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Gauge-invariant, nonperturbative approach to quark confinement and chiral-symmetry breaking in QCD

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A gauge-invariant, nonperturbative approach to quark confinement and chiral-symmetry breaking in the context of the Schwinger-Dyson equations and corresponding Slavnov-Taylor identities is presented. Making only one widely accepted assumption that the full gluon propagator becomes an infrared singularity like q^{-4} at small momenta, we obtain a nonperturbative, gauge-invariant, infrared-finite quark propagator, which has no pole (confinement-type solution) and implies chiral-symmetry breaking (dynamical quark mass generation), so that we establish a close connection between these nonperturbative phenomena. We discover that the ghost degrees of freedom play an essential role in the dynamics of chiral-symmetry breaking, but nevertheless the quark propagator we got was free of ghost complications. In addition to the infrared-finite solution, we find also two infrared-vanishing (after the removal of the infrared regulation parameter) solutions for the quark propagator. For the dynamical (nonperturbative) quark mass we derive the expression which exhibits an essential singularity in the coupling constant in accordance with renormalization-group solutions in the infrared region.

I. INTRODUCTION

It is well known that the infrared region is responsible for nonperturbative effects in quantum chromodynamics (QCD). The most important nonperturbative problems in QCD are quark confinement[1] and chiral-symmetry breaking (CSB).[2-4] Apparently, there is a close connection between these nonperturbative phenomena, Refs. 5–7 (see also, Ref. 14). One of the effective and adequate methods to studying these hypotheses is to investigate infrared singularities, the analytic structure, and asymptotic properties of the Schwinger-Dyson (SD) equations for the quark propagator with the help of the corresponding Slavnov-Taylor (ST) identities.[8-21] A popular approach to the dynamics of chiral-symmetry breaking is to write down a gap equation in some approximation (see, for example, Refs. 3, 4, and 7), but the SD equation for the quark proper self-energy $\Sigma(p)$ is the gap equation rewritten in a different notation.

In these investigations which have been performed in various gauges (covariant and noncovariant), dimensions, and other different approximations, confinement was implemented by assuming that the full gluon propagator becomes an infrared singularity such as $(q^2)^{-2}$ at small momenta,[8-23] providing a linearly rising quark-antiquark potential at large distances in the nonrelativistic limit. The cluster property of the Wightman functions in QCD fail[24] and this makes it possible to admit such singular behavior for the full gluon propagator in the infrared region. For this case one needs to introduce a small infrared regulation parameter ϵ in order to define exactly initial SD equations in the infrared region [postponing to Sec. II the precise definition of the distribution $(q^2)^{-2}$ in n dimensions]. Because of this, the quark propagator and other Green's functions become dependent in general on the infrared regulation parameter ϵ, which is to be set to zero at the end of computing ($\epsilon \to 0$). Evidently, there are only two different types in the behavior of the quark propagator with respect to ϵ in the $\epsilon \to 0^+$ limit. If the quark propagator does not depend on the ϵ parameter in the $\epsilon \to 0^+$ limit then one obtains the infrared-finite quark propagator. In this case quark confinement is understood as the disappearance of the quark propagator pole at the point $p^2 = m^2$ where m is a quark mass. A quark propagator may or may not be an entire function, but in any case the pole of the first order disappears. On the other hand, a quark propagator can vanish after the removal ($\epsilon \to 0^+$) of the infrared regulation parameter ϵ. The vanishing quark propagator is also a direct manifestation of the quark confinement.

Using the usual decomposition of the quark propagator

$$-iS(p) = \hat{p}A(-p^2) + B(-p^2) , \tag{1}$$

CSB can be implemented as satisfying the condition

$$\{S(p), \gamma_5\}_+ = 2i\gamma_5 B(-p^2) \neq 0 \tag{2}$$

so that the γ_5 invariance of the quark propagator (1) is broken. This condition leads to the zero-mass boson (Goldstone state) in the flavor axial-vector Ward-Takahasi-Fradkin identity. On the other hand, the quark must have a nonzero mass (nonperturbative) even if the bare mass of a quark is equal to zero (dynamical quark mass generation). For the quark propagator (1) which has no pole (for example, entire function) nonperturbative (effective) mass M is defined as[5]

$$M = -B^{-1}(-p^2 = 0) = [iS(0)]^{-1} \tag{3}$$

so that one needs to find regular solutions for the finite-quark propagator (1) in the infrared limit.

Confirmation of the quark confinement and CSB hypotheses on the basis of the infrared structure investigations of the SD equations and corresponding ST identities in an arbitrary covariant gauge is an extremely difficult problem because of the unknown ghost contributions in this gauge. Our primary aim in this paper is to propose and develop a general, gauge-invariant approach to the extraction of the infrared-finite Green's functions in QCD. Infrared finiteness of the Green's functions means that they do not depend on the infrared regular parameter ϵ in the $\epsilon \to 0^+$ limit. We will show that for the covariant gauges the complications due to ghost contributions can be considerable in our approach. Moreover, it is the consideration of ghost degrees of freedom that makes it possible to obtain the infrared-finite, gauge-invariant quark propagator which has no pole (confinement-type solution) and implies dynamical chiral-symmetry breaking (dynamical quark mass generation). For the dynamical (nonperturbative) quark mass we obtain the expression which exhibits an essential singularity in the coupling constant in accordance with renormalization-group solutions in the infrared region. In addition to the infrared-finite solution we discover two infrared-vanishing solutions for the quark propagator in the infrared region. Thus, we find three and only three confinement-type solutions for the quark propagator and clearly establish a close connection between quark confinement and CSB.

To recapitulate, the plan of this paper is as follows. In Secs. II and III we examine the covariant gauge quark and ghost propagators in the infrared region. The ST identity in the infrared region is investigated in Sec. IV. Conditions of cancellation of the infrared divergences in the quark, ghost SD equations, and ST identity are obtained in Secs. II, III, and IV, respectively. In Sec. IV finally we obtain three different systems (containing the quark SD equation and corresponding ST identity in each case), describing the quark propagator in the infrared region. In Sec. V we solve explicitly the SD equation with a corresponding ST identity for the infrared-finite quark propagator and in Sec. VI we apply an effective potential in order to determine completely this solution. We also derive the nonperturbative (effective) quark mass. Finally we summarize our results in Sec. VII.

