not have intrinsic transverse momentum. But, since the constituents may perform intrinsic random motion inside the proton, the momentum of a constituent in the transverse direction is in general nonzero. That is to say, the axis of the produced hadrons of the current jet may deviate from the direction of the virtual photon/vector boson. In fact, it has been shown by Cahn[12] and by König and Kroll[13] that such intrinsic motion of the constituents should cause asymmetry in azimuthal distributions of the produced hadrons around the momentum transfer $\mathbf{q}$ in deep-inelastic lepton-proton scatterings. Such effects have indeed been observed by the European Muon Collaboration[14] in the muon-proton, and by Mukherjee *et al.*

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(MIT–Fermilab–Michigan State Collaboration)[15] in neutrino-nucleon scattering processes. To be more precise, in the coordinate system in which the line $\phi=0$ (on the outgoing lepton side) and π is the lepton plane and this line is perpendicular to the momentum transfer **q** (see Fig. 1 of the second paper in Ref. 14), the existing data[14] shows that the azimuthal distributions $(1/N)dN/d\phi(\phi) = g(\phi)$ of the produced charged hadrons in the forward direction have a rather pronounced structure. It has a maximum at $\phi=\pi$ and a minimum at $\phi=0,2\pi$. (See, e.g., Fig. 2 of the second paper in Ref. 14.) If we plot, instead of $g(\phi)$, the combination[16]

$$h(\phi)=[g(\phi)+g(\phi+\pi)]/2 \quad (1)$$

against the variable ϕ, we see the following. The data for the ϕ distribution shows that $h(\phi)$ is approximately a horizontal straight line [see Fig. 1(a) of this paper]. This result agrees with the theoretical expectations[12,13] for cases in which Q^2 is large compared to the average transverse momentum squared $\langle k_\perp^2 \rangle$ of the constituents. In fact, by expressing $g(\phi)$ in the usual (see, e.g., Refs. 13 and 14 and the references given therein) way, $g(\phi)=A+B\cos\phi+C\cos2\phi+D\sin\phi$, all contributions—including the QCD contributions[13,17]—to the terms $B\cos\phi+D\sin\phi$ cancel one another in $h(\phi)$. The contributions to $C\cos2\phi$ are of higher order in $k_\perp^2/Q^2$, and this term is indeed negligibly small in practice.[14]

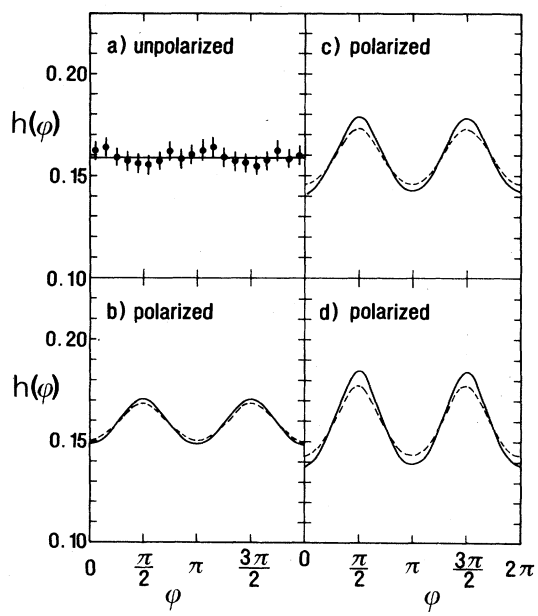

FIG. 1. Azimuthal distributions of charged hadrons produced by the current jet. (a) Experimental result for $h(\phi)$ in deep-inelastic μ-p scattering with an unpolarized target. Here, h is defined in Eq. (1) as a function of the azimuthal angle ϕ measured with respect to the lepton plane. Data are taken from the second paper of Ref. 14 (Fig. 2 for $x_F > 0.1$). Note that $g(\phi)$ in our plot is normalized to unity (b)–(d) Calculated result for $h_p(\varphi)$ in deep-inelastic lepton-nucleon scattering with polarized target. Here h_p is given in Eqs. (2), (7), and (9) as a function of the azimuthal angle φ measured with respect to the polarization plane. (b)–(d) correspond to the cases characterized by the α values 1.4, 1.8, and 2.2, respectively. The broken lines are the results obtained by setting the deviation angle $\delta=25°$. (See text for further details.)

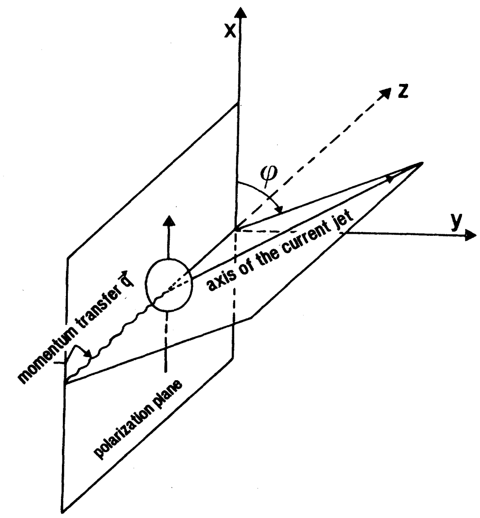

FIG. 2. Event geometry: The lepton plane is not shown.

