Cosmic strings and ultrahigh-energy cosmic rays

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We calculate the flux of ultrahigh-energy protons due to the process of "cusp evaporation" from cosmic-string loops. For the "standard" value of the dimensionless cosmic-string parameter $\epsilon \equiv G\mu \approx 10^{-6}$, the flux is several orders of magnitude below the observed cosmic-ray flux of ultrahigh-energy protons. However, the flux at any energy initially increases as the value of ϵ is decreased. This at first suggests that there may be a lower limit on the value of ϵ, which would imply a lower limit on the temperature of a cosmic-string-forming phase transition in the early Universe. However, our calculation shows that this is not the case—the particle flux at any energy reaches its highest value at $\epsilon \approx 10^{-15}$ and it then *decreases* for further decrease of the value of ϵ. This is due to the fact that for too small values of ϵ ($< 10^{-15}$) the energy loss of the loops through the cusp evaporation process itself (rather than gravitational energy loss of the loops) becomes the dominant factor that controls the behavior of the number density of the loops at the relevant times of emission of the particles. The highest flux at any energy remains at least 4 orders of magnitude below the observed flux. There is thus no lower limit on ϵ.

I. INTRODUCTION

Cosmic strings[1,2] (CS's), which could be formed as a result of certain symmetry-breaking phase transitions in the early Universe, have been studied vigorously for their possible important role in the formation of galaxies and the large-scale structure in the Universe.[2] Here we consider another aspect of CS's: namely, the production of high-energy particles from oscillating closed CS loops. CS's can be thought of as "made" of the quanta of the massive gauge and Higgs fields of the underlying spontaneously broken gauge theory. Under certain circumstances these and other particles are emitted from CS's. The decay products of these massive particles emitted from CS's would, in principle, be present in today's Universe in the form of (ultra)high-energy particles. It is of interest to have a quantitative estimate of the flux of these particles in relation to the flux of ultrahigh-energy (UHE) cosmic-ray particles. Clearly, for CS's not to be inconsistent with reality, the high-energy particle flux from CS's must not exceed the peak observed flux of these particles in UHE cosmic rays.

In this paper we estimate the flux of UHE protons in the present epoch due to one particular particle emission process involving CS's: namely, the so-called "cusp evaporation" process.[3] We find that the calculated proton flux is several orders of magnitude below the observed flux if the value of the dimensionless CS parameter, $\epsilon \equiv G\mu$, is 10^{-6}, which is the kind of value for ϵ envisaged[2] in the theory of galaxy formation with CS's. Here μ is the mass-energy per unit length of the string, which is fixed by the energy scale at which the CS-forming symmetry breaking takes place, and G is Newton's constant (we use natural units with $\hbar = c = 1$). On the other hand, it turns out that the particle flux increases as the value of ϵ is decreased. The reason for this is that, for smaller values of ϵ, the energy loss of the CS loops through gravitational radiation is less so that the loops survive longer giving a higher value for the number density of the loops at any time, which in turn gives higher particle flux. One might then expect that the particle flux from CS's with a sufficiently small value of ϵ would exceed the observed particle flux thereby giving a *lower limit* to ϵ, i.e., a lower limit to the temperature in the early Universe at which a CS-forming phase transition could take place. The detailed calculation described below, however, shows that this is not the case. The reason is interesting—what happens is that below a certain value of ϵ, the energy loss of the loops through gravitational radiation becomes so small that the energy loss through "cusp evaporation" itself becomes the dominant factor that controls the behavior of the number of density of the loops. When this happens, the CS loop number densities start decreasing again with decreasing values of ϵ, leading to a decreasing particle flux. This implies that the particle flux at any given energy has a peak as a function of ϵ, and the calculations below show that the peak flux, at all energies, remains below the observed flux.

Recently, MacGibbon and Brandenberger[4] have estimated the neutrino flux from CS cusp evaporation and obtained the lower limits, $\epsilon \gtrsim 10^{-17}$ and $\epsilon \gtrsim 10^{-15}$, for two different cases considered by them. They have, however, assumed that the CS loop number densities at all times are determined by gravitational radiation from the loops, irrespective of the value of ϵ—an assumption which, as we have mentioned above and shall discuss below, is not valid. We will report the explicit calculations for the case of neutrinos elsewhere, but from the results of Ref. 4 and the discussions given below, it already appears that the use of the correct formulas for the loop number densities would also eliminate the lower bounds on ϵ found in Ref. 4.

In Sec. II, we briefly describe the process of cusp evaporation from CS loops and estimate the number of primary particles emitted from the string per unit time. The UHE proton injection spectrum, resulting from the decay of the primary particles and the subsequent hadronization of the decay products, is estimated in Sec. III by using a suitable hadronic jet fragmentation distribution function. A general expression for the predicted flux in the present epoch is written down in Sec. IV. In Sec. V, we briefly discuss the main processes by which UHE protons lose energy during their propagation through the cosmic medium, and discuss how the effective maximum possible redshift of injection is determined for a given value of the energy of the proton in the present epoch. The CS loop length distribution function required for our calculation is obtained in Sec. VI. The main calculation of the flux is described in Sec. VII, and the results, discussions, and conclusions are presented in Sec. VIII.