II. THE SD EQUATION FOR THE QUARK PROPAGATOR

Let us consider the exact, unrenormalized SD equation for the quark propagator in momentum space (see Fig. 1):

$$S^{-1}(p)=S_0^{-1}(p) +g^2C_F\int \frac{d^nq}{(2\pi)^n}\Gamma_\mu(p,q)S(p-q)\gamma_\nu D_{\mu\nu}(q)\ , \quad (4)$$

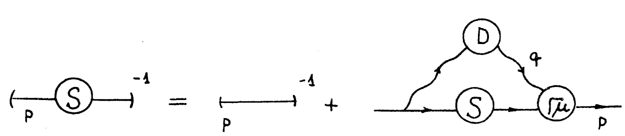

FIG. 1. The quark SD equation.

where C_F is the eigenvalue of the quadratic Casimir operator in the fundamental representation [for SU(N) in general, $C_F=(N^2-1)/2N=\frac{4}{3}$, $N=3$] and

$$S_0^{-1}(p)=-i(\hat{p}-m_0) \quad (5)$$

with m_0 a bare quark mass. $\Gamma_\mu(p,q)$ is a corresponding quark-gluon proper vertex function.

The full gluon propagator in an arbitrary covariant gauge is

$$D_{\mu\nu}(q)=-i\left[\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right]\frac{1}{q^2}d(-q^2,a)+a\frac{q_\mu q_\nu}{q^4}\right]\ , \quad (6)$$

where a is a gauge-fixing parameter ($a=0$, Landau gauge).

Assuming that, in the infrared region,

$$d(-q^2,a)=\left[\frac{\mu^2}{q^2}\right]+\beta(a)+O(q^2),\quad q^2\to 0\ , \quad (7)$$

where μ is the mass parameter characterizing the scale of confinement, we obtain the generally accepted form of the infrared singular asymptotics for the full gluon propagator[8–23,25]

$$D_{\mu\nu}(q)\sim(q^2)^{-2},\quad q^2\to 0\ . \quad (8)$$

In order to define exactly an initial SD equation (4) in the infrared region (at small momenta) let us apply the gauge-invariant dimensional regularization method in the limit $n=4+2\epsilon$, $\epsilon\to 0^+$. Here and below ϵ is a small infrared regulation parameter which is to be set to zero at the end of computing. In what follows we consider the SD equations and corresponding quark-gluon ST identity (see below) in Euclidean space ($d^nq\to id^nq_E$, $q^2\to -q_E^2$, $p^2\to -p_E^2$, but for simplicity Euclidean subscript E will be omitted).

Let us use in the sense of distribution theory the relation[26]

$$(q^2)^{-2+\epsilon}=\frac{\pi^2}{\epsilon}\delta^4(q)+(q^2)_+^{-2}+O(\epsilon),\quad \epsilon\to 0^+ \quad (9)$$

which implies that the full gluon propagator (6) in the infrared region behaves as

$$D_{\mu\nu}(q)=\epsilon^{-1}\bar{D}_{\mu\nu}(q),\quad \epsilon\to 0^+ \quad (10)$$

in the $\epsilon\to 0^+$ limit and $\bar{D}_{\mu\nu}(q)$ exists as $\epsilon\to 0^+$. Here and below $(q^2)_+^{-n}$ ($n=1,2$) are the functionals acting on the main (test) functions according to the standard formulas.[26] These formulas play no role in future analysis and therefore we will not write them down explicitly.

Substituting (7)–(9) into the SD equation (4) we obtain the quark propagator expansion in the infrared region (in Euclidean space):

$$S^{-1}(p)=S_0^{-1}(p)+\frac{1}{\epsilon}\tilde{g}^2\Gamma_\mu(p,0)S(p)\gamma_\mu+C_F g^2 i\int\frac{d^nq}{(2\pi)^n}\Gamma_\mu(p,q)S(p-q)\gamma_\nu[\mu^2(q^2)_+^{-1}+\beta(a)]D_{\mu\nu}^{(0)}(q)$$
$$+aC_F g^2\int\frac{d^nq}{(2\pi)^n}\Gamma_\mu(p,q)S(p-q)\gamma_\nu\frac{q_\mu q_\nu}{q^4}+O(\epsilon),\quad \epsilon\to 0^+\ , \tag{11}$$

where $\tilde{g}^2=C_F\frac{3}{4}g^2\mu^2\pi^2(2\pi)^{-4}$ and $D_{\mu\nu}^{(0)}(q)$ is a free gluon propagator ($a=0$).

In order to extract the finite Green's functions in the infrared region [and in accordance with (10)] it is sufficient and necessary to represent the quark-gluon vertex function $\Gamma_\mu(p,q)$ (including zero momentum transfer) and the quark propagator $S(p)$ as

$$\Gamma_\mu(p,q)=\epsilon^\delta\bar{\Gamma}_\mu(p,q),\quad \epsilon\to 0^+ \tag{12}$$

and

$$S(p)=\epsilon^\kappa\bar{S}(p),\quad \kappa\ge 0,\quad \epsilon\to 0^+\ , \tag{13}$$

respectively. Here $\bar{\Gamma}_\mu(p,q)$ and $\bar{S}(p)$ are the finite Green's functions and therefore do not depend on the ϵ parameter in the $\epsilon\to 0^+$ limit. In this paper we regard the infrared-finite Green's functions $\bar{\Gamma}_\mu(p,q)$, $\bar{G}_\mu(k,q)$, $\bar{G}(k)$, and $\bar{b}(k^2)$ (see below) as regular functions of the arguments k and q, respectively. Singular dependence leads to more complicated SD equations and requires special treatment.

It is easy to see that $\kappa>0$ corresponds to the vanishing quark propagator after the removal ($\epsilon\to 0^+$) of the infrared regulation parameter and $\kappa=0$ corresponds to the infrared-finite quark propagator. Let us point out that here and below δ and κ are real numbers.

