Neutral exotics in the composite model: Leptonic gluon

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In this and the subsequent papers we investigate the properties of neutral exotics, which survive any ordinary exotic-forbidding mechanisms, and provide us early signatures of composite models. In this paper, we concentrate on the leptonic gluon $G_\mu^{(l)}$, the vector boson made of the leptonic-color-carrying subquark $c_{(l)}$ and its antiparticle. Starting with a dynamical model for subquarks, we derive various physical quantities for the leptonic gluon. We also calculate cross sections for the processes mediated by the leptonic gluon.

I. INTRODUCTION

The composite models of quarks and leptons are based on the anticipation that the quantum number which repeatedly appears in the spectrum should be carried by some common subconstituent (called "subquark" or "preon").[1-6] From this point of view, the models are summarized as follows. Repetition of the color triplets or the SU(4) quartets including the leptonic color requires the subconstituent c_i ($i=1,2,3$) carrying the colors, and the subconstituent $c_{(l)}$ carrying leptonic color.[1] Repetition of the weak isodoublets requires the subconstituent w_j ($j=1,2$) carrying the weak isospin.[2] Also the generations possibly originate from the subconstituents h_k ($k=1,2,\ldots,N_g$; N_g is the number of the generations) carrying the "horizontal spin."[2] In terms of them, the quarks q and the leptons l are composed as

$$q \sim wc \quad \text{or} \quad whc, \quad l \sim wc_{(l)} \quad \text{or} \quad whc_{(l)} . \tag{1}$$

The former compositions in (1) without h require other mechanisms to give rise to the generations.[3] The weak bosons can be composite of the type[7]

$$W_\mu^i \sim \bar{w}_L \gamma^i \gamma_\mu w_L \quad (\text{or } w_L^\dagger \tau^i \overleftrightarrow{\partial}_\mu w_L) , \tag{2}$$

if w is a spinor (scalar) particle. The Higgs scalar can be composite:[2,8]

$$\phi \sim \bar{w}_L w_R \quad (\text{or } w_L^\dagger w_R) . \tag{3}$$

To be more radical, the gluons G_μ^a and even the photon A_μ are also taken as composite:[9]

$$G_\mu^a \sim \bar{c} \lambda^a \gamma_\mu c \quad (\text{or } c^\dagger \lambda^a \overleftrightarrow{\partial}_\mu c) , \tag{4}$$

$$A_\mu \sim \sum_s \bar{s} Q_s \gamma_\mu s \quad (\text{or } \sum_s s^\dagger Q_s \overleftrightarrow{\partial}_\mu s) , \tag{5}$$

where Q_s is the electric charge of the subconstituent s ($s=w,h,c,c_{(l)}$). If we require freedom from the anomaly of $SU(2)_L \times U(1)$ in the subquark level, we have $Q_{w_1} = -Q_{w_2} = \frac{1}{2}$ (Ref. 10). Then, in the models without the subquark h, the other Q's are uniquely fixed as $Q_c = \frac{1}{6}$ and $Q_{c_{(l)}} = -\frac{1}{2}$. In the models with h, if we take $Q_h = 0$, we get the same result. This is natural, since the subquarks w and c correspond to the actually observed symmetry structure of the standard model, while h does not. Hereafter, we adopt this "standard" charge assignment.

An immediate question in this type of model is why the other combinations ("exotics")[2,10] such as ww, wcc, etc., do not exist in the known low-energy spectrum. There should be some mechanisms to forbid them. As candidates, we can think of the strong U(1) forces,[11] confining subcolors,[12] the strong magnetic forces,[13] etc. Some of the exotics, however, are allowed by those mechanisms, and should be found in the future. In particular, the neutral exotics composed of a subquark and its antisubquark have the quantum number of the vacuum, and are expected to survive any ordinary mechanisms to forbid exotics. As examples of the neutral exotics, we have the leptonic gluon[14]

$$G_\mu^{(l)} \sim \bar{c}_{(l)} \gamma_\mu c_{(l)} \quad (\text{or } c_{(l)}^\dagger \overleftrightarrow{\partial}_\mu c_{(l)}) , \tag{6}$$

the color-singlet gluon[15]

$$G_\mu^0 \sim \bar{c} \gamma_\mu c \quad (\text{or } c^\dagger \overleftrightarrow{\partial}_\mu c) , \tag{7}$$

the isosinglet weak boson[16]

$$W_\mu^0 \sim \bar{w}_L \gamma_\mu w_L \quad (\text{or } w_L^\dagger \overleftrightarrow{\partial}_\mu w_L) , \tag{8}$$

and the horizontal-spin-singlet $h\bar{h}$ composite

$$H_\mu^0 \sim \bar{h} \gamma_\mu h \quad (\text{or } h^\dagger \overleftrightarrow{\partial}_\mu h) . \tag{9}$$

In general, neutral exotics with other spins are also expected. It is merely by analogy with the existing particles that we have mentioned only the spin-1 bosons.

Important signatures for compositeness would be brought by the new states such as the exotics[2,10] and the excited states,[17] which should be observed in certain regularities. In general, the excited states would have masses of the order of the compositeness scale. The present-day experiments on the anomalous magnetic moments of the leptons,[18] and the νe, νp, (Ref. 19), $e\bar{e}$ (Ref. 20), and $p\bar{p}$ scattering cross sections[21] constrain the compositeness scale to be larger than hundreds of GeV's. [Note that the values over TeV claimed in some experimental papers are obtained under the assumption that the coupling strengths are unity, which is not the case in many models. The more appropriate values of several hundred GeV are obtained by multiplying them by a factor of $O(\sqrt{\alpha})$, where α is the fine-structure constant.] On the other hand, to avoid the "unnaturalness" of finetuning in mass renormalization in the Higgs sector, the Fermi mass scale $\sqrt{G_F}$ should be related with some scale of new physics,[22] which, we assume here, is supplied by compositeness. Furthermore, if the weak bosons are really composite, it seems that the ground states are somewhat light compared with the compositeness scale. Thus, we expect that some neutral exotics should be in the region of hundreds of GeV's. It is worthwhile at present to investigate the expected properties of the neutral exotics in the composite models. They would be produced in $e\bar{e}$, $p\bar{p}$, and pp collisions, and decay into a lepton pair, a quark pair, or a W-boson pair. Among them, the leptonic gluon would couple with the leptons, and the color-singlet gluon would couple more strongly with the quarks. The isosinglet weak boson and the boson H^0 in Eq. (9) couple equally with the quarks and leptons, and the isosinglet weak boson couples also with the weak bosons. Their mixing with the photon and the Z boson, and mixing among themselves would modify the naive expectations. In principle, all those quantities are determined from the subquark dynamics. The purpose of this and forthcoming papers is to derive quantitative predictions on the properties of the neutral exotics. In particular, we concentrate on investigations of the leptonic gluons in this paper.

The plan of this paper is as follows. In Sec. II we specify the fundamental dynamics of subquarks, and derive the effective Lagrangian for composites. In Sec. III we investigate the compositeness conditions among coupling constants, and determine the coupling constant of the leptonic gluon. In Sec. IV mixing among the photon, the neutral weak boson, and the leptonic gluon is diagonalized, and the experimental bound on the mass of the diagonalized leptonic gluon is derived. In Sec. V couplings with quarks, leptons, and W bosons are investigated, the mass bounds from the neutrino scattering experiments are derived, and the scattering cross sections at high energies are calculated. In Sec. VI we discuss the distinction between the neutral exotics in the composite model and the extra U(1) gauge bosons in the grand unification or superstring model, and we give a brief summary of this paper.