Let us now turn our attention to the spin problem and ask what we expect to see if the orbital motion of the constituents of the proton indeed contributes to its spin. We first consider the case in which the proton is polarized perpendicular to the direction of the virtual photon/vector boson. In order to compare this with the usual (unpolarized) case, we adopt the same assumption, concerning the existence of pointlike constituents inside the proton, such that the only difference between the unpolarized and the polarized case is the following. The constituents in the latter case are performing an *ordered*—in addition to or instead of the *random*—motion.

Theoretically, the simplest and the most direct way to see the effect due to this difference is to consider those events in which the plane determined by the momentum transfer **q** and the polarization axis (i.e., the polarization plane shown in Fig. 2) coincides with the lepton plane (i.e., the plane determined by the incoming and the outgoing lepton, not shown in Fig. 2). In terms of the variables ϕ defined in the same way as that in the unpolarized case, the difference between the azimuthal distribution $g_p(\phi)$ for the polarized target and the corresponding distribution $g(\phi)$ for the unpolarized target can be most easily seen in the following way. The distribution

$$h_p(\phi)=[g_p(\phi)+g_p(\phi+\pi)]/2 \quad (2)$$

which corresponds to $h(\phi)$ for the unpolarized case as defined in Eq. (1), should have two maxima, one at $\phi=\pi/2$ and the other at $\phi=3\pi/2$. That is, the result should have qualitatively the form of one of the curves shown in Figs. 1(b)–1(d). (Details about these curves will be given below.) This is because the constituents which are rotating about the polarization axis (the x axis in the coordinate system shown in Fig. 2) have nonvanishing velocity components perpendicular to the polarization axis and perpendicular to the lepton plane (i.e., velocity components in the y direction of the chosen coordinate system shown in Fig. 2). This means the orbital motion of the constituents in the polarized proton causes the current jet to move either to $\phi=\pi/2$ or to $\phi=3\pi/2$.

The above-mentioned method, though theoretically simple and clean, is not very practical. This is because the criterion for event selection strongly restricts the number of useful events. This handicap can, however, be overcome in the following way. We recall the following. (i) In the measured azimuthal distribution $g(\phi)$ of the charged hadrons (or the energy flow) in the current jet, the azimuthal angle ϕ is defined according to the lepton plane on an event-to-event basis.[14] The observed structure in $g(\phi)$, in particular the maximum at $\phi=\pi$ and the minimum at $\phi=0$ and 2π, is in fact nothing else but a kinematical consequence of the random transverse motion of the constituent, when it interacts in a *pointlike* fashion with the virtual photon/vector boson. In terms of $h(\phi)$ [see Eq. (1)], the ϕ distribution of the produced hadrons in each of those events is essentially flat in every event in which Q^2 is large compared with $\langle k_\perp^2 \rangle$. (ii) Azimuthal distributions for the produced hadrons have already been measured with considerable precision in collisions with unpolarized proton targets.[14,15] The apparatus used by EMC (Ref. 14) consisted of a large open dipole spectrometer with proportional and drift chambers. In particular, it is known that the momenta and the direction of both the incident and the scattered muons can be well measured, giving accurate knowledge of the virtual-photon direction. Based on these facts, we propose the following.

First, polarize the proton target in a direction which is (approximately) transverse to the direction of the lepton beam. Measure the outgoing lepton to determine the momentum transfer $\mathbf{q}$ carried by the virtual photon/vector boson. (We recall that the angle between the direction of the virtual photon/vector boson and that of the lepton beam is usually very small, i.e., $\ll\pi/2$.) The straight line along the polarization direction and the straight line along $\mathbf{q}$ define a plane which we call the polarization plane. In the ideal case, $\mathbf{q}$ is exactly perpendicular to the polarization axis. We choose a right-handed rectangular coordinate system (xyz) in which the polarization axis is defined as the x axis, the direction perpendicular to the polarization plane is the y axis, while the direction of $\mathbf{q}$ is the z axis. In the general case, $\mathbf{q}$ is not perpendicular to the polarization axis although it is in the polarization plane (by definition). The direction of $\mathbf{q}$, which we call the z' axis, has a nonzero deviation-angle δ with respect to z in the polarization plane. It is convenient, as we shall see later on, to define another right-handed rectangular coordinate system $x'y'z'$ in which the y' axis is perpendicular to the polarization plane. (It is, of course, the $x'z'$ plane in this system.) The angle φ is the azimuthal angle around the direction $\mathbf{q}$, where the point $\varphi=0$ is fixed by the polarization plane on the side of the polarization axis. See Fig. 2. Note that such a choice of coordinate system is not only possible but also meaningful because of the facts mentioned in (i).

Second, we measure the φ distributions $g_p(\varphi)$ and evaluate $h_p(\varphi)$ in the same way as in Eq. (2), where φ is measured with respect to the polarization plane. Because of the fact that the effects due to the random motion of the constituents do not appear in $h_p(\varphi)$ for scattering events in which Q^2 is sufficiently large $[Q^2\gg\langle k_\perp^2\rangle]$ the orbital motion will cause the struck constituent to move either towards $\varphi=\pi/2$ or towards $\varphi=3\pi/2$. That is, we expect to see two distinct enhancements in $h_p(\varphi)$: one at $\varphi=\pi/2$ and the other at $\varphi=3\pi/2$, provided that the angle δ mentioned above is not too large.