From (11) and taking into account (12) and (13) one has, as $\epsilon\to 0^+$ for the case $\kappa=0$ ($S\equiv\bar{S}$),

$$S^{-1}(p)=S_0^{-1}(p)+\tilde{g}^2\bar{\Gamma}_\mu(p,0)S(p)\gamma_\mu \tag{14}$$

and, for the case $\kappa>0$,

$$\bar{S}^{-1}(p)=\tilde{g}^2\bar{\Gamma}_\mu(p,0)\bar{S}(p)\gamma_\mu\ , \tag{15}$$

respectively, if and only if (iff) a cancellation of the infrared divergences takes place:

$$-1+\delta+2\kappa=0\ . \tag{16}$$

This is a quark convergence condition. Because of (16) gauge-dependent terms in the SD equation (11) become ϵ-order terms. For this reason these noninvariant terms vanish in the $\epsilon\to 0^+$ limit and we obtain (14) and (15) as the infrared content of the quark SD equations for the cases $\kappa=0$ and $\kappa>0$, respectively. For the infrared-finite quark propagator ($\kappa=0$) the corresponding quark-gluon vertex function $\Gamma_\mu(p,q)$ behaves as ϵ in the $\epsilon\to 0^+$ limit ($\delta=1$). As will be shown such behavior, in particular, for zero momentum transfer ($q=0$)

$$\Gamma_\mu(p,0)=\epsilon\bar{\Gamma}_\mu(p,0),\quad \epsilon\to 0^+\ , \tag{16$'$}$$

is not trivial, because in this case the solution for the infrared-finite quark propagator $S(p)$ in (14) can never be reduced to the free quark propagator $S_0(p)$.

The information about the quark-gluon vertex function at zero momentum transfer ($q=0$) can be provided by the ST identity,[27] which contains unknown ghost contributions in the covariant gauge. For this reason let us consider in the next section the SD equation for the ghost self-energy.

III. THE SD EQUATION FOR THE GHOST SELF-ENERGY

The ghost self-energy $b(-k^2)$ also obeys a simple SD equation (see Fig. 2):

$$ik^2b(-k^2)=C_A g^2\int\frac{d^nq}{(2\pi)^n}G_\mu(k,q)G(k-q)_\nu D_{\mu\nu}(q)\ , \tag{17}$$

where C_A is the eigenvalue of the quadratic Casimir operator in the adjoint representation [for SU(N), in general, $C_A=N=3$]. The ghost propagator is

$$G(k)=\frac{i}{k^2[1+b(-k^2)]} \tag{18}$$

and

$$G_\mu(k,q)=k^\lambda G_{\lambda\mu}(k,q) \tag{19}$$

is the ghost-gluon vertex function ($G_{\lambda\mu}\equiv g_{\lambda\mu}$ in perturbation theory).

Substituting (7)–(9) into (17) we obtain the ghost self-energy expansion in the infrared region (in Euclidean space)

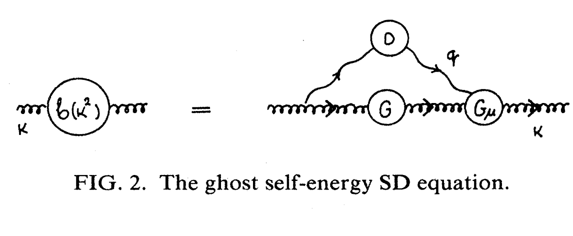

FIG. 2. The ghost self-energy SD equation.

$$-ik^2b(k^2)=\frac{1}{\epsilon}\tilde{g}_1^2G_\mu(k,0)G(k)k_\mu+C_Ag^2i\int\frac{d^nq}{(2\pi)^n}G_\mu(k,q)G(k-q)(k-q)_\nu D_{\mu\nu}^{(0)}(q)[\mu^2(q^2)_+^{-1}+\beta(a)]$$
$$+aC_Ag^2\int\frac{d^nq}{(2\pi)^n}G_\mu(k,q)G(k-q)(k-q)_\nu\frac{q_\mu q_\nu}{q^4}+O(\epsilon),\quad \epsilon\to 0^+\ , \tag{20}$$

where $\tilde{g}_1^2=(C_A/C_F)\tilde{g}^2$.

Similar to (10), (12), and (13) let us introduce the finite ghost self-energy

$$b(k^2)=\epsilon^\nu\bar{b}(k^2),\quad \epsilon\to 0^+ \tag{21}$$

and the ghost-gluon vertex function

$$G_\mu(k,q)=\epsilon^\beta\bar{G}_\mu(k,q),\quad \epsilon\to 0^+\ , \tag{22}$$

respectively, where $\bar{b}(k^2)$ and $\bar{G}_\mu(k,q)$ by definition are finite and do not depend on the ϵ parameter in the $\epsilon\to 0^+$ limit. According to (18) the finite ghost propagator is defined as

$$G(k)=\epsilon^\alpha\bar{G}(k),\quad \epsilon\to 0^+\ , \tag{23}$$

where

$$\alpha=\begin{cases}-\nu, & \nu<0\ ,\\ 0, & \nu\geq 0\ .\end{cases} \tag{24}$$

Here and below ν, β, and α are real numbers.

Substituting (21)–(23) into (20) we obtain, as $\epsilon\to 0^+$,

$$-ik^2b(k^2)=\tilde{g}_1^2\bar{G}_\mu(k,0)\bar{G}(k)k_\mu \tag{25}$$

as the infrared content of the ghost self-energy SD equation, iff a cancellation of the infrared divergences takes place in the $\epsilon\to 0^+$ limit

$$-1+\beta+\alpha-\nu=0\ . \tag{26}$$

This is the ghost self-energy convergence condition. Similar to a quark case, because of this condition gauge-dependent terms in the ghost self-energy expansion (20) are the terms of ϵ order $[O(\epsilon)]$ and therefore vanish in the $\epsilon\to 0^+$ limit. Thus, the finite ghost self-energy (25) does not depend on the gauge-fixing parameter a in the explicit form.

Defining (in Euclidean space)

$$\bar{G}_\mu(k,0)=k_\mu(-k^2)\bar{R}(k^2),\quad |\bar{R}(0)|<\infty\ , \tag{27}$$

where $\bar{R}(0)$ exists as $\epsilon\to 0^+$, for the case $\nu<0$ we can solve the SD equation (25) in the $k=0$ limit:

$$\bar{b}_\pm(0)=\pm\tilde{g}_1\sqrt{\bar{R}(0)}=\pm\tilde{b}(0) \tag{28}$$

and, for the case $\nu=0$,

$$\bar{b}_\pm(0)=-\tfrac{1}{2}\pm\sqrt{\tfrac{1}{4}+\tilde{g}_1^2\bar{R}(0)}\ , \tag{29}$$

respectively. It is easy to see that the full ghost propagator coincides with the free ghost propagator when $\nu>0$.