II. DYNAMICS

The dynamics of the composite quarks, leptons, and bosons should be derived from the fundamental one which is written in terms only of the subquarks. People considered two complementary types of dynamics; that of the Nambu–Jona-Lasinio type[23,24] and that with fundamental gauge interactions.[11–13] The former is perturbatively solvable for the composite states, while it requires explicit momentum cutoff, and is not renormalizable. On the other hand, the latter is renormalizable and confining under appropriate conditions, while it is difficult to get explicit solutions for the relativistic composite states. The former is somewhat phenomenological, since we need to prepare a fundamental interaction term in the Lagrangian per each composite state. It could be an intermediate effective theory of the more fundamental one, which, e.g., could be the latter type of theory, i.e., a gauge theory. In this paper we adopt the former type of theory for the phenomenological purpose to investigate the neutral exotics. The present authors[25] recently elaborated a natural and realistic subquark model with solvable dynamics of the Nambu–Jona-Lasinio type. We incorporate the leptonic gluon into the model in Ref. 25. The basic Lagrangian is obtained by adding the four-$c_{(l)}$ interaction term [the eighth term in Eq. (10) below] to that of the model in Ref. 25:

$$\mathcal{L}=\bar{w}(i\partial\!\!\!/-m_w)w+\bar{h}(i\partial\!\!\!/-m_h)h+\bar{c}(i\partial\!\!\!/-m_c)c+\bar{c}_{(l)}(i\partial\!\!\!/-m_{c_{(l)}})c_{(l)}+F_1\left[\sum_s \bar{s}\gamma_\mu Q_s s\right]^2$$
$$+F_2(\bar{w}_L\gamma_\mu\tau^i w_L)^2+F_3(\bar{c}\gamma_\mu\lambda^a c)^2+F_{(l)}(\bar{c}_{(l)}\gamma_\mu c_{(l)})^2+\sum_q F_q\bar{P}(w,h,c)P(w,h,c)+\sum_l F_l\bar{P}(w,h,c_{(l)})P(w,h,c_{(l)}) , \qquad (10)$$

where m_s ($s=w,h,c,c_{(l)}$) is the mass of the subquark s, F_1, F_2, F_3, $F_{(l)}$, F_q, and F_l are coupling constants, and $P(w,h,c)$ is the spin projection operator into a spin-$\frac{1}{2}$ state. The Lagrangian $\mathcal{L}$ is equivalent to

$$\mathcal{L}'=\bar{w}(iD\!\!\!\!/-m_w)w+\bar{h}(iD\!\!\!\!/-m_h)h+\bar{c}(iD\!\!\!\!/-m_c)c+\bar{c}_{(l)}(iD\!\!\!\!/-m_{c_{(l)}})c_{(l)}-\frac{1}{4F_1}(\tilde{A}_\mu)^2-\frac{1}{4F_2}(\tilde{W}^i_\mu)^2$$
$$-\frac{1}{4F_3}(\tilde{G}^i_\mu)^2-\frac{1}{4F_{(l)}}(\tilde{G}^{(l)}_\mu)^2+\sum_q \bar{\tilde{q}}P(w,h,c)+\text{H.c.}+\sum_l \bar{\tilde{l}}(w,h,c_{(l)})+\text{H.c.}-\sum_q\frac{1}{F_q}\bar{\tilde{q}}\tilde{q}-\sum_l\frac{1}{F_l}\bar{\tilde{l}}\tilde{l} \qquad (11)$$

with

$$D_\mu w=(\partial_\mu+iQ_w\tilde{A}_\mu+i\gamma_L\tau^j\tilde{W}^j_\mu)w , \qquad (12a)$$

$$D_\mu h=(\partial_\mu+iQ_h\tilde{A}_\mu)h , \qquad (12b)$$

$$D_\mu c=(\partial_\mu+iQ_c\tilde{A}_\mu+i\lambda^a\tilde{G}^a_\mu)c , \qquad (12c)$$

$$D_\mu c_{(l)}=(\partial_\mu+iQ_{c_{(l)}}\tilde{A}_\mu+i\tilde{G}^{(l)}_\mu)c_{(l)} . \qquad (12d)$$

where $\tilde{A}_\mu$, $\tilde{W}_\mu$, $\tilde{G}_\mu$, $\tilde{G}^{(l)}_\mu$, $\tilde{q}$, and $\tilde{l}$ are auxiliary fields.

The Lagrangians $\mathcal{L}$ and $\mathcal{L}'$ are equivalent, because their generating functionals of the Green's functions coincide with each other. We take the tight-binding limits F_1, F_3, F_q, $F_l \to \infty$, while F_2 and $F_{(l)}$ are taken as finite. The $U(1)_{em}\times SU(3)_c$ gauge symmetry is exact, but the $SU(2)_L$ and $U(1)_{G_{(l)}}$ symmetries are broken.

The kinetic and the interaction terms of the auxiliary fields are generated through the quantum effects of the subquarks. They are superficially divergent, but should be cut off by the finite-size effects. As an approximation, we adopt a regularization scheme which is invariant under the $SU(3)_c\times U(1)_{em}$ gauge transformations, and which recovers the chiral symmetry of w in the limit $m_w\to 0$. The lowest-order diagrams in Fig. 1 dominate over higher-loop diagrams because of a large number of subcolors (for a detailed argument, see Ref. 25). The explicit calculations (see the Appendix in Ref. 25) lead to the following corrections to the Lagrangian, where we retain only the most divergent terms in the bosonic and the fermionic sectors:

$$\Delta\mathcal{L} = -I_\gamma(\tilde{A}_{\mu\nu})^2 - I_w[(\tilde{W}^i_{\mu\nu}+\epsilon^{ij3}\tilde{A}_{[\mu}\tilde{W}^j_{\nu]})^2+\tilde{A}_{\mu\nu}\tilde{W}^3_{\mu\nu}-6(m_w\tilde{W}^i_\mu)^2]-2I_c(\tilde{G}^a_{\mu\nu})^2$$
$$-I_{c_{(l)}}[(\tilde{G}^{(l)}_{\mu\nu})^2+2Q_{c_{(l)}}\tilde{A}_{\mu\nu}\tilde{G}^{(l)}_{\mu\nu}]+\sum_q \bar{\tilde{q}}(J_q i\not{D}-K_q m_W)\tilde{q}+\sum_l \bar{\tilde{l}}(J_l i\not{D}-K_l m_w)\tilde{l} \quad (13)$$

with

$$\tilde{A}_{\mu\nu}=\partial_\mu\tilde{A}_\nu-\partial_\nu\tilde{A}_\mu\ , \quad (14a)$$

$$\tilde{W}^i_{\mu\nu}=\partial_\mu\tilde{W}^i_\nu-\partial_\nu\tilde{W}^i_\mu-2\epsilon^{ijk}\tilde{W}^j_\mu\tilde{W}^k_\nu\ , \quad (14b)$$

$$\tilde{G}^a_{\mu\nu}=\partial_\mu\tilde{G}^a_\nu-\partial_\nu\tilde{G}^a_\mu-2f^{abc}\tilde{G}^b_\mu\tilde{G}^c_\nu\ , \quad (14c)$$

$$\tilde{G}^{(l)}_{\mu\nu}=\partial_\mu\tilde{G}^{(l)}_\nu-\partial_\nu\tilde{G}^{(l)}_\mu\ , \quad (14d)$$

$$D_\mu\tilde{q}=(\partial_\mu+iQ_q\tilde{A}_\mu+i\gamma_L\tau^i\tilde{W}^i_\mu+i\lambda^a\tilde{G}^a_\mu)\tilde{q}\ , \quad (14e)$$

$$D_\mu\tilde{l}=(\partial_\mu+iQ_l\tilde{A}_\mu+i\gamma_L\tau^i\tilde{W}^i_\mu+i\tilde{G}^{(l)}_\mu)\tilde{l}\ . \quad (14f)$$

The I_s $(s=w,h,c,c_{(l)})$ in Eq. (13) are the logarithmically divergent coefficients of the s-loop diagrams [Figs. 1(a)–1(c)]:

$$I_s=\frac{N_s^{sc}}{24\pi^2}\ln\frac{\Lambda_s}{m_s}\quad (s=w,h,c,c_{(l)})\ , \quad (15)$$

where Λ_s is the effective cutoff and N_s^{sc} is the number of the subcolor. The I_γ is that with two external photon lines [Fig. 1(a)] and is written as

$$I_\gamma=[(Q_{w_1})^2+(Q_{w_2})^2]I_w+3(Q_c)^2I_c$$
$$+(Q_{c_{(l)}})^2I_{c_{(l)}}+N_g(Q_h)^2I_h\ , \quad (16)$$

where N_g is the number of generations. The J_q (J_l) and K_q (K_l) in (13) are the quartically divergent coefficients of the two-loop diagrams with w, u, and c $(c_{(l)})$ internal lines [Figs. 1(d) and 1(e)].

