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Some problems with extended inflation

Erick J. Weinberg
Physics Department, Columbia University, New York, New York 10027
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The recently proposed extended inflation scenario is examined. Upper bounds on the Brans-Dicke parameter ω are obtained by requiring that the recovery from the supercooled regime be such that the presently observed Universe could have emerged. These bounds are well below the present-day experimental limits, implying that one must use models which have a potential to fix the present value of the Brans-Dicke-like scalar field. The implications for extended inflation in such models are discussed.

I. INTRODUCTION

The inflationary universe model[1] was proposed some time ago as a solution to a number of cosmological puzzles, most notably the horizon and flatness problems. Its essential feature is a period of rapid expansion, during which the energy density of the Universe remains essentially constant, followed by a period of thermalization, in which this energy density is converted to radiation. A successful resolution of the cosmological problems requires that the expansion period be long enough that the cosmic scale factor $R(t)$ can increase by a factor of at least 10^{27}.

In "old inflation," the scenario which was originally suggested, the inflationary scheme was implemented by means of a first-order phase transition during which the Universe supercooled into a de Sitter-like regime where $R(t)$ grew exponentially with time. This inflationary period was to be terminated by the completion of the transition and the concomitant release and thermalization of the latent heat. However, it was shown that this completion never occurs.[2,3] The new phase, although occupying an ever-increasing fraction of space, never percolates, but instead remains confined to finite clusters of bubbles. Detailed analysis of the distribution of bubbles within these clusters shows that it is extremely unlikely that a homogeneous thermalized region of sufficient size to contain the observed Universe would emerge.

An exception occurs in models with unusually flat scalar potentials. In contrast with the usual case, these models allow the possibility of our Universe developing within the interior of a single bubble. The difficulty with this "new inflation"[4,5] scenario is that it appears to require extreme fine-tuning of the parameters of the particle physics theory responsible for the phase transition. Similar problems arise in the "chaotic inflation"[6] variation of this scheme.

La and Steinhardt[7,8] have recently proposed a new scenario, which they term "extended inflation." Like old inflation, it is based on a supercooled first-order transition which is globally completed throughout space. The essential difference is that gravity is described not by general relativity, but by the scalar-tensor Brans-Dicke theory.[9] The effect of this is that during the vacuum-dominated era R does not grow exponentially, but rather as $t^{\omega+1/2}$ (for large t), where ω is the dimensionless Brans-Dicke parameter. Since time-delay experiments[10] require that $\omega > 500$, the enormous increase in R needed for successful inflation can easily be achieved. At the same time, the change from exponential to power-law behavior is enough to ensure that percolation eventually occurs.

It is not necessary that the scalar-tensor gravity theory be fundamental. For example, such a theory could arise as an effective theory in the context of superstring models,[11] although typically one would then expect to obtain an ω of order unity. Furthermore, the extended inflation scheme can be readily adapted to models in which there is a nontrivial potential for the scalar field.

In this paper I examine this scenario in some detail, to see whether it really does offer the possibility of obtaining inflation without unnatural fine-tuning of parameters. I begin by reviewing some relevant facts concerning cosmological phase transitions and old inflation in Sec. II. In Sec. III, I outline the details of extended inflation and obtain some constraints on the bubble nucleation rate. The kinematics of bubble nucleation and growth in the supercooled regime are studied in Sec. IV, with particular emphasis on the implications for the thermalization and reheating process. I argue that this process could yield the homogeneity and isotropy of the present Universe, if at all, only with values of ω well below the astrophysical bounds. A discussion of these results and of possible variations on the scenario is contained in Sec. V. The Appendix contains some results concerning the Robertson-Walker solutions of the Brans-Dicke equations.

II. COSMOLOGICAL FIRST-ORDER PHASE TRANSITIONS

It is believed that as the early Universe expanded and cooled it experienced a series of phase transitions. Of interest here is the case of a first-order transition, in which the high-temperature phase remains metastable at temperatures below the critical temperature T_c. (This metastability may eventually disappear, or, as will be assumed for the remainder of this discussion, may survive down to

$T=0$.) Because the transition proceeds by the nucleation of bubbles of the low-temperature phase, it cannot occur instantaneously when the critical temperature is reached. Since the cosmic expansion continues to drive the temperature downward, the Universe enters a period of supercooling. The ultimate degree of this supercooling is determined by the competition between the rates of bubble nucleation and of the cosmic expansion.

The bubble nucleation rate per unit volume λ can be calculated by semiclassical methods. At high temperature, where bubble nucleation is driven by thermal fluctuations, these give[12,13]

$$\lambda(T)=A(T)e^{-E(T)/T}\,, \tag{2.1}$$

where $E(T)$ may be interpreted as the energy of a bubble of critical size. The prefactor $A(T)$ is equal to M^4 times terms expected to be of order unity, where $M\sim T_c$ is the mass scale characterizing the phase transition. As the temperature falls from T_c the critical bubble size, and thus $E(T)$, decreases, giving a rapid rise in λ. This effect is eventually offset by the reduction in thermal fluctuations, signaled by the growth of the $1/T$ factor in the exponent. The result is that, after reaching a value $\lambda_{\max}$ at a temperature T^*, the thermal nucleation rate decreases sharply. At low temperatures, the dominant mechanism for bubble nucleation is quantum-mechanical tunneling, the rate of which can be written in the form[14,15]

$$\lambda_0=A_0e^{-S}\,. \tag{2.2}$$

Here S is the action of the "bounce" solution of the Euclideanized field equations and again the prefactor is equal to M^4 times terms of order unity.

The velocity with which the bubbles expand after nucleation varies with temperature. For the low-temperature bubbles nucleated according to Eq. (2.2), which are the ones we will be primarily concerned with, this velocity rapidly approaches that of light.[14] This implies that the region outside the bubbles cannot be affected by them, and thus justifies the use of a Robertson-Walker metric to describe the old-phase regions even though their homogeneity is manifestly destroyed by the presence of bubbles.

