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Electromagnetic properties of neutrinos in a background of electrons

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Using covariant methods we calculate the neutrino electromagnetic vertex in a gas of electrons to lowest order in a loop expansion and to the lowest order in $1/M_W^2$. The new induced terms, while they are chirality preserving, yield additional contributions to the dipole moments in the nonrelativistic limit. These are identical for particles and antiparticles and so need not vanish for Majorana neutrinos. As applications of our formulas, the expression for the plasmon$\to\nu\bar{\nu}$ decay rate is rederived and the dispersion relation of a massless neutrino propagating in matter in the presence of an external magnetic field is determined. The possible implications of these effects are considered.

I. INTRODUCTION

The subject of the electromagnetic properties of neutrinos has been widely considered in the literature. At a fundamental level, it provides a well-defined setting for understanding some of the deep differences between Dirac and Majorana neutrinos. This was the subject of works cited in Ref. 1. For the case of neutrinos in a medium, the same issue was discussed in Ref. 2. With the insight gained from the latter, we can summarize the situation as follows.

In the vacuum, the electric- and magnetic-dipole-moment operators are odd under *CPT*. Thus, the dipole moments of a Dirac particle are opposite to those of its antiparticle while the dipole moments of a Majorana neutrino vanish because this particle is its own antiparticle.[1]

In a medium the situation can be drastically different.[2] In general, the effect of the medium can give rise to *CPT* asymmetries in the effective electromagnetic interactions. These asymmetries manifest themselves as new terms in the effective action which are identical for a particle and its antiparticle—and therefore can be nonzero even for a Majorana particle. The precise conditions under which these effects can appear were analyzed in Ref. 2.

In the present work we complement the analysis of Ref. 2 by presenting the results of detailed calculations of the background-dependent part of the $\nu\nu\gamma$ vertex for the case in which the medium consists of a gas of electrons. As usual, the electron gas is supposed to be embedded in a uniform positive-ion background. However, since the background-dependent terms turn out to be proportional to the inverse of the electron mass in the classical and nonrelativistic limit, the effect of the ions is negligible in most circumstances.

In Sec. II the calculation is described. To leading order in $1/M_W^2$, general formulas for the form factors are given in terms of integrals over the electron-positron energy distribution. The resulting formulas obtained in the static limit as well as in the long-wavelength limit are also given, some details of this calculation being given in the Appendix. Then in Sec. III we use these results as working ground to illustrate and complement the analysis of Ref. 2. In Sec. IV we consider two applications of our results. The first one illustrates how the formula for the plasmon decay rate can be easily rederived with our method. In addition, this example shows a well-defined path to follow if we want to calculate corrections to the classic results for such rates. In the second application, the correction to the index of refraction of neutrinos in matter, in the presence of an external magnetic field, is determined. Section V presents our conclusions.

II. CALCULATION OF THE VERTEX FUNCTION

The off-shell vertex function $\Gamma_\mu(k,k',v)$ is defined in such a way that

$$\langle \nu(k')|j_\mu^{\rm EM}(0)|\nu(k)\rangle=\bar{u}(k')\Gamma_\mu(k,k',v)u(k)\ . \tag{2.1}$$

We have explicitly indicated the dependence of Γ_μ on the four-velocity of the center of mass of the medium, v^μ. In the vacuum, the dependence on v^μ vanishes.

The diagrams that enter in the calculation of Γ_μ to lowest order are shown in Fig. 1. We assume that the temperature is such that there are no W bosons in the background. Therefore, only the electron propagator has a background-dependent term, and it is given by

$$S_F(p)=(\not{p}+m)\left[\frac{1}{p^2-m^2}+2\pi i\delta(p^2-m^2)\eta(p\cdot v)\right] \tag{2.2}$$

where

$$\eta(x)\equiv\frac{\theta(x)}{e^{\beta(x-\mu)}+1}+\frac{\theta(-x)}{e^{-\beta(x-\mu)}+1}\ . \tag{2.3}$$

Here, θ is the unit step function, $1/\beta$ is the temperature, and μ is the chemical potential of the electrons.

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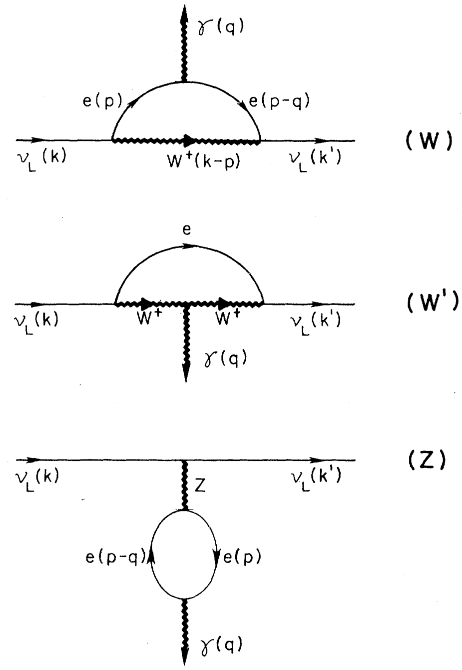

FIG. 1. The diagrams contributing to the vertex function $\Gamma(k,k',v)$ at the one-loop level.