In order to cast the kinetic terms into the standard forms, we rescale the fields as

$$A'_\mu=2\sqrt{I_\gamma}\,\tilde{A}_\mu\ , \quad (17a)$$

$$W^i_\mu=2\sqrt{I_w}\,\tilde{W}^i_\mu\ , \quad (17b)$$

$$G^a_\mu=2\sqrt{2I_c}\,\tilde{G}^a_\mu\ , \quad (17c)$$

$$G^{(l)}_\mu=2\sqrt{I_{c_{(l)}}}\,\tilde{G}^{(l)}_\mu\ , \quad (17d)$$

$$q=\sqrt{J_q}\,\tilde{q}\ , \quad (17e)$$

and

$$l=\sqrt{J_l}\,\tilde{l}\ , \quad (17f)$$

and we rewrite the constants as

$$e=\frac{1}{2\sqrt{I_\gamma}}\ , \quad (18a)$$

$$g=\frac{1}{\sqrt{I_w}}\ , \quad (18b)$$

$$g_s=\frac{1}{\sqrt{2I_c}}\ , \quad (18c)$$

$$g_{(l)}=\frac{1}{2\sqrt{I_{c_{(l)}}}}\ , \quad (18d)$$

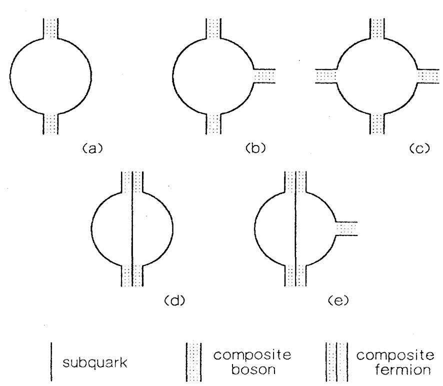

FIG. 1. Subquark-loop diagrams.

$$(M_W)^2=\frac{1}{8F_2I_w}+3(m_w)^2\ , \tag{18e}$$

$$(M_{G_{(l)}})^2=\frac{1}{8F_{(l)}I_{c_{(l)}}}\ , \tag{18f}$$

$$m_q=\frac{K_q m_w}{J_q}\ , \tag{18g}$$

$$m_l=\frac{K_l m_w}{J_l}\ . \tag{18h}$$

The effective Lagrangian $\mathcal{L}_{\rm eff}$ is obtained by adding to the $\mathcal{L}'$ the dominant contribution $\Delta\mathcal{L}$ from the quantum corrections. The kinetic and the interaction terms among composites are given by

$$\mathcal{L}_{\rm eff}^{\rm comp}=-\tfrac{1}{4}(A'_{\mu\nu})^2-\tfrac{1}{4}(W^i_{\mu\nu}+e\epsilon^{ij3}A'_{[\mu}W^j_{\nu]})^2-\tfrac{1}{2}\lambda_{\gamma W}A'_{\mu\nu}W^3_{\mu\nu}+\tfrac{1}{2}M_W^2(W^i_\mu)^2-\tfrac{1}{4}(G^a_{\mu\nu})^2-\tfrac{1}{4}(G^{(l)}_{\mu\nu})^2$$
$$-\tfrac{1}{2}\lambda_{\gamma G_{(l)}}A'_{\mu\nu}G^{(l)}_{\mu\nu}+\tfrac{1}{2}M^2_{G_{(l)}}(G^l_\mu)^2+\sum_q\bar q(i\not D-m_q)q+\sum_l\bar l(i\not D-m_l)l \tag{19}$$

with

$$A'_{\mu\nu}=\partial_\mu A'_\nu-\partial_\nu A'_\mu\ , \tag{20a}$$

$$W^i_{\mu\nu}=\partial_\mu W^i_\nu-\partial_\nu W^i_\mu-g\epsilon^{ijk}W^j_\mu W^k_\nu\ , \tag{20b}$$

$$G^a_{\mu\nu}=\partial_\mu G^a_\nu-\partial_\nu G^a_\mu-g_s f^{abc}G^b_\mu G^c_\nu\ , \tag{20c}$$

$$G^{(l)}_{\mu\nu}=\partial_\mu G^{(l)}_\nu-\partial_\nu G^{(l)}_\mu\ , \tag{20d}$$

$$D_\mu q=\left[\partial_\mu+ieQ_qA'_\mu+\frac{i}{2}g\gamma_L\tau^iW^i_\mu+\frac{i}{2}g_s\lambda^aG^a_\mu\right]q\ , \tag{20e}$$

$$D_\mu l=\left[\partial_\mu+ieQ_lA'_\mu+\frac{i}{2}g\gamma_L\tau^iW^i_\mu+ig_{(l)}G^{(l)}_\mu\right]\ , \tag{20f}$$

and with

$$\lambda_{\gamma W}=e/g \tag{21a}$$

and

$$\lambda_{\gamma G_{(l)}}=eQ_{c_{(l)}}/g_{(l)}\ . \tag{21b}$$

The $\mathcal{L}_{\rm eff}^{\rm comp}$ in (19) is the Lagrangian for the current mixing scheme[24] for photon A'_μ (to be diagonalized), weak boson W^i_μ, gluon G^a_μ, leptonic gluon $G^{(l)}_\mu$, quark q, and lepton l. The constants e, g, g_s, and $g_{(l)}$ are, respectively, interpreted as the coupling constants of the electromagnetic, the weak, the strong, and the leptonic gluon's interactions. The M_W, $M_{G_{(l)}}$, m_q, and m_l become the masses of the weak boson, the leptonic gluon, the quarks, and the leptons, respectively. The $\lambda_{\gamma W}$ and $\lambda_{\gamma G_{(l)}}$ are the mixing parameters between A'_μ and W^3_μ, and between A'_μ and $G^{(l)}_\mu$, respectively.

III. COMPOSITENESS CONDITIONS

The parameters in Eqs. (19) and (20a)–(20f) are related with the quantities in the subquark dynamics by Eqs. (15), (18a)–(18h), and further related by Eqs. (16), (21a), and (21b). Eliminating the sublevel quantities from them, we can get the "compositeness conditions" among the parameters at the composite level. Compositeness of W and $G_{(l)}$ leads to the relations (21a) and (21b), respectively. Among them, the relation (21a) is the "unification condition" of Ref. 26, which turned out to hold experimentally to a good accuracy. Under this condition, however, the current mixing scheme coincides with that in the standard model, except for the Higgs sector, and we cannot yet decide from this whether or not the weak bosons are composite. On the other hand, the relation (21b) would provide a test for compositeness of the leptonic gluon.

The compositeness condition of the photon (16) leads to the relation[25]

$$\frac{1}{\alpha_{\rm em}}=\frac{4[(Q_{w_1})^2+(Q_{w_2})^2]}{\alpha_2}+\frac{6(Q_c)^2}{\alpha_s}+\frac{(Q_{c_{(l)}})^2}{\alpha_{(l)}}+\frac{2N_g(Q_h)^2}{\alpha_h}\ , \tag{22}$$

where $\alpha_{\rm em}=e^2/4\pi$, $\alpha_2=g^2/4\pi$, $\alpha_s=g_s^2/4\pi$, $\alpha_{(l)}=g_{(l)}^2/4\pi$, $\alpha_h=g_h^2/4\pi$, and $g_h=1/\sqrt{2I_h}$. The g_h becomes the coupling constant for the (broken) horizontal gauge symmetry, when we incorporate the horizontal gauge boson $H^a_\mu\sim\bar h\lambda^a\gamma_\mu h$ into the model.[2] For $Q_h=0$, however, g_h is irrelevant to the relation (22), and we can determine the value of the coupling constant $\alpha_{(l)}$ for the leptonic gluon from the known values of $\alpha_{\rm em}$, α_2, and α_s. The relation (22) holds at the subquark scale $\Lambda_{\rm sub}$, up to which the coupling constants, we assume, to run with the scale μ according to the renormalization group equations:

$$\alpha_{\rm em}(\mu)^{-1}=\alpha_{\rm em}(0)^{-1}-\frac{2}{3\pi}\sum_\psi Q_\psi^2\ln\langle\mu/m_\psi\rangle+\frac{11}{3\pi}\ln\langle\mu/M_W\rangle\ , \tag{23a}$$

$$\alpha_2(\mu)^{-1}=\alpha_2(0)^{-1}-\frac{5}{3\pi}\ln\langle\mu/M_W\rangle+\frac{1}{4\pi}\ln\langle m_t/M_W\rangle\ , \tag{23b}$$

$$\alpha_s(\mu)^{-1}=\frac{9}{2\pi}\ln\langle\mu/\Lambda_{QCD}\rangle-\frac{1}{3\pi}\ln\langle\mu/m_c\rangle\langle\mu/m_b\rangle\langle\mu/m_t\rangle\ , \tag{23c}$$

where $\langle A\rangle=\max\{A,1\}$. For a given $\Lambda_{\rm sub}$, Eqs. (23a)–(23c) fix the $\alpha_{\rm em}(\Lambda_{\rm sub})$, $\alpha_2(\Lambda_{\rm sub})$, and $\alpha_s(\Lambda_{\rm sub})$.

