Possible effect of the local terrain on the Australian fifth-force measurement

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We believe that the local topography can account for most of the positive evidence for non-Newtonian gravity recently reported by Stacey and co-workers. We show that the Hilton mine site in Queensland, Australia, is effectively in a valley and speculate on how this feature could have been missed in the original analysis.

In 1984 Holding and Tuck reported evidence for non-Newtonian gravity. They made density and gravity measurements at the Hilton mine in Queensland, Australia. Using these they found a value of G which is 0.8% higher than the laboratory value. They concluded that "it [is] implausible to explain the high value of G in terms of an inadequate knowledge of density. The possible effect of regional gravity anomalies is not as securely discounted."[1]

In several subsequent publications[2-5] Stacey and co-workers have added data and related the results to the fifth-force controversy.[6] In their latest publication[5] they have also responded to both general[7] and specific criticisms.[8] Their conclusion, however, remains unaltered: the only plausible Newtonian explanation for their observations is an unspecified regional irregularity.

We believe that the local terrain ($r < 10$ km) is the specific irregularity that caused most of the observed anomalous variation of g with depth at the Hilton mine. Ridges of the Selwyn range lie to the west of the mine; to the east there is only a gradual decline. Thus the mine is effectively in a valley. (See Fig. 1.) The mean elevation at a distance of 3 km from the relevant P49 shaft is 24 m higher than the top of the shaft. This feature is equivalent to an extra ring of mass which opposes the normal increase of g with depth. The amount of the reduction is comparable to the residuals of the Australian observations.

To calculate the expected effect of the terrain $h(r,\phi)$, we first average h over azimuthal angle ϕ, thus obtaining $h(r)$, where r is the distance to the mine shaft. The excess height of the terrain above the top of the mine shaft, $\Delta h(r) = h(r) - h(0)$, serves as a source mass for generating "topographical" gravity $g_t(d)$ at a depth d below the top of the shaft.

We treat all source masses as though they were on the plane $d = 0$. (This is a reasonable approximation since when viewed from the top of the mine shaft no topographical feature makes an angle of more than 6° from the horizontal.) The contribution of these source masses to the vertical component of gravity $g_{tz} \simeq g_t$ satisfies Laplace's equation below as well as above ground. Thus we can treat these source masses as the boundary condition in a Bessel expansion to the gravity underground:[9]

$$g_t(d,r) = \int A(k) e^{-kd} J_0(kr) dk \ .$$

In principle all wave numbers k contribute to the integral. In fact the local terrain can be approximated by a single wave number, $h = \text{const} + h_0 J_0(kr)$. Consequently, the predicted variation of g down the mine is simply

$$g_t(d,0) = A e^{-kd} \ ,$$

where $A = 2\pi G \rho h_0$ and the superficial rock density ρ is assumed to be a constant.

Our analysis of the terrain (out to $r = 45$ km) used a circular grid over four topographical maps of the Australian 1:100 000 series.[10] Generally elevations were taken at radii satisfying

$$r = 2^{n/4} \text{ km for } -2 \le N \le 22$$

and at 15° intervals in ϕ. At three of the larger radii,

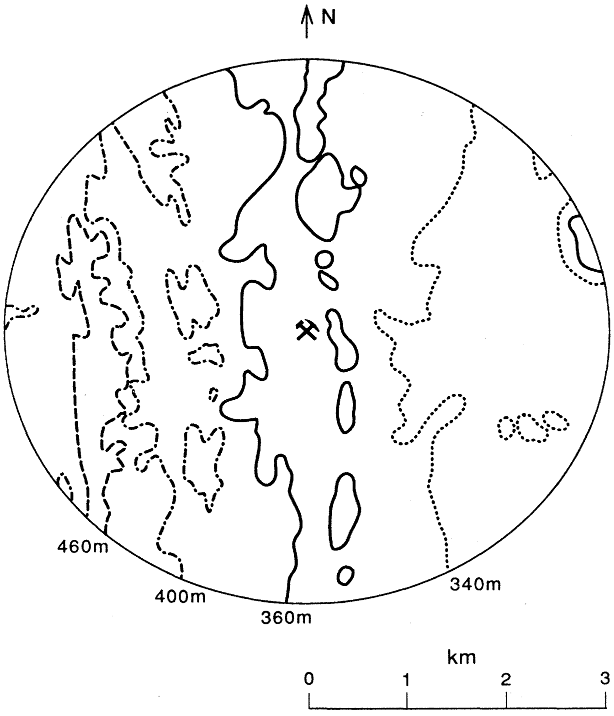

FIG. 1. Topography within 3 km of the P49 shaft of the Hilton mine.