We did a simple model calculation to estimate the significance of this effect quantitatively. In this calculations, we first discuss the case $\delta=0$, which is evidently the simplest. In order to compare this effect with that due to the *random* motion of the constituents discussed by Cahn,[12] we use exactly the same ansatz as Cahn for the distribution of the transverse momentum of the observed hadron:

$$D(\xi,\mathbf{p}_\perp)\propto e^{-bp_\perp^2}\ , \tag{3}$$

where $\mathbf{p}_\perp$ is the transverse momentum of the observed hadron produced by the struck constituent, b and ξ are constants. The latter takes the fact into account that even the fastest observed hadron cannot carry all the momentum of the struck constituent but just some fraction $\xi<1$ of it. (See Ref. 12 and/or Appendix A for further details.) Furthermore, we note the following. (i) The transverse-momentum distribution $P(k_x^2,k_y^2)$ due to the random intrinsic motion of the constituents has been successfully described[12,13,14] by the product of two Gaussians,

$$P(k_x^2,k_y^2)\propto\exp[-a(k_x^2+k_y^2)]\ , \tag{4}$$

with the same variance $(2a)^{-1}=\langle k_i^2\rangle$ $(i=x,y)$. The empirical value[14,15] for $\langle k_\perp^2\rangle^{1/2}=\langle k_x^2+k_y^2\rangle^{1/2}$ ranges from 0.4 to 0.7 GeV/c. (ii) Since rotating constituents about the polarization axis (which is the x axis in this case) will in general cause an asymmetry in the variables k_x and k_y, the corresponding transverse-momentum distribution $P_p(k_x^2,k_y^2)$ is expected to change its form. A simple way[18] to incorporate this effect is to generalize the ansatz (4) to

$$P_p(k_x^2,k_y^2)\propto\exp(-a_xk_x^2-a_yk_y^2)\ , \tag{5}$$

where $(2a_x)^{-1}=\langle k_x^2\rangle$, $(2a_y)^{-1}=\langle k_y^2\rangle$, with $a_x\geq a_y$. Thus the ratio

$$\alpha=\left[\frac{\langle k_y^2\rangle}{\langle k_x^2\rangle}\right]^{1/2}=\left[\frac{a_x}{a_y}\right]^{1/2} \tag{6}$$

can be used as a measure for the intensity of the effect caused by the rotating constituents about the polarization (the x) axis. The above-mentioned φ distribution $h_p(\varphi)$ can be readily obtained. (See Appendix A for details). It is

$$h_p(\varphi)=N(c_x,c_y)j(\varphi)\int_0^1 d\xi\frac{[(\xi^2+c_x)(\xi^2+c_y)]^{1/2}}{\xi^2+j(\varphi)}\ , \tag{7}$$

where $c_x=a_x/b$, $c_y=a_y/b$,

$$j(\varphi)=\frac{c_x}{1+(c_x/c_y-1)\cos^2\varphi}\ , \tag{8}$$

and $N(c_x,c_y)$ is a normalization constant.

In our calculation, we adopted the same ansatz, $a=b$, as that given in Ref. 12 for the unpolarized case, and generalized it correspondingly for the polarized case, which means $1/(2a_x)+1/(2a_y)=1/a=1/b$. Furthermore, we used the same numerical values as those given in Ref. 14, so that the only unknown is the ratio α defined in Eq. (6). Now, since our main concern is to find out how significant this effect must be such that the above-mentioned enhancements in $h_p(\varphi)$ at $\varphi=\pi/2$ and $3\pi/2$ can be observed, we begin with a simple guess. Let us assume that the orbital motion around the axis causes an increase in $\langle p_y^2\rangle$ such that $\langle p_y^2\rangle$ becomes twice as large as $\langle p_x^2\rangle$. This gives $\alpha\approx1.4$ and the corresponding curve for $h_p(\varphi)$ is shown in Fig. 1(b). Comparison between the results given in Figs. 1(a) and 1(b) shows that the effect should be visible in this case, provided that the statistics in the experiments with polarized target is comparable with that of the unpolarized target. Besides, it is rather amusing to see that the usual[19] semiclassical relationship between angular momentum (spin), the mean intrinsic transverse momentum $\langle k_\perp\rangle$ of the constituents, and the radius R of the proton (assuming homogeneous distribution of the constituents inside a sphere) gives $\alpha=1.4$ for $\langle k_\perp\rangle=0.4$ GeV/c and $R=0.8$ fm. More detailed dynamical models[20] seem to suggest that α should be larger. Hence, we also carried out calculations for $\alpha=1.8$ and for $\alpha=2.2$, which are shown in Figs. 1(c) and 1(d), respectively. Furthermore, although the φ distribution for protons and that for neutrons may be different because their electromagnetic form factors and thus their charge distributions are different, it is yet probably still too early to discuss these effects.