IV. THE ST IDENTITIES IN THE INFRARED REGION

Let us consider the ST identity for the quark-gluon vertex function $\Gamma_\mu(p,q)$:

$$-ik_\mu\Gamma_\mu(p,k)[1+b(-k^2)]$$
$$=[1-B(p,k)]S^{-1}(p+k)-S^{-1}(p)[1-B(p,k)^2]\ , \tag{30}$$

where $b(-k^2)$ is the ghost self-energy and $B(p,k)$ is the ghost-quark scattering kernel[4,9,28] (a skeleton expansion of this kernel is shown in Fig. 3). From (30) one recovers the standard QED-type Ward-Takahashi-Fradkin identity in the $b\equiv B=0$ limit.

In the Landau gauge ($a=0$) Taylor[27] has shown that

$$B(p,0)=0\ . \tag{31}$$

In the framework of our approach to the extraction of the infrared-finite Green's functions the terms, depending explicitly on a gauge-fixing parameter a (a terms) and violating Taylor's condition (31) are the terms of ϵ order $[O(\epsilon)]$. For this reason they are vanishing in the $\epsilon\to 0^+$ limit and therefore the Taylor condition (31) holds in our approach.

In order to check this explicitly it is sufficient to investigate the first term $B_1(p,k)$ of the $B(p,k)$ skeleton expansion (Fig. 3):

$$B_1(p,k)=-\tfrac{1}{2}g^2C_A\int\frac{d^nq}{(2\pi)^n}S(p+q)\Gamma_\nu(p+q,q)$$
$$\times G_\mu(k-q,-q)G(k-q)D_{\mu\nu}(q)\ , \tag{32}$$

where C_A is the quadratic Casimir operator in the adjoint representation. With the help of (7)–(9), similar to (11) and (20), we obtain (in Euclidean space)

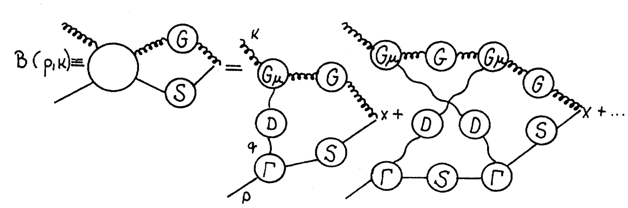

FIG. 3. Skeleton expansion for the ghost-quark scattering kernel.

$$B_1(p,k)=-\tfrac{1}{2}\tilde{g}\,_1^2\frac{1}{\epsilon}S(p)\Gamma_\mu(p,0)G_\mu(k,0)G(k)$$
$$+g^2\frac{C_F}{C_A}i\int\frac{d^nq}{(2\pi)^n}S(p+q)\Gamma_\nu(p+q,q)G_\mu(k-q,-q)G(k)[\mu^2(q^2)_+^{-1}+\beta(a)]D_{\mu\nu}^{(0)}(q)$$
$$+ag^2\frac{C_F}{C_A}\int\frac{d^nq}{(2\pi)^n}S(p+q)\Gamma_\nu(p+q,q)G_\mu(k-q,-q)G(k-q)\frac{q_\mu q_\nu}{q^4}+O(\epsilon),\quad \epsilon\to 0^+ . \tag{33}$$

Proceeding to the finite functions, which were defined by the relations (12), (13), and (21)–(23) we obtain ($B_1\equiv\bar{B}_1$)

$$B_1(p,k)=-\tfrac{1}{2}\tilde{g}\,_1^2\bar{S}(p)\bar{\Gamma}_\mu(p,0)\bar{G}_\mu(k,0)\bar{G}(k) , \tag{34}$$

iff a cancellation of the infrared divergences takes place in the $\epsilon\to 0^+$ limit:

$$\gamma\equiv -1+\kappa+\delta+\beta+\alpha=0 . \tag{35}$$

Similar to the quark (16) and the ghost (26) conditions of cancellation of the infrared divergences, and because of this relation the gauge-dependent terms (a terms) in the expansion of the first term B_1 becomes the terms of ϵ order $[O(\epsilon)]$. For this reason these terms vanish in the $\epsilon\to 0^+$ limit.

Using (25) from (34) we obtain

$$B_1(p,k)=-\tfrac{1}{2}i\bar{S}(p)\bar{\Gamma}_\mu(p,0)k_\mu\bar{b}(k^2) . \tag{36}$$

Taking into account that $\bar{b}(k^2)$ is finite at zero momentum $k^2=0$, Eq. (28), we conclude that the diagram for the B_1 is of $O(k)$. In the same way it is possible to show that the second term B_2 of the skeleton expansion for the ghost-quark scattering kernel $B(p,k)$ is of $O(k^2)$ in the arbitrary covariant gauge and the condition of a cancellation of the infrared divergences coincides with (35) within our approach. These arguments are valid term by term in the skeleton expansion for the ghost-quark scattering kernel. Thus, we have the estimation

$$B_n(p,k)=O(k^n),\quad k\to 0^+ \tag{37}$$

and the condition of cancellation of the infrared divergences again coincides with (35), so that Taylor's result (31) takes place in our approach and (35) is the ST identity convergence condition.