With gravity described by general relativity, the growth of the cosmic scale factor is controlled by the energy density $\rho(T)$. As the Universe cools below T_c, the radiation component of ρ is soon dominated by the vacuum contribution ρ_{vac}, and $R\approx R(0)\exp(\chi t)$, where

$$\chi=\left[\frac{8\pi\rho_{\mathrm{vac}}}{3M_P^2}\right]^{1/2} \tag{2.3}$$

and $t=0$ is defined to be the beginning of the vacuum-dominated era.

One measure of the progress of the transition is the fraction of space which remains in the high-temperature phase at time t. If the t_B denotes the time at which bubble nucleation begins, this is given by[16]

$$p(t)=\exp\left[-\int_{t_B}^{t}dt'\lambda(t')R^3(t')\frac{4\pi}{3}r^3(t,t')\right]\,, \tag{2.4}$$

where $r(t,t')$ is the coordinate radius at time t of a bubble which was nucleated at t'. It is relatively straightforward to reexpress this formula in terms of temperature. For $T<T^*$, one obtains[17]

$$p(T)=\exp\left\{-\left[c_1\frac{\lambda_{\max}}{\chi^4}\left[\frac{T^*-T}{T^*}\right]^3+c_2\frac{\lambda_0}{\chi^4}\ln\left[\frac{T_c}{T}\right]\right]\right\}\,, \tag{2.5}$$

where c_1 and c_2 are factors of order unity.

Two cases are clearly distinguished, depending on whether $\lambda_{\max}/\chi^4$ is much greater or much less than unity. In the former case, which might be termed a "fast transition," the Universe will be essentially all converted to true vacuum before T^* is reached, and there will be no extreme supercooling. The transition proceeds very rapidly once $\lambda(T)$ (which increases rapidly for $T>T^*$) becomes greater than χ^4. Many bubbles nucleate, expand, and coalesce on a time scale much shorter than that of the cosmic expansion, making the transition very much like a noncosmological one.

The latter, "slow transition," case is the one envisioned in old inflation. Although initially very small, the exponent in Eq. (2.5) does eventually become large, and $p(t)$ asymptotically approaches zero. However, the transition is never quite completed. One symptom of this is the failure of the new phase to percolate, i.e., to form a bubble cluster of infinite extent. Specifically, it can be shown[2] that percolation fails to occur if

$$\epsilon_0\equiv\frac{\lambda_0}{\chi^4} \tag{2.6}$$

is less than a critical value ϵ_{cr} which lies in the range $0.24>\epsilon_{\mathrm{cr}}>1.1\times10^{-6}$. (Somewhat less rigorous arguments[18] suggest $\epsilon_{\mathrm{cr}}\approx0.03$.)

The explanation of this result lies in the existence of event horizons in an exponentially expanding Universe. Even expanding at the speed of light, a bubble which nucleates at a time t can only grow to a finite comoving radius $r_{\mathrm{as}}(t)\sim\chi^{-1}\exp(-\chi t)$. One consequence is that if the separation between two bubbles at time t is greater than $2r_{\mathrm{as}}(t)$, the bubbles will never meet. Therefore the bubbles nucleated in a time interval of duration of, say, χ^{-1} can never fill space by themselves, but instead only occupy a fraction of order ϵ_0 of the region which remained in the old phase at the time that they were nucleated. Although bubble nucleation continues indefinitely, the bubbles produced have smaller and smaller comoving volume and so can fit in the remaining regions of old phase without overlapping. Indeed, the physical volume of the old phase region, which is proportional to $R^3(t)p(t)$, is an increasing function of time.

Given that the new phase never percolates, one must address the possibility that the observed Universe could have developed within either a single bubble or a finite cluster of bubbles. The difficulty with the former is that the energy released as the expanding bubble converts old phase to new is not distributed throughout the bubble, but is instead carried forward in a thin shell at the bubble wall.[14] The bubble interior itself remains a region of essentially pure true vacuum until collisions with other bubbles cause the bubble walls to dissolve. Only in the case of very exceptional sorts of dynamics (those giving rise to new inflation) can the interior of an uncollided bubble be consistent with the observed universe.

In finite bubble clusters, some of the latent heat is distributed through the interior. The difficulty lies in achieving a homogeneous and isotropic distribution of this energy over a region large enough to develop into the presently observed Universe. We might expect homogeneous thermalization to occur if many bubbles of comparable size coalesced within a short period of time. This is precisely what happens in a fast cosmological (or almost any noncosmological) first-order transition, where essentially all of the bubbles are produced within a short spurt of nucleation. In a slow transition, where the bubbles are nucleated steadily over a long period of time, there is instead a very broad distribution of bubble sizes. Analysis of the kinematics of bubble collisions[2] shows that these are overwhelmingly likely to be between bubbles of very disparate size, and that any cluster is dominated by a single bubble much larger than all the rest.[19] One might guess, and detailed examination verifies, that such unequal collisions do not yield the required homogeneity and isotropy.

III. INFLATION IN A BRANS-DICKE CONTEXT

La and Steinhardt[7,8] suggest that these problems might be evaded by working in the context of Brans-Dicke theory.[9] The action for this theory is

$$S=\int d^4x\sqrt{g}\left[-\Phi R+\omega\frac{(\partial_\mu\Phi)^2}{\Phi}+16\pi\mathcal{L}_{\text{matter}}\right]. \tag{3.1}$$

Here Φ is a scalar field which plays the role of a time-dependent gravitational coupling; it is normalized so that its present value is equal to the square of the Planck mass, M_P^2. The interactions of the other fields are described by $\mathcal{L}_{\text{matter}}$; these are assumed to be such that there is a first-order phase transition which can give rise to the supercooling needed to drive the inflation. The dimensionless parameter ω characterizes the theory; in the limit $\omega\to\infty$, the theory reduces to general relativity. As noted earlier, experiment requires that $\omega>500$.