Except where otherwise stated, we use natural units, $\hbar=c=1$, so that $\sqrt{G}=M_{\rm Pl}^{-1}=t_{\rm Pl}$, where $M_{\rm Pl}$ is the Planck mass and $t_{\rm Pl}$ is the Planck time. The Hubble constant is $H_0=100h$ km s^{-1}Mpc^{-1}, and we use $h=0.75$. Also $t_{\rm eq}$ is the time of equal matter and radiation energy density, $z_{\rm eq}$ is the corresponding redshift and t_0 is the present age of the Universe. We assume a $\Omega_0=1$ universe.

II. CUSP EVAPORATION

A non-self-intersecting,[5,6] freely oscillating CS loop has one or more points which momentarily achieve the speed of light once during every oscillation period. These points called "cusps" appear[6] if the motion of the loop is described by the Nambu action, which is valid for infinitely thin strings. In reality, CS's have a finite width, and so the Nambu action is, strictly speaking, not valid for CS's and true cusps may not form. Nevertheless, "near cusp" points are likely to occur where the string moves with very high Lorentz factor. At a cusp, two string segments overlap, and it has been pointed out[3] that interactions of the underlying fields lead to "evaporation" of the overlapped region whereby the energy contained in the overlapped region of the loop is released in the form of particles, thus smoothing out the cusp. New cusps continue to form and evaporate during each period of oscillation of the loop. The length of the cusp region of the loop can be estimated[3] as $l_{\rm cusp}\sim L^{2/3}\omega^{1/3}$, where L is the total length of the loop and $\omega\sim\mu^{-1/2}$ is the width of the string. (The length L of the string is defined such that μL is equal to the total energy of the string.) The energy released due to cusp evaporation will be in the form of bursts with time scale $\Delta t_{\rm burst}\sim l_{\rm cusp}$. The period of oscillation $T_{\rm ocs}$ for a loop of length L is[5] $L/2$. Thus, $\Delta t_{\rm burst}/T_{\rm osc}\sim(\omega/L)^{1/3}\ll 1$. Thus the rate of energy released due to cusp evaporation, obtained by averaging over a period of oscillation of the loop is given by

$$\left[\frac{dE}{dt}\right]_{\rm cusp}=-\gamma_c\mu^{5/6}L^{-1/3}\ , \tag{1}$$

where γ_c is a numerical factor of order unity which parametrizes our lack of precise knowledge about the efficiency of the cusp evaporation process. The primary particles emitted from the cusps will presumably be the massive gauge bosons, Higgs bosons, and/or heavy fermions coupled to the string-forming Higgs field.[7] In the absence of a detailed knowledge of the type and the energy-spectrum of the emitted particles, we shall generically call them X particles. We denote the average energy of each emitted particle by E_X and obtain, from Eq. (1), the number of X particles emitted per unit time from a loop of length L as

$$\frac{dN_X}{dt}=\gamma_c E_X^{-1}\mu^{5/6}L^{-1/3}\ . \tag{2}$$

The quantity E_X is expected to be of order $\mu^{1/2}$, this being the intrinsic energy scale of the particles in the problem. We will write

$$E_X=f\mu^{1/2}=f\epsilon^{1/2}M_{\rm Pl}\ , \tag{3}$$

where f is a constant (free parameter) of order unity.

III. DECAY OF X PARTICLES, HADRONIZATION OF THE DECAY PRODUCTS, AND THE INJECTION SPECTRUM OF UHE PROTONS

The X particles will decay[8] presumably into three-body final states involving two conventional quarks and a lepton.[11,12] The two quarks will hadronize[13] and produce two jets of hadrons with maximum energy of any hadron in a jet, $E_{\rm max}\le\frac{1}{3}E_X$, assuming that the three particles in the decay products of each X share energies roughly equally. Most of the hadrons in a jet will be pions; a small fraction will be nucleons. The neutrons will ultimately end up as protons after β decay. The protons lose energy as they propagate through the cosmic medium and appear today with degraded energy. The leptons in the decay products of the X particles will give rise to electromagnetic cascades leading to a γ-ray background today. There will also be high-energy neutrinos resulting from the direct production of them by the decay of the X particles as well as from the decay of the pions in the hadronic jets. An electromagnetic component of electrons and γ rays will also develop due to the indirect process of energy loss of the protons in collision with the background photons. All these processes remain to be studied. Here we shall only consider the case of protons.

