Dissipation of quantum fields from particle creation

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We discuss the nature and origin of the dissipation of quantum fields due to the back reaction of particle creation. We derive the effective action of a scalar $g\phi^3$ theory in the closed-time-path-integral formalism. From the real and causal equation of motion for the background field we deduce a dissipative function for this process and for the cosmological anisotropy damping problem studied earlier. This model illustrates that the appearance of dissipative behavior from the back reaction of particle creation in quantum fields is a general feature. It also suggests that the role of gravity in the display of dissipative behavior in semiclassical processes is not unique.

In this paper we ask the question: Is there dissipation associated with particle creation[1] and its back-reaction effects[2] depicted in quantum field theory, specifically in the effective-action[3] formalism? A well-known example is Euler and Heisenberg's 1936 semiclassical calculation and Schwinger's 1951 field-theoretical calculation of electron pair creation in a strong electromagnetic field.[4] Our interest in problems of this nature stemmed from earlier work in the back-reaction effect of particle creation in cosmological spacetimes, especially the so-called "anisotropy dissipation" problem.[5] Despite our ability to do full-scale calculations and produce the by now well-known results, the nature of "dissipation" in its most foundational and elemental statistical-mechanical sense remains elusive. This prompted one of us to begin questioning the notion of entropy defined for quantum fields,[6,7] and the entropy generated in particle production processes.[8–10] The recent work of Hu, Kandrup, Sorkin, and others[6,10] provided much insight into the role played by correlation[9] and coherence[10] in the statistical nature of dynamical quantum fields. However, because the processes studied by these authors are modeled by time-dependent coupled harmonic-oscillator systems and particle production is mimicked by the process of parametric amplification, only entropy change from changes in the correlations due to *interactions* or from the changes in the *phase relation* of the states (coherence) of the system as it evolves are considered. The problem of "dissipation" of a field due to the back reaction of created particles requires instead a full quantum-field-theoretical description. In Ref. 11 we have studied in detail the problem of back reaction of particle creation for free conformal fields in an anisotropic (Bianchi type-I) universe[5] by way of the closed-time-path-integral formalism.[12] We were able to deduce a real, causal equation of motion for the background field and from it identify the source of dissipation. In this paper we choose to analyze one of the simplest model fields theories, the $g\phi^3$ theory, under the simplest conditions—i.e., static, flat-space background. Putting aside the effect of dynamics and curvature enables us to focus only on the field-theoretical and statistical properties of quantum processes. We want to see if there is similar dissipative behavior in this back-reaction process and what are the basic assumptions entering into the analysis which leads to such behavior.

The *criteria* we use to determine whether a system shows dissipative behavior are based on first principles in statistical mechanics.[13,14] They involve (a) the separation of a part whose behavior we are interested in, which we call the *system*, and the rest, which we call the *bath* or the environment, and their coupling (in the language of subsystems—the relevant and irrelevant parts); (b) the choice of *boundary conditions* (e.g., the in-out vacuum persistence amplitude or the in-in vacuum expectation value) which includes the stipulation of the phase relation of the initial states (such as pure, random, or thermal states); and (c) a way to average out some degrees of freedom of the bath, known as *coarse graining.* All three steps need be stipulated for one to see dissipative behavior in the system. These points can be illustrated by a simple example, that of coupled harmonic oscillators.[13] Referring to a single oscillator as the system and the rest as the bath, by introducing certain averaging measure (coarse graining) in the bath variables and choosing some specific initial condition, one can see the dissipative behavior arising from the otherwise time-reversal-invariant dynamics of a classical system. Equivalently, one can use an effective action to describe these results, following the same set of procedures as notably demonstrated recently by Caldeira and Leggett based on earlier work of Feynman and Vernon.[14] Dissipation in interacting quantum field theory was also discussed in a recent paper of ours.[15] There we presented the field-theoretical version of the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy and used these criteria to show how dissipative effects appear when this series is truncated and certain averaging conditions on the higher-order correlation functions are introduced.