For a homogeneous and isotropic universe described by a Robertson-Walker metric, the field equations of this theory reduce to[20]

$$\left[\frac{\dot R}{R}\right]^2=\frac{8\pi\rho}{3\Phi}-\frac{k}{R^2}+\frac{\omega}{6}\left[\frac{\dot\Phi}{\Phi}\right]^2-\frac{\dot R}{R}\frac{\dot\Phi}{\Phi} \tag{3.2}$$

and

$$\ddot\Phi+3\frac{\dot R}{R}\dot\Phi=\frac{8\pi(\rho-3p)}{2\omega+3}, \tag{3.3}$$

where $k=1$, 0, or -1 for a closed, flat, or open universe. These equations require three boundary conditions. One is fixed, as in general relativity, by the definition of the zero of time. A second follows from the requirement that the present value of Φ be equal to M_P^2. The third boundary condition must be treated as a free parameter. Thus, for a particular form of ρ and a fixed value of k there will be a one-parameter family of solutions, rather than the single solution of general-relativistic cosmology.

Since the curvature term in Eq. (3.2) is negligible compared to the energy-density term for small R, it is sufficient to consider only $k=0$. For this case there are power-law solutions both for a matter-dominated and for a radiation-dominated universe. While these correspond to special values of the initial conditions, all other solutions approach these at large time.[21] The matter-dominated solution is

$$R(t)\sim t^{(2\omega+2)/(3\omega+4)}, \tag{3.4a}$$

$$\Phi(t)\sim t^{2/(3\omega+4)}, \tag{3.4b}$$

while for the radiation-dominated case the power-law solution is the same as in general relativity: namely,

$$R(t)\sim t^{1/2}, \tag{3.5a}$$

$$\Phi=\text{const}. \tag{3.5b}$$

There is also a power-law solution for the $k=0$ vacuum-dominated case.[22] If $t=0$ is defined to be the beginning of the vacuum-dominated era, and the arbitrariness in the scale of R is fixed by setting $R(0)=1$, this solution takes the form

$$R(t)=(1+Bt)^{\omega+1/2}, \tag{3.6a}$$

$$\Phi=\Phi(0)(1+Bt)^2, \tag{3.6b}$$

where

$$B=\left[\frac{32\pi\rho_{\text{vac}}}{(6\omega+5)(2\omega+3)\Phi(0)}\right]^{1/2}. \tag{3.7}$$

In the general relativity limit, $\omega\to\infty$, the scale factor becomes an exponential, $R(t)\sim\exp(\tilde\chi t)$, where $\tilde\chi$ is given by Eq. (2.3), but with M_P^2 replaced by $\Phi(0)$. For large but finite ω, one can distinguish an exponential regime, $t\lesssim B^{-1}$, and a power-law regime, $t\gtrsim B^{-1}$. The approach of other solutions to this power-law form is discussed in the Appendix.

The Hubble parameter is not constant in a vacuum-dominated universe, as in general relativity, but varies with time according to

$$H(t)\equiv\frac{\dot R}{R}=\frac{(\omega+\frac{1}{2})B}{1+Bt} \tag{3.8}$$

so that the constant ϵ_0 of Eq. (2.6) is replaced by the time-dependent quantity

$$\epsilon(t)\equiv\frac{\lambda_0}{H(t)^4}=\frac{\lambda_0}{[(\omega+\frac{1}{2})B]^4}(1+Bt)^4. \tag{3.9}$$

With a suitably small nucleation rate λ_0, ϵ can be small enough at the beginning of the vacuum-dominated era to give a long period of supercooling. During the course of the transition it will increase, so that eventually, after a time of order $\omega\lambda_0^{-1/4}$, the criterion for percolation will be met. Similarly, one finds that the physical volume of old phase per unit comoving volume, $p(t)R^3(t)$, initially increases, but then begins to decrease after a time of order $\omega\lambda_0^{-1/4}$. It therefore seems reasonable to expect the phase transition to be completed at a time

$$t_{\rm end}=q(\omega-\tfrac{1}{2})\lambda_0^{-1/4}\ , \qquad (3.10)$$

where q is of order unity.

During the course of the transition the scale factor will increase by a factor

$$\frac{R(t_{\rm end})}{R(0)}=\left[\frac{\Phi(t_{\rm end})}{\Phi(0)}\right]^{(\omega+1/2)/2} . \qquad (3.11)$$

The growth of $\Phi(t)$ in any subsequent radiation- or matter-dominated eras will be small [see Eq. (A9)], so $\Phi(t_{\rm end})\approx M_P^2$. In order to justify the neglect of quantum gravity effects, we must require that the initial value of Φ be greater than M^2, where $M\sim T_c\sim\rho_{\rm vac}^{1/4}$ is the mass scale characterizing the phase transition. Requiring that the scale factor grow by at least 27 orders of magnitude yields the inequality

$$10^{27}<\frac{R(t_{\rm end})}{R(0)}<\left[\frac{M_P}{M}\right]^{\omega+1/2} . \qquad (3.12)$$

For $M\sim10^{14}$ GeV, this only requires that $\omega\gtrsim6$, a far less stringent restriction than the experimental bound.

Almost all of the transition takes place during the power-law regime, $t\gtrsim B^{-1}$. This can be seen by combining Eqs. (3.8) and (3.10) to yield

$$Bt_{\rm end}=q\left[\frac{\omega-\frac{1}{2}}{\omega+\frac{1}{2}}\right]\left[\frac{H^4(0)}{\lambda_0}\right]^{1/4}\gg\left[\frac{H^4(0)}{\lambda_{\rm max}}\right]^{1/4}\gtrsim1\ . \qquad (3.13)$$

The second relationship is obtained by noting that the zero-temperature bubble nucleation rate is less than the maximum value of the high-temperature nucleation rate; because both quantities vary exponentially [see Eqs. (2.1) and (2.2)], the inequality is almost certainly strong. The final inequality follows from the very fact that the Universe has reached a strongly supercooled state; if it were not satisfied, the transition would have been "fast" and would have been completed by high-temperature bubble nucleation.

