Pion-pair contribution to the decay $K_L \to \pi^0\gamma\gamma$

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A simple prediction for the decay $K_L \to \pi^0\gamma\gamma$, obtained from the rescattering to two photons of the $\pi^+\pi^-$ intermediate state in the process $K_L \to \pi^+\pi^-\pi^0$, is compared with a recent calculation based on chiral perturbation theory. The $\pi^+\pi^-$ rescattering model predicts a two-photon effective mass spectrum peaked at slightly higher $m_{\gamma\gamma}$ than in chiral perturbation theory, and a branching ratio $B(K_L \to \pi^0\gamma\gamma) \approx 7.5\times10^{-7}$. Implications are discussed for the corresponding decay of the charged kaon.

The experimental study of neutral-kaon decays[1-4] has reached levels of sensitivity which permit detection of several new processes involving photons. Evidence for the decay $K_L \to \gamma\gamma$ has existed for quite some time.[4-6] More recently, the decay $K_S \to \gamma\gamma$ has been seen,[4] at a level corresponding to expectations.[7-9] It now appears that a substantial improvement can be made[10] in the sensitivity of searches for $K_L \to \pi^0\gamma\gamma$, for which the previous upper limit[5,11] is a branching ratio of less than 2.4×10^{-4} (90% C.L.) This process is of particular interest because it can contribute as an intermediate state to the decay $K_L \to \pi^0 e^+ e^-$, for which improved experimental upper limits have become available recently.[2] In this process, the single-photon and electroweak-boson-loop contributions, though *CP*-violating, are most likely more important than the two-photon one,[12-18] but the question remains open as long as $K_L \to \pi^0\gamma\gamma$ has not been measured directly.

The most complete calculation[16] of the process $K_L \to \pi^0\gamma\gamma$ (with intermediate states restricted to the pion octet) is based on chiral perturbation theory. In this paper we compare that result with similar ones from models proposed some time ago,[7,9,19] in which the decay is dominated by a $\pi^+\pi^-$ intermediate state which rescatters to two photons. Although these rescattering models do not have all the features of chiral perturbation theory, they seem to hold both for the decay $K_S \to \gamma\gamma$ and for two-photon production of two neutral pions via the charged-pion-pair intermediate state. In these last two examples, the model's predictions[9] are slightly different from those of chiral perturbation theory,[8,20,21] but experiment cannot tell the difference at present. We wish to see whether one can tell the difference between the two approaches on the basis of the process $K_L \to \pi^0\gamma\gamma$. We shall comment at the end on the additional possible contributions of vector-meson intermediate states, which can be appreciable.

We shall present an expression for the two-charged-pion contribution to the decay $K_L \to \pi^0\gamma\gamma$. We then calculate the two-photon effective mass spectrum using the observed Dalitz-plot distribution in $K_L \to \pi^+\pi^-\pi^0$, and obtain a total decay rate by integrating this spectrum. We compare our results with those of Ref. 16, and conclude with some remarks on the decay $K^\pm \to \pi^\pm\gamma\gamma$.

We first recapitulate some results of Ref. 9 for the decay $K_S \to \gamma\gamma$ in order to calibrate the accuracy expected from the rescattering model. The relation derived in Ref. 9 between the amplitude $A_{\gamma\gamma}$ for $K_S \to \gamma\gamma$ and that $(A_{\pi\pi})$ for the decay $K_S \to \pi^+\pi^-$ is

$$A_{\gamma\gamma} = \alpha A_{\pi\pi}[F(m_K^2) + \Delta R_0(m_K^2)]/2 + s\bar{A}_{\gamma\gamma}(s) , \quad (1)$$

where

$$F(s) = \tfrac{1}{2}h_1 f_0 - h f_0 + \frac{\pi m_\pi^2}{s} - \frac{1}{\pi} , \quad (2)$$

$$h_1(s) \equiv (\beta/2\pi)\ln[(1+\beta)/(1-\beta)] , \quad (3)$$

$$f_0 \equiv \frac{1-\beta^2}{\beta}\ln\frac{1+\beta}{1-\beta} , \quad (4)$$

and

$$h(s) = h_1(s) - i\beta/2 . \quad (5)$$

Here s is the square of the total center-of-mass energy, while $\beta \equiv (1-4m_{\pi^+}^2/s)^{1/2}$ is the velocity of each pion in the $\pi\pi$ center of mass. The term $\Delta R_0(s)$ represents the effect of a polynomial ambiguity[22] in the description of the *S*-wave amplitude for $\gamma\gamma \to \pi\pi$ in a state of isospin $I=0$. One expects $\Delta R_0(s) = O(s/m_\rho^2)$. The remaining term in Eq. (1), $s\bar{A}_{\gamma\gamma}(s)$, represents that portion of the $\gamma\gamma$ decay amplitude which is not dominated by the two-pion intermediate state. For the observed values of particle masses, we find[9]

