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Evolution of the cosmic density matrix

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The dynamics of a reduced cosmic density matrix for minisuperspace is derived by projecting out inhomogeneous modes. This projection procedure roughly represents the intrinsic loss of information beyond the horizon of the local observer. Even if the wave function of the whole Universe exists, it will be unobservable and therefore should be projected onto actually accessible dynamics. We use a simple model of the homogeneous Robertson-Walker universe with an inhomogeneous scalar field. The path-integral expression for the density matrix is derived and evaluated by the steepest-descent method. We show that the cosmic expansion reduces the quantum coherence of the scale factor and simultaneously that of the canonically conjugate momentum.

I. INTRODUCTION

The dynamics of the cosmic evolution must ultimately be described by quantum theory. Especially when we trace back to much earlier stages of the cosmic evolution, the effect of quantum fluctuations becomes more prominent. For example, quantum fluctuations themselves may be the origin of the large-scale structure presently observed, and they may remove the classical space-time singularity.[1]

One of the extreme ideas to describe the Universe by quantum theory is the introduction of the wave function of the Universe. This idea has been widely studied.[2–4] Although it is possible to calculate the wave function of the Universe from a simple initial condition and a minisuperspace model (spatially uniform matter and space-time), we have to deduce some observable predictions from the wave function. The deduction is highly nontrivial and is always indispensable since the observable variables are drastically different from the original variables of the wave function. The logic is quite the same as in statistical mechanics; the contraction of information leads to a variety of phenomena even if the basic dynamics is only QED. The deduced results may depend on the deduction methods. However, a universal property in this deduction process exists: the destruction of quantum coherence. This will be the origin of why the various cosmological variables we observe now are well described by classical mechanics. We are going to study this decoherence property in this paper. We emphasize here that the interpretation of the whole wave function of the Universe, without any deduction, will cause various difficulties as long as we do not invent any nonstandard interpretation of the wave function.[4] One of those difficulties will be the problem of dispersion. Even if we can set a definite initial condition, almost all the dispersion of physical variables seems to become larger in the course of cosmic expansion.[5] It means that a tiny observation of the late state of the Universe changes the entire Universe drastically, which is, of course, inconsistent with the present structure of the Universe.

In this paper, we consider the dynamics of decoherence of homogeneous subdynamics by projecting out the inhomogeneous matter fields. At first we remember that in many cosmological models, homogeneity and isotropy of the matter distribution and that of background space-time (minisuperspace model) are assumed from the beginning. There, the restriction of the model to the minisuperspace variables is just an assumption. Here, we would like to consider a more active role of this restriction procedure.

If we consider the whole Universe, we cannot observe all the dynamical variables in the Universe. When we observe the Universe and interpret the results, we only treat just a few variables, which can be directly accessible variables, and neglect the other huge number of variables. This partial observation is an intrinsic nature in spite of the ambiguous separation of the variables into observable and unobservable. For example, the particles which go out beyond the past light cone of the observer in the course of cosmic expansion will never be observed by the observer. Those particles are represented by the inhomogeneous modes of the scalar field. Here, we remember the eternal inflationary scenario by Linde.[6] There, the self-reproducing domains appear due to the inhomogeneity of the scalar field. Those domains become causally separated and the information in the original inhomogeneity is lost to the causally inaccessible region of the Universe. What we can observe is the remaining almost homogeneous portion of the Universe. This scenario may justify our projection procedure of the inhomogeneous mode of the scalar field onto the homogeneous modes. The qualification of this mechanism will be discussed separately elsewhere. At this point, there naturally arises a density-matrix description[7,8] of the Universe that we will study in this paper. We emphasize that what we want to do is not an *a priori* introduction of a wave function of the minisuperspace,[2–4] but the projection of the wave function of the entire Universe onto the density matrix of the minisuperspace.

The merits of the reduced description are the following. First, the density matrix is directly related to each local observation and there is no need to introduce any nonstandard interpretation of quantum theory. Second, it predicts the phase transition from the quantum regime of the Universe to the classical one. Actually one of our main conclusions is that the phase transition does exist for the expanding universes. The degree of classicality is indicated by a quantum coherence width of the density matrix, which we calculate in a simple model. The idea is very similar to that of Ref. 9. (The *environment* there corresponds to the unobservable inhomogeneous modes in our case and *unavoidable measurement* to the back reaction of the particle production.) Third, the subdynamics description necessarily introduces partially stochastic evolution which is favorable for the generation of various structures of the Universe especially in the inflationary stage.