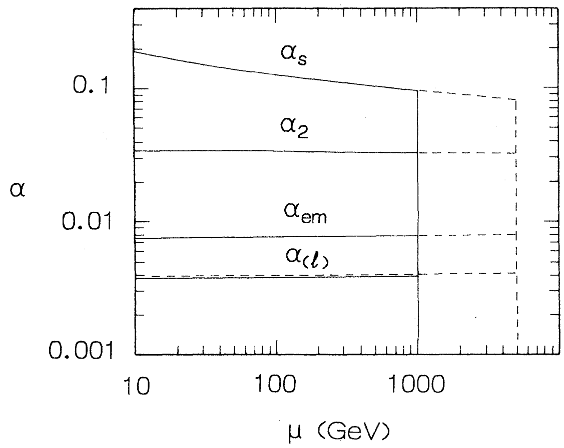

FIG. 2. The running coupling constants $\alpha_{\rm em}$, α_2, α_s, and $\alpha_{(l)}$ vs the scale μ. The solid (dashed) lines are those for $\Lambda_{\rm sub}=1$ TeV (5 TeV).

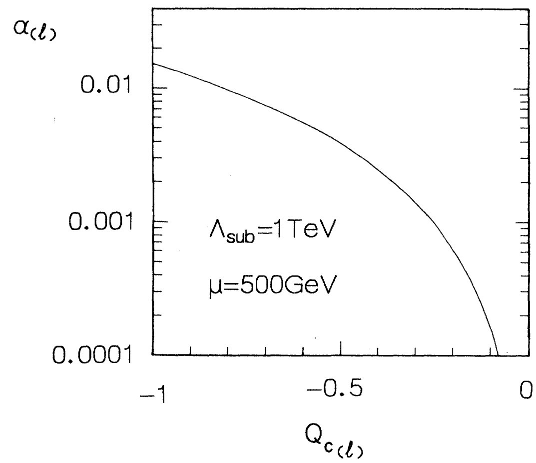

FIG. 3. Dependence of $\alpha_{(l)}$ on $Q_{c_{(l)}}$ with $Q_h=0$.

Then, we apply Eq. (22) to get $\alpha_{(l)}(\Lambda_{\rm sub})$, and we finally get $\alpha_{(l)}(\mu)$ at the relevant scale μ by

$$\alpha_{(l)}(\mu)^{-1}=\alpha_{(l)}(\Lambda_{\rm sub})^{-1}-\frac{4}{\pi}\ln(\mu/\Lambda_{\rm sub}) \ . \tag{24}$$

This procedure is illustrated in Fig. 2. The obtained $\alpha_{(l)}(\mu)$ is almost independent of the scale μ and choice of $\Lambda_{\rm sub}$, as far as we fix the subquark charge assignment. We later use the values $\alpha_{(l)}=\frac{1}{261}$ at $\mu=20$ GeV for the low-energy neutrino scattering, $\alpha_{(l)}=\frac{1}{259}$ at $\mu=80$ GeV for comparison with the weak-boson masses, and $\alpha_{(l)}=\frac{1}{257}$ at $\mu=500$ GeV for the high-energy predictions. (These values are obtained by using $\Lambda_{\rm sub}=1$ TeV, the QCD scale $\Lambda_{\rm QCD}=0.2$ GeV, and the top-quark mass $m_t=40$ GeV. Change in $\Lambda_{\rm sub}$ to 5 TeV causes 4% increase in $\alpha_{(l)}$, change in $\Lambda_{\rm QCD}$ from 0.1 to 0.3 GeV causes 1% decrease in $\alpha_{(l)}$, and change in m_t to 200 GeV causes 0.5% decrease in $\alpha_{(l)}$. Note that $\Lambda_{\rm sub}$ cannot be too large in our model, since it is taken as something related with the weak-interaction scale. See Ref. 25.) On the other hand, the $\alpha_{(l)}$ depends strongly on choice of charge assignment of the subquarks (Fig. 3). As is stated in Sec. I we adopt the standard assignment $Q_{w_1}=-Q_{w_2}=-Q_{c_{(l)}}=\frac{1}{2}$, $Q_c=\frac{1}{6}$, $Q_h=0$, which is free from $SU(2)_L\times U(1)$ anomaly at the subquark level.

One may wonder if the photon is really a composite. In the models of this type, each of the other composite bosons can be composed of a single pair of a subquark and its antisubquark [see Eqs. (2)–(4) and (6)–(9)]. The photon, however, should be the linear combination of all the charged subquark pairs with the weight of the *a priori* assigned charges [see Eq. (5)]. It is not sufficiently simple, and there remains the question why the other linear combinations are not composed. Instead of the above model, one can consider a model where the photon is taken as elementary. Instead of the Lagrangian (10) we start with

$$\begin{aligned}\mathcal{L}=&\bar{w}(i\not{D}^A-m_w)w+\bar{h}(i\not{D}^A-m_h)h+\bar{c}(i\not{D}^A-m_c)c+\bar{c}_{(l)}(i\not{D}^A-m_{c_{(l)}})c_{(l)}-\tfrac{1}{4}(\partial_{[\mu}A_{\nu]})^2+F_2(\bar{w}_L\gamma_\mu\tau^i w_L)^2\\&+F_3(\bar{c}\gamma_\mu\lambda^a c)^2+F_{(l)}(\bar{c}_{(l)}\gamma_\mu c_{(l)})^2+\sum_q F_q\bar{P}(w,h,c)P(w,h,c)+\sum_l F_l\bar{P}(w,h,c_{(l)})P(w,h,c_{(l)}) \ ,\end{aligned} \tag{25}$$

where

$$D_\mu^A s=(\partial_\mu+ieQ_s A_\mu)s \quad (s=w,h,c,c_{(l)}) \ . \tag{26}$$

In this case we again arrive at the same effective Lagrangian as Eq. (19) with the parameters satisfying (21a) and (21b). However, the sum rule (22) does not hold, since the photon is not composite. Then, the coupling constant $g_{(l)}$ of the leptonic gluon becomes a free parameter. In the following, we fix $\alpha_{(l)}$ at the values of the photon compositeness condition, except for the case where we intend to show the $\alpha_{(l)}$ dependence of the quantities.

IV. BOSON MIXING

In general, the current and mass mixing among the n vector bosons $\mathbf{V}_\mu=(V_\mu^1,V_\mu^2,\ldots,V_\mu^n)^t$ with the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}\partial_{[\mu}\mathbf{V}_{\nu]}^t\mathbf{K}\partial_{[\mu}\mathbf{V}_{\nu]}+\tfrac{1}{2}\mathbf{V}_\mu^t\mathbf{M}^2\mathbf{V}_\mu \tag{27}$$

(the $\mathbf{K}$ and $\mathbf{M}$ are constant $n\times n$ matrices) is diagonalized as follows. Let $\mathbf{U}$ be the unitary matrix to diagonalize the matrix $\mathbf{M}\mathbf{K}^{-1}\mathbf{M}$, and $\mathbf{M}'^2$ be its diagonalized form. Then, the transformation

$$\mathbf{V}'_\mu = \mathbf{T}\mathbf{V}_\mu \ , \tag{28}$$

with

$$\mathbf{T} = \mathbf{M}'^{-1}\mathbf{U}\mathbf{M} \tag{29}$$

casts the Lagrangian (27) into the diagonalized form

$$\mathcal{L} = -\tfrac{1}{4}\partial_{[\mu}\mathbf{V}'^t_{\nu]}\partial_{[\mu}\mathbf{V}'_{\nu]} + \tfrac{1}{2}\mathbf{V}'^t_\mu \mathbf{M}'^2 \mathbf{V}'_\mu$$
$$= -\tfrac{1}{4}\sum_i (\partial_{[\mu}V'^i_{\nu]})^2 + \tfrac{1}{2}\sum_i (M'^i V'^i_\mu)^2 \ . \tag{30}$$

For pure mass mixing $\mathbf{K}=\mathbf{1}$, $\mathbf{T}$ becomes unitary ($\mathbf{T}=\mathbf{U}$), while in general, $\mathbf{T}$ involves rescaling of the fields. When one of the eigenvalues of $\mathbf{M}'$ tends to zero (let $M'_1 \to 0$, without loss of generality) the form of $\mathbf{T}$ in (29) is singular. But $\mathbf{T}$ itself can be regular by taking infinitesimal $\mathbf{U}$. In this case, $\mathbf{T}$ becomes a triangular matrix with $T_{21}=T_{31}=\cdots=T_{n1}\to 0$.