We now discuss the influence of the deviation angle δ on the azimuthal distribution $h_p(\varphi)$. That is, we consider the general case in which the angle between the direction of the virtual photon/vector boson and the polarization direction (which is shown as the x axis in Fig.2) is not $\pi/2$ (which is shown as the z axis in Fig. 2), but $\pi/2+\delta$. (This direction is called the z' axis in the right-hand rectangular $x'y'z'$ system. Recall that the $x'z'$ plane coincides with the xy plane.) What does the distribution $P_p'(k_x'^2,k_y'^2)$ of the corresponding transverse momenta (k_x',k_y') look like? What is the relationship between this and the distribution given in Eq. (5) that corresponds to the case $\delta=0$? Since the coordinate systems (xyz) and $(x'y'z')$ are related to each other through a rotation in the xz (which is identical with the $x'z'$) plane, where the rotation angle is δ. It can be readily shown that

$$P_p'(k_x'^2,k_y'^2)\propto\exp\left[-\frac{a_xa_y}{(a_x-a_y)\sin^2\delta+a_y}k_x'^2-a_yk_y'^2\right]\ . \tag{9}$$

Here we again use the same value as that in the unpolarized case[12,13] (see Appendix A for details). Hence, the influence in $g_p(\varphi)$ caused by the deviation angle δ can be calculated provided that δ is known.

In Figs. 1(b)–1(d) we show (by the broken lines) the corresponding results we obtained by setting $\delta=25°$ instead of $\delta=0°$. The deviation angle $\delta=25°$ is of particular interest because it is the value we obtained by inserting the limiting Q^2 and E' values in the EMC experiments[14] in which the azimuthal distributions for the unpolarized target have been measured.

Third, we rotate the polarization axis (of the target) 90° to (or against) the lepton beam, and measure again the azimuthal distribution $g_p(\varphi)$. Here, the angle φ is defined in the same way as before. To be more precise, it is the azimuthal angle around the direction of the virtual photon/vector boson, where $\varphi=0$ is again fixed by the polarization plane. We note, also in this case, the polarization plane is fixed by the following two straight lines: the direction of the target polarization and the direction of the momentum transfer $\mathbf{q}$ via the virtual photon/vector boson. Because of the facts that have already been mentioned in the case in which the target is transversely polarized, the corresponding φ distribution $h_p(\phi)=[g_p(\varphi)+g_p(\varphi+\pi)]/2$ in this case should be flat. Furthermore it can be shown (see Appendix A) that the effect due to the deviation angle δ is also negligibly small in this case. Hence, we are led to the conclusion that measurements of the φ distribution $g_p(\varphi)$ in deep-inelastic lepton-nucleon scattering with transversely and longitudinally polarized target can be used to find out whether the constituents of a polarized proton are indeed rotating around the polarization axis. In principle, such experiments can be done by replacing the unpolarized target by a polarized one in those experiments[14,15] which have already been successfully performed. Our quantitative estimates shown in this paper suggest that the number of events expected in such experiments will be sufficient to see the effects caused by rotating constituents if they indeed exist.

It would be interesting to perform the proposed polarization experiments also for small values of Q^2. For small Q^2, the interaction between the virtual photon/vector boson and the polarized nucleon is no longer pointlike and hence the individual constituents, in particular their motion inside the nucleon, are no longer relevant for the "detecting device." (Recall that a real or almost real high-energy photon behaves like a hadron.) Hence, we expect that this kind of effect will disappear.

Experiment B. Measurement of the average transverse momentum of the timelike virtual photon in lepton-pair production in collisions of polarized proton (or antiproton) beams with polarized proton targets in the longitudinal direction. Here, we compare the result in the case in which the beam and the target particles are polarized parallel to one another with that in which they are antiparallel. The reasons why we think such measurements should be helpful are given below.

We recall the following. The lepton-pair production processes in high-energy hadron-hadron collisions can be understood, according to Drell and Yan,[21] as annihilation of quark-antiquark pairs, where the quarks and the antiquarks are considered as pointlike constituents of the colliding hadrons. The average transverse momentum, usually given in the squared values $\langle p_t^2 \rangle$ of such lepton pairs, has been measured[22] in various processes at different energies. Based on the fact that lepton-pair production experiments can be described[22] by the quark-antiquark annihilation picture, and that the quark-helicity does not depend very much on the proton polarization,[1] we expect to see the following, provided that the constituents are rotating about the polarization axis. The transverse momentum of the produced lepton pair depends very much on the polarization directions. To be more precise, we expect to see that the difference

$$\Delta\langle p_t^2 \rangle = \langle p_t^2(+) \rangle - \langle p_t^2(-) \rangle \qquad (10)$$

between the average of the transverse momentum squared in the antiparallel case (denoted by $+$) and that in the parallel case (denoted by $-$) should be greater than zero.[23] Note that by considering the difference between the spin-antiparallel and the spin-parallel cases, we can simply neglect the contributions due to the random motion of the annihilation quark-antiquark pairs.

