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Futility of high-precision SO(10) calculations

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In grand unified models, there are a large number of scalar bosons with masses of the order of the unification scale. Since the masses could be an order of magnitude or so above or below the vector-boson masses, they will affect the beta functions and thus low-energy predictions; the lack of knowledge of the masses translates into an uncertainty in these predictions. Although the effect is very small for a single scalar field, SO(10) models have hundreds of such fields, leading to very large uncertainties. We analyze this effect in SO(10) models with intermediate scales, and show that all such models have an *additional* uncertainty which can be as large as 4 orders of magnitude in the proton lifetime and as large as a factor of 0.02 in $\sin^2\theta_w$. In models with 210-dimensional representations, the weak mixing angle is uncertain by as much as 0.06. As a result, we argue that precise calculations in SO(10) models with intermediate scales may not be possible.

The realization that the simplest grand unified theories predicted proton decay at an experimentally accessible rate has led many authors to attempt to calculate the proton lifetime, as well as $\sin^2\theta_w$, as precisely as possible.[1] Such calculations include threshold effects (at both the electroweak and grand unification scales), two-loop effects, etc. Coupled with the increasing lower bound to the proton lifetime, these calculations sufficed to rule out the minimal (nonsupersymmetric) SU(5) model. Since SO(10) models have a much richer phenomenology, including n-$\bar{n}$ oscillations, right-handed currents, neutrino masses, $K \rightarrow \mu e$, etc., there have also been attempts to calculate the various mass scales of these models to high precision. In particular, Chang *et al.*[2] calculated two-loop corrections to a large number of symmetry-breaking chains, and Wang[3] extended this work to models with intermediate-scale axions. In these works, many interesting chains were "ruled out" by the two-loop analysis.

In this paper, we point out that these calculations ignored an effect which drastically increases the uncertainty of the results, by an amount which may exceed the two-loop effects considered. In the absence of a much more fundamental theory, this uncertainty cannot be reduced. As a result, we argue that high-precision calculations in SO(10) models are impossible.

The source of this uncertainty is the effect of superheavy scalars on the beta functions used to calculate the weak mixing angle and unification scale. All grand unified models contain many scalars whose masses are $O(M_X)$. If the masses were *precisely* M_X, then they would have no effect on the calculations. However, the masses could easily be an order of magnitude or two lower or greater than M_X; thus, they would, for values of the energy scale greater than their masses, affect the beta functions. Since the masses are determined by unknown (and unknowable) parameters of the scalar potential, our lack of knowledge of the masses translates into an uncertainty in the calculations. Of course, since the effect on the beta functions exists only for a decade or two in energy scale (and the calculations usually cover 15 decades), this uncertainty, *for a single scalar,* is very small. However, SO(10) models typically have hundreds of such scalars, and hundreds of small uncertainties may add up to a very large uncertainty.

The effect of superheavy scalars on calculations in grand unified theories was first considered by Cook, Mahanthappa, and Sher.[4] They considered the effects of the scalars in a 45-dimensional representation of SU(5), by numerical integration of the renormalization-group equations. A much more thorough analysis, using analytic expressions, was performed by Hall,[5] who gives general expressions for the effects of superheavy scalars [for models which break into $SU(3)\times SU(2)\times U(1)$]. More recently, Parida[6] looked at the effects in various SU(5) models and in the simplest SO(10) model, but did not consider the most interesting SO(10) models (such as those with intermediate scales). None of these works considered supersymmetric models.