$$F(m_K^2) = -0.21 + 0.37i . \quad (6)$$

If $\bar{A}_{\gamma\gamma}(s)$ is very small so that two-pion states dominate, then[9]

$$\frac{\Gamma(K_S\to\gamma\gamma)}{\Gamma(K_S\to\pi^+\pi^-)}=\frac{\alpha^2}{4\beta(m_K^2)}|F(m_K^2)+\Delta R_0(m_K^2)|^2 \tag{7}$$

$$=2.94\times10^{-6} \quad \text{for } \Delta R_0(m_K^2)=0\ . \tag{8}$$

Experimentally[4]

$$\frac{\Gamma(K_S\to\gamma\gamma)}{\Gamma(K_S\to\pi^+\pi^-)}=\frac{(2.4\pm1.2)\times10^{-6}}{0.686}$$

$$=(3.5\pm1.8)\times10^{-6}\ . \tag{9}$$

It appears self-consistent to assume that $K_S\to\gamma\gamma$ indeed is dominated by the two-pion intermediate state, and that $\Delta R_0(m_K^2)$ is not too large. In what follows we shall neglect $\Delta R_0(s)$.

Results corresponding to the central experimental value quoted in Eq. (9) are obtained from chiral perturbation theory.[8] Here the amplitude is predicted to contain an additional factor of $1-m_\pi^2/m_K^2$, and the contribution of kaon loops is included. Present experimental data cannot distinguish between the predictions of chiral perturbation theory and the rescattering model [Eq. (8)].

We may apply a similar approach to relate the decay $K_L\to\pi^0\gamma\gamma$ to $K_L\to\pi^+\pi^-\pi^0$. This last process is dominated by a contribution $\Gamma^{(S)}$ in which the charged pions are in a relative S wave, while there is a small additional contribution from a D-wave amplitude, visible only in its interference with the S wave. (A P-wave amplitude violates CP.) We shall be concerned with the contribution of the S-wave charged pion pair's annihilation into the two-photon final state. We then find

$$\frac{d\Gamma(K_L\to\pi^0\gamma\gamma)}{dm_{\gamma\gamma}}=\frac{\alpha^2|F(m_{\gamma\gamma}^2)|^2}{\beta}\frac{d\Gamma^{(S)}(K_L\to\pi^+\pi^-\pi^0)}{dm_{\pi^+\pi^-}}\ , \tag{10}$$

where $\beta\equiv(1-4m_{\pi^+}^2/\hat{s})^{1/2}$, $\hat{s}\equiv m_{\pi^+\pi^-}^2$, and the last term on the right-hand side of Eq. (10) is to be evaluated at $m_{\pi^+\pi^-}=m_{\gamma\gamma}$.

The expression (10) is straightforward for $m_{\gamma\gamma}\geq 2m_\pi$. The Dalitz-plot distribution for the decay $K_L\to\pi^+\pi^-\pi^0$ may be expressed in terms of the square of a covariant matrix element $\mathcal{M}$:

$$\frac{d\Gamma(K_L\to\pi^+\pi^-\pi^0)}{d\hat{\Omega}\,dm_{\pi^+\pi^-}}=\text{const}\times\beta\hat{s}p_{\pi^0}|\mathcal{M}|^2\ , \tag{11}$$

where $\hat{\Omega}$ denotes the direction of the π^+ relative to the neutral pion in the $\pi^+\pi^-$ center of mass, and p_{π^0} is the magnitude of the neutral pion's three-momentum in the rest frame of the kaon:

$$p_{\pi^0}=\lambda^{1/2}(m_K^2,m_{\pi^0}^2,\hat{s})/(2m_K)\ , \tag{12}$$

with

$$\lambda(a,b,c)\equiv a^2+b^2+c^2-2ab-2ac-2bc\ . \tag{13}$$

The square of the matrix element $|\mathcal{M}|^2$ may be parametrized[5] as

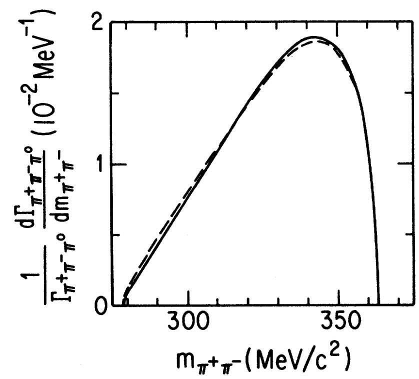

FIG. 1. Normalized spectrum in $m_{\pi^+\pi^-}$ for the decay process $K_L\to\pi^+\pi^-\pi^0$. The solid line is based on the squared matrix element $|\mathcal{M}^{(S)}|^2$ defined on the right-hand side of Eq. (17b); the dashed line comes from using the definition (21).