Here we shall use these basic criteria to show that the back reaction of particle creation exhibits dissipative behavior even in the simplest setting. In a background-field splitting one separates the quantum field into a background field which is our system, and a fluctuation field which acts as the bath. In setting up the problem one gives the appropriate boundary conditions (in-out or in-in), e.g., by choosing the appropriate propagators which contain the causal information of the system. Then in calculating the effective action one implicitly introduces a coarse-graining procedure when the quantum fluctuations are integrated out.[16] We shall illustrate how these steps are taken in a typical field-theoretical calculation. This analysis serves to clarify the origin of dissipation in these field-theoretical processes. We find that both the anisotropy-damping problem and the ϕ^3 theory embody the same dissipative mechanism. In so doing we ascribe the dissipation in the cosmological anisotropy-damping problem to the same set of basic assumptions and measures introduced in a much broader context than that associated with gravitational fields. In fact, we see that the role of gravity in these semiclassical processes is not more than providing the dynamics of the background field and the energy for particle creation.[17]

In this calculation we shall derive the effective action in the closed-time-path-integral formalism.[12] The energy density associated with particle production is measured by the expectation value of the energy-momentum tensor with respect to the same "in" vacuum. The advantage is that the equation of motion for the background field is real and causal. When written in the frequency representation the equation of motion for the system has the form of a damped oscillator system or an *LCR* circuit. One can thus easily identify the respective dissipative and reactive components. So, as a by-product we also derive the dissipative function (or the related spectral density) for the $g\phi^3$ theory which has a structure similar to that of the cosmological anisotropy-damping process studied before.

The classical action of a scalar field with $g\phi^3$ interaction is

$$S[\Phi]=\int d^4x[\tfrac{1}{2}(\partial\Phi)^2-\tfrac{1}{2}m^2\Phi^2-\tfrac{1}{6}g\Phi^3-J\Phi]\ , \quad (1)$$

where J is an external source. In a background-field decomposition $\Phi=\phi+\varphi$ we may view the background field ϕ as our system and the fluctuation field φ as our bath variables. The closed-time-path effective action is obtained after integrating out the fluctuating field

$$\Gamma[\phi^+,\phi^-]=-i\ln\int D\varphi^+ D\varphi^- \exp\{i(S[\phi^+ +\varphi^+] - S[\phi^- +\varphi^-])\}\ , \quad (2)$$

where the $\pm$ superscripts on ϕ and φ denote the positive and negative time branches. To order g^2, only one-loop graphs contribute which contain only linear fluctuations around ϕ^+ and ϕ^-:

$$\Gamma[\phi^+,\phi^-]=S[\phi^+]-S[\phi^-]-i\ln\int D\varphi^+ D\varphi^- \left[\exp\left[\frac{i}{2}\int d^4x\{[(\partial\varphi^+)^2-m^2(\varphi^+)^2]-[(\partial\varphi^-)^2-m^2(\phi^-)^2]\}\right]\right]$$
$$\times\left[1-\frac{i}{2}\int d^4x\, g[\phi^+(\varphi^+)^2-\phi^-(\varphi^-)^2]-\tfrac{1}{8}\int d^4x\, d^4x' g^2[\phi^+(\varphi^+)^2-\phi^-(\varphi^-)^2](x)\right.$$
$$\left.\times[\phi^+(\varphi^+)^2-\phi^-(\varphi^-)^2](x')\right]\ . \quad (3)$$

Since the measure is Gaussian, we may use the Wick theorem to get

$$\Gamma[\phi^+,\phi^-]=S[\phi^+]-S[\phi^-]-\frac{g}{2}\int d^4x[\phi^+\langle(\varphi^+)^2\rangle-\phi^-\langle(\varphi^-)^2\rangle]$$
$$+(ig^2/8)\int d^4x\, d^4x'\langle[\phi^+(\varphi^+)^2-\phi^-(\varphi^-)^2](x)[\phi^+(\varphi^+)^2-\phi^-(\varphi^-)^2](x')\rangle_{\text{connected}}\ . \quad (4)$$

The Feynman rules are

$$\langle\varphi^+(x)\varphi^+(x')\rangle=i\int[d^4p/(2\pi)^n]e^{ip(x-x')}/(p^2-m^2+i\epsilon)=\langle 0|T[\varphi(x)\varphi(x')]|0\rangle\ , \quad (5a)$$

$$\langle\varphi^+(x)\varphi^-(x')\rangle=-i\int[d^np/(2\pi)^n]e^{ip(x-x')}2\pi i\delta(p^2-m^2)\theta(p^0)=\langle 0|\varphi(x')\varphi(x)|0\rangle\ . \quad (5b)$$

The part which contributes to the linearized equations for ϕ is then Γ_{quad}:

$$\Gamma_{\text{quad}}[\phi^+,\phi^-]=-\tfrac{1}{2}\int d^4x\,\phi^+(x)(\Box+m^2)\phi^+(x)$$
$$-\frac{ig^2}{4}\int d^4x\, d^4x'\phi^+(x)\left[\phi^+(x')\left[\int\frac{d^np}{(2\pi)^n}e^{ip(x-x')}\int\frac{d^nk}{(2\pi)^n}\frac{1}{(k^2-m^2+i\epsilon)[(p-k)^2-m^2+i\epsilon]}\right]\right.$$
$$-2\phi^-(x')\left[\int\frac{d^np}{(2\pi)^n}e^{ip(x-x')}\int\frac{d^nk}{(2\pi)^n}[2\pi i\delta(k^2-m^2)\theta(k^0)]\right.$$
$$\left.\left.\times[2\pi i\delta((p-k)^2-m^2)\theta((p-k)^0)]\right]\right]\ . \quad (6)$$