In order to make a quantitative estimate, we completely neglect the contributions due to the random motion of the annihilating quark-antiquark pairs and consider the effects due to the rotation only. Let us denote the overlapping domain of the two colliding hadrons P and T by D. Every point in D can be characterized by the impact parameter b and the two angles θ_P, θ_T, shown in Fig. 3. The transverse momentum of the lepton pair created by the quark antiquark at the point (b,θ_P,θ_T) is obtained from the vector sum of the momenta $\mathbf{k}_{PR}$ and $\mathbf{k}_{TR}$ of the rotating constituents which annihilate each other. That is,

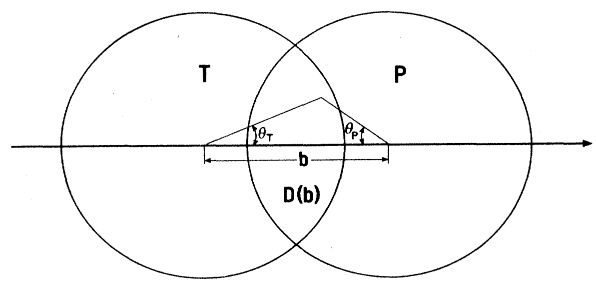

FIG. 3. Collision geometry: The collision axis is pointing towards the reader.

$$p_t^2(b,\theta_P,\theta_T;\pm) = |\mathbf{k}_{PR}|^2 + |\mathbf{k}_{TR}|^2 \pm 2|\mathbf{k}_{PR}||\mathbf{k}_{TR}|\cos(\theta_P+\theta_T) , \qquad (11)$$

where $+$ $(-)$ refers to the case in which P and T are antiparallel (parallel) polarized. The observed average transverse momentum in the case in which the polarizations of P and T are antiparallel, and that in the case in which they are parallel are

$$\langle p_t^2(\pm) \rangle = \int_0^{2R} \langle p_t^2(b;\pm) \rangle b\, db \Big/ \int_0^{2R} b\, db , \qquad (12)$$

respectively. Here, $\langle p_t^2(b;\pm) \rangle$ is the mean value of $\langle p_t^2(b,\theta_P,\theta_T;\pm) \rangle$ [given in Eq. (11)] averaged over all possible angles θ_P and θ_T for a given impact parameter b. That is,

$$\langle p_t^2(b,\pm) \rangle = \frac{\int_{D(b)} \langle p_t^2(b,\theta_P,\theta_T;\pm) \rangle F(b,\theta_P,\theta_T) d\theta_P d\theta_T}{\int_{D(b)} F(b,\theta_P,\theta_T) d\theta_P d\theta_T} , \qquad (13)$$

where we have denoted the overlapping domain of P and T, for the corresponding b, by $D(b)$. $F(b,\theta_P,\theta_T)$ stands for the product of a given Jacobian and the two-dimensional density functions of the quarks/antiquarks inside the colliding hadrons. (See Appendix B for details.) It is clear that, in contrast with $\langle p_t^2(\pm) \rangle$, neither $\langle p_t^2(b,\pm) \rangle$ nor $\langle p_t^2(b,\theta_P,\theta_T;\pm) \rangle$ can be directly measured. But they are extremely useful for the present calculation because the impact-parameter concept plays an essential role in discussing orbital motion of the constituents. Furthermore, since we are primarily interested in the difference of the average transverse momentum squared in the antiparallel-spin case and that in the parallel-spin case, it is also useful to consider

$$\Delta\langle p_t^2(p) \rangle = \langle p_t^2(b,+) \rangle - \langle p_t^2(b,-) \rangle \qquad (14)$$

and note that it follows from Eqs. (11), (13), and (14) that

$$\Delta\langle p_t^2(b) \rangle = \frac{4\int_{D(b)} |\mathbf{k}_{PR}||\mathbf{k}_{TR}|\cos(\theta_P+\theta_T) F(b,\theta_P,\theta_T) d\theta_P d\theta_T}{\int_{D(b)} F(b,\theta_P,\theta_T) d\theta_P d\theta_T} . \qquad (15)$$

The difference $\Delta\langle p_t^2 \rangle$ mentioned in Eq. (10) between the directly measured quantities $\langle p_t^2(+) \rangle$ and $\langle p_t^2(-) \rangle$ can then be expressed as

$$\Delta\langle p_t^2 \rangle = \int_0^{2R} \Delta\langle p_t^2(b) \rangle b\, db/(2R^2) , \qquad (16)$$

where the integrand $\Delta\langle p_t^2(b) \rangle$ is given in Eq. (15).

For an exact evaluation of the right-hand side of Eq. (15) and thus that of Eq. (16) we need to know $|\mathbf{k}_{PR}|$ and $|\mathbf{k}_{TR}|$, which are in general functions of the variables b, θ_P, and θ_T. This means for such calculation we need a detailed dynamical model in which the orbital motion of the constituents are precisely specified. As a rough estimate we assume that the dependence of $|\mathbf{k}_{PR}|$ and $|\mathbf{k}_{TR}|$ on b, θ_P, and θ_T is very weak, so weak that we can replace them by their average values $\langle |\mathbf{k}_{PR}| \rangle = \langle |\mathbf{k}_{TR}| \rangle$

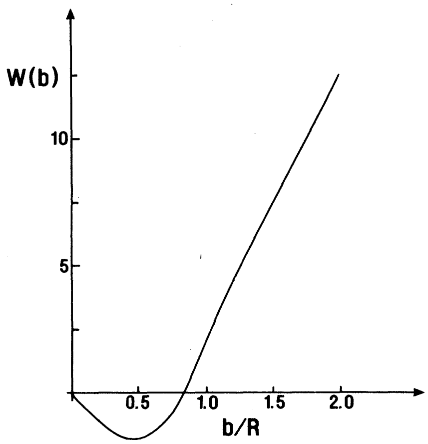

FIG. 4. Geometrical weighting factor $W(b)$ as a function of the impact parameter b. Here b is shown in unit of R, the proton radius. (Note that the result is independent of the absolute value of the nucleon radius.)