Now, a quark in the decay product of X will fragment and produce a jet of hadrons. We first want the fragmentation distribution function (FDF) of a jet, i.e., the number N of hadrons carrying a fraction x of the total energy in the jet. Unfortunately, the precise nature of the fragmentation process is not known and no "first-principles" derivation of an FDF is available. However, models yielding FDF consistent with QCD expectations have been studied. Following Ref. 11, we shall use here a simple FDF formula that roughly reproduces the particle multiplicity growth as seen in GeV–TeV jets in colliders. This gives[11]

$$\frac{dN}{dx}=\tfrac{15}{16}x^{-3/2}(1-x)^2\ , \tag{4}$$

where

$$x=\frac{E}{E_{\rm jet}}=\frac{E}{E_X/3}\le 1 \ ,$$

E being the energy of a hadron in the jet. A small fraction ($\sim 3\%$) (Ref. 11) of the hadrons in the jet will be nucleons and antinucleons which ultimately end up as protons and antiprotons. Observationally, since the primary particles at the high energies involved here are not detected directly, one cannot distinguish between protons and antiprotons. We shall, therefore, in the following, collectively refer to them simply as protons. Let $\Phi(E_i,t_i)$ denote the injection spectrum of the protons, i.e., the number density of injected protons per unit energy interval at an injection energy E_i per unit time at an injection time t_i due to cusp evaporation from all CS loops. Then using Eqs. (4) and (2) we get

$$\Phi(E_i,t_i)\simeq 2\times 0.03\times \tfrac{15}{16}x^{-3/2}(1-x)^2\frac{3}{E_X}\gamma_c E_X^{-1}\mu^{5/6} \times \int dL\frac{dn}{dL}(L,t_i)L^{-1/3} \ , \quad (5)$$

where $x=3E_i/E_X=3E_i f^{-1}\epsilon^{-1/2}M_{\rm Pl}^{-1}$ and $dn(L,t_i)/dL$ is the CS loop length distribution function, i.e., $dn(L,t_i)$ is the number density of CS loops with lengths in the interval $[L,L+dL]$ at the time t_i. The factor of 2 in Eq. (5) takes care of the fact that we have assumed two quarks in the decay products of each X and each quark produces one hadronic jet. Thus Eq. (4) yields an injection spectrum $\propto E_i^{-3/2}$ for $E_i \ll E_X$.

IV. GENERAL EXPRESSION FOR THE FLUX

Let $j(E_0)$ denote the number of protons per unit energy interval at energy E_0 in the present epoch (t_0) crossing per unit area per unit solid angle per unit time due to the source $\Phi(E_i,t_i)$. Then, assuming an isotropic distribution of the CS loops in an Einstein–de Sitter "flat" ($\Omega_0=1$) universe, we get

$$j(E_0)=\frac{1}{4\pi}\int_0^\infty 4\pi a^3(t_i)r^2dr[(1+z_i)^{-1}\Phi(E_i,t_i)] \times \left[\frac{dE_i}{dE_0}\right]_{E_0}\frac{1}{4\pi a^2(t_0)r^2} \ , \quad (6)$$

where t_i is the injection time, z_i is the corresponding redshift, $E_i\equiv E_i(E_0,t_i)$ is the energy at the time of injection t_i, $a(t)$ is the scale factor of the Universe, and r is the comoving radial coordinate of the source. The factor $(1+z_i)^{-1}=a(t_i)/a(t_0)$ in Eq. (6) is due to the cosmological "redshift" of the frequency of emission.[14] Now for a $\Omega_0=1$ universe, $r=c\int_{t_i}^{t_0}dt/a(t)$ (assuming that the particles are ultrarelativistic, so that they travel almost with the speed of light, c), so that $a(t_i)dr=-c\,dt_i$. Furthermore, $t_i>t_{\rm eq}$ (in fact, as we shall see below, for all values of energy E_0, all injection times t_i satisfy $t_i\gg t_{\rm eq}$) so that $(1+z_i)^{-1}=a(t_i)/a(t_0)=(t_i/t_0)^{2/3}$, giving $a(t_i)dr=-c\,dt_i=\frac{3}{2}ct_0(1+z_i)^{-5/2}dz_i$. Putting all these together, Eq. (6) becomes

$$j(E_0)=\frac{3}{8\pi}ct_0\int_0^\infty dz_i(1+z_i)^{-11/2} \times\left[\frac{dE_i(E_0,z_i)}{dE_0}\right]_{E_0}\Phi(E_i,z_i) \ . \quad (7)$$

Equation (7) is valid for any general source.

V. ENERGY-LOSS PROCESSES

Now, a proton of energy E propagating through the cosmic medium at an epoch of redshift z loses energy primarily[15] through three processes: (i) cosmological redshift, due to the expansion of the Universe, (ii) e^+e^- pair production ($p+\gamma\to p+e^++e^-$), and (iii) photopion production ($p+\gamma\to\pi+N$), where γ in the processes (ii) and (iii) are the background photons at the epoch z. Assuming the energy loss to be continuous, which for our purpose is a reasonably good approximation[17] at the energies of interest, we define

$$\beta(E,z)\equiv-\frac{1}{E}\frac{dE}{dt} =\beta_{\rm rsh}(E,z)+\beta_{\rm pair}(E,z)+\beta_{\rm pion}(E,z) \ , \quad (8)$$

where $\beta_{\rm rsh}$, $\beta_{\rm pair}$, and $\beta_{\rm pion}$ refer, respectively, to the energy loss due to the three processes mentioned above. We have

$$\beta_{\rm rsh}(E,z)=H_0(1+z)^{3/2} \ , \quad (9)$$

$$\beta_{\left\{\substack{\rm pair\\ \rm pion}\right\}}(E,z)=(1+z)^3\beta_{0,\left\{\substack{\rm pair\\ \rm pion}\right\}}((1+z)E) \ , \quad (10)$$

where, in Eq. (10), $\beta_0(E)$ refers to the energy loss suffered by a proton of energy E in the *present epoch* ($z=0$) due to the processes indicated. Equation (10) follows from the fact that the number density of the background photons was higher by a factor $(1+z)^3$ and the energy of each photon higher by a factor $(1+z)$ at the epoch with redshift z, compared to the respective values of these quantities in the present epoch. Equations (8)–(10) give, after changing variable from t to z,

$$\frac{1}{E}\frac{dE}{dz}=(1+z)^{-1}+H_0^{-1}(1+z)^{1/2}[\beta_{0,\rm pair}((1+z)E) +\beta_{0,\rm pion}((1+z)E)] \ . \quad (11)$$

