# Breaking of SU(3) in vector-meson radiative decays

A. Bramon and M. D. Scadron*
*Grup de Física Teórica, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*
(Received 28 July 1989)

The radiative decays of charged and neutral $K^*$ vector mesons are discussed in a general quark-model context. Their ratio is found to be particularly appropriate for the analysis of SU(3) breaking in terms of the ratio $\overline{M}/M$ of strange-to-nonstrange (constituent) quark masses. The standard value $\overline{M}/M \simeq 1.5$ is clearly favored. Other $V \to P\gamma$ decays are discussed.

The vast migration of the Particle Data Group[1] branching ratios and decay rates for the vector-to-pseudoscalar-meson radiative transitions $V \to P\gamma$ and also $P \to V\gamma$ over the last decade has left our belief in SU(3) symmetry and its breaking pattern somewhat in confusion.[2] On the one hand, the new results[1] $\Gamma(\omega \to \pi\gamma)/\Gamma(\rho \to \pi\gamma) = 9.9 \pm 1.9$ and $B(\eta' \to \omega\gamma/\rho\gamma) = 0.10 \pm 0.01$ are now close to the predicted SU(3) ratios

$$\frac{\Gamma(\omega \to \pi\gamma)}{\Gamma(\rho \to \pi\gamma)} = 9\left[\frac{p_\omega}{p_\rho}\right]^3 = 9.5\ ,$$
$$B\left[\frac{\eta' \to \omega\gamma}{\eta' \to \rho\gamma}\right] = \frac{1}{9}\left[\frac{p_\omega}{p_\rho}\right]^3 \approx 0.093\ , \tag{1}$$

obtained assuming ideally mixed $\omega$ and $\phi$ mesons. On the other hand, however, the measured $K^*$ branching ratio[1]

$$B\left[\frac{K^{*0} \to K^0\gamma}{K^{*+} \to K^+\gamma}\right] = 2.34 \pm 0.31 \tag{2}$$

remains quite far from the exact SU(3) ratio of 4, thus requiring substantial symmetry breaking.

In this paper we observe that this situation is really much better than it appears in the latter case. Although the $\omega,\rho \to \pi\gamma$ and also $\eta' \to \omega\gamma,\rho\gamma$ rates in (1) all involve exclusively nonstrange $u$- and $d$-quark radiative transitions, the $K^* \to K\gamma$ rates in (2) excite both strange and nonstrange quarks as well. Therefore $K^* \to K\gamma$ decays naturally should be expected to break SU(3) to the order of the strange-to-nonstrange- (constituent-) quark mass ratio even if symmetric couplings $g_P$ and $g_V$ are employed at quark-meson vertices.

With reference to the constituent-quark triangle graphs of Fig. 1, the $K^* \to K\gamma$ (and other $V \to P\gamma$) decay amplitudes are found to be given by the expression

$$A(K^* \to K\gamma) = -N_C g_V g_P e Q_M \kappa^\mu \epsilon^\nu \int \frac{d^4p}{(2\pi)^4}\, \mathrm{Tr}[\gamma^\nu(\not{p} - M)^{-1}\gamma_5(\not{p} + \not{P} - \not{k} - \overline{M})^{-1}\gamma^\mu(\not{p} - \not{k} - M)^{-1}] + (M \leftrightarrow \overline{M})\ , \tag{3}$$

where $\kappa^\mu$ and $\epsilon^\nu$ ($P$ and $k$) are the $K^*$ and $\gamma$ polarization vectors (four-momenta) and $eQ_{M,\overline{M}}$ are the electric charges of the quarks having (constituent) masses $M,\overline{M}$. The two terms of the amplitude (3) correspond to the two graphs of Fig. 1, where the roles played by the nonstrange (mass $M$) and strange (mass $\overline{M}$) quarks have been interchanged. These two Feynman graphs contain all the relevant dynamics in the context of the colored ($N_C = 3$) constituent quark model. The successes of this model have been known for years and its importance has been more recently emphasized by Manohar and Georgi.[3]

The amplitude (3) has the general covariant form

$$A(K^* \to K\gamma) = i g_{K^*K\gamma} \epsilon_{\alpha\beta\mu\nu} P^\alpha k^\beta \kappa^\mu \epsilon^\nu\ , \tag{4}$$

leading to the decay width $\Gamma(K^* \to K\gamma) = g^2_{K^*K\gamma} k^3/12\pi$. Then the radiative coupling constant is

$$g_{K^*K\gamma} = \frac{N_C g_P g_V e}{4\pi^2}\left[\frac{1}{M} Q_M J_M + \frac{1}{\overline{M}} Q_{\overline{M}} J_{\overline{M}}\right]. \tag{5}$$

Here $J_M$ comes from the Feynman loop integration in Fig. 1 giving after straightforward mathematical manipulations the expression[4]

$$J_M \equiv M \int_0^1 dx \int_0^{1-x} dy \frac{M + (\overline{M} - M)x}{M^2 + (\overline{M}^2 - M^2)x - P^2 xy}$$
$$= \frac{M^2}{P^2}\left[\delta + \sum_{i=1,2}[\mathrm{Li}_2(v_i) - \delta(1 - 1/v_i)\ln|1 - v_i|] - \mathrm{Li}_2(-2\delta) + (\delta - \tfrac{1}{2})\ln|1 + 2\delta|\right] \tag{6}$$