The outline of this paper is as follows. In Sec. II a method to project the inhomogeneous variables onto the homogeneous ones is shown. Then using a simple model of universe and matter, we derive the density matrix for the minisuperspace variables in a path-integral form. In Sec. III the density matrix is evaluated by the steepest-descent method and the destruction of the quantum coherence is shown as a reduction of the quantum coherence width of the density matrix. Then the density matrix is transformed into a Wigner function and the similar destruction of the coherence for the conjugate momentum is derived. Conclusions and further problems are in Sec. IV.

II. MINISUPERSPACE AS A REDUCED DYNAMICS

The deduction process mentioned in the previous section is in general highly nontrivial since we cannot construct a realistic wave function of the whole Universe and we cannot definitely specify the individual observational variables. Therefore, we will investigate the deduction process by utilizing the following very simple model. Space-time is represented by a spatially uniform and isotropic metric (scale factor a). The remaining inhomogeneous modes are discarded for simplicity. Matter is represented by a scalar field ϕ which couples to the metric almost conformally invariantly. The zero-mode condensation is disregarded also for simplicity. Terms which break the conformal invariance are treated as a perturbation. Our basic strategy is to construct the effective dynamics of the minisuperspace variable a by projecting out all the other inhomogeneous modes.[10,11]

The metric we consider is

$$ds^2=a^2(\eta)[N^2(\eta)d\eta^2-d\mathbf{x}^2]\ , \tag{2.1}$$

where N is the lapse function, which represents an arbitrariness of the time reparametrization. The Einstein action for this metric becomes

$$S_g[a]=\frac{-V}{16\pi G}\int d\eta\left[6\frac{a'^2}{N}+2\Lambda a^4 N\right]\ , \tag{2.2}$$

where a prime denotes a differentiation with respect to η and Λ is the cosmological constant. The choice of a spatially flat metric as in Eq. (2.1) forces us to restrict our consideration within a finite constant coordinate three-volume V in order to make any calculation finite. The matter action becomes

$$S_m[\phi]=\tfrac{1}{2}\int d^4x\, N\left[N^{-2}\left[\frac{\partial\phi}{\partial\eta}\right]^2-\left[\frac{\partial\phi}{\partial\mathbf{x}}\right]^2\right]\ , \tag{2.3}$$

where ϕ is a rescaled field from the original scalar field $\bar{\phi}$ as $\phi(x)=a(\eta)\bar{\phi}(x)$. A small mass term and a small coupling term to the scalar curvature are regarded as perturbations:

$$S_{\text{int}}=-\tfrac{1}{2}\int d^4x\, h(x)\phi^2(x) \tag{2.4}$$

with

$$h(x)=N\left[m^2a^2+(6\xi-1)\frac{1}{aN}\left[\frac{a'}{N}\right]'\right]\ . \tag{2.5}$$

The total action S is the summation of the above terms:

$$S[a,\phi]=S_g[a]+S_m[\phi]+S_{\text{int}}[a,\phi]\ . \tag{2.6}$$

The total density matrix is given by

$$\begin{aligned}\bar{\rho}[a_+,\phi_+,;a_-,\phi_-]&\equiv\langle a_-,\phi_-|\bar{\rho}|a_+,\phi_+\rangle\\&=\int da'_+d\phi'_+da'_-d\phi'_-\int_{a'_+}^{a_+}\mathcal{D}a_+\int_{\phi'_+}^{\phi_+}\mathcal{D}\phi_+\int_{a'_-}^{a_-}\mathcal{D}a_-\int_{\phi'_-}^{\phi_-}\mathcal{D}\phi_-e^{i(S[a_+,\phi_+]-S[a_-,\phi_-])}\\&\qquad\times\rho[a'_+,\phi'_+;a'_-,\phi'_-]\ .\end{aligned} \tag{2.7}$$

From this, we define a reduced density matrix ρ for the minisuperspace variables:

$$\rho[a_+,a_-]\equiv\int d\phi_+\int d\phi_-\bar{\rho}[a_+,\phi_+;a_-,\phi_-]\delta(\phi_+-\phi_-)\ . \tag{2.8}$$