Proceeding to the finite Green's functions in the ST identity (30) and taking into account the convergence conditions (16), (26), and (35) we obtain ($B\equiv\bar{B}$)

$$-ik_\mu\bar{\Gamma}_\mu(p,k)[\epsilon^{1-\kappa}+\bar{b}(-k^2)]$$
$$=[1-B(p,k)]\bar{S}^{-1}(p+k)-\bar{S}^{-1}(p)[1-B(p,k)] . \tag{38}$$

Differentiating (38) with respect to k_μ, passing to the limit $k=0$ and taking into account (36) and (37), finally one obtains the identity

$$\bar{\Gamma}_\mu(p,0)[\epsilon^{1-\kappa}+\tfrac{1}{2}\bar{b}(0)]$$
$$=i\partial_\mu\bar{S}^{-1}(p)-\tfrac{1}{2}\bar{b}(0)\bar{S}(p)\bar{\Gamma}_\mu(p,0)\bar{S}^{-1}(p) , \tag{39}$$

where

$$\partial_\mu\bar{S}^{-1}(p)=\frac{\partial\bar{S}^{-1}(p)}{\partial p_\mu} . \tag{40}$$

Thus, this identity depends only on a quark propagator power number κ, which by definition is $\kappa\geq 0$ as in Eq. (13). From (39) and (13) we obtain

$$0\leq\kappa\leq 1 \tag{41}$$

so that there are three and only three different types in the quark propagator (13) behavior in the $\epsilon\to 0^+$ limit.

Let us write down these different possibilities for the quark propagator SD equation with corresponding ST identity,

$$\text{(I)}\quad \kappa=1,\quad S(p)=\epsilon\bar{S}(p),\quad \Gamma_\mu(p,0)=\epsilon^{-1}\bar{\Gamma}_\mu(p,0),\quad \bar{S}^{-1}(p)=\tilde{g}^2\bar{\Gamma}_\mu(p,0)\bar{S}(p)\gamma_\mu ,$$
$$[1+\tfrac{1}{2}\bar{b}(0)]\bar{\Gamma}_\mu(p,0)=i\partial_\mu\bar{S}^{-1}(p)-\tfrac{1}{2}\bar{b}(0)\bar{S}(p)\bar{\Gamma}_\mu(p,0)\bar{S}^{-1}(p) . \tag{42}$$

This system has been investigated by Pagels in his pioneer work on nonperturbative QCD,[9] who found some particular solutions to this system.

$$\text{(II)}\quad 0<\kappa<1,\quad S(p)=\epsilon^\kappa\bar{S}(p),\quad \Gamma_\mu(p,0)=\epsilon^{1-2\kappa}\bar{\Gamma}_\mu(p,0),\quad \bar{S}^{-1}(p)=\tilde{g}^2\bar{\Gamma}_\mu(p,0)\bar{S}(p)\gamma_\mu ,$$
$$\tfrac{1}{2}\bar{b}(0)\bar{\Gamma}_\mu(p,0)=i\partial_\mu\bar{S}^{-1}(p)-\tfrac{1}{2}\bar{b}(0)\bar{S}(p)\bar{\Gamma}_\mu(p,0)\bar{S}^{-1}(p) . \tag{43}$$

These two possibilities lead to the vanishing quark propagator ($0<\kappa\leq 1$).

$$\text{(III)}\quad \kappa=0,\quad S(p)\equiv\bar{S}(p),\quad \Gamma_\mu(p,0)=\epsilon\bar{\Gamma}_\mu(p,0),\quad S^{-1}(p)=S_0^{-1}(p)+\tilde{g}^2\bar{\Gamma}_\mu(p,0)S(p)\gamma_\mu ,$$
$$\tfrac{1}{2}\bar{b}(0)\bar{\Gamma}_\mu(p,0)=i\partial_\mu S^{-1}(p)-\tfrac{1}{2}\bar{b}(0)S(p)\bar{\Gamma}_\mu(p,0)S^{-1}(p) . \tag{44}$$

This system leads to the infrared-finite quark propagator.[13,14]

For the ghost degrees of freedom these possibilities imply

$$
\begin{aligned}
&\text{(I)}\quad \kappa=1,\quad G_\mu(k,0)=\epsilon\bar{G}_\mu(k,0),\\
&\qquad G(k)\equiv\bar{G}(k)\ ,\\
&\qquad b(k^2)\equiv\bar{b}(k^2)\ ,\\
&\text{(II)}\quad 0<\kappa<1,\quad G_\mu(k,0)=\epsilon^{-1+2\kappa}\bar{G}_\mu(k,0)\ ,\\
&\qquad G(k)=\epsilon^{1-\kappa}\bar{G}(k)\ ,\\
&\qquad b(k^2)=\epsilon^{-1+\kappa}\bar{b}(k^2)\ ,\\
&\text{(III)}\quad \kappa=0,\quad G_\mu(k,0)=\epsilon^{-1}\bar{G}_\mu(k,0)\ ,\\
&\qquad G(k)=\epsilon\bar{G}(k)\ ,\\
&\qquad b(k^2)=\epsilon^{-1}\bar{b}(k^2)\ .
\end{aligned}
\tag{45}
$$

We obtain Pagels's solution I as a particular case of our gauge-invariant approach to quark confinement in QCD. His system has a regular limit $\bar{b}(k^2)=0$, while systems II and III do not have this limit.

It is necessary to point out here that, only in our approach, the ghost-quark scattering kernel $B(p,k)$ does not depend on the ϵ parameter (each term of its skeleton expansion does not depend on this parameter) and is compatible with the corresponding SD equations for the quark propagator and the ghost self-energy. There is a consistency between the SD equations, the ST identity, the relations (10), (12), (13), and auxiliary *Ansätze* (21)–(24) in the infrared region. In any other case this consistency would be destroyed. The most interesting solution for the quark propagator is, of course, the solution for the infrared-finite quark propagator. The possibility of the existence of the infrared-finite quark propagator has been mentioned in Ref. 13 and briefly discussed in Ref. 14, respectively.

Let us note that the final results are expressed only in terms of the quark propagator power number κ. It means that the ghost degrees of freedom play an auxiliary role and therefore we avoid the difficulties due to the Ward identity for the ghost-gluon vertex function $G_\mu(k,q)$ in the infrared region.[9,29] At the same time, it is the consideration of the ghost contributions that makes it possible to obtain the infrared-finite, gauge-invariant quark propagator. We see that for covariant gauges the complications due to ghosts can be considerable in our approach.

Although our consideration was carried out in the arbitrary covariant gauge, effectively only the transverse part of the full gluon propagator $D_{\mu\nu}(q)$ Eq. (7) makes sense in our approach. The longitudinal part (which depends on a gauge-fixing parameter a) becomes of ϵ-parameter order within our approach and therefore vanishes in the $\epsilon\to 0^+$ limit. It is possible to say that the Landau gauge ($a=0$) is preferable at least in the infrared region.