We apply the above diagonalization procedure to mixing among A'_μ, W^3_μ, and $G^{(l)}_\mu$. It is achieved by considering the terms quadratic in these fields:

$$\mathcal{L}_{\rm quad} = -\tfrac{1}{4}(\partial_{[\mu}A'_{\nu]})^2 - \tfrac{1}{4}(\partial_{[\mu}W^3_{\nu]})^2 - \tfrac{1}{4}(\partial_{[\mu}G^{(l)}_{\nu]})^2 - \tfrac{1}{2}\lambda_{\gamma W}\partial_{[\mu}A'_{\nu]}\partial_{[\mu}W^3_{\nu]} - \tfrac{1}{2}\lambda_{\gamma G_{(l)}}\partial_{[\mu}A'_{\nu]}\partial_{[\mu}G^{(l)}_{\nu]}$$
$$+\tfrac{1}{2}M_W^2(W^3_\mu)^2 + \tfrac{1}{2}M^2_{G_{(l)}}(G^{(l)}_\mu)^2 \ . \tag{31}$$

First we transform A'_μ into A_μ by

$$A'_\mu = A_\mu - \lambda_{\gamma W}W^3_\mu - \lambda_{\gamma G_{(l)}}G^{(l)}_\mu \tag{32}$$

to get

$$\mathcal{L}_{\rm quad} = -\tfrac{1}{4}(\partial_{[\mu}A_{\nu]})^2 - \frac{a}{4}(\partial_{[\mu}W^3_{\nu]})^2 - \frac{b}{4}(\partial_{[\mu}G^{(l)}_{\nu]})^2 - \frac{c}{2}\partial_{[\mu}W^3_{\nu]}\partial_{[\mu}G^{(l)}_{\nu]} + \tfrac{1}{2}M_W^2(W^3_\mu)^2 + \tfrac{1}{2}M^2_{G_{(l)}}(G^{(l)}_\mu)^2 \ , \tag{33}$$

where

$$a = 1-(\lambda_{\gamma W})^2, \quad b = 1-(\lambda_{\gamma G_{(l)}})^2, \quad c = -\lambda_{\gamma W}\lambda_{\gamma G_{(l)}} \ . \tag{34}$$

Then, we transform W^3_μ and $G^{(l)}_\mu$ into Z_μ and X_μ by

$$\begin{bmatrix} W^3_\mu \\ G^{(l)}_\mu \end{bmatrix} = \begin{bmatrix} M_W^{-1} & \\ & M^{-1}_{G_{(l)}} \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} M_Z & \\ & M_X \end{bmatrix} \begin{bmatrix} Z_\mu \\ X_\mu \end{bmatrix} \tag{35}$$

to get the diagonalized form

$$\mathcal{L}_{\rm quad} = -\tfrac{1}{4}(\partial_{[\mu}A_{\nu]})^2 - \tfrac{1}{4}(\partial_{[\mu}Z_{\nu]})^2$$
$$-\tfrac{1}{4}(\partial_{[\mu}X_{\nu]})^2 + \tfrac{1}{2}M_Z^2(Z_\mu)^2 + \tfrac{1}{2}M_X^2(X_\mu)^2 \ , \tag{36}$$

where

$$M_W^2(M_Z^{-2}\cos^2\phi + M_X^{-2}\sin^2\phi) = a = 1-(\lambda_{\gamma W})^2 \ , \tag{37a}$$

$$M^2_{G_{(l)}}(M_Z^{-2}\sin^2\phi + M_X^{-2}\cos^2\phi) = b = 1-(\lambda_{\gamma G_{(l)}})^2 \ , \tag{37b}$$

and

$$M_W M_{G_{(l)}}(M_z^{-2} - M_X^{-2})\sin\phi\cos\phi = c = -\lambda_{\gamma W}\lambda_{\gamma G_{(l)}} \ . \tag{37c}$$

These equations can be solved for the unknown variables M_X, $M_{G_{(l)}}$, and ϕ in terms of the known variables:

$$M_X^2 = \frac{M_Z^2 - M_W^2 b/\Delta}{aM_Z^2/M_W^2 - 1} \ , \tag{38a}$$

$$M^2_{G_{(l)}} = \frac{M_Z^2\Delta - M_W^2 b}{a - M_W^2/M_Z^2} \ , \tag{38b}$$

and

$$\tan\phi = -\frac{1}{c}\{[1-a(M_Z/M_W)^2][b(M_W/M_Z)^2 - \Delta]\}^{1/2} \tag{38c}$$

with a, b, and c in Eq. (34) and

$$\Delta = ab - c^2 = 1-(\lambda_{\gamma W})^2 - (\lambda_{\gamma G_{(l)}})^2 \ . \tag{39}$$

Note that the diagonalized mass M_X is directly observable, while the $M_{G_{(l)}}$ is not.

If we know, in the near future, sufficiently accurate values of the M_W and M_Z from experiment, we would be able to predict the mass M_X, the angle ϕ, and all the concerning physical quantities with no free parameter. At present, however, the experimental values only impose the lower bound on M_X, and are consistent with the limit $M_X \to \infty$, where the $\mathcal{L}^{\rm comp}_{\rm eff}$ coincides with that of the standard model (apart from the Higgs sector). In Fig. 4(a) we show the contours with fixed M_X due to Eq. (38a) in the M_W-M_Z plane (for $\alpha_{(l)} = \frac{1}{259}$, the value from photon compositeness), together with the region indicated by experiment[27] at present, where we have adopted the values

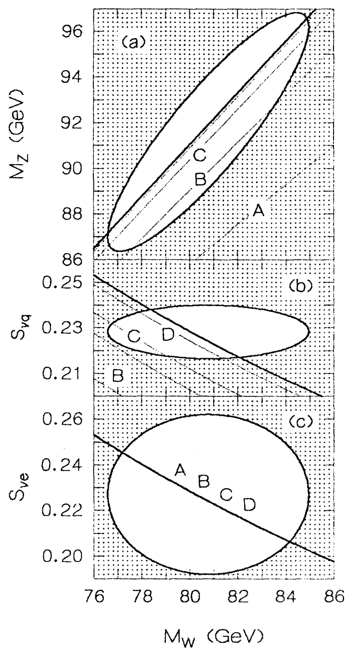

FIG. 4. Contours with fixed M_X in (a) M_W-M_Z plane [Eq. (38a)], (b) M_W-S_{vq} plane [Eq. (51a)], and (c) M_W-S_{ve} plane [Eq. (51b)]. *A*, *B*, *C*, and *D* indicate those for $M_X=200$, 300, 500, and 1000 GeV, respectively. The thick lines are contours for $M_X=\infty$. The shaded regions are experimentally excluded (95% C.L.).