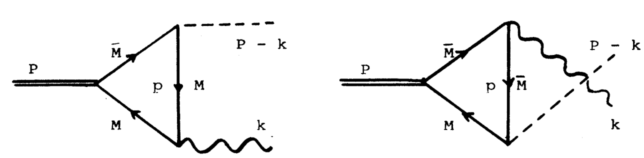


FIG. 1. Diagrams contributing to $K^* \to K\gamma$ (or $V \to P\gamma$) decays. The quark masses in the loop are $M$ and $\overline{M}$.

with $P^2=m_{K^*}^2 \gg m_K^2$, $\delta \equiv (\overline{M}-M)/M$, $J_{\overline{M}}$ corresponds to $J_M$ in Eq. (6) with $M \leftrightarrow \overline{M}$, and

$$v_{1,2} \equiv \frac{P^2}{2M^2}-\delta \pm \left[ \left[ \frac{P^2}{2M^2}-\delta \right]^2 - \frac{P^2}{M^2} \right]^{1/2} .$$

With the standard values for the nonstrange and strange constituent quark masses,[5] $M \simeq \frac{1}{2}m_{\rho,\omega} \simeq 0.34$ GeV and $\overline{M} \simeq \frac{1}{2}m_\phi \simeq 0.51$ GeV, one predicts, from Eqs. (5) and (6),

$$g_{K^{*0}K^0\gamma}/g_{K^{*+}K^+\gamma} = -1.51 , \tag{7}$$

which is fully consistent with the experimental ratio deducible from (2)

$$|g_{K^{*0}K^0\gamma}/g_{K^{*+}K^+\gamma}|_{\text{expt}} = 1.53 \pm 0.11 . \tag{8}$$

It is important to notice that the cancellation of the (unknown) strong coupling constants $g_P$ and $g_V$ when calculating the ratio (7) for neutral over charged $K^*$ meson amplitudes is valid up to negligible SU(2) [not SU(3)] violations. Our prediction (7) depends almost exclusively on the SU(3)-breaking quark mass ratio $\overline{M}/M$. Here the experimental error in (8) allows us to fix $\overline{M}/M$ in the range $1.3 \lesssim \overline{M}/M \lesssim 1.6$, i.e., around the standard value.[5]

Alternatively, in the limit of vanishing $K^*$ masses our expression for the ratio of radiative coupling constants reduces to

$$|g_{K^{*0}K^0\gamma}/g_{K^{*+}K^+\gamma}|_{m_{K^*}^2=0} \simeq -2(1-\delta/2) \simeq -1.5 , \tag{9}$$

in agreement with previous work.[6] The consistency of our prediction (7) and (9), somehow linked to the conventional successes of vector-meson dominance, further justifies the chiral limit $m_K^2 \ll m_{K^*}^2$ in Eq. (6) and confirms the essential role that SU(3) breaking through $M \neq \overline{M}$ plays in the ratio of $K^*$ radiative decays. This latter aspect is clearly shown in Eq. (9) where the exact SU(3) value $-2$ is recovered when breaking effects are turned off, $\delta \to 0$.

Similar effects can be studied comparing $K^* \to K\gamma$ with $\omega,\rho \to \pi\gamma$ radiative decays. In the latter cases, where only nonstrange quarks [with degenerated masses $M$ in the SU(2) limit] appear in Fig. 1, Eqs. (5) and (6) mathematically reduce to

$$\begin{aligned} g_{\omega\pi\gamma} &= 3g_{\rho\pi\gamma} \\ &= \frac{N_C g_P g_V e}{4\pi^2}\frac{2}{\xi^2}\left\{ \frac{\pi^2}{4} - \ln^2\left[ \frac{\xi}{2} + \left[ \frac{\xi^2}{4}-1 \right]^{1/2} \right] \right\}, \end{aligned} \tag{10}$$

where $\xi \equiv M/m_{\omega,\rho}$ and the pion mass has been neglected. For $M=0.34$ GeV, $\overline{M}=0.51$ GeV and the experimental[1] vector-meson masses one finds $\xi \simeq 0.43$ so that the ratio of Eqs. (5) and (10) predicts

$$g_{K^{*0}K^0\gamma}/g_{\omega\pi^0\gamma} = -0.52 , \tag{11}$$

in reasonable agreement with the corresponding experimental[1] ratio $0.56 \pm 0.04$. Notice, however, that in this case the ratios of the coupling constants $g_K/g_\pi$ and $g_{K^*}/g_\omega$ tend to unity only in the exact-SU(3) limit, thus making our prediction (11) less accurate than (7).