$\equiv\langle k_R\rangle$ (which should be the same because of the obvious symmetry). In this case, Eq. (16) becomes

$$\Delta\langle p_t^2\rangle=\frac{4\langle k_R\rangle^2}{\pi(2R)^2}\int_0^{2R}W(b)db\ . \tag{17}$$

Here $W(b)$ is a pure geometrical weighting factor:

$$W(b)=\frac{2\pi b\int_D\cos(\theta_P+\theta_T)F(b,\theta_P,\theta_T)d\theta_P d\theta_T}{\int_D F(b,\theta_P,\theta_T)d\theta_P d\theta_T}\ . \tag{18}$$

The dependence of this factor on b is shown in Fig. 4. This impact-parameter dependence implies in particular that the chance to observe Drell-Yan pairs in the polarized case should be comparable with that in the unpolarized case, provided that the degree of polarization is sufficiently high. The integral on the right-hand side of Eq. (17) can be evaluated numerically. The result is

$$\Delta\langle p_t^2\rangle\approx1.9\langle k_R\rangle^2\ , \tag{19}$$

provided that the proton/antiproton are taken as homogeneous spheres. By assuming that $\langle k_R\rangle$ is of the same order of magnitude as $\langle k_\perp\rangle$, the average transverse momentum due to the random intrinsic motion of the constituents, we have

$$\Delta\langle p_t^2\rangle\approx\begin{cases}(0.6\ \text{GeV}/c)^2 & \text{for } \langle k_\perp\rangle=0.4\ \text{GeV}/c\ ,\\(1\ \text{GeV}/c)^2 & \text{for } \langle k_\perp\rangle=0.7\ \text{GeV}/c\ .\end{cases} \tag{20}$$

Compared with the corresponding values for $\langle p_t^2\rangle$ obtained in unpolarized hadron-hadron collisions,[22] we are led to the conclusion that the expected difference mentioned in Eq. (10) should be observable at the presently available energies.

In conclusion, the purpose of this paper is to show that the question "Are there rotating constituents inside a polarized nucleon?" can be answered by performing different kinds of experiments. In particular, a deep-inelastic lepton-nucleon scattering experiment is proposed in which neither polarized beam, nor spin-dependent structure functions, nor sum rules are needed. Hence, the result of such experiments are expected to give an independent check on the current picture for hadron structure in general, and that for proton spin in particular.

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APPENDIX A

We consider the momentum distribution of the constituents inside a nucleon, and discuss the effects caused by their *orbital* motion. In particular, we show what happens when the deviation angle δ (defined in the text) is different from zero when the target nucleon is polarized in such a deep-inelastic lepton-nucleon scattering experiment. In order to do this, we consider the Lorentz frame in which the (either polarized or unpolarized) nucleon is at rest. (The constituents we discuss in this paper should not be identified with objects which are only defined in the infinite-momentum frame of the nucleon.)

In a given Cartesian coordinate system (xyz) in which the momentum of a constituent is denoted by (k_x,k_y,k_z), the momentum distribution of the constituents due to random intrinsic motion can be considered as the product of three Gaussians:

$$P(k_x^2,k_y^2,k_z^2)\propto\exp(-a_xk_x^2-a_yk_y^2-a_xk_z^2)\ , \tag{A1}$$

where $a_i=1/(2\langle k_i^2\rangle)$, $i=x,y,z$. Since none of the directions is distinguished, we have

$$a_x=a_y=a_z\equiv a^{\text{unpol}}\ . \tag{A2}$$

Hence, if we choose the z axis as the direction of the virtual photon/vector meson, the distribution of the transverse momentum $|\mathbf{k}_\perp|\equiv(k_x^2+k_y^2)^{1/2}$ is

$$P(|\mathbf{k}_\perp|)\equiv P(k_x^2,k_y^2)\propto\exp[-a^{\text{unpol}}(k_x^2+k_y^2)]\ . \tag{A3}$$

The polarization along a given axis will in general *break part of the symmetry* mentioned in Eq. (A2). In particular, the relation given in Eq. (A2) should be replaced by

$$a_x^\perp\equiv a_x\geq a_y=a_z\equiv a^\perp \tag{A4}$$

if the nucleon is polarized along the x axis, and it should be replaced by

$$a_z^\parallel\equiv a_z\geq a_x=a_y\equiv a^\parallel \tag{A5}$$

if the nucleon is polarized along the z axis.