$$|\mathcal{M}|^2=1+g\frac{\hat{s}-s_0}{m_{\pi^+}^2}+h\left[\frac{\hat{s}-s_0}{m_{\pi^+}^2}\right]^2+k\left[\frac{s_2-s_1}{m_{\pi^+}^2}\right]^2\ . \tag{14}$$

Here

$$s_0\equiv(m_K^2+2m_{\pi^+}^2+m_{\pi^0}^2)/3,\quad s_{2,1}\equiv m_{\pi^\pm\pi^0}^2\ . \tag{15}$$

The coefficients in Eq. (14) have the values[5]

$$g=0.670\pm0.014,\quad h=0.079\pm0.007\ ,$$
$$k=0.0098\pm0.0018\ . \tag{16}$$

Here we have neglected a CP-violating term, for which there are only upper limits.[5]

The coefficients g and h contribute only to $\Gamma^{(S)}$, while k contributes both to $\Gamma^{(S)}$ and to an interference term between the S- and D-wave amplitudes. This interference term vanishes when we integrate over $\hat{\Omega}$, and we find

$$\frac{d\Gamma(K_L^{(S)}\to\pi^+\pi^-\pi^0)}{dm_{\pi^+\pi^-}}=\text{const}\times\beta\hat{s}p_{\pi^0}|\mathcal{M}^{(S)}|^2\ , \tag{17a}$$

$$|\mathcal{M}^{(S)}|^2\equiv1+g\frac{\hat{s}-s_0}{m_{\pi^+}^2}+h\left[\frac{\hat{s}-s_0}{m_{\pi^+}^2}\right]^2+4k\frac{(\hat{s}-4m_{\pi^+}^2)\hat{p}_{\pi^0}^2}{3m_{\pi^+}^4}\ . \tag{17b}$$

Here $\hat{p}_{\pi^0}$ is the three-momentum of the neutral pion in the rest frame of the charged-pion pair: $\hat{p}_{\pi^0}=(m_K/m_{\pi^+\pi^-})p_{\pi^0}$. This spectrum, normalized to the total $\pi^+\pi^-\pi^0$ width, is shown as the solid line in Fig. 1.

In order to continue the prediction for the process $K_L\to\pi^0\gamma\gamma$ below $m_{\gamma\gamma}=2m_\pi$, one must show that all the quantities in Eq. (10) are well defined. The function $|F(s)|^2$, as defined in Eqs. (2)–(5), is well behaved down to $s=0$. For $0\leq s\leq 4m_{\pi^+}^2$, one finds

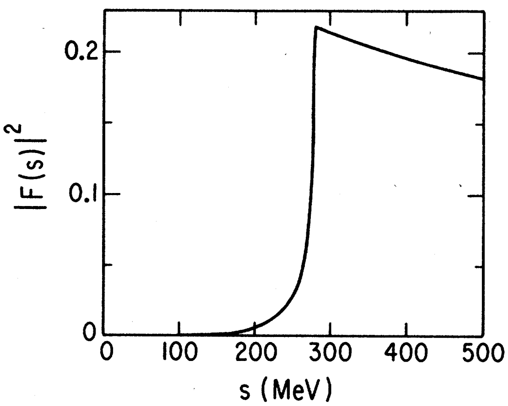

FIG. 2. Squared magnitude of the function $F(s)$ defined in Eqs. (2)–(5) and (18) and (19).

$$F(s)=\frac{4m_{\pi^+}^2}{s}A\left[\frac{A}{\pi}-1\right]+\frac{\pi m_{\pi^+}^2}{s}-\frac{1}{\pi}\ , \quad (18)$$

$$A\equiv\arctan\left[\frac{4m_{\pi^+}^2}{s}-1\right]^{1/2}\ . \quad (19)$$

The function $|F(s)|^2$ is plotted in Fig. 2. It has a cusp at two-pion threshold, above which it varies slowly.