We will follow the procedure of Hall.[5] The reader is referred to Ref. 5 for details. Hall defines matching functions λ_i, which relate the various gauge couplings g_i to the grand unified coupling g_G:

$$\frac{1}{g_i^2(\mu)}=\frac{1}{g_G^2(\mu)}-\lambda_i(\mu)\ . \tag{1}$$

The effects of superheavy scalars, vectors, fermions, etc., are all included in the λ_i. Hall shows that the λ_i can be written as

$$\lambda_i(\mu)=B_i+2b_{iG}\ln\frac{M_X}{\mu}+2b_{iS}\ln\frac{M_S}{\mu}+2b_{iF}\ln\frac{M_F}{\mu}\ , \tag{2}$$

where B_i is a constant, b_{iG}, b_{iS}, and b_{iF} are the beta functions for the superheavy vector boson, scalar boson, and fermion, respectively (the sum over all such particles is understood). In this work, only the b_{iS} term will concern us. From the three equations in Eq. (1), one can derive expressions for the unification scale and for $\sin^2\theta_w$ in the usual way. Ignoring two-loop beta functions, Hall finds [for the case where the grand unified model breaks into $SU(3)\times SU(2)\times U(1)$]

$$\ln\frac{M_X}{\bar{\mu}}=\frac{1}{5b_1+3b_2-8b_3}\times\left[\frac{3/\alpha-8/\alpha_s}{8\pi}+\tfrac{1}{2}[5\lambda_1(M_X)+3\lambda_2(M_X)-8\lambda_3(M_X)]\right],$$
$$\sin^2\theta_w=\frac{1}{5b_1+3b_2-8b_3}\times\left[3(b_2-b_3)+5(b_1-b_2)\frac{\alpha}{\alpha_s}+10\pi\sum_{ijk}\epsilon_{ijk}(b_i-b_j)\lambda_k(M_X)\right], \tag{3}$$

where $\bar{\mu}=80$ GeV. From these, we can find a simple expression for the effects of a superheavy scalar on the prediction of M_X and of $\sin^2\theta_w$. For the case in which the gauge group breaks directly into $SU(3)\times SU(2)\times U(1)$, we obtain, for the uncertainty in M_X and $\sin^2\theta_w$,

$$\Delta(\ln M_X)=-3.36(5b_1+3b_2-8b_3)\ln\frac{M_X}{M_S},$$
$$\Delta\sin^2\theta_w=-(0.054b_1-0.160b_2+0.106b_3)\ln\frac{M_X}{M_S}, \tag{4}$$

where the subscript S on the b_i has been dropped. The procedure is simple. For a scalar with given $SU(3)\times SU(2)\times U(1)$ quantum numbers and a given mass one substitutes the appropriate values of the b_i and M_S into the above equations and thus calculates the effect on the standard predictions.

In the most interesting SO(10) models,[7] the SO(10) gauge group breaks into $SU(4)\times SU(2)_L\times SU(2)_R$. From the evolution equations[8] one can easily see that the corresponding formulas for this case can be obtained by substituting $b_1\to\frac{2}{5}b_4+\frac{3}{5}b_{2R}$ and $b_3\to b_4$, with corresponding changes for the λ_i [see, for example, Eq. (9) of Ref. 8].

What is a reasonable range of superheavy scalar masses to consider? In Ref. 4, it was assumed that the masses lie in the range αM_X to M_X; in Ref. 5, it was assumed that they lie in the range $0.001M_X$ to $1000M_X$. We will be guided by the expected range of scalar masses in the standard model, but will give results in terms of $a\equiv\log_{10}(M_S/M_X)$. Since the range of Higgs-boson masses in the standard model spans 3 orders of magnitude, we will consider the most reasonable range of a to be 0 ± 1.5.

Our results are given in Tables I and II. We consider the effects of the most common representations of $SU(3)\times SU(2)\times U(1)$ and of $SU(4)\times SU(2)_L\times SU(2)_R$, respectively, and list results for nonsupersymmetric models. Since the only effect of supersymmetry is to triple the beta functions (the Higgs fermions are degenerate in mass with the Higgs scalars), the results for supersymmetric models are found simply by replacing a with $3a$. We see that the uncertainty in $\sin^2\theta_w$ in models which break to $SU(3)\times SU(2)\times U(1)$ is (remembering that $a=\pm1.5$ is the preferred range) typically ~0.002 for each scalar representation. Even with ten representations [a **45** of SU(5) has six], the uncertainty would not exceed 0.02. However, in supersymmetric models, the uncertainty is three times larger, and this effect can be quite significant. We will discuss specific models shortly.