The energy-loss functions $\beta_{0,\rm pair}(E)$ and $\beta_{0,\rm pion}(E)$ have been calculated by several authors.[15] For a nice summary, see Fig. 1 of Ref. 17. Here we only note the following. For $E\lesssim 6\times10^{19}$ eV, $\beta_{0,\rm pair}(E)$ dominates over $\beta_{0,\rm pion}(E)$; $\beta_{0,\rm pair}^{-1}(E)$ decreases from $\sim 10^{11}$ years at $E\simeq 10^{18}$ eV to $\sim 7.8\times10^9$ years at $E\simeq 4.6\times10^{18}$ eV. For 5×10^{18} eV $\lesssim E\lesssim 6\times10^{19}$ eV, $\beta_{0,\rm pair}^{-1}(E)$ has a weak energy dependence—it can be taken to be roughly constant at $\sim 5\times10^9$ yr. At $E\gtrsim 6\times10^{19}$ eV, $\beta_{0,\rm pion}$ becomes dominant and it rises very steeply with increasing E; $\beta_{0,\rm pion}^{-1}$ decreases from $\sim 4.7\times10^9$ years at $E\simeq 6\times10^{19}$ eV to $\sim 7.9\times10^7$ years at $E\simeq 2\times10^{20}$ eV. By Eq. (10), protons at earlier epochs enter the regime of photopion energy-loss dominance at even smaller values of energy. The "lifetime" $\beta_{\rm pion}^{-1}(E,z)$, of a proton in the photopion energy-loss regime decreases exponentially with increasing energy. These facts imply that the spectrum of UHE protons today should show the onset of a cutoff at $E\sim 6\times 10^{19}$ eV, unless the sources are so nearby that the propagation times are short compared to $\beta_{\rm pion}^{-1}$. This is the well-known Greisen-Zatsepin-Kuz'min[18] cutoff prediction. The observations of UHE cosmic rays, while not entirely devoid of controversies, do seem to indicate[19] the existence of a cutoff as predicted.

Now, given the full knowledge of the energy-loss functions $\beta_{0,\rm pair}(E)$ and $\beta_{0,\rm pion}(E)$, one can solve Eq. (11) numerically to find the energy E_i of a proton at any injection redshift z_i corresponding to a given value of its energy in the present epoch (E_0). One can then evaluate the injection spectrum $\Phi(E_i,z_i)$ using Eq. (5) (with a given CS loop length distribution function; see Sec. VI) and obtain the flux by evaluating the z_i integral in Eq. (7). The full numerical calculation according to this procedure is described in Ref. 20 in the context of another particle production process involving CS's. Here we undertake an approximate calculation which essentially yields the same result, but it allows us to avoid the full numerical solution of Eq. (11). The approximation is based on the use of the arguments that lead to the prediction of the Greisen-Zatsepin-Kuz'min[18] cutoff mentioned above.

To see this, let us consider the energy-range 5×10^{18} eV $\lesssim E_0 \lesssim 6\times 10^{19}$ eV, in which, as mentioned above, $\beta_{0,\rm pair}$ is dominant over $\beta_{0,\rm pion}$ and the former is weakly energy dependent remaining roughly constant at $\beta_0 \approx 2.13\times 10^{-10}$ yr^{-1}. In this case, as long as $(1+z_i)E_i < 6\times 10^{19}$ eV, Eq. (11) has the analytic solution: namely,

$$E_i(z_i)=E_0(1+z_i)\exp\left[\frac{2}{3}\frac{\beta_0}{H_0}[(1+z_i)^{3/2}-1]\right]$$

$$\text{for } 5\times 10^{18}\ \text{eV} \lesssim E_0 \lesssim 6\times 10^{19}\ \text{eV}\ . \quad (12)$$

Thus in the above energy range, if we consider a proton at energy E_0 today, its energy E_i at any injection redshift z_i rises exponentially with z_i. If for any given value of E_0 we define the injection redshift $z_{i,\max}$ such that

$$(1+z_{i,\max})E_i(z_i=z_{i,\max},E_0)\simeq 6\times 10^{19}\ \text{eV}\ , \quad (13)$$

then for $z_i \ge z_{i,\max}$, the proton would be in the photopion energy-loss regime. In this regime the energy-loss itself rises sharply (roughly[17] exponentially) with energy and so the energy E_i of the proton at the injection redshifts $z_i > z_{i,\max}(E_0)$ rises even faster[21] with increasing values of z_i. As a result, the rapid fall of the injection spectrum $\Phi(E_i,z_i)$ (which goes as $\sim E_i^{-3/2}$) with increasing value of z_i dominates over the power-law rise of Φ with z_i coming from the fact that the number density of the CS loops increases with redshift (see Secs. VI–VII). This in fact ensures that the z_i integral in Eq. (7) converges fast. In other words, for a given value of E_0, the contributions to the flux $j(E_0)$ of Eq. (7) from injection redshifts $z_i > z_{i,\max}(E_0)$ are negligible compared to those from the injection redshifts $z_i < z_{i,\max}(E_0)$. The quantity $z_{i,\max}$