We assume the matter unperturbed state is the conformal vacuum. Then the reduced density matrix is expressed as

$$\rho[a_+,a_-]=\int da'_+\int da'_-\int_{a'_+}^{a_+}\mathcal{D}a_+\int_{a'_-}^{a_-}\mathcal{D}a_-\rho[a'_+,a'_-]\exp(i\tilde{S}[a_+,a_-])\ , \tag{2.9}$$

where

$$\exp(i\tilde{S}[a_+,a_-])=\mathcal{F}[a_+,a_-]\exp\{i(S_g[a_+]-S_g[a_-])\} \tag{2.10}$$

and

$$\begin{aligned}\mathcal{F}[a_+,a_-]=\int\mathcal{D}\phi_p\exp\{i(&S_m[\phi_+]+S_{\text{int}}[\phi_+,a_+]\\&-S_m[\phi_-]-S_{\text{int}}[\phi_-,a_-])\}\ .\end{aligned} \tag{2.11}$$

In Eq. (2.11), ϕ_p means a pair of scalar fields ϕ_+ and ϕ_-

and its path integration is performed on the conformal vacuum. This is the in-in formalism of quantum field theory which is natural and useful for calculating density matrixes.[12–14] It is straightforward to evaluate Eq. (2.11) in perturbation series in the interaction. In two-by-two matrix representation, it becomes

$$\mathcal{F}=\exp\left\{-\tfrac{1}{2}\mathrm{Tr}\ln\left[-\Box_p+\begin{bmatrix}-h_+ & 0\\ 0 & h_-\end{bmatrix}\right]\right\}$$
$$\approx -\tfrac{1}{4}\int d^4x\int d^4x'[h_\Delta(x)\Pi_1(x,x')h_\Delta(x')+h_\Delta(x)\Pi_2(x,x')h_c(x')]+O(h_\pm^3)\ , \tag{2.12}$$

where

$$h_\pm=m^2a_\pm^2+(6\xi-1)\left[\frac{a_\pm''}{a_\pm}\right]\ ,\qquad h_\Delta=h_+-h_-,\quad 2h_c=h_++h_- \tag{2.13}$$

and

$$\frac{1}{\Box_p}=i\begin{bmatrix}\langle T\phi(x)\phi(x')\rangle & \langle\phi(x')\phi(x)\rangle\\ \langle\phi(x)\phi(x')\rangle & \langle\bar{T}\phi(x)\phi(x')\rangle\end{bmatrix}\ , \tag{2.14}$$

$$\Pi_1(x,x')=\mathrm{Re}\langle T\phi(x)\phi(x')\rangle^2\ ,\qquad \Pi_2(x,x')=2i\theta(x^0-x^{0'})\mathrm{Im}\langle T\phi(x)\phi(x')\rangle^2\ . \tag{2.15}$$

In deriving Eq. (2.12), we have set $N_\pm=1$ and assumed the regularization: $\langle\phi^2(x)\rangle=0$. Fourier transform of the square of the Feynman propagator becomes

$$\int d^nx\,\exp(ipx)\langle T\phi(x)\phi(0)\rangle^2=\frac{-i}{4\pi^2}\left[\frac{1}{\epsilon}+(2-\gamma+\ln 4\pi)+\pi i-\ln p_0^2+O(\epsilon)\right]\ , \tag{2.16}$$

where $\epsilon=n-4$ and γ is Euler's constant. In deriving Eq. (2.16), we have already confined ourselves to the minisuperspace description and have picked up a spatially uniform component ($\mathbf{p}=0$) because within the volume V, the perturbations $h_\pm$ are assumed to be homogeneous. A real term in Eq. (2.16) appears in the process of the analytic continuation from Euclidean (p_E) to Lorentzian (p_M) momentum: $p_M=(-p_E^0,\mathbf{p}_E)$ and represents a ϕ-particle production rate, which is therefore necessarily positive definite. In this paper we do not touch upon divergent terms since their proper treatment is not yet clear in the formalism of subdynamics. They may be absorbed into renormalized quantities by introducing conformally invariant counterterms which are quadratic in the curvature tensor as in Ref. 15. Then, Eq. (2.12) becomes

$$\mathcal{F}[a_+,a_-]=\exp\left[-\frac{V}{16\pi}\int d\eta\,h_\Delta^2(\eta)-\frac{iV}{8\pi^2}\int d\eta\int d\eta'\theta(\eta-\eta')h_\Delta(\eta)\times\ln(\eta-\eta')h_c(\eta')\right]\ . \tag{2.17}$$

The first term of the exponent in Eq. (2.17) arises as a back reaction of ϕ-particle production and will cause diffusion of quantum coherence of the scale factor. The second term of the exponent in Eq. (2.17) is very similar to one that appeared in Ref. 15. It represents a dispersive back reaction of the ϕ field and is needed for the concrete evaluation of the diagonal elements of the density matrix. However, it does not have any relation to a reduction of classical properties of the density matrix.