TABLE I. Effects of heavy scalars on the predictions of M_X and $\sin^2\theta_w$ for models which break into $SU(3)\times SU(2)\times U(1)$. The first column gives the quantum numbers of the representation under $SU(3)\times SU(2)\times U(1)$. The second and third give the changes in M_X and $\sin^2\theta_w$ as a function of $a\equiv\log_{10}(M_X/M_S)$. The value of a will be different for each representation, and is expected to be in the range -1.5 to 1.5. For example, a model with an (8,1,0) with $a=1.5$ and a (1,3,0) with $a=-1.5$ will have total $\Delta\sin^2\theta_w=0.002\,25$.

Representation	M_X/M_X^0	$\Delta\sin^2\theta_w$
$(3,1,-\frac{1}{3})$	0.97^a	$0.0003a$
$(3,3,-\frac{1}{3})$	1.02^a	$-0.0013a$
$(3,1,\frac{4}{3})$	1.30^a	$0.0017a$
$(3,2,\frac{7}{6})$	1.53^a	$0.0015a$
$(6,1,-\frac{1}{3})$	0.83^a	$0.0015a$
$(8,2,\frac{1}{2})$	1.02^a	$0.0010a$
$(3,1,-\frac{2}{3})$	1.03^a	$0.0006a$
$(3,2,\frac{1}{6})$	1.02^a	$-0.0018a$
$(1,2,\frac{1}{2})$	1.07^a	$-0.0006a$
(8,1,0)	0.87^a	$0.0007a$
(1,3,0)	1.03^a	$-0.0008a$

TABLE II. Same as Table I for models which break into $SU(4)\times SU(2)\times SU(2)$.

Representation	M_X/M_X^0	$\Delta\sin^2\theta_w$
(15,1,1)	0.76^a	$0.0025a$
(15,1,3)	1.23^a	$0.0120a$
(15,3,1)	1.23^a	$-0.0150a$
(10,2,2)	0.87^a	$0.0012a$
(15,2,2)	0.94^a	$0.0006a$
(10,3,1)	1.07^a	$-0.0100a$
(6,1,1)	0.94^a	$0.0006a$
(20,1,1)	0.57^a	$0.0005a$
(6,2,2)	1.15^a	$-0.0012a$
(1,3,3)	1.51^a	$-0.0037a$
(1,1,3)	1.23^a	$0.0010a$
(1,3,1)	1.23^a	$-0.0046a$

The most interesting SO(10) models break into $SU(4)\times SU(2)_L \times SU(2)_R$, and here we see potentially large uncertainties. For example, note the effect on the unification scale from a (20,1,1) representation. For $a=0\pm1.5$, we see an uncertainty in M_X of 2.3. For a (1,3,3) representation, the uncertainty in M_X is 1.8. Thus, any model with both representations has an uncertainty in M_X which can be as large as a factor of 4, which would translate into an uncertainty in the proton lifetime of over 2 orders of magnitude. This representation occurs in the decomposition of the **54** of SO(10); thus all SO(10) models with a **54** breaking the gauge symmetry will have an *additional* uncertainty in the proton lifetime which can be as large as 2 orders of magnitude. Even worse are the effects on $\sin^2\theta_w$ in models with (15,1,3) and (15,3,1) representations, which occur in the decomposition of the **210** of SO(10). Here, the uncertainty in $\sin^2\theta_w$ from both representations is enormous, and can be as large as 0.04. Thus, all SO(10) models with a **210** breaking the gauge symmetry may have an *additional* uncertainty in $\sin^2\theta_w$ of 0.04. Since most intermediate scale SO(10) models use either a **54** or a **210** to break the gauge symmetry, one can see that precise calculations in such models may not be possible. In supersymmetric SO(10) models, the uncertainties are ridiculously large; the proton lifetime could be changed by over 6 orders of magnitude in models with a **54**, and $\sin^2\theta_w$ could be changed by over 0.12 in models with a **210**. This would seem to render even the tree-level calculations in such models meaningless. However, one should note that the Higgs potential in supersymmetric models is very highly restricted; thus, the mass range allowed may be much smaller (the masses may even be completely fixed by the gauge couplings); the calculation of such effects is clearly essential to make any meaningful predictions in supersymmetric SO(10) models.