The right-hand side of Eq. (10) also contains the combination

$$\frac{1}{\beta}\frac{d\Gamma(K_L^{(S)}\rightarrow\pi^+\pi^-\pi^0)}{dm_{\pi^+\pi^-}}=\text{const}\times\hat{s}p_{\pi^0}|\mathcal{M}^{(S)}|^2\ , \quad (20)$$

which is nonsingular below $2m_\pi$. Here $|\mathcal{M}^{(s)}|^2$ denotes the quantity on the right-hand side of Eq. (17b). This quantity becomes negative below $\hat{s}\approx(255\text{ MeV})^2$. For the sake of simplicity and in order to obtain a sensible result, we adopt an alternative parametrization[23]

$$\mathcal{M}^{(S)}=1+0.314\frac{\hat{s}-s_0}{m_{\pi^+}^2}\ , \quad (21)$$

which fits the $K_L\rightarrow\pi^+\pi^-\pi^0$ data almost as well. (The spectrum in $m_{\pi^+\pi^-}$ resulting from this fit is shown as the dashed line in Fig. 1.)

The predicted two-photon effective mass spectrum for the decay $K_L\rightarrow\pi^0\gamma\gamma$, normalized to the total rate for $K_L\rightarrow\pi^+\pi^-\pi^0$, is shown as the solid line in Fig. 3. It is peaked at $m_{\gamma\gamma}\approx 337\text{ MeV}/c^2$ and is negligible below 200 MeV/c^2. When integrated over the two-photon invariant mass and multiplied by the branching ratio[5] $B(K_L\rightarrow\pi^+\pi^-\pi^0)=(12.37\pm0.18)\%$, the spectrum yields the estimate

$$B(K_L\rightarrow\pi^0\gamma\gamma)=7.5\times10^{-7}\ . \quad (22)$$

The results from the $\pi^+\pi^-$ scattering model just described may be compared with those obtained in Ref. 16 on the basis of chiral perturbation theory, which leads to the mass spectrum shown as the dashed line in Fig. 3. Here we have evaluated the expression in Ref. 16 using

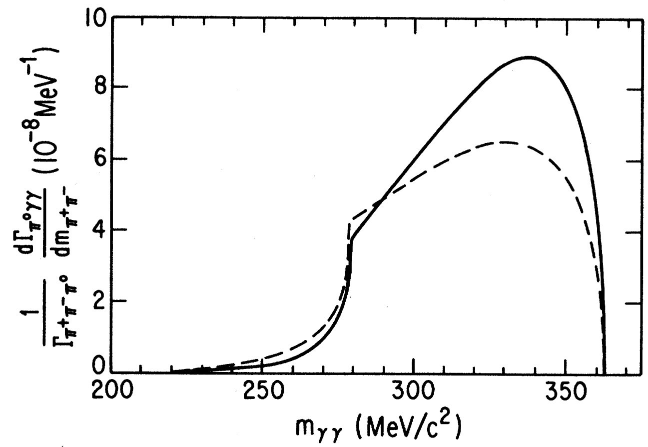

FIG. 3. Predicted spectra in two-photon effective mass $m_{\gamma\gamma}$ for the decay $K_L\rightarrow\pi^0\gamma\gamma$, normalized to the total width for $K_L\rightarrow\pi^+\pi^-\pi^0$. Solid line: $\pi^+\pi^-$ rescattering model; dashed line; chiral perturbation theory.

the charged-pion mass for the calculation of the pion loop, leading to a slightly smaller rate than estimated in Ref. 16. The integral of the mass spectrum under the dashed curve gives $B(K_L\rightarrow\pi^0\gamma\gamma)=6.3\times10^{-7}$.

The pion-loop contribution of Ref. 16 is similar in form to the result based on $\pi^+\pi^-$ rescattering. A factor of $\hat{s}-m_\pi^2$ is contained in the amplitude of Ref. 16, whereas in the rescattering model the matrix element, taken from a fit to experimental data, is linear in $\hat{s}$ but vanishes near $\sqrt{\hat{s}}=200$ MeV. As a consequence, the rescattering model predits a two-photon mass spectrum peaked more strongly toward the high end. The chiral perturbation theory approach also predicts a small contribution from kaon loops, not considered here.