It is easy to see that the second k integral vanishes if $p^0<0$, and that $p^0-k^0>0$ for $p^0>0$ if the arguments of both δ functions are to vanish. For $p^0>0$, the second integral is given by twice the imaginary part of the first.[18] The first integral is evaluated by introducing a Feynman integration parameter x, rotating k^0 to Euclidean space, developing in powers of $\epsilon = n-4$, and choosing the cut along the negative real axis to ensure positivity in the argument of the logarithm. We get the one-loop *effective action* to order g^2:

$$\Gamma_2^{(1)} = -\tfrac{1}{2}\int d^4x\, \phi^+(x)(\Box + m^2)\phi^+(x) - \frac{g^2}{4(4\pi)^2}\int d^4x\, d^4x' \phi^+(x)\int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')}\int_0^1 dx \left[\phi^+(x')\left[\ln\frac{|m^2-p^2x(1-x)|}{4\pi\mu^2} - i\pi\theta(p^2x(1-x)-m^2)\right] - 2\phi^-(x')[-2i\pi\theta(p^2x(1-x)-m^2)]\theta(p^0)\right] . \quad (7)$$

The *equation of motion* is found by taking the variation with respect to $\phi^+(x)$ and identifying ϕ^+ and ϕ^- (see Ref. 11). In terms of the Fourier function $\phi(p) = \int d^4x\, e^{-ipx}\phi(x)$ we get

$$\left[p^2 - m^2 - \frac{g^2}{(4\pi)^2}\int_0^1 dx \left[\tfrac{1}{2}\ln\frac{|m^2-p^2x(1-x)|}{4\pi\mu^2} + \tfrac{1}{2}i\pi\theta(p^2x(1-x)-m^2)\mathrm{sgn}(p^0)\right]\right]\phi(p) = -J(p) . \quad (8)$$

With the above choice of the branch for the argument of a complex number, we can write ($\omega = p^0$)

$$\left[\omega^2 - |\mathbf{k}|^2 - m^2 - \frac{g^2}{(4\pi)^2}\int_0^1 dx \ln\left[\frac{\{m^2-[(\omega-i\epsilon)^2-|\mathbf{k}|^2]x(1-x)\}^{1/2}}{\sqrt{4\pi}\mu}\right]\right]\phi(p) = -J(p) . \quad (9)$$

We see that the equation for $\phi(p)$ has an imaginary term given by

$$\frac{g^2}{(4\pi)^2}\left[\frac{i\pi}{2}\right]\mathrm{sgn}(\omega)\left[1-\frac{4m^2}{p^2}\right]^{1/2}\theta(p^2-4m^2) . \quad (10)$$

This corresponds to a viscous force term $F_v = \gamma\dot{\phi}$ in the equation of motion for a damped harmonic oscillator, or the resistance term in a *LCR* circuit which signifies the appearance of dissipation. In fact, if we write the imaginary term as $\gamma(p)\omega\phi(p)$, the dissipative function γ is given by

$$\gamma(p) = [\pi g^2/(4\pi^2)][\beta(p)/|\omega|]\theta(p^2-4m^2) , \quad (11)$$

where $\beta(p) = [1-(4m^2/p^2)]^{1/2}$.

Dissipation arises because by looking at the expectation value of the field we are only considering one-particle states. (The expectation value of the field is related through the reduction formula to the amplitude for the source to emit *one* particle.) But the source can emit also pairs, triplets, etc., whose energy are not accounted for if only the background-field evolution is observed. This is related to the explanation of dissipation arising from truncating the higher-order correlation functions given in Ref. 15. We can show that the energy dissipated in the background field as represented by this complex term is exactly equal to the energy of the coherent pairs emitted by the source.

The total energy dissipated over the whole history is $\mathcal{E} = -\int_{-\infty}^{\infty} dt\, F_v\dot{\phi}$, where F_v is the dissipative force. In Fourier representation $\mathcal{E} = \int d\omega\, \omega\phi^*(\omega)\mathrm{Im}\Delta^{-1}(\omega)\phi(\omega)$, where Δ is the propagator in Eq. (9). Using the optical theorem one can show that

$$\mathcal{E} = \tfrac{1}{2}\int d\omega(2\omega)\sum_a |A(1\to a)\Delta(\omega)\phi(\omega)|^2 , \quad (12)$$

where $A(1\to a)$ is the transition amplitude from a one-particle state to any pair state a. The integrand gives the total probability for the creation of a particle pair with energy 2ω, which is the only mode up to order g^2.