III. STEEPEST-DESCENT EVALUATION OF THE DENSITY MATRIX

In the preceding section, we have derived the path-integral expression Eq. (2.9) for the density matrix of the minisuperspace variable. We evaluate this by the steepest-descent method; the path-integral is approximated by the integrand with the extremum path (classical solution). As far as the lowest order of the perturbation is concerned, perturbation-free classical solutions are sufficient to the evaluation of the integrand. The perturbation-free action for the density matrix is given by

$$\tilde{S}_0=S_g[a_+]-S_g[a_-]=\frac{-V}{8\pi G}\int d\eta\left[N_+\left[3\frac{a_+'^2}{N_+^2}+\Lambda a_+^4\right]-N_-\left[3\frac{a_-'^2}{N_-^2}+\Lambda a_-^4\right]\right]\ , \tag{3.1}$$

where we have chosen a matter vacuum state in accordance with the preceding section. The classical solution must satisfy the extremum condition

$$0=\frac{\delta\tilde{S}_0}{\delta a_\pm}\propto a_\pm''-2H^2a_\pm^3 \tag{3.2}$$

and the constraint equation which comes from the lapse-function independence of the density matrix

$$0=\frac{\delta\tilde{S}_0}{\delta N_\pm}\propto a_\pm'^2-H^2a_\pm^4\ , \tag{3.3}$$

where $H=\sqrt{\Lambda/3}$. Here we try to impose the boundary condition: $a_\pm\to 0$ for $\eta\to-\infty$. Then the classical solution is determined as

$$a_\pm(\eta)=-[H(\eta-\eta_\pm^*)]^{-1}\ . \tag{3.4}$$

In the gauge $N_\pm=1$, the perturbation becomes

$$h_\pm(\eta)=\left[m^2a_\pm^2+(6\xi-1)\frac{a_\pm''}{a_\pm}\right]=[m^2+(6\xi-1)2H^2]a_\pm^2\equiv M^2a_\pm^2\ . \tag{3.5}$$

We put the classical solution Eq. (3.4) into the total action Eq. (2.10) and expand it as a series in the difference of arguments of the density matrix a_+-a_- ($\equiv a_\Delta$) assuming smallness of a_Δ: $|a_\Delta/a_c|\ll 1$ ($2a_c\equiv a_++a_-$). The result becomes

$$\tilde{S}=-\frac{VH}{4\pi G}(a_+^3-a_-^3)-\frac{VM^4}{12\pi^2}a_c a_\Delta[\tfrac{1}{3}-\ln(Ha_c)]+\frac{iM^4V}{20\pi H}a_c a_\Delta^2 , \tag{3.6}$$

which gives an approximate density matrix:

$$\rho[a_+,a_-]\approx\mathcal{N}\exp\left[-\frac{iVH}{4\pi G}(a_+^3-a_-^3)-\frac{iVM^4}{12\pi^2}a_c a_\Delta[\tfrac{1}{3}-\ln(Ha_c)]-\frac{M^4V}{20\pi H}a_c a_\Delta^2\right] . \tag{3.7}$$

Here we have taken each lowest contribution of real and imaginary terms. From Eq. (3.7), the dispersion (the width of the quantum coherence) σ of this density matrix is given by

$$\sigma^2=\frac{10\pi H}{M^4V}a_c^{-1} . \tag{3.8}$$

We should evaluate the degree of decoherence by the quantity which is almost independent of a normalization of variables:

$$\frac{\sigma}{a_c}=\left[\frac{10\pi H}{M^4V}\right]^{1/2}a_c^{-3/2} . \tag{3.9}$$

This quantity manifestly reduces with the cosmic expansion: $a_c\to\infty$. This means that the density matrix diagonalizes in the course of cosmic expansion. Since the diagonalization means the reduction of quantum coherence among different values of the scale factor a, we can conclude that the Universe we observe (minisuperspace) transforms from the quantum era to the classical era in this sense. Furthermore, we can show that even if we take another boundary condition $a_\pm\to 0$ for $\eta\to+\infty$, we also get the same expression for the dispersion. Therefore, we can claim as follows: in a large universe, the quantum coherence is small in different values of the scale factor a while in a small universe, there is a strong coherence. The loss of quantum coherence may be caused by a diffusion of information from the homogeneous minisuperspace into inhomogeneous modes through particle production. This classicalization corresponds to the destruction of the quantum coherence among various universes and is different from that claimed in Refs. 3 and 4: the appearance of a peak in the wave function of the Universe. These two classicalization properties are discussed in the next section.