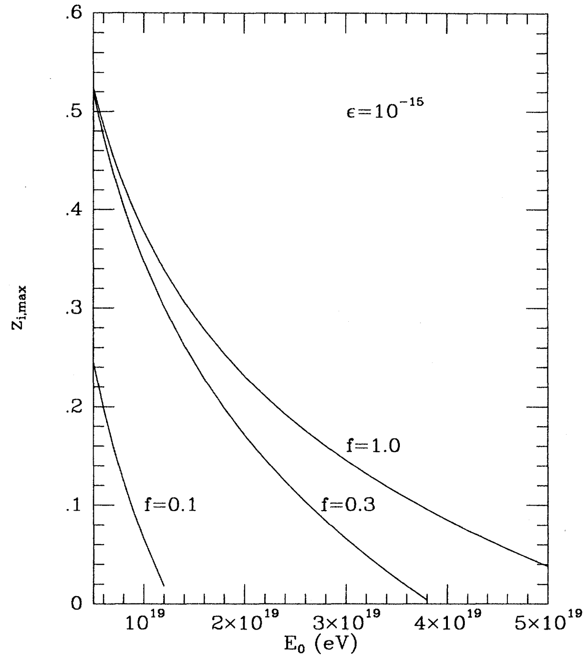

FIG. 1. The maximum possible injection redshift $z_{i,\max}$ as a function of energy (E_0), for $\epsilon = 10^{-15}$ and three different values of f. The curves are obtained by solving Eq. (14).

defined by Eq. (13) can, therefore, be taken as an effective cutoff for the integral in Eq. (7). Actually, since the maximum energy of a particle in our case cannot exceed $\frac{1}{3}E_X$, the cutoff redshift should be determined from the condition

$$(1+z_{i,\max})E_i(z_i=z_{i,\max},E_0)$$
$$=\min\{\tfrac{1}{3}E_X(1+z_{i,\max}),\ 6\times 10^{19}\ \text{eV}\}\ , \quad (14)$$

where E_X is given by Eq. (3). Equations (11)–(14) provide us with a consistent set of equations for determining the cutoff $z_{i,\max}$ for any given value of E_0. Figure 1 shows the behavior of $z_{i,\max}(E_0)$. All $z_{i,\max}$ are $\ll z_{\rm eq}$ and so we need to be concerned with cusp evaporation occurring in the matter-dominated era only.

VI. THE LENGTH DISTRIBUTION FUNCTION (LDF) FOR CS LOOPS

What remains now is to specify the CS loop LDF, $dn(L,t_i)/dL$. This is known from numerical simulations[22] of evolution of CS, which yield a so-called "scaling solution"[23,22] according to which, on the average, a number β of non-self-intersecting loops of length $L_f=\alpha t_f$ are formed per horizon volume per expansion time at any time t_f, which gives a LDF at any formation time t_f as

$$\frac{dn}{dL_f}(L_f,t_f)=\beta\alpha^3 L_f^{-4}\ . \quad (15)$$

The exact values of the quantities β and α are not certain, but representative values[22] are $\beta\sim 10$ and $\alpha\sim 0.01$. After formation, the loops oscillate, radiate energy and, as a result, shrink in length. The loop LDF at any arbitrary time t is determined by the energy-loss rate. The primary mode of energy loss of oscillating loops is gravitational radiation, which occurs at a rate[6,24]

$$\left[\frac{dE}{dt}\right]_{\rm grav} = -\Gamma\epsilon\mu \ , \tag{16}$$

where Γ is a constant which is independent of the length of the loop but depends on its shape. Typically,[6,24] $\Gamma \sim 100$. Now, comparing (16) with (1) we see that loops of length $L < \gamma_c^3 \Gamma^{-3}\epsilon^{-7/2} M_{\rm Pl}^{-1}$ lose energy primarily through cusp evaporation rather than through gravitational radiation. Let us consider all loops which survive at least one expansion time $\sim t$ at any time t. For the loops losing energy primarily through gravitational radiation, the 'lifetime" of a loop of length L is, from Eq. (16), $\tau_{\rm grav} \sim (\Gamma\epsilon)^{-1}L$, and the minimum length of a loop which survives one expansion time scale at time t is $L_{\rm min}^{\rm grav}(t) \sim \Gamma\epsilon t$. If cusp evaporation dominates, the corresponding quantities are $\tau_{\rm cusp} \sim \gamma_c^{-1}\mu^{1/6}L^{4/3}$ and $L_{\rm min}^{\rm cusp}(t) \sim \gamma_c^{3/4}\epsilon^{-1/8}t^{3/4}t_{\rm Pl}^{1/4}$. Thus[10] there is a time t_* given by

$$t_* = \gamma_c^3 \Gamma^{-4}\epsilon^{-9/2} t_{\rm Pl} \ , \tag{17}$$

such that, for $t < t_*$, the loop LDF is primarily determined by cusp evaporation rather than gravitational radiation. From these considerations and taking into account the depletion of the number density of the loops due to expansion of the Universe subsequent to their formation, one gets from (15) the following LDF at any time t.