We now turn to specific models for a discussion of the size of the uncertainties, again choosing the range of a, for definitiveness, to be 0 ± 1.5. It is crucial to determine which scalar multiplets have masses of $O(M_X)$ and which do not. In models without intermediate scales, this is straightforward, but there is much ambiguity in models with intermediate scales. We will adopt the *extended survival hypothesis* of Mohapatra and Senjanovic,[9] in which it is assumed that only the scalars necessary for breaking a symmetry have masses of the order of that breaking, and that the others have masses of $O(M_X)$. All nonsupersymmetric grand unified models have to be fine-tuned; this hypothesis minimizes the amount of fine-tuning needed. In this work, we are only considering the uncertainties caused by scalars of mass $O(M_X)$, and ignoring the uncertainties caused by scalars with lower masses. Should the extended survival hypothesis be false, then this would simply transfer the uncertainty from the unification scale to the intermediate scale and would not appreciably affect the overall uncertainty (in fact, it would probably increase it significantly since these scalars would affect the evolution over a larger range). This hypothesis gives a well-defined algorithm for determining which scalars have masses of $O(M_X)$. See Ref. 9 for details.

In determining the uncertainty of a specific model, we have taken the a's for the various representations to have values which maximize the deviation of M_X and $\sin^2\theta_w$. One might argue that the uncertainties for each of the scalars should be added in quadrature, resulting in a smaller net uncertainty. It is difficult to be precise about the word "uncertainty" when dealing with a range of possible parameters. Our philosophy is simply to find a range of values of M_X and $\sin^2\theta_w$ which can be accommodated by choosing reasonable values of the parameters and to take the size of the range as a measure of the "uncertainty." The reader may prefer another definition, in which case Tables I and II will provide the necessary information.

Let us restate this crucial point. One might argue that we have overstated the size of the uncertainty by considering the extreme values of parameter space, and that a Monte Carlo procedure would yield much smaller uncertainties. Suppose, however, that some quantity in the standard electroweak model depended in a complicated manner on the Higgs-boson mass, which is between 1 and 1000 GeV. How would our lack of knowledge of the Higgs-boson mass translate into an uncertainty in this quantity? One could do a Monte Carlo calculation which would give an idea of the most likely range of the quantity, but no physicist would consider the standard model to be ruled out until the entire range had been considered. Similarly, if one wishes to rule out an SO(10) model by doing a high-precision calculation, then the extreme values should be considered, as we have done. On the other hand, if one wished to find the most likely prediction of a model (to determine the *approximate* value of neutron oscillations in a model, for example), then a Monte Carlo calculation would give a better idea of the most likely range. Since most high-precision calculations do attempt to rule out models, we have taken the uncertainty to be defined by the extreme range of parameter space.

The results for specific models are given in Table III. First, the maximum uncertainties in models in which SU(5) breaks to the standard model are given [models (a)–(e)]. Model (a) is the minimal model; the uncertainties are quite small. Model (b) is the minimal supersymmetric model; the uncertainties are small, but perhaps not negligible. Models (c) and (d) extend the Higgs sector to include a 45-dimensional representation; the maximum uncertainty in the nonsupersymmetric case is fairly large (and in agreement with the calculations of Refs. 4 and 5), and in the supersymmetric case is so large as to make any serious calculation hopeless. Finally, model (e) is the popular flipped SU(5) model; since the Higgs sector of the model is so economical, the uncertainties can be ignored.