We have to come back to the validity of the perturbation calculation which led to the above result. Since the perturbative calculation breaks down for large $h(x)$, we cannot conclude the final decoherence of the expanding Universe only from the above argument. Furthermore, at the late stage of the cosmic expansion, the produced particles may change the quantum state for the scalar field from the conformal vacuum. In order to conclude the final decoherence, some other approximation method such as an adiabatic approximation will be needed, or otherwise, we have to go over to the third quantization of the Universe where the interaction term Eq. (2.4) goes over to the *time*- and *space*-dependent *mass* term in the new action for the field of Universe. These subjects are beyond the scope of this paper and will be studied elsewhere.

Now let us study what is going on in the other representation of the cosmic density matrix. A special interest is the representation in the conjugate momentum of the scale factor. For this purpose, we transform the density matrix into a Wigner function W, which is a distribution function of the variables a_c ($\equiv a$) and the momentum of a_Δ ($\equiv p$):

$$W(a,p)=\int da_\Delta \exp(ia_\Delta p)\rho[a_+,a_-] . \tag{3.10}$$

For large p, the Wigner function, except the prefactor, becomes

$$W\approx\exp\left[-\frac{(p-p_0)^2}{2\sigma_p^2}\right] , \tag{3.11}$$

where

$$p_0\approx\frac{3VH}{4\pi G}a^2+\frac{VM^4}{12\pi^2}a[\tfrac{1}{3}-\ln(Ha)] , \quad \sigma_p^2\approx\frac{M^4V}{10\pi Ha} . \tag{3.12}$$

At first glance, the uncertainty in the variable p (σ_p^2) seems to increase with cosmic expansion. However, we should as before, evaluate the quantity which is almost independent of a normalization of variables as a measure of uncertainty. It becomes, for large a,

$$\frac{\sigma_p}{\langle p\rangle}\approx\frac{4GM^2}{3}\left[\frac{\pi}{10VH^3}\right]^{1/2}a^{-3/2} , \tag{3.13}$$

where the average of the variable p with respect to W ($\langle p\rangle$) is approximately given by p_0. This shows that the uncertainty of the momentum decreases with cosmic expansion. However, we should be careful that this result does not necessarily indicate that the coherence in momentum is also deduced by the cosmic expansion. Note that the average p_0 asymptotically becomes the classical value of the canonical momentum of the scale factor.

IV. CONCLUSIONS

A reduced density matrix for a scale factor is derived by integrating unobservable inhomogeneous modes of a scalar matter field. The path integral form is derived and evaluated by the steepest-descent approximation. The width of quantum coherence of the density matrix turned out to reduce as the Universe expands. This means the destruction of quantum coherence between universes with different values of scale factor. Moreover, by transforming the density matrix into the Wigner function, we found that the uncertainty in canonical momentum of the scale factor also reduces in the course of cosmic expansion.

This destruction of the quantum correlation is a different property from the appearance of a peak in the wave function of the Universe.[4] Although we have not emphasized the latter in this paper, both properties seem to be needed for the complete classicalization of the Universe: the destruction of the quantum coherence (which is indicated by the diagonalization of the density matrix) and the creation of strong correlation between a detector and the Universe (which is indicated by the sharp peak in the distribution function). If the former property were missing, the remaining quantum coherence among macroscopic systems would introduce a strange correlation which is incompatible with the self-independence of macroscopic objects. On the other hand, if the latter property were missing, the theory cannot predict anything definite. The complete description of the classicalization based on the above two properties is in progress by the author and will be reported in a separate paper.

Finally we will mention several comments. The intrinsic limitation of the observation should more precisely be considered in an actual cosmological situation. Probably, the degree of decoherence will change if we change the reduced variables. However, we do not think that this leads to an ambiguity of our approach. It just indicates that the degree of decoherence depends on the energy scale of the system. We expect that the decoherence will vanish for high-energy scale and approaches some constant for small-energy scale. To clarify this expectation is our future problem. Effects of the spatial curvature, inhomogeneous modes of gravity, zero modes of matter fields, etc., should be clarified in order to check the generality of the present results. In the course of cosmic expansion, the scale factor a becomes classical in the sense that the quantum coherence width reduces. Simultaneously, the distribution of the conjugate momentum p forms a peak at its classical value. In this sense, both the variables a and p become definite. We will, in the next step, check the generality of this result extending the present model. After this work was completed, the author noticed papers which discuss the same problem with different methods.[16]

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