(i) For $t < t_*$,

$$dn(L,t) \simeq \begin{cases} \beta\alpha^3 L^{-4}\left[\dfrac{a(L/\alpha)}{a(t)}\right]^3 dL \quad \text{if } L_{\rm min}^{\rm cusp}(t) \le L \le \alpha t \ , \\ 0 \quad \text{if} \quad L < L_{\rm min}^{\rm cusp}(t) \ . \end{cases} \tag{18}$$

(ii) For $t > t_*$,

$$dn(L,t) \simeq \begin{cases} \beta\alpha^3 L^{-4}\left[\dfrac{a(L/\alpha)}{a(t)}\right]^3 dL \quad \text{if } \Gamma\epsilon t \le L \le \alpha t \ , \\ \beta\alpha^3 (\Gamma\epsilon t)^{-4}\left[\dfrac{a(\Gamma\epsilon t/\alpha)}{a(t)}\right]^3 dL \quad \text{if } L_{\rm min}^{\rm cusp}(t) \le L < \Gamma\epsilon t \ , \\ 0 \quad \text{if } L < L_{\rm min}^{\rm cusp} \ . \end{cases} \tag{19}$$

In deriving Eqs. (18) and (19) we have assumed that the loops survive with their lengths essentially unchanged till the end of their lifetime at which they instantaneously disappear.

VII. CALCULATION OF THE FLUX

We are now ready to evaluate the flux from Eq. (7). First let us define

$$I_L \equiv t_0^{10/3} \int dL \frac{dn}{dL}(L,t_i) L^{-1/3} \ .$$

Using (18) and (19) together with the appropriate forms for the scale factor of the Universe in the matter- and radiation-dominated epochs, these L integrals are easily evaluated. After some algebra, and expressing t_i in terms of z_i by the relation $t_i = t_0(1+z_i)^{-3/2}$, we get the following.

(i) For $t_i < t_*$,

$$I_L = \begin{cases} \kappa_1(1+z_i)^{9/2} - \kappa_2(1+z_i)^5 \quad \text{for } (1+z_i) \le Z_1 \ , \\ \kappa_3(1+z_i)^{81/16} + \kappa_4(1+z_i)^3 - \kappa_5(1+z_i)^5 \quad \text{for } (1+z_i) > Z_1 \ , \end{cases} \tag{20}$$

where $Z_1 = \gamma_c^{2/3}\epsilon^{-1/9}\alpha^{-8/9}(t_{\rm Pl}/t_0)^{2/9}(1+z_{\rm eq})^{4/3}$, and

$$\begin{aligned} &\kappa_1 = \tfrac{3}{4}\beta\alpha\gamma_c^{-1}\epsilon^{1/6}\left[\frac{t_{\rm Pl}}{t_0}\right]^{-1/3}, \quad \kappa_2 = \tfrac{3}{4}\beta\alpha^{-1/3}, \quad \kappa_3 = \tfrac{6}{11}\beta\alpha^{3/2}\gamma_c^{-11/8}\epsilon^{11/48}\left[\frac{t_{\rm eq}}{t_0}\right]^{1/2}\left[\frac{t_{\rm Pl}}{t_0}\right]^{-11/24}, \\ &\kappa_4 = \tfrac{9}{44}\beta\alpha^{-1/3}\left[\frac{t_{\rm eq}}{t_0}\right]^{-4/3}, \quad \kappa_5 = \kappa_2 \ . \end{aligned} \tag{21}$$

(ii) For $t_i \geq t_*$,

$$I_L = \begin{cases} A_1(1+z_i)^5 - A_2(1+z_i)^{21/4} & \text{for } (1+z_i) \leq Z_2 , \\ B_1(1+z_i)^{23/4} - B_2(1+z_i)^6 + B_3(1+z_i)^3 - B_4(1+z_i)^5 & \text{for } (1+z_i) > Z_2 , \end{cases} \quad (22)$$

where $Z_2 = (\Gamma\epsilon/\alpha)^{2/3}(1+z_{\rm eq})$, and

$$A_1 = \tfrac{9}{4}\beta\alpha(\Gamma\epsilon)^{-4/3}, \quad A_2 = \tfrac{3}{2}\beta\alpha\gamma_c^{1/2}\Gamma^{-2}\epsilon^{-25/12}\left[\frac{t_{\rm Pl}}{t_0}\right]^{1/6}, \quad B_1 = \tfrac{45}{22}\beta\alpha^{3/2}(\Gamma\epsilon)^{-11/6}\left[\frac{t_{\rm eq}}{t_0}\right]^{1/2},$$
$$B_2 = \tfrac{3}{2}\beta\alpha^{3/2}\gamma_c^{1/2}\Gamma^{-5/2}\epsilon^{-31/12}\left[\frac{t_{\rm eq}}{t_0}\right]^{1/2}\left[\frac{t_{\rm Pl}}{t_0}\right]^{1/6}, \quad B_3 = \tfrac{9}{44}\beta\alpha^{-1/3}\left[\frac{t_{\rm eq}}{t_0}\right]^{-4/3}, \quad B_4 = \tfrac{3}{4}\beta\alpha^{-1/3} . \quad (23)$$

One may explicitly check that the I_L's given above are positive definite, as they should be, in the respective domains of their validity as defined above.