From this inequality, together with Eqs. (3.6b) and (3.10), it follows that

$$\Phi(t_{\rm end})=\frac{32\pi q^2(\omega-\frac{1}{2})^2\rho_{\rm vac}}{(6\omega+5)(2\omega+3)\lambda_0^{1/2}}\approx\frac{8\pi q^2\rho_{\rm vac}}{3\lambda_0^{1/2}} \qquad (3.14)$$

with the second line being valid for large ω. Since $\Phi(t_{\rm end})\approx M_P^2$,

$$\lambda_0=\left[\frac{8\pi}{3}\right]^2 q^4\frac{\rho_{\rm vac}^2}{M_P^4}\ . \qquad (3.15)$$

Alternatively, Eq. (2.2), with $A_0\approx M^4$, gives

$$e^S\approx\left[\frac{M_P}{M}\right]^4 . \qquad (3.16)$$

A further condition follows from the last inequality in Eq. (3.10). This implies that $\lambda_{\rm max}\lesssim[\rho_{\rm vac}/\Phi(0)]^2$, and hence that

$$\frac{\lambda_{\rm max}}{\lambda_0}\lesssim\frac{M_P^4}{\Phi(0)^2}\lesssim\left[\frac{M_P}{M}\right]^4 . \qquad (3.17)$$

Note that Eqs. (3.15) and (3.16) involve neither ω nor $\Phi(0)$. The only adjustable parameters are those of the particle physics theory, which determine both $\rho_{\rm vac}$ and λ_0. These parameters must be chosen to be such that these equations are satisfied. This is not fine-tuning in the technical sense, since in the Brans-Dicke context the Planck mass is simply the evolved value of the gravitational coupling, and Eq. (3.15) describes the outcome of that evolution. Nevertheless, the fact that M_P has a particular experimentally determined value severely constrains the choice of models for extended inflation. In particular, the fact that S typically varies inversely with coupling constants makes it likely that these couplings must be at least moderately strong if Eq. (3.16) is to be satisfied with a plausible value for M (Ref. 23).

IV. RECOVERY FROM THE SUPERCOOLED REGIME

The failure of old inflation is often attributed simply to the nonpercolation of the new phase. However, percolation by itself would not be sufficient to ensure successful inflation. The latent heat, which is released as the bubble walls dissolve in bubble collisions, must be converted to radiation in thermal equilibrium, and this thermalization must occur at a temperature high enough that the successful features of the standard cosmology can be recovered. Furthermore, this reheating process must yield a universe which is homogeneous and isotropic over a region large enough to contain the presently observed Universe. In this section I will examine the question of whether these additional requirements can be met within the context of extended inflation.

For this analysis we will need to know the densities of bubbles of various sizes, as well as some statistics which describe the collisions between various bubbles. When deriving these, it is most convenient to work in terms of comoving coordinates and volumes, with the time coordinate being the Robertson-Walker time in the old phase region. (Because bubble nucleation and expansion can be described "from the outside," the metric inside the bubbles is not yet relevant.) The conversion to physical dimensions can always be done by multiplying by appropriate powers of $R(t)$. In particular, the nucleation rate per unit comoving volume is $\lambda_0 R^3(t)$.

To begin, consider the growth of a single bubble. It is a good approximation to treat this as nucleating with zero radius at a time t_0 and then expanding along lightlike geodesics. Assuming that it has not lost its identity through collisions with other bubbles, its coordinate radius at some later time t will be

$$r(t,t_0)=\int_{t_0}^{t}\frac{dt'}{R(t')}$$
$$=\frac{1}{B(\omega-\frac{1}{2})}\left[\frac{1}{(1+Bt_0)^{\omega-1/2}}-\frac{1}{(1+Bt)^{\omega-1/2}}\right] . \tag{4.1}$$

As $t\to\infty$, this approaches a finite asymptotic value

$$r_{as}(t_0)=\frac{1}{B(\omega-\frac{1}{2})}\frac{1}{(1+Bt_0)^{\omega-1/2}} . \tag{4.2}$$

For later reference, note that solving these equations for t_0 gives

$$1+Bt_0=(1+Bt)\left[\frac{r_{as}(t)}{r+r_{as}(t)}\right]^{1/(\omega-1/2)}$$
$$=(1+Bt)\left[\frac{r_{as}(t)}{r_{as}(t_0)}\right]^{1/(\omega-1/2)} . \tag{4.3}$$

A convenient reference point for setting the scale of bubble sizes is $r_{as}(t_{end})\equiv r_0$. At the completion of the transition this corresponds to a physical distance

$$r_0R(t_{end})=\frac{1+Bt_{end}}{B(\omega-\frac{1}{2})}$$
$$\approx q\lambda_0^{-1/4}$$
$$\approx\left[\frac{3}{8\pi}\right]^{1/2}\frac{M_P}{\rho_{vac}^{1/2}} . \tag{4.4}$$

This should be compared to the radius r_{Univ} of the region which evolved into the presently observed portion of the Universe. If the subsequent expansion was approximately adiabatic, then $r_{Univ}R(t_{end})T(t_{end})\approx r_{Univ}R(t_{now})T_{now}\approx 10^{29}$, where $T(t_{end})$ is the temperature established at the completion of the transition. Assuming this to be of order $\rho_{vac}^{1/4}\sim M$,

$$r_{Univ}\approx 10^{29}\frac{M}{M_P}r_0 . \tag{4.5}$$

The fraction of space still remaining in the old phase at time t is given by Eq. (2.4). Substituting Eqs. (3.6) and (4.1) into this expression, and using Eq. (3.15) to eliminate λ_0, yields

$$p(t)=\exp\left[-\frac{\pi q^4(\omega-\frac{1}{2})}{3(Bt_{end})^4}[(1+Bt)^4g(\omega)-1+O((1+Bt)^{-(\omega-1/2)})]\right] , \tag{4.6}$$

where

$$g(\omega)=1-\frac{24}{2\omega+7}+\frac{12}{2\omega+3}-\frac{8}{6\omega+5} . \tag{4.7}$$

For Bt and ω both large, this reduces to

$$p(t)\approx\exp\left[-\frac{\pi}{3}q^4\omega\left[\frac{t}{t_{end}}\right]^4\right]$$
$$\approx\exp\left[-\frac{\pi}{3}q^4\omega\left[\frac{r_0}{r_{as}(t)}\right]^{\delta}\right] \tag{4.8}$$

with

$$\delta\equiv\frac{4}{\omega-\frac{1}{2}} . \tag{4.9}$$

[It may seem a bit disturbing that although $p(t)$ can become quite small, it never quite vanishes. This is inevitable, given the random nature of bubble nucleation. However, once $p(t)$ becomes sufficiently small there is a high probability of finding a suitably large region—much greater than the presently observed Universe—which is entirely in the new phase, and this is all that is really needed.]