Hence, if the z axis is again defined as the direction of the virtual photon/vector meson, the corresponding transverse-momentum distributions should be as follows. In the case in which the nucleon is polarized along the x axis we have

$$P_p^{\perp}(k_x^2,k_y^2)\propto \exp(-a_x^{\perp}k_x^2-a^{\perp}k_y^2)\ , \tag{A6}$$

where $a_x^{\perp}=1/(2\langle k_x^2\rangle)\geq 1/(2\langle k_y^2\rangle)=a^{\perp}$, and in the case in which the nucleon is polarized along the z axis we have

$$P_p^{\parallel}(k_x^2,k_y^2)\propto \exp[-a^{\parallel}(k_x^2+k_y^2)]\ , \tag{A7}$$

where $a^{\parallel}=1/(2\langle k_x^2\rangle)=1/(2\langle k_y^2\rangle)$.

We now consider the Cartesian coordinate system (x',y',z'), which is related to (xyz) in the following way. The $x'z'$ plane coincides with the xz plane such that y' axis is the same as the y axis. The angle between the z' and the z axis (and thus also the angle between the x' and the x axis) is δ. We wish to know the relationship between the coefficients a_x, a_y, and a_z in the momentum distribution $P(k_x^2,k_y^2,k_z^2)$ given in Eqs. (A1)–(A7) and the corresponding coefficients a_x', a_y', and a_z' in

$$P(k_x'^2,k_y'^2,k_z'^2)\propto \exp(-a_x'k_x'^2-a_y'k_y'^2-a_z'k_z'^2)\ , \tag{A1$'$}$$

where $a_i'=1/(2\langle k_i'^2\rangle)$, $i=x,y,z$. Here, (k_x',k_y',k_z') is the momentum of the above-mentioned constituent in the $(x'y'z')$ frame.

It is obvious that we should have

$$a_x'=a_y'=a_z'=a^{\text{unpol}} \tag{A2$'$}$$

if the nucleon is not polarized. Hence, if we choose the z' axis as the direction of the virtual photon/vector meson, the transverse-momentum distribution is

$$P(k_x'^2,k_y'^2)\propto \exp[-a^{\text{unpol}}(k_x'^2+k_y'^2)]\ . \tag{A3$'$}$$

The corresponding transverse-momentum distributions for the polarized targets can be written as follows. In the case in which the nucleon is polarized along the x axis, we have

$$P_p^{\perp}(k_x'^2,k_y'^2)\propto \exp(-a_x'^{\perp}k_z'^2-a_y'^{\perp}k_y'^2)\ , \tag{A6$'$}$$

where

$$a_x'^{\perp}=\frac{a_x^{\perp}a^{\perp}}{(a_x^{\perp}-a^{\perp})\sin^2\delta+a^{\perp}}\ ,\qquad a_y'^{\perp}=a^{\perp}\ . \tag{A4$'$}$$

In the case in which the nucleon is polarized along the z axis, we have

$$P_p^{\parallel}(k_x'^2,k_y'^2)\propto \exp(-a_x'^{\parallel}k_x'^2-a_y'^{\parallel}k_y'^2)\ , \tag{A7$'$}$$

where

$$a_x'^{\parallel}=\frac{a^{\parallel}a_z^{\parallel}}{(a^{\parallel}-a_z^{\parallel})\sin^2\delta+a_x^{\parallel}}\ ,\qquad a_y'^{\parallel}=a^{\parallel}\ . \tag{A5$'$}$$

In order to estimate the order of magnitude of the deviation angle δ, we consider the case in which the nucleon target is polarized perpendicular to the lepton beam. Denoting the energy-momentum vector of the incoming lepton by $(E,\mathbf{k})$, that of the outgoing lepton by $(E',\mathbf{k}')$, and that of the virtual photon/vector boson by $(q_0,\mathbf{q})$, it follows from energy-momentum conservation that δ, the angle between the incoming lepton and the virtual photon-vector boson, is

$$\tan\delta=\frac{E'\sin\theta}{E-E'\cos\theta}\ , \tag{A8}$$

where θ is the laboratory-frame scattering angle. Here we have neglected the lepton mass. Taken together with the well-known relationship between θ and $Q^2=-q^2$,

$$Q^2=4EE'\sin^2(\theta/2)\ , \tag{A9}$$

Eq. (A8) can be rewritten as

$$\tan\delta=\frac{\sqrt{Q^2(4EE'-Q^2)}}{2E^2-2EE'+Q^2}\ . \tag{A10}$$

By inserting the maximum values for E' and Q^2 obtained in the EMC experiments (Ref. 14) into Eq. (A10), we obtain $\delta=25.5°$. This is the largest deviation angle we would have if we would polarize the target used in Ref. 14 transverse to the lepton beam. The effect of this deviation angle to the azimuthal distribution $h(\varphi)$ is shown as dashed lines in Figs. 1(b)–1(d).