In conclusion to this section let us investigate the behavior in the infrared region of the triple gauge field vertex $T_{\lambda\mu\nu}(k,q,\Gamma)$ with the help of a corresponding ST identity. The identity is[4,9,28]

$$
\begin{aligned}
[1+b(-k^2)]k^\lambda T_{\lambda\mu\nu}(k,q,\Gamma)
&=d^{-1}(-q^2)G_{\lambda\nu}(q,k)(g^{\lambda\mu}q^2-q^\lambda q^\mu)\\
&\quad+d^{-1}(-\Gamma^2)G_{\lambda\mu}(\Gamma,k)(g^{\lambda\nu}\Gamma^2-\Gamma^\lambda\Gamma^\nu)\ ,
\end{aligned}
\tag{46}
$$

where $k+q+\Gamma=0$ and $d^{-1}(-q^\lambda)$ are defined in (8). $G_{\lambda\nu}(q,k)$ is the ghost gluon vertex function (19). Let us define the infrared-finite triple vertex $\bar{T}_{\lambda\mu\nu}$ by the relation

$$T_{\lambda\mu\nu}(k,q,\Gamma)=\epsilon^\rho\bar{T}_{\lambda\mu\nu}(k,q,\Gamma),\quad \epsilon\to 0^+\ , \tag{47}$$

where ρ is a real number. Using now auxiliary relations (45), one obtains the following [$d^{-1}(-q^2)$ does not depend on the ϵ parameter at all].

(I) $\kappa=1$, Pagels's solution:

$$T_{\lambda\mu\nu}(k,q,\Gamma)=\epsilon\bar{T}_{\lambda\mu\nu}(k,q,\Gamma),\quad \epsilon\to 0^+ \tag{48}$$

and the ST identity for $\bar{T}_{\lambda\mu\nu}$ is

$$[1+b(-k^2)]k^\lambda\bar{T}_{\lambda\mu\nu}(k,q,\Gamma)=d^{-1}(-q^2)\bar{G}_{\lambda\nu}(q,k)(g^{\lambda\mu}q^2-q^\lambda q^\mu)+d^{-2}(-\Gamma^2)\bar{G}_{\lambda\mu}(\Gamma,k)(g^{\lambda\nu}\Gamma^2-\Gamma^\lambda\Gamma^\nu)\ . \tag{49}$$

(II) $0<\kappa<1$:

$$
\begin{aligned}
&T_{\lambda\mu\nu}(k,q,\Gamma)=\epsilon^\kappa\bar{T}_{\lambda\mu\nu}(k,q,\Gamma),\quad \epsilon\to 0^+\ ,\\
&\bar{b}(-k^2)k^\lambda\bar{T}_{\lambda\mu\nu}(k,q,\Gamma)=d^{-1}(-q^2)\bar{G}_{\lambda\nu}(q,k)(g^{\lambda\mu}q^2-q^\lambda q^\mu)+\alpha^{-1}(-\Gamma^2)\bar{G}_{\lambda\mu}(\Gamma,k)(g^{\lambda\nu}\Gamma^2-\Gamma^\lambda\Gamma^\nu)\ .
\end{aligned}
\tag{50}
$$

(III) $\kappa=0$:

$$T_{\lambda\mu\nu}(k,q,\Gamma)\equiv\bar{T}_{\lambda\mu\nu}(k,q,\Gamma),\quad \epsilon\to 0^+\ , \tag{51}$$

and the corresponding ST identity coincides with (50).

Thus we obtain that the behavior of the triple gauge field vertex function $T_{\lambda\mu\nu}(k,q,\Gamma)$ in the infrared region is determined again by the quark propagator power number κ ($\rho=1,\kappa,0$); i.e., this vertex behaves like the quark propagator $S(p)$ in the $\epsilon\to 0^+$ limit.

The quark propagator of the vanishing type (systems I and II) is not so interesting, because after the removal

($\epsilon\rightarrow 0^+$) of the infrared regulation parameter in (3) the γ_5 invariance of the quark propagator will be restored; i.e., chiral symmetry is preserved for these solutions. These solutions also can be used in the Bethe-Salpeter equations. Let us consider in Sec. V the most interesting case of the infrared-finite quark propagator (III).

V. SOLUTION OF THE SD EQUATION FOR THE INFRARED-FINITE QUARK PROPAGATOR

In order to solve the system III Eq. (44) it is convenient to represent the finite quark-gluon vertex function at zero momentum transfer as

$$\overline{\Gamma}_\mu(p,0)=F_1(p^2)\gamma_\mu+F_2(p^2)p_\mu+F_3(p^2)p_\mu\hat{p}+F_4(p^2)\hat{p}\gamma_\mu \ . \tag{52}$$

Obviously, (52) is a four-vector decomposition on independent matrix components. Substituting this representation to the ST identity (44) one can express the scalar functions $F_i(p^2)$ $(i=1,2,3,4)$ in terms of the quark propagator (2), scalar functions $A(p^2)$ and $B(p^2)$, respectively. Let us point out that the ST identity (44) can be solved explicitly without the help of (52) (Ref. 13):

$$F_1(p^2)=-\frac{1}{\bar{b}}\frac{A(p^2)}{E(p^2)},\quad F_4(p^2)=\frac{1}{\bar{b}}\frac{A^2(p^2)B^{-1}(p^2)}{E(p^2)} \ ,$$

$$F_2(p^2)=-\frac{2}{\bar{b}}\frac{1}{E^2(p^2)}[B(p^2)E(p^2)]'-F_4(p^2) \ , \tag{52a}$$

$$F_3(p^2)=\frac{2}{\bar{b}}\frac{1}{E^2(p^2)}[A(p^2)E(p^2)]' \ ,$$

where a prime denotes a differential with respect to the Euclidean momentum variable p^2,

$$E(p^2)=p^2A^2(p^2)+B(p^2) \ , \tag{52b}$$

and $\bar{b}\equiv\bar{b}(0)$ is the ghost self-energy at the zero point (28).