however, the sampling interval was decreased to 7.5°. The origin for the grid was the P49 shaft which is located at a latitude of 20° 34′ 10″ S and a longitude of 139° 28′ 32″ E (Ref. 11). The head of the shaft is 355.48 m above sea level.[12]

We find that for $r < 10$ km the azimuthally averaged terrain can be fit by the single Bessel function $h(r)=370-18J_0(kr)$, where h is in meters and $k=1.20$ km^{-1}. (See Fig. 2.) The error in average height for each of the 16 measured radii was taken as the standard error of the mean as determined from breaking the measurements at each radius into four interleaved groups.[13] This error varied from 1.4 to 6.4 m. We assigned a minimum error of $\Delta h(r)=3$ m at all radii as an estimate of the accuracy of the average elevations on the maps relative to that of the head of the mine. With these error assignments the fit of the points to the curve yields a χ^2 of 14.5 for 12 degrees of freedom.

The best Bessel function fit to the topography thus gives $k=0.0012=m^{-1}$ and $H_0=-18$ m. Using $\rho=2.75$ g/cm^3 for the average rock density,[5] we find $A=2\pi G\rho h_0=2.07$ mgal (1 gal=1 cm/s^2). Using these values for the wave number k and the amplitude A we predict the variation of gravity down the mine, $g_t=Ae^{-kd}$. This prediction is compared to the Australian data in Fig. 3.

We note that the local topography does give a fair, though not complete, fit. We do not argue for an exact fit. To do so would be to claim that there is no fifth force and further that the original experimenters accounted correctly for all Newtonian forces except the topography which they missed completely. We are, however, concerned by the original treatments of topography. Initially Holding and Tuck wrote, "The topographic effects are insignificant."[1] However, our analysis here indicates that they are important. In a subsequent paper Holding, Stacey, and Tuck made surface gravity measurement and added density measurements. They state explicitly that "Surface terrain corrections were applied to all values [of gravity readings]."[2] But if terrain corrections were applied in Ref. 2, but not in Ref. 1, it is then surprising that there is such a small difference between these data in the mine residuals of Fig. 3. We have communicated these concerns to the authors and are confident that they will be answered.

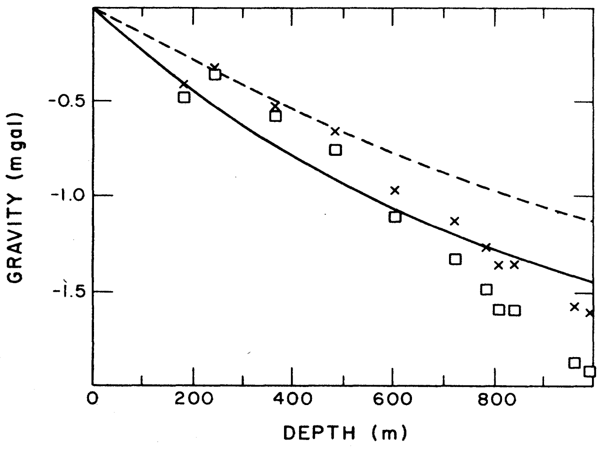

FIG. 3. Mine residuals [g(observed)$-g$(Newtonian)] vs depth d from Refs. 1 and 2 compared to prediction $g_t(d)-g_t(0)$ from single Bessel function fit to local terrain. Squares=residuals from Ref. 1; crosses=residuals from Ref. 2. Also shown is prediction from direct integration of terrain (dashed curve).

Finally we observe that the agreement of the above Bessel function to the topography breaks down for $10<r<20$ km. (See Fig. 4.) In this region the actual elevation exceeds the Bessel function by an average of 11 m. We estimate the effect of this surplus mass by considering the gradient in the gravitational field produced at the center of a ring of width Δr and thickness t:

$$dg_z/dz=2\pi G\rho\Delta rt/r^2=0.06\ \text{mgal/km}\ .$$

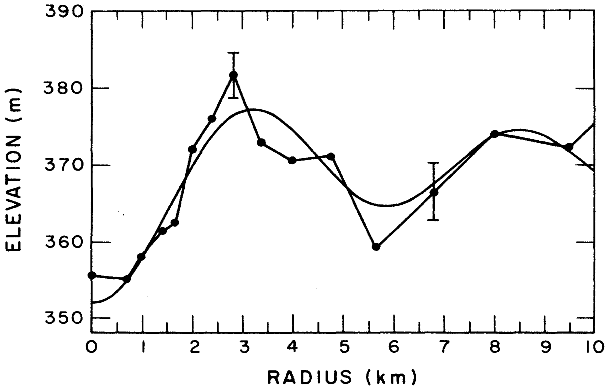

FIG. 2. Elevation (averaged over azimuth) $h(r)$ vs distance to mine shaft r. Solid points with typical errors and connected by line segments are from topographical maps. The smooth curve is the best fitting Bessel function.