The same happens when extending the above analysis to other closely related $V \to P\gamma$ or $P \to V\gamma$ processes such as those quoted in Eqs. (1). In most of these latter processes the $\eta$-$\eta'$ mixing angle $\theta_P$, which introduces a third source of SU(3) breaking, turns out to play an important role. As a result one can achieve a globally satisfactory and consistent description of all these decays[2,7] for $\theta_P \simeq -14°$, although then the superposition of independent SU(3)-breaking effects (different quark masses, coupling constants, and mixing angles) is under much less control.

By contrast, the branching ratio between $K^*$ radiative decays (2) offers a unique opportunity for testing basic aspects of SU(3)-breaking mechanisms. In this paper and in the spirit of the constituent-quark model, those mechanisms simply reduce to an approximate quark mass ratio $\overline{M}/M \simeq 1.5$ (or, taking experimental uncertainties into account, in the range $1.3 \lesssim \overline{M}/M \lesssim 1.6$). This mass ratio is fully compatible with what is needed to explain vector-meson and baryon masses[5,8] or the observed baryon magnetic moments, such as[5] $\mu_p/\mu_\Lambda \simeq -2\overline{M}/M = -2.93$ leading to $\overline{M}/M \simeq 1.47$. Likewise the measured decay-constant ratio $f_K/f_\pi \simeq 1.25$ obtained from $K_{l2}$ and $\pi_{l2}$ decays combined with the Goldberger-Treiman relations at the quark level give the (constituent) mass ratio[9] $\overline{M}/M \simeq 2f_K/f_\pi - 1 \simeq 1.5$. Finally, the experimental ratios among meson charge radii calculated via analogous quark triangle loops can be expressed in terms of a power series in $\delta$ as[10]

$$\begin{aligned} \langle r_{K^+}^2 \rangle / \langle r_{\pi^+}^2 \rangle &\simeq 1 - \tfrac{5}{6}\delta + \tfrac{3}{5}\delta^2 + \cdots \simeq 0.70 \pm 0.12 , \\ \langle r_{K^0}^2 \rangle / \langle r_{\pi^+}^2 \rangle &\simeq -\tfrac{1}{3}\delta + \tfrac{1}{2}\delta^2 + \cdots \simeq 0.12 \pm 0.06 , \\ \langle r_{K\pi}^2 \rangle / \langle r_{\pi^+}^2 \rangle &\simeq 1 - \tfrac{3}{4}\delta + \tfrac{3}{10}\delta^2 + \cdots \simeq 0.8 \pm 0.1 , \end{aligned} \tag{12}$$

and, respectively, corresponds to $\overline{M}/M \equiv 1+\delta \simeq 1.6$, 1.25, and 1.3.

In summary, the $K^* \to K\gamma$ decays have been shown to offer a particularly clear method for testing the standard SU(3)-breaking mechanism in quark triangle diagrams through the strange-to-nonstrange (constituent) mass ratio $\overline{M}/M$. The widely accepted conventional value $\overline{M}/M \simeq 1.5$ is found to explain the data in a direct and fully satisfactory way.

*Note added in proof.* In passing we note that a simpler (non-quark-triangle) $M1$ magnetic-moment model has long been used to describe $V \to P\gamma$ decays. This approach now predicts[11] $\overline{M}/M = 1.24 \pm 0.08$ for the present experimental $K^*$ amplitude ratio (8), slightly below our preferred value of $\overline{M}/M \approx 1.5$. However the $K^*K\gamma/\omega\pi\gamma$ ratio cannot be uniquely determined unless an additional parameter indicating the degree of wave-function overlap is included. Contrast this with our quite reasonable quark-triangle prediction, Eq. (11).

Thanks are due to C. Ayala for discussions and Comisión Interministerial de Ciencia y Tecnología and the U.S. Department of Energy for partial financial support. M.D.S. appreciates the hospitality of the Universitat Antònoma de Barcelona.

*Permanent address: Physics Department, University of Arizona, Tucson, AZ 85721.

[1]Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).

[2]R. L. Thews, Phys. Rev. D **14**, 3021 (1976); **17**, 3038 (1978); J. P. O'Donnell, Rev. Mod. Phys. **53**, 673 (1981).

[3]A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).

[4]The dilogarithm function is defined by

$$\mathrm{Li}_2(z)=\sum_{\nu=1}^{\infty}\nu^{-2}z^{\nu}=-\int_0^z x^{-1}\ln(1-x)dx \ .$$

[5]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).

[6]Ll. Ametller and A. Bramon, Ann. Phys. (N.Y.) **15**, 308 (1984). In this reference we used $\delta m/m$ for our present $\delta\simeq 0.5$.

[7]A. Bramon and M. D. Scadron (unpublished).

[8]N. Isgur and G. Karl, Phys. Rev. D **20**, 1191 (1979).

[9]See, e.g., L. H. Chan, Phys. Rev. Lett. **39**, 1125 (1977); M. D. Scadron, Rep. Prog. Phys. **44**, 213 (1981).

[10]C. Ayala and A. Bramon, Europhys. Lett. **4**, 777 (1987).

[11]E. Sucipto and R. L. Thews, Phys. Rev. D **36**, 2074 (1987).