Proceeding now to the dimensionless variables

$$A(p^2)=\mu^{-2}A(t),\quad B(p^2)=\mu^{-1}B(t),\quad t=\frac{p^2}{\mu^2} \ , \tag{53}$$

and doing some algebra, the initial system (44) can be rewritten as (normal form)

$$A'=-2\left[\frac{1}{t}+\frac{1}{\lambda}\right]A-\frac{2}{\lambda}\frac{1}{t}-\frac{2}{\lambda}\frac{1}{t}m_0B \ , \tag{54}$$

$$B'=-\tfrac{3}{2}A^2B^{-1}+\frac{2}{\lambda}(m_0A-B) \ , \tag{55}$$

where $A\equiv A(t)$, $B\equiv B(t)$, and a prime denotes a differential with respect to the Euclidean dimensionless momentum variable t. The coupling constant is

$$\lambda=g^2[\tilde{b}(0)]^{-1}(2\pi)^{-2} \ , \tag{56}$$

where $\tilde{b}(0)$ is the finite ghost self-energy at the zero point (28). Thus our system (54) and (55) does not depend on this parameter in the explicit form. Instead of the two various parameters, the initial coupling constants g^2 and $\tilde{b}(0)$, we have only one parameter—the coupling constant λ (56), and therefore we avoid the difficulties associated with the unknown ghost contributions in the covariant gauge. Moreover, it is the consideration of those ghost degrees of freedom that makes it possible to obtain the infrared-finite, gauge-invariant quark propagator. As it will be shown later, this system always has a chiral-symmetry-breaking solution ($m_0=0$, $B\neq 0$). Singular dependence of the coupling constant (56) on the ghost self-energy at the zero point indicates the essential role of the ghost degrees of freedom in the dynamics of chiral-symmetry breaking.

The system (54) and (55) cannot be solved exactly in the general case ($m_0\neq 0$), but it is possible to show that the solutions of this system cannot have polelike singularities in any finite point $t=t_0$ on the real axis in the whole t-complex plane.[13] It is a direct manifestation of quark confinement. Our system (54) and (55) excludes the trivial solution $A=B=0$. Any nontrivial solution automatically breaks the γ_5 invariance of the quark propagator (1) and (2) and therefore leads to spontaneous chiral-symmetry breakdown [$m_0=0$, $B(t)\neq 0$, dynamical quark mass generation].

Let us consider the case $m_0=0$. In this case, the initial system (54) and (55) can be solved exactly. The regular solution for the differential equation (54) is

$$A(t)=\left[-\frac{\lambda}{2}\right]t^{-2}e^{-(2/\lambda)t}\left[1-e^{(2/\lambda)t}\left[1-\frac{2}{\lambda}t\right]\right] . \tag{57}$$

Asymptotic expansions of this solution are

$$A(t)=-\frac{1}{\lambda}+\frac{2}{3}\frac{1}{\lambda^2}t+O(t^2),\quad t\rightarrow 0 \tag{58}$$

and

$$A(t)\sim-\frac{1}{t}+\frac{2}{\lambda}\frac{1}{t^2},\quad t\rightarrow+\infty \ , \tag{59}$$

respectively. Thus, exact solution (57) has correct asymptotic properties (is regular at small t and asymptotically approaches the free propagator at infinity).

Let us consider Eq. (55) for $B(t)$ when $m_0=0$. Defining

$$B(t)=\pm\phi^{1/2}(t) \tag{60}$$

an exact solution can be expressed as

$$\phi(t)=B^2(t)=3e^{-(4/\lambda)t}\int_t^{t_0}e^{(4/\lambda)t'}A^2(t')dt' \ , \tag{61}$$

where t_0 is an arbitrary constant of integration. In the explicit form we have

$$\phi(t)=3e^{-(4/\lambda)t}[C(t_0)-\bar{\phi}(t)] \ , \tag{62}$$

where

$$\bar{\phi}(t)=\left[\frac{\lambda}{2}\right]\left[-\frac{1}{3}t^{-3}+\frac{2}{3}\left[\frac{2}{\lambda}\right]^3\mathrm{Ei}\left[\frac{2}{\lambda}t\right]-\frac{1}{12}\left[\frac{4}{\lambda}\right]^3\mathrm{Ei}\left[\frac{4}{\lambda}t\right]-\frac{2}{3}\left[\frac{2}{\lambda}\right]^2t^{-1}e^{(2/\lambda)t}-\frac{2}{3}\left[\frac{2}{\lambda}\right]t^{-2}e^{(2/\lambda)t}\right.$$
$$\left.+\frac{2}{3}t^{-3}e^{(2/\lambda)t}+\frac{1}{12}\left[\frac{4}{\lambda}\right]^2t^{-1}e^{(4/\lambda t)}+\frac{1}{3}\left[\frac{4}{\lambda}\right]t^{-2}e^{(4/\lambda)t}-\frac{1}{3}t^{-3}e^{(4/\lambda)t}\right] \quad (63)$$

and

$$C(t_0)=\bar{\phi}(t_0) \ . \quad (64)$$

Here Ei(z) is the exponential integral function.

Asymptotic expansions of this solution are

$$\phi(t)=B^2(t)$$
$$=C\left[1-\frac{4}{\lambda}\left[1+\frac{3}{4}\frac{1}{\lambda C}\right]t+O(t^2)\right], \quad t\to 0 \ , \quad (65)$$

where C is slightly different from $C(t_0)$ and

$$\phi(t)=B^2(t)$$
$$=-\tfrac{3}{4}\lambda t^{-2}[1-\tfrac{1}{2}\lambda t^{-1}+O(t^{-2})], \quad t\to+\infty \ , \quad (66)$$

respectively. The real part of the exact solution for $B(t)$ at infinity is identically equal to zero, according to $m_0=0$.

Thus, the solution for $B(t)$ as well as the solution for $A(t)$ have correct asymptotic behavior at small t and at infinity. Exact solution (61)–(64) for $B(t)$ depends on the one arbitrary constant of integration, which will be determined by the effective potential method for a composite operator[3,30] in the next section.

Exact solutions for $A(t)$ in Eq. (57) and $B(t)$ in Eqs. (61)–(64) do not have polelike singularities at any finite point on the real axis in the whole t-complex plane and, therefore, the exact solution for the infrared-finite quark propagator $S(p)$ in this case ($m_0=0$) is a confinement-type solution.