Another way to see the dissipative effect of particle creation is to examine the impulsive response of the system to see if the background field will damp away in time. Thus, we introduce a δ-function source into the equation of motion (9) and consider the behavior of any spatial Fourier component of the field $\phi_k(t)$ (system being spatially homogeneous):

$$\left[\omega^2 - \omega_k^2 - \frac{g^2}{(4\pi)^2}\int_0^1 dx \ln\left[1 - \frac{[(\omega-i\epsilon)^2-|\mathbf{k}|^2]x(1-x)}{m^2}\right]^{1/2}\right]\phi_k(\omega) = -1 , \quad (13)$$

where $\omega_k^2 = |\mathbf{k}|^2 + m^2$. The short-time behavior of $\phi_k(t)$ is determined by the large-ω behavior of $\phi_k(\omega)$. For large ω ($\omega^2 - |\mathbf{k}|^2 \gg 4m^2$), the inverse propagator (9) acquires an imaginary part $i\gamma \sim [g^2/2(4\pi)^2]\mathrm{sgn}(\omega)$. We may approximate Eq. (9) as[18]

$$\{[\omega_k - (i\gamma/2\omega_k)]^2 - \omega_k^2\}\phi_k(\omega) = -1 , \quad (14)$$

which leads immediately to

$$\phi_k(t) \sim (\sin\omega_k t/\omega_k)e^{-\gamma t/2\omega_k} , \quad (15)$$

valid for $t \ll (4\omega_k/g^2)$. We see that $\phi_k(t)$ is indeed damped, at least for short times. At late times, the transient contribution dies out entirely, and $\phi_k(t)$ becomes a simple oscillation with frequency ω_k (we are neglecting the difference between the minimally substracted mass m^2 and the physical mass). In this regime

$$\phi_k(t) \sim B^{-1}(\sin\omega_k t/\omega_k) , \tag{16}$$

where $B = 1 + C(g^2/m^2)$ and C is a positive constant. B^{-1} is the residue of the propagator at the pole $p^2 \sim m^2$, being related to the dissipative function through the dispersion relation for the propagator. $B > 1$ implies that the system is damped, and vice versa.

Using the same line of reasoning, one can also deduce the dissipative function for the cosmological anisotropy damping problem. Earlier, we derived an in-in effective action and a real, causal equation of motion for the rate of change $\kappa_{ij} \equiv \beta'_{ij}$ (with respect to conformal times) of anisotropy β_{ij} in the metric [Eq. (3.24) of Ref. 11]. In the frequency domain, if we write the imaginary term $F_v(\omega)_{ij} = i\gamma(\omega)\kappa_{ij}(\omega)$, we can identify the dissipative function $\gamma(\omega)$ to be

$$\gamma(\omega) = [\pi/60(4\pi)^2]|\omega|^3 . \tag{17}$$

Since the conformal scalar field is massless there is no threshold and thus this term is nonzero for all frequencies. We can show that the total anisotropy energy dissipated using the expression for damped harmonic oscillators

$$E = \int_0^\infty dt\, F_{vij}(t)\kappa'_{ij} = \int_{-\infty}^{\infty} (d\omega/2\pi)(\omega\kappa^*_{ij})(\gamma\omega\kappa^{ij}) \tag{18}$$

is indeed equal to the total energy of the particle pairs created [Eq. (3.29) of Ref. 11]. This provides a clear physical meaning of dissipation of anisotropy by particle creation.

In this paper, through a simple model problem, we have demonstrated the following points.

(1) The existence of dissipation to the background field due to particle creation. The back reaction of particles created tends to diminish the source where they are created; this form of Lenz-law[1,2,6,19] behavior noticed in cosmological back-reaction problems thus acquires a more general field-theoretical explanation.

(2) A theoretical explanation of the nature of dissipation in these processes in terms of basic statistical-mechanical premises. Their physical meaning is clarified by relating the amount of particle creation to the energy dissipation. As a concrete result the dissipative functions of both the $g\phi^3$ model field theory and the anisotropy-damping problem are derived.

(3) The dissipative nature of back-reaction problems in field theory is not special to gravitational fields. This example helps to correctly identify the role of gravity in the class of semiclassical theories.[16]

Further discussion on the theoretical implication of these and related issues can be found in Ref. 20. Application of this method to a derivation of the viscosity function associated with particle creation in the reheating epoch of the new inflationary cosmology is being carried out by Stylianopoulos.[21]

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