We recall that, for sufficiently large Q^2, the effects due to the random motion of the constituents do not appear in $h_p(\varphi)$. Hence the φ dependence of $h_p(\varphi)$ can be obtained by folding the transverse-momentum distribution $P_p(k_x^2,k_y^2)$ [such as that given in Eq. (5) or that in Eq. (9)] with the distribution $D(\xi,\mathbf{p}_\perp)$ [given in Eq. (3)] which describes the distribution of the transverse momentum given to the observed hadron by the struck constituent. That is, the distribution of the transverse momentum (p_x,p_y) of the hadrons produced by the struck constituent which carries a fraction ξ of its momentum [the transverse part of which is (k_x,k_y)] can be written as

$$\int dk_x dk_y \exp\{-b[(p_x-\xi k_x)^2+(p_y-\xi k_y)^2]\}\exp(-a_x k_x^2-a_y k_y^2)$$
$$=2\pi[(a_x+b\xi^2)(a_y+b\xi^2)]^{-1/2}\exp\left[-b\left[\frac{p_x^2}{a_x+b\xi^2}+\frac{p_y^2}{a_y+b\xi^2}\right]\right] \tag{A11}$$

provided that the longitudinal momentum of the struck constituent is much larger than $p_\perp$ and $k_\perp$. By inserting $(p_\perp\cos\varphi, p_\perp\sin\varphi)=(p_x,p_y)$ in the expression given in Eq. (A11), and integrating it over $p_\perp$, we obtain

$$h_p(\varphi|\xi)\propto\frac{[(a_x+b\xi^2)(a_y+b\xi^2)]^{1/2}}{b\{a_xa_y+[a_y+(a_y-a_x)\cos^2\varphi]b\xi^2\}} . \quad \text{(A12)}$$

This is the contribution to $h_p(\varphi)$ from those hadrons which carry a fraction ξ of the momentum of the struck constituent. Thus, the right-hand side of Eq. (7) in the text is nothing else but the integral of $h_p(\varphi|\xi)$ over ξ. The corresponding expression for $h(\varphi)$ in the case $\delta\neq 0$ can be obtained from Eq. (9) in a similar manner.

APPENDIX B

We connect the center of P with that of T by a straight line; and consider the plane which contains this line and which is perpendicular to the collision axis. This plane is shown in Fig. 3 together with $D(b)$, the domain of intersection, and the set of variables (b,θ_P,θ_T) which is used to characterize an arbitrary point in $D(b)$. The variables b,θ_P,θ_T are related to the usual Cartesian coordinate in a simple manner. In fact, by choosing the center of T as the origin of the coordinate system (x,y) and by choosing the straight line between T and P as the positive x axis, we have

$$x=b\frac{\sin\theta_P\cos\theta_T}{\sin(\theta_P+\theta_T)} ,$$

$$y=b\frac{\sin\theta_P\sin\theta_T}{\sin(\theta_P+\theta_T)} ; \quad \text{(B1)}$$

$$dx\,dy=J(b,\theta_P,\theta_T)d\theta_P d\theta_T , \quad \text{(B2)}$$

where $J(b,\theta_P,\theta_T)$ is the Jacobian

$$J(b,\theta_T,\theta_T)=b^2\frac{\sin\theta_P\sin\theta_T}{\sin^3(\theta_P+\theta_T)} . \quad \text{(B3)}$$

Let us now consider the density distributions of the colliding hadrons P and T. In terms of the above-mentioned Cartesian coordinate system in which the z axis is the direction pointing towards the reader, the two-dimensional density distribution $\sigma_T(x,y)$ of T is defined as the integral of the density distribution $\rho(x,y,z)$ over z:

$$\sigma_T(x,y)=\int dz\,\rho_T(x,y,z) . \quad \text{(B4)}$$

The corresponding two-dimensional density distribution $\sigma_P(x,y)$ for P can be obtained from $\rho_P(x,y,z)$ in a similar way. Obviously, the density functions σ_T and σ_P can be expressed in terms of the variables (b,θ_P,θ_T). For example, in the case in which T is a homogeneous sphere of radius R, we have

$$\sigma_T(b,\theta_P,\theta_T)=\frac{3}{2\pi R^3}\left[R^2-b^2\frac{\sin^2\theta_P}{\sin^2(\theta_P+\theta_T)}\right]^{1/2} . \quad \text{(B5)}$$

The function $F(b,\theta_P,\theta_T)$ in Eqs. (13), (15), and (18) is the product of $J(b,\theta_P,\theta_T)$, $\sigma_T(b,\theta_P,\theta_T)$, and $\sigma_P(b,\theta_P,\theta_T)$. That is

$$F(b,\theta_P,\theta_T)=J(b,\theta_P,\theta_T)\sigma_P(b,\theta_P,\theta_T)\sigma_T(b,\theta_P,\theta_T) , \quad \text{(B6)}$$

where $\sigma_P(b,\theta_P,\theta_T)$ is defined in a similar way as $\sigma_T(b,\theta_P,\theta_T)$ given in Eq. (B5).

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*Permanent address: Hua-Zhong Normal University, Wuhan, China.

†On leave from Shandong University, Jinan, China.

‡On leave from East China Normal University, Shanghai, China.

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$$A_{\text{orb}}=\frac{\langle p_t(+)\rangle-\langle p_t(-)\rangle}{\langle p_t(+)\rangle+\langle p_t(-)\rangle} .$$