Now let $\rho(r,t)dr$ be the number of bubbles per unit coordinate volume whose radii lie in the range r to $r+dr$. These bubbles were nucleated at a time t_0 given by Eq. (4.3). The nucleation rate per unit coordinate volume at that time was $\lambda_0R^3(t_0)$, while the fraction of space available for bubble nucleation was $p(t_0)$. Therefore,

$$\rho(r,t)=\lambda_0R^3(t_0)p(t_0)\frac{dt_0}{dr}$$
$$=\lambda_0R^4(t_0)p(t_0) . \tag{4.10}$$

Using Eqs. (3.6), (3.10), (4.2), and (4.3), we find

$$\rho(r,t)=q^4\left[\frac{1+Bt}{Bt_{end}}\right]^4\left[\frac{1}{r+r_{as}(t)}\right]^4$$
$$\times\left[\frac{r_{as}(t)}{r+r_{as}(t)}\right]^{\delta}p(t_0(r)) . \tag{4.11}$$

In particular, at $t=t_{end}$,

$$\rho(r,t_{end})=q^4\left[\frac{1+Bt_{end}}{Bt_{end}}\right]^4\left[\frac{1}{r+r_0}\right]^4\left[\frac{r_0}{r+r_0}\right]^{\delta}p(t_0(r))$$
$$\approx q^4\left[\frac{1}{r+r_0}\right]^4\left[\frac{r_0}{r+r_0}\right]^{\delta}$$
$$\times\exp\left[-\frac{\pi}{3}q^4\omega\left[\frac{r_0}{r+r_0}\right]^{\delta}\right] . \tag{4.12}$$

This expression represents the average of the number density over all of space. When examined on distance scales of order r, this density varies considerably from place to place. It vanishes in regions occupied by bubbles much larger than r, while in the spaces between such bubbles the suppression by $p(t_0(r))$ (i.e., the exponential factor in the second line) should be omitted. It follows that, given any bubble of radius r, there will typically be a bubble of equal or greater radius within a distance of order $rq^{-4/3}(r/r_0)^{\delta/3}$.

In addition to the number density, we will also need to know the volume occupied by large bubbles, in particular the volume fraction $\mathcal{V}_>(r,t_{end})$ contained in bubbles greater than a given size r at $t_{\rm end}$. Multiplying the expression in Eq. (4.12) by $(4\pi/3)r^3$ and then integrating would give an overestimate, since regions lying in the overlaps of two or more bubbles would be counted twice. These overlaps can be taken into account by using Eq. (2.4), but with λ_0 replaced by $\lambda_0\theta(t^*-t)$, where t^* is the nucleation time corresponding to bubble radius r at $t_{\rm end}$. Thus,

$$\mathcal{V}_>(r,t_{\rm end})=1-\exp\left[-\int_0^{t^*}dt'\lambda_0 R^3(t')\frac{4\pi}{3}r^3(t,t')\right]$$
$$\approx 1-\exp\left[-\frac{\pi}{3}q^4\left[\omega-\frac{1}{2}\right]\left[\frac{r_0}{r}\right]^{\delta}\right], \quad (4.13)$$

where the second of these equalities is valid for $Bt^*\gg 1$ and $r\gg r_0$. For r large enough that $\mathcal{V}_>(r,t_{\rm end})$ is small, the expansion of the exponential, together with Eq. (4.8), gives

$$\mathcal{V}_>(r,t_{\rm end})\approx \ln[p^{-1}(t_{\rm end})]\left[\frac{r_0}{r}\right]^{\delta}. \quad (4.14)$$

The bubble distribution described by Eqs. (4.12) and (4.13) depends on a scale r_0, but that dependence is rather weak. Once r is large enough that $r+r_0\approx r$, the distribution is essentially scale invariant until $(r/r_0)^\delta$ begins to differ significantly from unity. With large ω, and thus small δ, this deviation from a scale-invariant distribution—such as one had with old inflation—begins only at enormous values of r. It is only when one examines the distribution on such scales that the suppression of large bubbles can be seen. Thus, if we want a randomly chosen region to neither lie within a single bubble nor have any large fraction within a single bubble, we must require that radius d of the region be such that $(d/r_0)^\delta$ is not too close to unity. Requiring, for example, that the presently observed portion of the Universe should be such a region would place an upper bound on ω; some considerations leading to stronger bounds will be pursued below.

Now consider the set of bubbles which collide with a given reference bubble, which is nucleated at a time t. Without any loss of generality, the center of the reference bubble can be chosen to be at $r=0$. A bubble which nucleates at a point with radial coordinate r' at a time t' can collide with the reference bubble if

$$r'<r_{\rm as}(t)+r_{\rm as}(t') . \quad (4.15)$$

In order that spacetime point where the younger of the two bubbles nucleates not be already within the older bubble, we must also require that

$$r'>|r_{\rm as}(t)-r_{\rm as}(t')| . \quad (4.16)$$

The probability that the reference bubble will collide with a bubble formed in the interval t' to $t'+dt'$ is equal to $\lambda_0 R^3(t')dt'$ times the coordinate volume of the region defined by Eqs. (4.15) and (4.16). In particular, the expected number of older bubbles which collide with the reference bubble is

$$N_-(t)=\int_0^t dt'\lambda_0 R^3(t')\frac{4\pi}{3}\{[r_{\rm as}(t')+r_{\rm as}(t)]^3-[r_{\rm as}(t')-r_{\rm as}(t)]^3\} . \quad (4.17)$$