In model (f), SO(10) breaks directly to the standard model. Again, the uncertainties are large enough that they should certainly be considered in any comparison with experiment; likewise, the supersymmetric case has extremely large uncertainties.

Finally, we consider models with intermediate scales. Model (g) is a model analyzed by del Aguila and Ibanez,[8] which contains an intermediate $SU(4)\times SU(2)_L \times SU(2)_R$ group. The additional uncertainty in the proton lifetime is 3 orders of magnitude, and in $\sin^2\theta_w$ is 0.025. The

TABLE III. Uncertainty in the proton lifetime and $\sin^2\theta_w$ for various models (described in the text). The models, relevant Higgs representations, and the uncertainties are given. It is assumed that a, for each representation, lies in the range from -1.5 to 1.5.

Model	Representations	Uncertainty in proton lifetime	Uncertainty in $\sin^2\theta_w$
(a) minimal SU(5)	(24,5)	2.8	0.0027
(b) minimal SUSY SU(5)	$(24,5,\bar{5})$	40	0.0096
(c) extended SU(5)	(24,45)	300	0.011
(d) extended SUSY SU(5)	$(24,45,\overline{45})$	10^{12}	0.065
(e) flipped SU(5)	$(10,\overline{10})$	2.7	0.003
(f) minimal SO(10) (no intermediate scale)	(45,16,10)	50	0.013
(g) minimal SO(10) [with intermediate SU(4)×SU(2)×SU(2) group]	(54,126,10)	500	0.024
(h) Mohapatra and Senjanovic model	(54,45,126,10)	60 000	0.019
(i) many other intermediate scale models	(210,126,10, . . .)	10 000	0.064

model of Mohapatra and Senjanovic,[9] model (h), has an additional $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$ intermediate group, and has very similar uncertainties. Chang *et al.*[2] consider a huge number of possible chains of symmetry breaking. We note that all of their "acceptable" models contain a **210**, **126**, and a **10** of Higgs fields. We will thus consider the allowed range due to the scalars in these representations only; the actual range will be much wider (there may be slight differences due to the extended survival hypothesis, but as discussed above, this should not affect the total uncertainty). These models all have an uncertainty which can be as large as 4 orders of magnitude in the proton lifetime and as large as 0.06 in $\sin^2\theta_w$. This makes clear our assertion that precise calculations in these models are not possible.

It has been claimed[10] that the conventional model in which SO(10) breaks into a left-right-symmetric group cannot accommodate a low-energy right-handed W without too large a value of $\sin^2\theta_w$. The uncertainty discussed here would seem to allow such models, although one would have to stretch the uncertainties as far as possible (obviously, by introducing a large number of **210**'s, for example, one could get *any* desired result for the low-energy predictions). In calculating the uncertainties, we have assumed that all Higgs scalars are at the extreme end of the range (with those that, say, increase the proton lifetime at one end and those that decrease the proton lifetime at the other). It is doubtful that such conditions actually occur, the effects are probably thus somewhat smaller, but without information about the Higgs potential, one cannot be certain.

Thus, we conclude that the lack of knowledge of the precise values of the scalar masses at the unification scale introduces very large uncertainties into SO(10) calculations. In models with intermediate scales, this *additional* uncertainty can be as large as 3 orders of magnitude in the proton lifetime calculation and can be as large as 0.02 (and in models with 210-dimensional representations, as large as 0.06) in the weak mixing angle calculation. Since the masses depend on unmeasureable parameters of the Higgs potential, we conclude that precise predictions in these models are impossible. In supersymmetric models, the uncertainties are even larger, although the restricted nature of the Higgs potential might make determination of the scalar masses possible.

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[10]See Ref. 2 for a proof and list of references.