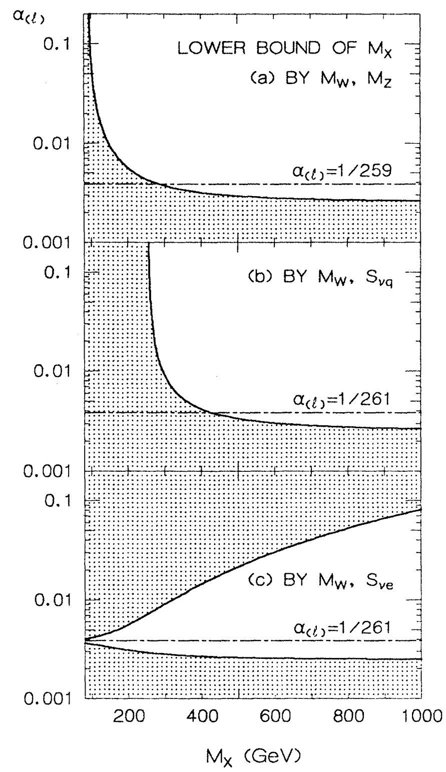

FIG. 5. The lower bounds of M_X for fixed $\alpha_{(l)}$ by the experimental values of (a) M_W and M_Z, (b) M_W and S_{vq}, and (c) M_W and S_{ve}. The dash-dotted lines indicate the value of $\alpha_{(l)}$ determined from the photon compositeness condition. The shaded regions are experimentally excluded (95% C.L.).

$$M_W=80.76\pm1.71 \text{ GeV}$$

and (40)

$$M_Z=91.59\pm2.14 \text{ GeV} ,$$

with the correlation $r=0.879$, following Costa *et al.*[28] The lower bound on M_X (for $\alpha_{(l)}=\frac{1}{259}$) to the 95% C.L. is given by

$$M_X > 290 \text{ GeV} . \tag{41}$$

In Fig. 5(a) the bounds on M_X are plotted as a function of the coupling strength $\alpha_{(l)}$. The lower M_X is allowed for the larger $\alpha_{(l)}$, because the mixing parameter $\lambda_{\gamma G_{(l)}}$ becomes small for the larger $\alpha_{(l)}$. Note that the mixing parameter $\lambda_{\gamma G_{(l)}}$ is proportional to $1/g_{(l)}$ [see Eq. (21b)]. On the other hand, $\alpha_{(l)}<\alpha Q^2_{c_{(l)}}/(1-e^2/g^2)$ is forbidden, because mixing becomes too strong for Eqs. (37a)–(37c) to have a solution with positive M_X^2.

We can eliminate from the M_X, $M_{G_{(l)}}$, and ϕ the factor

$$\rho-1=M_W^2/M_Z^2 a-1 , \tag{42}$$

which is the source of the large relative error, and get the more accurate relations

$$M_{G_{(l)}}=M_Z M_X\sqrt{\Delta}/M_W , \tag{43a}$$

$$\sin 2\phi=\frac{2M_X M_Z}{M_X^2-M_Z^2}\,\frac{\lambda_{\gamma W}\lambda_{\gamma G_{(l)}}}{\sqrt{\Delta}} . \tag{43b}$$

These relations enable us to calculate all the physical quantities as functions of the parameter M_X. The values of ϕ according to Eq. (43b) are plotted against M_X in Fig. 6, for various values of $\alpha_{(l)}$.

V. COUPLING WITH QUARKS, LEPTONS, AND W BOSONS

The full expression of $\mathcal{L}_{\text{eff}}^{\text{comp}}$ under the transformations (32) and (35) is given by

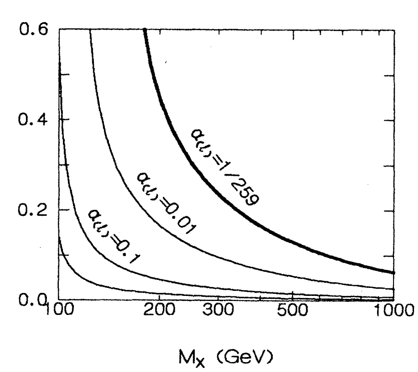

FIG. 6. The mixing angle ϕ vs M_X due to Eq. (43b).

$$\mathcal{L}_{\rm eff}^{\rm comp}=-\tfrac{1}{4}(\hat{A}_{\mu\nu})^2-\tfrac{1}{4}(\hat{Z}_{\mu\nu})^2-\tfrac{1}{4}(\hat{X}_{\mu\nu})^2-\tfrac{1}{2}D_{[\mu}W^+_{\nu]}D_{[\mu}W^-_{\nu]}-\tfrac{1}{4}(G_{\mu\nu})^2+\tfrac{1}{2}M_Z^2(Z_\mu)^2+\tfrac{1}{2}M_X^2(X_\mu)^2$$
$$+M_W^2W^+_\mu W^-_\mu+\sum_\psi \bar\psi(i\not{D}-m_\psi)\psi \quad (\psi=q_L,q_R,l_L,l_R)\,, \tag{44}$$

where

$$W^\pm_\mu=(W^1_\mu\mp iW^2_\mu)/\sqrt{2}\,, \tag{45a}$$
$$\hat{A}_{\mu\nu}=\partial_{[\mu}A_{\nu]}+if_{\gamma WW}W^+_{[\mu}W^-_{\nu]}\,, \tag{45b}$$
$$\hat{Z}_{\mu\nu}=\partial_{[\mu}Z_{\nu]}+if_{ZWW}W^+_{[\mu}W^-_{\nu]}\,, \tag{45c}$$
$$\hat{X}_{\mu\nu}=\partial_{[\mu}X_{\nu]}+if_{XWW}W^+_{[\mu}W^-_{\nu]}\,, \tag{45d}$$
$$D_\mu W^\pm_\nu=(\partial_\mu\pm ieA_\mu\pm ig_{ZWW}Z_\mu\pm ig_{XWW}X_\mu)W^\pm_\nu\,, \tag{45e}$$
$$D_\mu\psi=(\partial_\mu+ieQ_\psi A_\mu+ig_{Z\psi\psi}Z_\mu+ig_{X\psi\psi}X_\mu)\psi\,, \tag{45f}$$

$(\psi=q_L,q_R,l_L,l_R)$ with

$$f_{\gamma WW}=e\,, \tag{46a}$$
$$f_{ZWW}=gM_W\cos\phi/M_Z\,, \tag{46b}$$
$$f_{XWW}=-gM_W\sin\phi/M_X\,, \tag{46c}$$
$$g_{ZWW}=M_z[(g-e\lambda_{\gamma W})\cos\phi/M_W+e\lambda_{\gamma G_{(l)}}\sin\phi/M_{G_{(l)}}]\,, \tag{46d}$$
$$g_{XWW}=M_X[(g-e\lambda_{\gamma W})\sin\phi/M_W-e\lambda_{\gamma G_{(l)}}\cos\phi/M_{G_{(l)}}]\,, \tag{46e}$$
$$g_{Z\psi\psi}=M_Z[(gT_\psi-eQ_\psi\lambda_{\gamma W})\cos\phi/M_W-(g_{(l)}L_\psi-eQ_\psi\lambda_{\gamma G_{(l)}})\sin\phi/M_{G_{(l)}}]\,, \tag{46f}$$
$$g_{X\psi\psi}=M_X[(gT_\psi-eQ_\psi\lambda_{\gamma W})\sin\phi/M_W+(g_{(l)}L_\psi-eQ_\psi\lambda_{\gamma G_{(l)}})\cos\phi/M_{G_{(l)}}]\,. \tag{46g}$$

The Q_ψ, T_ψ, and L_ψ in Eqs. (46a)–(46g) are, respectively, the electric charge, the third component of the weak isospin, and the lepton number of the fermion $\psi=q_L$, q_R, l_L, and l_R. If we use the relation (21a) of our model, which also holds experimentally to a good accuracy, we have

$$f_{ZWW}=g_{ZWW} \quad\text{and}\quad f_{XWW}=g_{XWW}\,. \tag{47}$$

In Fig. 7, the values of the coupling constants in Eqs. (46b)–(46g) are plotted against M_X and against $\alpha_{(l)}$. They approach their finite limiting values as $M_X\to\infty$, and are practically constant above $M_X\approx 300$ GeV. In particular, the coupling constants for Z tend to their values in the standard model. At $\alpha_{(l)}=\frac{1}{261}$, the value from the photon compositeness condition, the X on the whole couples stronger with the right-handed components of the fermions than the left-handed ones. The coupling with quarks tends to zero with increasing $\alpha_{(l)}$, while couplings with leptons increase, and tend to vectorlike coupling. This is because the mixing becomes the weaker for the larger $g_{(l)}$ [see Eq. (21b)], and the X increases its leptonic-gluon likelihood. It can be seen more clearly in Fig. 8, where their trajectories with varying $\alpha_{(l)}$ are plotted in the $g_{X\psi_L\psi_L}$-$g_{X\psi_R\psi_R}$ plane.