Now, using Eqs. (7), (5), (3), and (12), and reinstating c's and $\hbar$'s in proper units, we get the expression for the flux $j(E_0)$ as

$$j(E_0) \simeq 3.793\times 10^{-17}\gamma_c f^{-1/2}\epsilon^{7/12}\left[\frac{3E_0}{\rm eV}\right]^{-3/2} \times \int_0^{z_{i,\max}(E_0)} dz_i (1+z_i)^{-6}\exp\left[-\frac{1}{3}\frac{\beta_0}{H_0}[(1+z_i)^{3/2}-1]\right]$$
$$\times\left[1-\left[\frac{3E_0\epsilon^{-1/2}}{fM_{\rm Pl}}\right](1+z_i)\exp\left[\frac{2}{3}\frac{\beta_0}{H_0}[(1+z_i)^{3/2}-1]\right]\right]^2 I_L(\epsilon,z_i)({\rm eV}^{-1}\,{\rm s}^{-1}\,{\rm m}^{-2}\,{\rm sr}^{-1}) , \quad (24)$$

where $I_L(\epsilon,z_i)$ is given by Eqs. (20)–(23), and $\beta_0/H_0 \approx 2.79$.

Notice that I_L is a function of ϵ and z_i. For any given injection time $t_i \geq t_*$, in which case gravitational radiation from the loops govern the form of their LDF, we see from Eqs. (22)–(24) that the flux increases as ϵ is decreased—the dominant contribution to the flux at any given energy behaves as $\epsilon^{-1.25}$ for $t_i < \alpha t_{\rm eq}(\Gamma\epsilon)^{-1}$ and as $\epsilon^{-0.75}$ for $t_i \geq \alpha t_{\rm eq}(\Gamma\epsilon)^{-1}$. On the other hand, we see from Eq. (17) that the time t_* increases as ϵ is decreased. So for a sufficiently small value of ϵ, a given injection time will eventually satisfy $t_i < t_*$. When this happens, the energy loss in the form of cusp evaporation itself governs the behavior of the loop LDF, and we see from Eqs. (20), (21), and (24) that the contribution to the present-day flux from that injection time decreases with further decrease of ϵ—the dominant contribution to the flux goes as $\epsilon^{13/16}$ for those t_i's which satisfy $L_{\min}^{\rm cusp}(t_i) < \alpha t_{\rm eq}$, and as $\epsilon^{0.75}$ otherwise. Normally, for the "standard" value of $\epsilon \sim 10^{-6}$, we have $t_* \approx 5.4\times 10^{-25}$ s (with $\gamma_c = 1$, $\Gamma = 100$) so that all relevant injection times t_i satisfy $t_i > t_*$. On the other hand, for a sufficiently small value of ϵ, say, $\epsilon = 10^{-16}$, we have $t_* \simeq 5.4\times 10^{20}$ s $\gg t_0$, so that all injection times satisfy $t_i < t_*$, in which case the flux decreases with further decrease of ϵ. In our calculation of the flux, we have used the appropriate form of the loop LDF given by Eq. (18) or (19) depending on the values of ϵ and t_i.

VIII. RESULTS, DISCUSSIONS, AND CONCLUSIONS

The integrals over z_i in Eq. (24) are the exponential integral functions which are easily evaluated. The value of $z_{i,\max}$ for any given value of E_0 is found by solving Eq. (14) (see Fig. 1) once we fix the values of f and ϵ. Our results are shown in Fig. 2 for the case when $f=1$, i.e., $E_X = \mu^{1/2}$, and for various values of ϵ. We have taken $\gamma_c = 1$, $\alpha = 0.01$, $\beta = 10$, and $\Gamma = 100$. For $\epsilon = 10^{-6}$, the flux is at least 12 orders of magnitude below the flux of UHE protons observed by the Fly's Eye Group,[19] for example. However, at any given value of the energy E_0, the flux initially increases as the value of ϵ is made smaller. The flux is highest at $\epsilon \approx 10^{-15}$ (at $\epsilon \approx 8.7\times 10^{-16}$, to be very accurate), and it then decreases with further decrease of ϵ. The peak flux remains at least four orders of magnitude below the observed flux. There is thus no lower limit on ϵ.