VI. THE EFFECTIVE POTENTIAL AND THE NONPERTURBATIVE (EFFECTIVE) QUARK MASS

The effective potential method for a composite operator[3,30] can be applied for the choice of the correct "physical" solution in the problem of dynamical breakdown of chiral symmetry in the framework of the SD equations (see, for example, Refs. 3, 12, and 31).

The effective potential is

$$V(S)=-i\int\frac{d^np}{(2\pi)^n}\mathrm{Tr}[\ln(S_0^{-1}S)-(S_0^{-1}S)+1]+V_2(S) \ , \quad (67)$$

where $V_2(S)$ is the sum of all two-particle-irreducible vacuum diagrams. If $S(p)$ is a solution of the SD equation, then the effective potential can be evaluated as

$$V(S)=-i\int\frac{d^np}{(2\pi)^n}\mathrm{Tr}[\ln(S_0^{-1}S)-\tfrac{1}{2}S_0^{-1}S+\tfrac{1}{2}] \ . \quad (68)$$

Going to Euclidean space ($d^np\to id^np$, $p^2\to -p^2$) and dimensionless variables (53), finally we obtain

$$V(A,B)$$
$$=\frac{\mu^4}{2(2\pi)^4}\int_0^\infty dt\, t\{\ln t[tA^2(t)+B^2(t)]+tA(t)+1\} \ . \quad (69)$$

Solution for $A(t)$ is completely determined and the solution for $B(t)$, Eq. (61), depends on the one arbitrary constant t_0. As was pointed out in the previous section, the exact solutions for $A(t)$ and $B(t)$, respectively, asymptotically approach the free propagator solution. For this reason the effective potential (69) diverges in the ultraviolet limit. In order to define exactly the effective potential in the ultraviolet region it is necessary to introduce an ultraviolet cutoff Λ. It is easy to see that the effective potential (69) has a minimum in the region $t_0\geq\Lambda$ at the point $t_0=\Lambda$. We obtain that the arbitrary constant of integration t_0 must be identified with the ultraviolet cutoff Λ. It should be pointed out here that in order to calculate any physical quantities such as quark condensates, the pion decay constant, meson mass spectrum via the Bethe-Salpeter equation, and so on, one certainly needs to introduce an ultraviolet cutoff Λ.

So our quark propagator contains two natural (intrinsic) parameters: the coupling constant λ and the mass parameter μ, characterizing the scale of confinement, and the third parameter t_0 (arbitrary constant of integration) must be identified as the ultraviolet cutoff Λ via the effective potential method.

Let us now calculate the nonperturbative (effective) quark mass M defined in (3). Restricted by the main term in the asymptotic expansion of the $\phi(0,t_0)$ at large t_0 in (62)–(64), finally we obtain

$$M=\frac{2\mu}{\sqrt{3\lambda}}t_0e^{-(2/\lambda)t_0} \ , \quad (70)$$

the well-known formula, which exhibits an essential singularity in the coupling constant λ, in accordance with the renormalization-group solutions in the infrared region.[9,28] As is well known this nonanalytic dependence on the coupling constant Λ is a characteristic feature for the nonperturbative solutions in QCD at large distances (infrared region). In (70) we observe an interesting connection between the infrared (nonperturbative parameter μ) and the ultraviolet (perturbative cutoff $\Lambda=t_0$) regions, respectively. In the models[11] which are characterized by the existence of the boundary-value momentum k_0, separating the infrared (nonperturbative) from the ultraviolet (perturbative) regions, respectively, our parameter t_0 can be identified with this boundary value k_0.

Concluding this section let us point out that chiral symmetry is automatically broken at any (nonzero) value of the coupling constant λ in (56). In the infrared region the coupling constant is already strong enough and therefore without fail exceeds the possible critical value, provided by the contributions to CSB from intermediate and ultraviolet regions (see, for example, Refs. 32 and 33).

VII. CONCLUSIONS

Making only one widely accepted assumption that the full gluon propagator becomes infrared singular as q^{-4} at small momenta in the arbitrary covariant gauge, we propose and develop a general, gauge-invariant, and nonperturbative approach to the extraction of the infrared-finite Green's functions in the context of the SD equations, completed by the corresponding ST identities in the infrared region in QCD. The precise definition of the singularity q^{-4} in the sense of distribution theory in the framework of our approach allows us to prove quark confinement (in other words, we prove that there are no free quark states in nature) and chiral-symmetry breaking in the infrared region. We find three and only three confinement-type solutions for the quark propagator and clearly establish a close connection between quark confinement and chiral-symmetry breaking in the infrared region. Two of these solutions are of vanishing-type solutions and one solution is the infrared-finite solution. We discover the essential role of the ghost degrees of freedom in the dynamics of chiral-symmetry breaking. We show that for the covariant gauges the complications due to ghost contributions can be considerable in our approach. Moreover, it is the consideration of ghost contributions that makes it possible to obtain the infrared-finite, gauge-invariant, nonperturbative quark propagator, which has no pole (confinement-type solution) and implies chiral-symmetry breaking (dynamical quark mass generation). The infrared-finite quark propagator depends only on two natural (intrinsic) parameters: the coupling constant λ and the mass parameter μ, characterizing the scale of confinement, and the third parameter t_0 (arbitrary constant of integration) can be identified as the ultraviolet cutoff Λ in momentum space via the effective potential method. For the dynamical (nonperturbative) quark mass we derive the expression which exhibits an essential singularity in the coupling constant in accordance with the renormalization-group solutions in the infrared region. We also point out that a gauge-invariant, nonperturbative approximation to the bound-state problem is within our approach in the context of the BS equation. Evidently, that proposed quark propagator can be applied to the evaluation of the various physical quantities, such as quark condensates, the pion decay constant, meson mass spectrum, and so on.

Concluding this section it is necessary to emphasize that the problem of the consistency of our solutions with the full apparatus of the SD equations requires a separate consideration. In this connection let us point out that our solutions correspond to a pure gluon dynamics in the infrared region (the quark and ghost loops behave as ϵ in the $\epsilon \rightarrow 0^+$ limit in the context of our solutions). The infrared singular asymptotics (8) for the full gluon propagator have been obtained exactly in this approximation.

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