(This is actually an overestimate, since the fact that these bubbles cannot form within one another has been ignored.) By changing variables to $y=r_{\rm as}(t)/r_{\rm as}(t')$ and using Eqs. (3.6), (3.10), (3.13), and (4.2), this can be rewritten as

$$N_-(t)=A(t)I_-(t) , \quad (4.18)$$

where

$$A(t)=\frac{4\pi}{3}\lambda_0 r_{\rm as}^4(t)R^4(t)=\frac{4\pi}{3}q^4\left[\frac{r_0}{r_{\rm as}(t)}\right]^{\delta} \quad (4.19)$$

and

$$I_-=2\int_{r_{\rm as}(t)/r_{\rm as}(0)}^{1}dy\,(3+y^2)y^{\delta}$$
$$=\frac{20}{3}+O(\delta)+O\left[\left[\frac{r_{\rm as}(t)}{r_{\rm as}(0)}\right]^{1+\delta}\right]. \quad (4.20)$$

The expected volume in these bubbles, measured in units of the asymptotic volume of the reference bubble $r_{\rm as}(t)$, and with the overlaps between bubbles double counted, is $\mathcal{V}_-(t)=A(t)J_-(t)$, where

$$J_-=2\int_{r_{\rm as}(t)/r_{\rm as}(0)}^{1}dy\,(3+y^2)y^{-(3-\delta)}$$
$$\approx 3\left[\frac{r_{\rm as}(0)}{r_{\rm as}(t)}\right]^{2-\delta}. \quad (4.21)$$

Similar calculations for the younger bubbles give $N_+(t)=A(t)I_+(t)$ and $\mathcal{V}_+(t)=A(t)J_+(t)$, with

$$I_+=2\int_{r_0/r_{\rm as}(t)}^{1}dy\,(3+y^2)y^{-(3+\delta)}$$
$$\approx 3\left[\frac{r_{\rm as}(t)}{r_0}\right]^{2+\delta}, \quad (4.22)$$

$$J_+=2\int_{r_0/r_{\rm as}(t)}^{1}dy\,(3+y^2)y^{-\delta}$$
$$=\frac{20}{3}+O(\delta)+O\left[\left[\frac{r_0}{r_{\rm as}(t)}\right]^{1-\delta}\right]. \quad (4.23)$$

Just as in old inflation, a bubble will collide with many more younger than older bubbles, although the latter will be far greater in total volume. For very large bubbles (those large enough to see the departure from a scale-invariant bubble distribution) $A(t)$ is small, so that most such bubbles collide only with smaller bubbles. Comparison of Eqs. (4.13) and (4.19) shows that most of the volume is occupied by bubbles with small values of $A(t)$.

Let us now turn to the thermalization process. As noted previously, the latent heat of the phase transition is initially stored in the walls of the expanding bubbles;[14] these "walls" are essentially thin regions in which the scalar field responsible for the transition varies rapidly between its new phase and old phase values. When two bubbles collide, the bubble walls separating them disappear and the stored energy can begin to spread through the bubble interior. This energy initially appears as coherent scalar field waves propagating into the bubble interior,[3] but is presumably converted to incoherent radiation through the interactions of the scalar field with itself and with the other matter fields. This radiation must then evolve into a homogeneous and isotropic thermal distribution.

How long does this process take within any given bubble? A clear lower bound is a few times the time required for light to cross the bubble. To calculate this we need to know the details of the metric inside the bubble. Before the bubble has undergone any collisions, its interior is essentially a pure vacuum of the new phase, and so has a metric which is either de Sitter-like or flat, depending on the vacuum energy density of the new phase. Once the latent energy of the transition starts propagating into the bubble interior this simple picture is lost and the best we can hope for is a rough approximation to the metric. A plausible guess is that the energy released as the waves in the scalar field propagate through the interior gives a region which expands at roughly the same rate as a radiation-dominated Robertson-Walker universe. This gives a picture in which the larger bubbles, where the thermalization and homogenization of the latent heat is still taking place, expand at the same rate as the surrounding regions, evolved from smaller bubbles, which have already entered the standard radiation-dominated Robertson-Walker scenario. The region within a bubble of coordinate radius r will not complete the recovery from the supercooled regime until after the horizon distance $d_H(T) \sim M_P/T^2$ has become equal or greater than $rR(t)$. If $R(t) \sim R(t_{\rm end})T(t_{\rm end})/T(t) \sim R(t_{\rm end})M/T(t)$, the temperature of the Universe will have fallen by that time to Mr_0/r.

There will clearly be conflicts with the standard cosmology if too many large bubbles are still completing the thermalization process at nucleosynthesis, or even later, times. Such considerations can be used to place upper bounds on ω. If we require that the fraction of space in such regions be less than 10^{-n} when the temperature is T, then

$$\omega < \frac{4\log_{10}(M/T)}{n+\log_{10}\ln[p^{-1}(t_{\rm end})]} < \frac{4}{n}\log_{10}\left[\frac{M}{T}\right], \qquad (4.24)$$

where the second inequality follows from the fairly mild assumption that $p(t_{\rm end}) < 1/e$. Given the quantitative success of the predictions for the nucleosynthesis of the light elements, it is hardly extreme to require that no more than 10% of space be still undergoing thermalization when $T \approx 100$ keV. If $M \approx 10^{14}$ GeV, this translates into the bound $\omega < 76$. A more stringent bound can be obtained by considering the Universe at the time of recombination. To maintain the bounds on the anisotropy of the microwave background, let us require that no more than 10^{-4} to 10^{-3} of space be still undergoing thermalization at $T \approx 4000$ K. This gives an upper bound on ω which ranges from 24 to 32.

While these bounds are certainly not precise, it is clear that values of ω large enough to meet the present-day experimental limits cannot be accommodated. Furthermore, even meeting these bounds would hardly be sufficient to assure a satisfactory reheating process, since the arguments above have clearly underestimated the difficulties in obtaining large-scale homogeneous thermalization. This is particularly true of the largest bubbles, which tend to collide primarily with much smaller bubbles, since we should expect the energy released in such collisions to primarily flow outward from the larger bubble.