A function identical to $F(s)$ of Eqs. (2)–(5) and (18) and (19) (aside from an overall factor) appears in many other treatments (e.g., Ref. 16) in the equivalent form $(z\equiv s/m_\pi^2)$

$$-\pi F(s)=\begin{cases}1-\dfrac{4}{z}\left[\arcsin\dfrac{\sqrt{z}}{2}\right]^2 & (z\leq4)\ ,\\[2ex] 1-\dfrac{\pi^2}{z}+\dfrac{1}{z}\ln^2\dfrac{1+\beta}{1-\beta}-\dfrac{2i\pi}{z}\ln\dfrac{1+\beta}{1-\beta} & (z\geq4)\ .\end{cases} \quad (23)$$

The inclusion of vector-meson contributions as intermediate states in the decay $K_L\rightarrow\pi^0\gamma\gamma$ can affect the predicted rate and two-photon mass spectrum substantially.[15,17,24] One of the most appealing interpretations of deviations from the predictions presented here, in fact, would be that such contributions are important. The study of such effects is continuing.[24]

Chiral perturbation theory does not give a unique prediction[16] for the corresponding process with charged kaons and pions, $K^\pm\rightarrow\pi^\pm\gamma\gamma$. Our model predicts the contribution for the two-pion rescattering to two photons from the decay $K^\pm\rightarrow\pi^\pm\pi^+\pi^-$. Since the two pions which rescatter are in an $I=0$ state, which we assume to be an S wave, the assumption that the nonleptonic weak interaction is dominated by a $\Delta I=\frac{1}{2}$ transition leads to the relation

$$\Gamma(K^{\pm}\to\pi^{\pm}\gamma\gamma)|_{\pi^+\pi^-\to\gamma\gamma}=\Gamma(K_L\to\pi^0\gamma\gamma) . \quad (24)$$

The corresponding branching-ratio prediction is

$$B(K^{\pm}\to\pi^{\pm}\gamma\gamma)|_{\pi^+\pi^-\to\gamma\gamma}=(\tau_{K^+}/\tau_{K_L})B(K_L\to\pi^0\gamma\gamma)$$
$$=1.8\times10^{-7} . \quad (25)$$

This is about a factor of 2 smaller than the lower bound in Ref. 16, and about a factor of 5 smaller than the estimate in Ref. 19. We suspect that the reason for the discrepancy with Ref. 19 is the approximation made there of a constant matrix element for the decay $K^{\pm}\to\pi^{\pm}\pi^+\pi^-$.

There will be additional long-distance contributions related to the $K^{\pm}\to\pi^+\pi^+\pi^-$ decay in which a P-wave virtual $\pi^+\pi^-$ pair annihilates to a photon, and the other photon is radiated by the $K^{\pm}$ or remaining $\pi^{\pm}$. These contributions are not expected to lead to the distinctive mass spectrum of Fig. 3. However, a calculation in which such effects are taken into account (see the second of Ref. 16, especially Fig. 9 there) shows that a high-mass dipion peak persists under quite a range of parameters. One does expect such a spectrum shape from the $\pi^+\pi^-\to\gamma\gamma$ rescattering contribution leading to Eqs. (24) and (25). Experiments now in progress[25] searching for the decay $K^+\to\pi^+\nu\bar{\nu}$ may be sensitive to the process $K^+\to\pi^+\gamma\gamma$ for certain ranges of $m_{\gamma\gamma}$ where backgrounds are low.

To summarize, we have compared the predictions of chiral perturbation theory for the decay $K_L\to\pi^0\gamma\gamma$ with those of a simple model involving the rescattering of pions from the process $K_L\to\pi^+\pi^-\pi^0$. Detailed measurements of the shape of the two-photon spectrum will be needed to distinguish between the two approaches. Both treatments predict a branching ratio in the vicinity of 7×10^{-7}.

The model discussed here has some, but not all, contributions which would be associated with higher orders in a chiral expansion. In fact, the distinction made here between predictions of chiral perturbation theory and phenomenological final-state interaction theory for $K_L\to\pi^0\gamma\gamma$ is very similar to that drawn in the second of Ref. 9 with regard to the reaction $\gamma\gamma\to\pi^0\pi^0$. Although additional contributions due to vector mesons may be important for both reactions,[15,17,24] the final-state interaction model in the second of Ref. 9 accounts for the rate and spectrum shape in $\gamma\gamma\to\pi^0\pi^0$ without any need for vector-meson contributions.

Very recently, the upper bound $B(K_L\to\pi^0\gamma\gamma)\leq2.7\times10^{-6}$ (90% C.L.) has been set in Ref. 10. This represents an improvement by nearly a factor of 90 over the previous experimental limit, and suggests that tests of the distinction noted here may not lie too far in the future.

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