From the Lagrangian (44), we can deduce the effective Lagrangian for the neutral-current interactions:

$$\mathcal{L}_{\rm NC}=\sum_{\psi,\psi'}P_{\psi\psi'}(t)(\bar\psi\gamma_\mu\psi)(\bar\psi'\gamma_\mu\psi') \quad (\psi,\psi'=q_L,q_R,l_L,l_R) \tag{48}$$

with

$$P_{\psi\psi'}(t)=\frac{e^2Q_\psi Q_{\psi'}}{t}+\frac{g_{Z\psi\psi}g_{Z\psi'\psi'}}{t-M_Z^2+iM_Z\Gamma_Z}+\frac{g_{X\psi\psi}g_{X\psi'\psi'}}{t-M_X^2+iM_X\Gamma_X}\,, \tag{49}$$

where t is the exchanged invariant mass squared, and Γ_Z and Γ_X are the decay widths of Z and X, respectively. The Γ's should be set to zero below the threshold. For neutrino scattering at low energies $|t|\ll M_Z^2$, we have

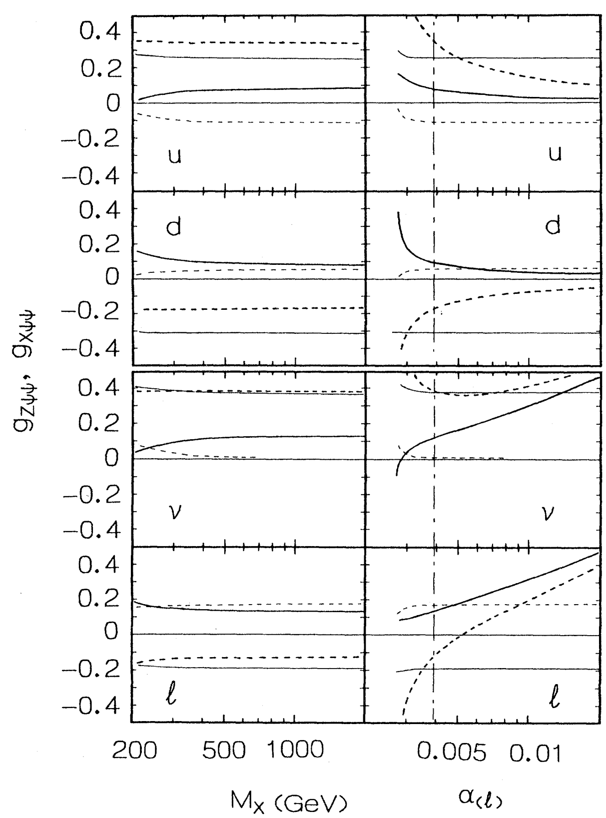

FIG. 7. The coupling constants $g_{B\psi\psi}$ of the boson $B=X$ (thick lines) and Z (thin lines) with the currents $\bar\psi_L\gamma_\mu\psi_L$ (solid lines) and $\bar\psi_R\gamma_\mu\psi_R$ (dashed lines) $(\psi=u,d,\nu,l)$ vs M_X (for $\alpha_{(l)}=\frac{1}{257}$) and vs $\alpha_{(l)}$ (for $M_X=500$ GeV). The dash-dotted lines indicate the value of $\alpha_{(l)}$ determined from the photon compositeness condition $\alpha_{(l)}=\frac{1}{257}$.

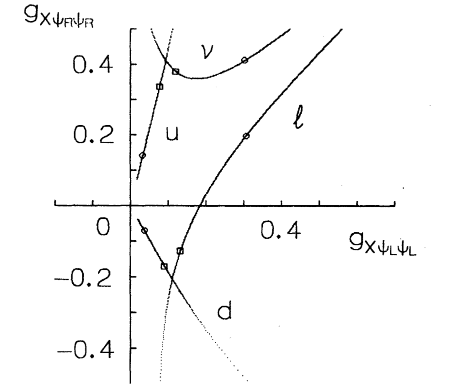

FIG. 8. Contours in the $g_{X\psi_L\psi_L}$-$g_{X\psi_R\psi_R}$ plane with varying $\alpha_{(l)}$. The squares (circles) indicate the points with $\alpha_{(l)}=\frac{1}{257}$ ($\frac{1}{100}$).

$$P_{\nu\psi}(t)=\frac{1}{2}g^2(T_\psi-Q_\psi S_{\nu\psi})/M_W^2 \quad (\psi=q_L,q_R,l_L,l_R)\ , \tag{50}$$

where $S_{\nu\psi}$ are independent of the chirality of ψ, and given by

$$S_{\nu q}=(e/g)^2(1+2Q_{c_{(l)}}M_W^2/M_{G_{(l)}}^2)\ , \tag{51a}$$

$$S_{\nu e}=(e/g)^2[1+2(Q_{c_{(l)}}+g_{(l)}^2/e^2)M_W^2/M_{G_{(l)}}^2]\ . \tag{51b}$$

The $S_{\nu q}$ and $S_{\nu e}$ are, respectively, determined by νp- and νe-scattering experiments.[19] They are nothing but the "$\sin^2\theta_W$" determined by assuming the standard model. Following Costa *et al.*,[28] we adopt the values

$$S_{\nu q}=0.2283\pm0.0048\ ,\qquad S_{\nu e}=0.2271\pm0.0143\ . \tag{52}$$

On the other hand, g in the expressions in Eqs. (51a) and (51b) depends on M_W, since $g^2=4\sqrt{2}\,G_F M_W^2$. In Figs. 4(b) and 4(c), the contours with fixed M_X due to Eqs. (51a) and (51b) (for $\alpha_{(l)}=\frac{1}{261}$) are plotted in the M_W-$S_{\nu q}$ and M_W-$S_{\nu e}$ planes, together with region indicated by the experimental data at present [Eqs. (40) and (52)]. The lower bound of M_X (for $\alpha_{(l)}=\frac{1}{261}$) by the νp scattering experiments is

$$M_X>440\ \mathrm{GeV}\ , \tag{53}$$

and no significant bound is obtained from the νe scattering experiments. In Figs. 5(b) and 5(c), the lower bounds are plotted as functions of $\alpha_{(l)}$. By the νp scattering experiments, the lower M_X is allowed for the larger $\alpha_{(l)}$ because the leptonic gluon $G_{(l)}$ before mixing decouples from quarks q, and because the mixing parameter $\lambda_{\gamma G_{(l)}}$ becomes small for the larger $\alpha_{(l)}$. On the other hand, the lower bound on M_X from the νe scattering experiments becomes the larger for the larger $\alpha_{(l)}$, because the leptonic gluon $G_{(l)}$ does couple with leptons with the strength $\alpha_{(l)}$ even before mixing. The $\alpha_{(l)}$ smaller than $\alpha Q_{c_{(l)}}^2/(1-e^2/g^2)$ is again forbidden, because mixing becomes too strong for Eqs. (37a)–(37c) to have a solution with positive M_X^2.

Now it is straightforward to calculate the scattering cross sections of various processes. In Fig. 9(a) we show the integrated cross section σ of $e\bar{e}\to\mu\bar{\mu}$ versus the invariant mass $\sqrt{s}$ of the $e\bar{e}$ system. In Figs. 9(b) and 9(c) we show the cross sections $s\,d\sigma/ds$ of $p\bar{p}\to l\bar{l}+$anything vs $\sqrt{s}$ at $\sqrt{s_0}=2$ TeV, and of $pp\to l\bar{l}+$anything vs $\sqrt{s}$ at $\sqrt{s_0}=40$ TeV, where s and s_0 are the invariant masses squared of the $l\bar{l}$ and $p\bar{p}$ (or pp) systems, respectively. In the numerical calculation, we have used the parton distribution Set I of Ref. 29. In both figures we can see clear resonance peaks standing about a hundred times higher than the background continuity. More detailed analysis on the decay widths, the cross sections, and asymmetries will appear in the subsequent paper.[30]

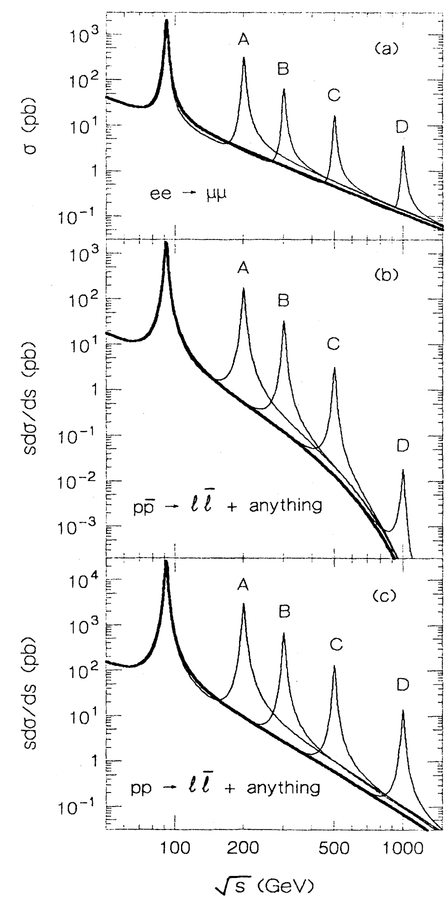

FIG. 9. (a) The integrated cross section σ of $e\bar{e}\to\mu\bar{\mu}$ vs $\sqrt{s}$, and the cross sections $s\,d\sigma/ds$ of (b) $p\bar{p}\to l\bar{l}+$anything vs $\sqrt{s}$ at $\sqrt{s_0}=2$ TeV, and (c) $pp\to l\bar{l}+$anything vs $\sqrt{s}$ at $\sqrt{s_0}=40$ TeV. *A*, *B*, *C*, and *d* indicate those for $M_X=200$, 300, and 500, and 1000 GeV, respectively. The thick lines are those for $M_X=\infty$.