Another aspect of the reheating process also leads one to low values of ω. In a general Robertson-Walker universe there is a preferred choice of the time coordinate, namely, that in which hypersurfaces of constant time are homogeneous and isotropic. In the process of reestablishing a hot Robertson-Walker universe from the aftermath of the supercooled inflationary period, there must be some means of ensuring that the same preferred coordinate frame is established everywhere.

Some sense of the obstacles to this can be gained by considering the situation in old inflation, where the vacuum-dominated universe locally approximates a de Sitter spacetime. Because the O(4,1) de Sitter symmetry includes Lorentz-like transformations which mix space and time coordinates, there is no uniquely preferred coordinate frame. As bubble clusters form during the course of the (never-ending) transition, the random nucleation of bubbles picks out certain choices of the time coordinate in some regions (e.g., in the overlap of two bubbles, those choices which make the two bubbles have the same age), but there is no correlation of these choices between different clusters.

In extended inflation the bubble clusters meet to form an infinite region of new phase, but the problem of obtaining the same distinguished coordinate system in widely separated regions still remains. It is clear from causality considerations that this frame cannot be newly established after the inflationary period. Instead, there must be some mechanism for retaining a memory of the original Robertson-Walker frame.[24] In a transition which was completed at high temperature the necessary information could be carried by the temperature, but this surely plays no role in an inflationary transition which supercools to essentially $T=0$. Instead, we must look to either the record of the time evolution of R, as reflected in the distribution of bubble sizes, or to the value of the Brans-Dicke scalar field Φ; these are essentially equivalent, since in the old phase regions $H=\dot{R}/R \sim \Phi^{-1/2} \sim t^{-1}$. Since the case of constant H and Φ corresponds to the de Sitter case, where there is no distinguished frame, we must require that there be a significant variation of these quantities. The time scale over which we should look is that relevant for the presently observed Universe, i.e., the interval from $t(r_{\rm Univ})$, when bubbles with asymptotic coordinate size equal to that of the observed Universe were being nucleated, until $t_{\rm end}$. In fact, since the homogeneity and isotropy were certainly established by the time of recombination, and most likely well before nucleosynthesis took place, the relevant interval is even shorter. From Eqs. (3.8) and (4.3),

$$\frac{H(t)}{H(t_{\rm end})}=\left[\frac{r_{\rm as}(t)}{r_0}\right]^{1/(\omega-1/2)}\approx\left[\frac{M}{T}\right]^{1/(\omega-1/2)}, \quad (4.25)$$

where T is the temperature corresponding to $r_{\rm as}(t)$. A variation of at least one order of magnitude in H over the interval defined by the nucleosynthesis temperature would then require that $\omega < 18+\log_{10}(M/10^{14}\ \text{GeV})$. While it is not certain that a variation of this magnitude would be enough to differentiate the situation from the de Sitter case (it might conceivably even be a bit more than is needed), it is clear that we are once again being led to values of ω which are far too low to be consistent with present-day observation.

V. CONCLUDING REMARKS

We have seen that the extended inflation scenario with a pure Brans-Dicke theory can yield a cosmologically acceptable outcome only if ω is not too great. The existence of such an upper bound should not be surprising, since in the $\omega\to\infty$ limit, where Brans-Dicke theory reduces to general relativity, extended inflation should coincide with old inflation, which is known to fail. The difficulty lies not with the existence of an upper bound on ω, but rather with its value.

For a phase transition characterized by a mass scale M of the order of 10^{14} GeV, the arguments of Sec. IV give upper bounds on ω of several tens, certainly well below 100. These bounds become stronger as M is decreased. Increasing M weakens the bounds slightly, but the requirement that $M<M_P$, together with the restrictions [(3.15)–(3.17)] on the bubble nucleation rate, preclude any significant gain from moving in this direction. Furthermore, the true bound on ω is probably even lower than the values cited here, for two reasons. First, the difficulties of converting latent heat to homogeneous and isotropic thermal energy, either within a single bubble or over a much larger region, were certainly underestimated. Second, these bounds were obtained by requiring consistency with the standard cosmology at nucleosynthesis and later times. Obtaining a satisfactory scenario at earlier times (e.g., up to electroweak or higher temperatures) and requiring sufficient baryogenesis might well place further restrictions on ω. In fact, it is quite plausible that there is no value of ω which is large enough to give sufficient inflation and yet low enough to give a satisfactory reheating process.

In any case, ω must certainly lie below the present-day experimental limits. We must therefore abandon the pure Brans-Dicke theory and work with models which have a potential to fix the present value of Φ at M_P^2. The simplest example one could imagine would have a $V(\Phi)$ which was essentially flat up to some value $\Phi_0<M_P^2$ and then had a minimum at $\Phi=M_P^2$. Assuming that $V(\Phi)$ was small compared to the vacuum energy of the matter fields, the scenario would proceed very much as in the pure Brans-Dicke theory provided that the transition was completed before Φ was trapped at its present value. (If the transition were not completed by this point, the scenario would pass over into the general relativistic old inflation.) This requirement would imply a restriction of the form $\lambda_0<M^8/M_P^4$ [cf. Eq. (3.15)], as well as the inequality (3.17). Because M_P is an input parameter of this model, rather than simply the outcome of the Brans-Dicke evolution, any fine-tuning needed to satisfy these inequalities should be viewed as unnatural.

One can easily envision more complicated models.[25] The general requirements are clear. One wants the effective gravitational coupling to evolve at the same time that the matter fields are undergoing a first-order phase transition. This should occur in such a manner that the Hubble parameter, while initially large enough that $\lambda_{\max}/H^4\lesssim 1$, eventually decreases to the point that $\lambda_0/H^4>\epsilon_{\rm cr}$. This evolution should be relatively prolonged, so that there will be sufficient inflation. Finally, to obtain a satisfactory thermalization and reheating process, the final decrease in H should be relatively rapid, probably faster than the $1/t$ behavior of the pure Brans-Dicke theory. Showing that such a model can be constructed, preferably without unnaturally fine-tuned parameters, remains an outstanding challenge.