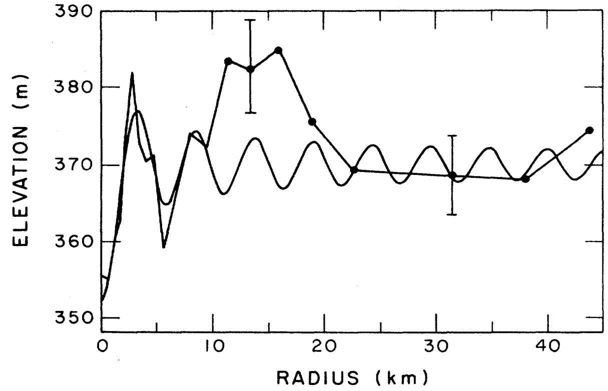

FIG. 4. Elevation $h(r)$ vs distance for $r<45$ km. The smooth curve is the same Bessel function as shown in Fig. 2.

The strong dependence on the inverse square of the distance renders this contribution negligible compared to the initial slope already seen in the curve on Fig. 3: $dg_z(0)/dz = Ak = 2.5$ mgal/km.

By contrast, the departure of the single Bessel function from the actual terrain may be significant in the nearby region ($r < 0.7$ km). At $r = 0$ the Bessel fit is 3 m below the top of the mine shaft. Because of this discrepancy a direct numerical integration of the terrain shown in Fig. 4 reduces the predicted effect of the topography on the mine residuals by about 25%. (See the dashed line in Fig. 3.)[14]

Alternatively the 3-m discrepancy could arise from the fact that the datum for the elevation of the mine shaft comes from Mt. Isa Mines Ltd., whereas all other elevations come from a different source.[10] The resolution of this discrepancy awaits a more detailed survey of the nearby topography than we can make.

We also do not analyze the effect of the topography beyond 45 km. This is because of the phenomenon of isostatic compensation.[15] At long wavelengths elements of the deep crust and mantle yield over time to compensate the changing topography.

Earlier Stacey and co-workers had investigated gravity in the Mt. Isa mine which is 18 km south of the Hilton mine.[16] We do not discuss the topography around that site except to observe that, for $r > 0.3$ km, the Mt. Isa topography appears quite similar to Hilton's. The nearby terrain at Hilton, however, is much less complicated than at Mt. Isa. This feature was a dominant reason for the move to Hilton.[17]

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[9] See the "RET method" of D. H. Eckhardt, C. Jekeli, A. R. Lazarewicz, A. J. Romaides, and R. W. Sands, Phys. Rev. Lett. **60**, 2568 (1988).

[10] Mt. Isa, Kennedy Gap, Prospector, and Mary Kathleen quadrangles (Australian Surveying and Land Information Group, Belconnen, 1971–1973). All maps have stated elevation accuracies of ±5 m; the first two (which include the mine itself) have contour intervals of 20 m; the last two have intervals of 40 m.

[11] "Geological Map of Mt. Isa District," ed 2, scale 1:50 000 (Mt. Isa Mines Ltd., Mt. Isa, 1973), confirmed by G. Tuck (private communication).

[12] G. Tuck (private communication). Note that this elevation is misprinted (as 386 m) in Ref. 1.

[13] The four interleaved groups were (0°,60°,120°,. . .), (15°,75°,. . .), (30°,. . .), and (45°,. . .).

[14] D. Eckhardt originally made this observation. He also has provided us with a program to make the numerical integration.

[15] See W. A. Heiskanen and F. A. Venig Meinesz, *The Earth and Its Gravity Field* (McGraw-Hill, New York, 1958), pp. 124–215.

[16] F. D. Stacey, G. J. Tuck, S. C. Holding, A. R. Mahler, and D. Morris, Phys. Rev. D **23**, 1683 (1981).

[17] F. Stacey (private communication).