VI. DISCUSSIONS AND SUMMARY

Now we briefly comment on distinction of the neutral exotics in the composite models from the extra U(1) gauge bosons[31] (extra Z bosons) in the grand unification models[32,33] or in the superstring models.[34,35] From the phenomenological points of view, the neutral exotics resemble the extra Z bosons in giving resonance peaks in the $l\bar{l}$, $q\bar{q}$, and W^+W^- channels. There exist, however, important differences. First, the neutral exotics couple with the photon and the ordinary Z (not extra) boson via current mixing due to the subquark-loop effects, while the extra Z bosons couple via mass mixing due to the Higgs mechanism. As for the ordinary Z (not extra) boson itself in the Glashow-Salam-Weinberg (GSW) model, the predictions at low energies coincide with those from current mixing of the composite weak bosons, and we cannot yet discriminate them by experiments. However, this equivalence between the current and the mass mixing schemes no longer persists for the neutral exotics and the extra Z bosons. They coincide only for accidental choices of the parameters.

Another difference is in their coupling with quarks and leptons. The leptonic gluon $G_\mu^{(l)}$ couples purely with the lepton-number current

$$J_\mu^l = \bar{l}\gamma_\mu l \; , \tag{54}$$

and the color-singlet gluon G_μ^0 couples with the pure quark-number current

$$J_\mu^q = \bar{q}\gamma_\mu q \; . \tag{55}$$

The boson H_μ^0 in Eq. (9) couples with their sum $J_\mu^{q+l} = J_\mu^q + J_\mu^l$, and the isosinglet weak boson W_μ^0 couples with the left-handed quark-plus-lepton-number current

$$J_\mu^{q+l,L} = \bar{q}\gamma_\mu q_L + \bar{l}\gamma_\mu l_L \; . \tag{56}$$

On the other hand, the extra Z bosons couple with quarks and leptons as follows. The extra Z boson Z_{B-L} in the model with the breaking, $SU(4)_C \to SU(3)_C \times U(1)_{B-L}$, couples with the baryon-minus-lepton-number current

$$J_\mu^{B-L} = \tfrac{1}{3} J_\mu^q - J_\mu^l \; . \tag{57}$$

The extra Z boson Z_{LR} in the model with the breaking, $SU(2)_L \times SU(2)_R \times U(1) \to SU(2)_L \times U(1)_Y \times U(1)_{LR}$, couples with the current J_μ^{LR} which is a linear combination of J_μ^{B-L} and

$$J_\mu^{R3} = \tfrac{1}{2}(\bar{u}\gamma_\mu u_R - \bar{d}\gamma_\mu d_R + \bar{\nu}\gamma_\mu \nu_R - \bar{e}\gamma_\mu e_R) \; . \tag{58}$$

The extra Z boson Z_χ in the model with the breaking, $SO(10) \to SU(5) \times U(1)_\chi$, couples with the current

$$J_\mu^\chi = -\bar{q}\gamma_\mu q_L + \bar{u}\gamma_\mu u_R - 3\bar{d}\gamma_\mu d_R + 3\bar{l}\gamma_\mu l_L + \bar{e}\gamma_\mu e_R \; . \tag{59}$$

(We retain only the part concerned with the light quarks and leptons, because we are, for the time being, interested in experimental tests in terms of them.) The extra Z boson Z_ψ in the model with breaking $E_6 \to SO(10) \times U(1)_\psi$ couples with the current

$$J_\mu^\psi = \bar{q}\gamma_\mu q_L - \bar{q}\gamma_\mu q_R + \bar{l}\gamma_\mu l_L - \bar{e}\gamma_\mu e_R \; . \tag{60}$$

The extra Z boson Z_η in the model with breaking $E_6 \to SU(5) \times U(1)_\eta$ couples with the current J_μ^η which is a linear combination of J_μ^χ and J_μ^ψ. None of J_μ^l, J_μ^q, J_μ^{q+l}, and $J_\mu^{q+l,L}$ which couple with the neutral exotics coincide with any of J_μ^{B-L}, J_μ^{LR}, J_μ^χ, J_μ^ψ, and J_μ^η which couple with the extra Z bosons. Furthermore, we can explicitly show linear independence of (i) the weak hypercharge current J_μ^Y; (ii) the neutral component of the weak isospin current J_μ^{L3}; (iii) any one of J_μ^l, J_μ^q, J_μ^{q+l}, and $J_\mu^{q+l,L}$; and (iv) any one of J_μ^{B-L}, J_μ^{LR}, J_μ^χ, J_μ^ψ, and J_μ^η. This means that the couplings of the neutral exotics never coincide with those of the extra Z bosons even after mixing with the photon and Z boson, and we can, in principle, distinguish them by detailed experimental analyses. Although the final judgment should be formed from the more general points of view including the full pattern of the spectra and the substructure effects (the form factors,[36] subquark jets,[37] etc.).

In summary, we have investigated the properties of the leptonic gluon $G_\mu^{(l)}$ [Eq. (6)], the vector boson made of the leptonic-color-carrying subquark $c_{(l)}$ and its antiparticle. Such neutral exotics as the leptonic gluon survive any ordinary exotic-forbidding mechanisms, and would provide us early signatures of composite models. We started from a dynamical model for subquarks [Eq. (10)], and derived the effective Lagrangian for composites including the leptonic gluon [Eq. (19)]. The subquark-loop effects (Fig. 1) cause current mixing among the photon, the neutral weak boson, and the leptonic gluon. The parameters in the effective Lagrangian for composites are related to the quantities in the subquark dynamics [Eqs. (15) and (18a)–(18h)]. Compositeness of the photon, the weak boson, and the leptonic gluon impose the relations among their coupling constants and the mixing parameters [Eqs. (21a), (21b), and (22)]. Using them we determined the coupling constant of the leptonic gluon (Fig. 2). Mixing among the neutral bosons is diagonalized by a nonunitary transformation involving rescaling of the fields [Eqs. (32) and (35)]. In principle, from deviations of the weak-boson masses from their standard-model values, we can uniquely predict the mass and the mixing angle of the leptonic gluon [Eqs. (38a) and (38c)]. At present, however, the experimental uncertainty does not allow it [Fig. 4(a)]. We can only get the bound on the (diagonalized) leptonic gluon mass [Eq. (41) and Fig. 5(a)]. Then, we investigated the leptonic-gluon coupling with the quarks, leptons, and W bosons [Eqs. (46a)–(46g) and Fig. 7]. Under the photon compositeness condition, it is more like a right-handed one coupling equally to the quarks and leptons, while it approaches pure leptonic vector coupling with increasing $\alpha_{(l)}$ (Fig. 8). Applying them to the experimental results on neutrino scattering, we obtained a more severe bound on the (diagonalized) leptonic gluon mass [Eq. (53) and Figs. 5(b) and 5(c)]. We also calculated the cross sections at high energies [Figs. 9(a) and 9(b)], where we can see clear resonance peaks a hundred times higher than the background. Further detailed analysis on the cross sections will appear in a separate paper.[30] We hope that these predictions will be tested at Fermilab Tevatron, the Superconducting Super Collider and $e\bar{e}$ colliders in the TeV region.

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