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APPENDIX

Consider first the case of a radiation-dominated $k=0$ universe. As in general relativity,

$$\rho R^4\equiv A=\text{const.} \quad (A1)$$

Since $\rho=3p$, Eq. (3.3) implies that

$$\dot\Phi R^3\equiv B=\text{const.} \quad (A2)$$

These results, together with Eq. (3.2), give

$$\dot R=\frac{1}{2\psi}\left\{-B\pm\left[\left[\frac{2\omega+3}{3}\right]B^2+\frac{32\pi}{3}A\psi\right]^{1/2}\right\}, \quad (A3)$$

where

$$\psi\equiv\Phi R^2\,. \quad (A4)$$

From Eqs. (A2)–(A4) it follows that

$$\dot\psi=\pm\frac{1}{R}\left[\left[\frac{2\omega+3}{3}\right]B^2+\frac{32\pi}{3}A\psi\right]^{1/2} \quad (A5)$$

and hence that

$$\frac{dR}{d\psi}=\frac{\dot R}{\dot\psi}=\frac{R}{2\psi}\left[1\mp\frac{B}{\left[\left[\frac{2\omega+3}{3}\right]B^2+\frac{32\pi}{3}A\psi\right]^{1/2}}\right]. \quad (A6)$$

This equation is readily integrated, and one finally obtains

$$\Phi=C\left[\frac{\left[\dfrac{32\pi A\Phi R^2}{(2\omega+3)B^2}+1\right]^{1/2}-1}{\left[\dfrac{32\pi A\Phi R^2}{(2\omega+3)B^2}+1\right]^{1/2}+1}\right]^{\pm\sqrt{3/(2\omega+3)}}, \quad \text{(A7)}$$

where C is a constant of integration. For either choice of sign, Φ tends toward this constant as R becomes large. Equation (3.2) then reduces to the usual general-relativistic equation, whose solution is Eq. (3.5).

We can estimate the growth of Φ after an inflationary era by matching this solution onto a vacuum-dominated solution of the form of Eq. (3.6). Assuming for simplicity a sharp transition from the vacuum-dominated to the radiation-dominated regime at $t=t_{\text{end}}$, with $\rho_{\text{rad}}=\rho_{\text{vac}}$ and with R, Φ, and $\dot{\Phi}$ all continuous, we would have

$$\frac{32\pi A\Phi(t_{\text{end}})R^2(t_{\text{end}})}{(2\omega+3)B^2}=\frac{32\pi\rho(t_{\text{end}})\Phi(t_{\text{end}})}{(2\omega+3)\dot{\Phi}^2(t_{\text{end}})}=\frac{6\omega+5}{4}. \quad \text{(A8)}$$

Substituting this into Eq. (A7), and using $\Phi(\infty)=C$, we obtain

$$\frac{\Phi(\infty)}{\Phi(t_{\text{end}})}=\left[\frac{\sqrt{6\omega+9}-2}{\sqrt{6\omega+9}+2}\right]^{\mp\sqrt{3/(2\omega+3)}}=1\pm\frac{2}{\omega}+O(\omega^{-2}). \quad \text{(A9)}$$

Next consider a vacuum-dominated universe, also with $k=0$. As usual, $p_{\text{vac}}=-\rho_{\text{vac}}$. Solving Eq. (3.2) for $\dot{R}/R$ and then substituting the result into Eq. (3.3) yields

$$\frac{\dot{R}}{R}=\frac{\dot{\Phi}}{2\Phi}\left\{-1\pm\left[\left[\frac{2\omega+3}{3}\right]+\frac{32\pi\rho_{\text{vac}}\Phi}{3\dot{\Phi}^2}\right]^{1/2}\right\}, \quad \text{(A10)}$$

$$\ddot{\Phi}+\frac{3\dot{\Phi}^2}{2\Phi}\left\{-1\pm\left[\left[\frac{2\omega+3}{3}\right]+\frac{32\pi\rho_{\text{vac}}\Phi}{3\dot{\Phi}^2}\right]^{1/2}\right\}-\frac{32\pi\rho_{\text{vac}}}{2\omega+3}=0. \quad \text{(A11)}$$

Now consider the possibilities for the large-time behavior of $z\equiv\dot{\Phi}^2/\Phi$; to have gravity attractive, let us assume that Φ, hence z, are positive (note that Φ cannot change sign without the solution becoming singular). I will also assume that $\omega>0$. If z tends toward a constant value, the upper choice of sign gives a solution which approaches that of Eq. (3.6); the lower sign leads to a solution with negative Φ. If z were to approach 0, the first and last terms in Eq. (A11) would dominate; these would imply $\Phi\sim t^2$ and hence $z\sim$const, contradicting the assumption $z\rightarrow 0$. Finally, the hypothesis that z becomes arbitrarily large leads to $z\sim t^{-(2-2/p)}$, with $p=-1\pm\sqrt{6\omega+9}$; for either sign choice this is a decreasing function, again contradicting the hypothesis.

Thus, any solution with positive Φ should tend toward the form of Eq. (3.6) at large time. To verify this, at least for nearby solutions, let $\Phi(t)=\Phi_0(t)+\delta\Phi(t)$, where $\Phi_0(t)$ is given by Eq. (3.6). To first order in $\delta\Phi$, Eq. (A11) (with the upper choice of sign) becomes

$$\delta\ddot{\Phi}+\left[\frac{6\omega+1}{2}\right]\left[B\frac{\delta\dot{\Phi}}{1+Bt}-\frac{B^2\delta\Phi}{(1+Bt)^2}\right]=0. \quad \text{(A12)}$$

The general solution of this is

$$\delta\Phi=c_1(1+Bt)^{-(6\omega+1)/2}+c_2(1+Bt). \quad \text{(A13)}$$

The first term on the right-hand side falls rapidly with time, as expected. The second term grows linearly, but can be eliminated by shifting the zero of time, i.e., by the transformation $t\rightarrow t+c_2/[2B\Phi(0)]$.

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