One might think, by looking at Eq. (24), that decreasing the value of f [i.e., E_X; see Eq. (3)] may give a higher value of the flux at any given energy. This is true as long as the values of f and ϵ are such that $E_X > 6\times 10^{19}$ eV. However, if f is made too small, eventually one gets

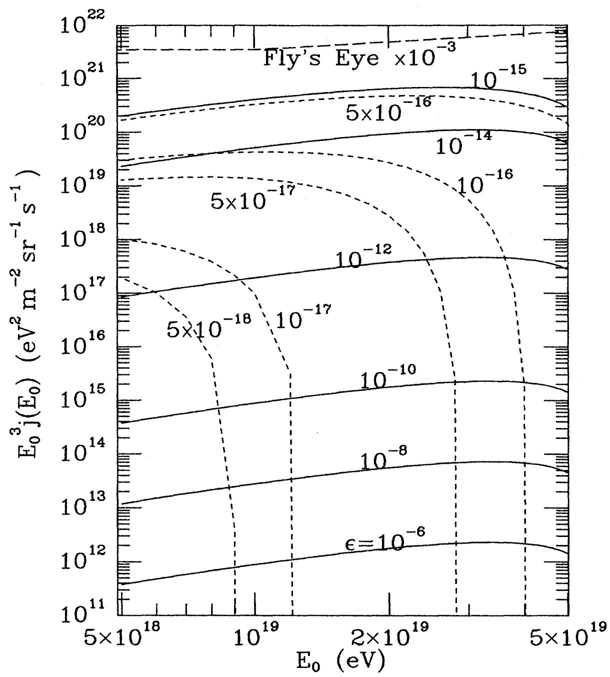

FIG. 2. The predicted flux of ultrahigh-energy protons as a function of energy (E_0) for the case $f=1$ (i.e., $E_X=\epsilon^{1/2}M_{\rm Pl}$, E_X being the energy of a primary X particle emitted from the string) and for various values of the CS parameter ϵ. The values of other constants are $\alpha=0.01$, $\beta=10$, $\gamma_c=1$, and $\Gamma=100$. For clarity of display, the curves for $\epsilon \lesssim 5\times10^{-16}$ are short dashed; the value of ϵ for each of these curves is such that the flux at any given energy decreases if the value of ϵ is decreased further, because the loop LDF (at the injection times which give the dominant contribution to the flux) for these values of ϵ is determined by the energy loss of the loops through cusp evaporation rather than gravitational radiation. The upper dashed line ($j\propto E_0^{-3}$ for $E_0\lesssim 10^{19}$ eV and $j\propto E_0^{-2.5}$ for 10^{19} eV $<E_0\lesssim 5\times10^{19}$ eV) represents the UHE cosmic-ray proton spectrum ($\times10^{-3}$) observed by the Fly's Eye group (Ref. 19).

$E_X < 6\times10^{19}$ eV, in which case Eq. (14) yields a smaller value of $z_{i,\max}$ (than what one would get for the case $E_X > 6\times10^{19}$ eV; see Fig. 1) resulting in a smaller flux. Moreover, for too small values of f and ϵ one gets $E_X < E_0$, in which case, obviously, no particles of the given energy can be produced in the first place. Explicit calculation shows that the peak flux always remains below the value obtained with $f=1$ and $\epsilon\approx10^{-15}$.

Note also that in all the above calculations we have assumed that the cusp evaporation process occurs at the maximum efficiency ($\gamma_c=1$). If $\gamma_c \ll 1$, then all the above values of the fluxes will be correspondingly lower.

Now, consider the case of neutrinos. First note that for $\epsilon \lesssim 5.43\times10^{-16}$, Eq. (17) gives $t_* \gtrsim t_0 \approx 2.67\times10^{17}$ s (for $\Omega_0=1$, $h=0.75$). In this case, obviously, all the injection times t_i satisfy $t_i < t_*$ irrespective of whether one is considering neutrinos or protons. Equations (20) and (21) then imply that the values of the flux at all energies will decrease with further decrease of the value of ϵ for $\epsilon \lesssim 5.43\times10^{-16}$. So the lower limit $\epsilon \gtrsim 10^{-17}$ found in Ref. 4 will probably disappear when the correct form of the loop LDF is used. Similarly, for the case $\epsilon=10^{-15}$, we have $t_* \approx 1.7\times10^{16}$ s, and with $E_X=10^{15}$ GeV and for $E_0=10^{19}$ eV, say, we have for neutrinos[4,25] $1+z_{i,\max}=E_X/E_0=10^5$, implying that the earliest possible time of injection ($t_{i,\min}$) satisfies $t_{i,\min} \ll t_*$. So, the contribution to the present-day flux from the t_i's in the range $t_{i,\min}\le t_i \le t_*$, when calculated by using the loop LDF as determined by cusp evaporation itself [Eqs. (20) and (21)], will give a lower value of the flux (at the given energy) than what is obtained in Ref. 4. Further reduction of the value of ϵ will then reduce the flux further. Thus the lower bound, $\epsilon \gtrsim 10^{-15}$, found in Ref. 4 will also, it seems, disappear, unless the values of some other parameters (e.g., β) are significantly different from their currently favored values.

In summary, we have estimated the UHE proton flux resulting from the CS cusp evaporation process and found that the flux at all energies remains below the observed flux, and that there is no lower limit on the temperature of a CS-forming phase transition in the early Universe as far as high-energy particle production from CS cusp evaporation is concerned.

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