A few observations help us to simplify the calculation of the background-dependent part of Γ_μ, which we denote by Γ'_μ. Since the integrals involved in the calculation of Γ'_μ are cut off by the electron-positron distributions, the diagram marked as (W') gives a contribution to Γ'_μ which is suppressed by an extra power of $1/M_W^2$ relative to the other two diagrams. Therefore, to leading order in $1/M_W^2$, only the diagrams marked as (W) and (Z) need to be calculated. In the 't Hooft–Feynman gauge, the diagrams in which the W's are replaced the unphysical Higgs fields are also suppressed by extra powers of $1/M_W^2$ and therefore do not contribute to Γ'_μ to the leading order in $1/M_W^2$.

With these preliminaries, our task is reduced to the calculation of the following quantities:

$$\begin{aligned}-i\Gamma_\mu^{(W)} &= \tfrac{1}{2}eg^2 \int \frac{d^4p}{(2\pi)^4}\gamma^\alpha L iS_F(p-q)\\ &\quad\times\gamma_\mu iS_F(p)\gamma_\alpha L\frac{1}{(k-p)^2-M_W^2}\,,\\ -i\Gamma_\mu^{(Z)} &= -\frac{eg_Z^2}{q^2-M_Z^2}\gamma^\alpha L\\ &\quad\times\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[iS_F(p-q)\gamma_\mu iS_F(p)\\ &\quad\times\gamma_\alpha(a_Z+b_Z\gamma_5)]\end{aligned} \tag{2.4}$$

which are the contributions to Γ_μ from diagrams (W) and (Z). We have written the couplings to the Z in the form

$$\mathcal{L}_Z = -g_Z A^\mu[\bar{e}\gamma_\mu(a_Z+b_Z\gamma_5)e+\bar{\nu}_L\gamma_\mu\nu_L]\,. \tag{2.5}$$

In the standard model,

$$\begin{aligned}g_Z &= g/(2\cos\theta_W)\,,\\ a_Z &= -\tfrac{1}{2}+2\sin^2\theta_W\,,\\ b_Z &= \tfrac{1}{2}\,.\end{aligned} \tag{2.6}$$

Finally, $L=\frac{1}{2}(1-\gamma_5)$ as usual and e is the magnitude of the electric charge of the electron. Also, notice that the coupling of the electron neutrino to the photon is given by $\Gamma_\mu^{(W)}+\Gamma_\mu^{(Z)}$, whereas only $\Gamma_\mu^{(Z)}$ contributes for the other flavors of neutrinos. In view of this, we keep the calculations of $\Gamma_\mu^{(W)}$ and $\Gamma_\mu^{(Z)}$ separate from one another. We also make the local approximation, i.e., neglect the momentum dependence in the W and the Z propagators from now on.

When the electron propagator from Eq. (2.2) is substituted in Eq. (2.4), several terms are produced. The terms that are independent of the electron-positron distribution functions contribute only to the standard vacuum part and we drop them. The rest of the terms contain either one or two factors of $\eta(p\cdot v)$ and therefore contribute to Γ'_μ. However, the terms with two factors of η contribute only to the absorptive part of the amplitude. In this work we will calculate only the dispersive part of the form factors and hence we also drop the terms with two factors of η (Ref. 3).

The expression for $\Gamma_\mu^{(W)}$ in Eq. (2.4) can be rewritten by using the Fierz-type identity

$$\gamma^\alpha L A\gamma_\alpha L = -(\mathrm{Tr}\,A\gamma_\alpha L)\gamma^\alpha L\,, \tag{2.7}$$

which is valid for any 4×4 matrix A. It then follows that we can write both the expressions in Eq. (2.4) in the form

$$\Gamma_\mu'^{(W,Z)} = \mathcal{T}_{\mu\nu}^{(W,Z)}\gamma^\nu L\,, \tag{2.8}$$

where

$$\begin{aligned}\mathcal{T}_{\mu\nu}^{(Z)} &= \frac{eg_z^2}{M_Z^2}\int\frac{d^4p}{(2\pi)^3}\mathrm{Tr}[(\not{p}-\not{q}+m)\gamma_\mu(\not{p}+m)\\ &\quad\times\gamma_\nu(a_Z+b_Z\gamma_5)]\\ &\quad\times\left[\frac{\delta[(p-q)^2-m^2]\eta[(p-q)\cdot v]}{p^2-m^2}\right.\\ &\quad\left.+\frac{\delta(p^2-m^2)\eta(p\cdot v)}{(p-q)^2-m^2}\right]\end{aligned} \tag{2.9}$$

and $\mathcal{T}_{\mu\nu}^{(W)}$ is given by an expression that can be obtained from this by making the following replacements:

$$\frac{g_Z^2}{M_Z^2}\to\frac{g^2}{2M_W^2}\,,\quad a_Z\to\tfrac{1}{2}\,,\quad b_Z\to-\tfrac{1}{2}\,. \tag{2.10}$$

In view of this, we henceforth concentrate on the evaluation of $\Gamma_\mu^{(Z)}$ only. The results for $\Gamma_\mu^{(W)}$ can easily be recovered at any stage by using the substitutions given in Eq. (2.10).

Making the change of variable $p\to p+q$ in the first integral of Eq. (2.9) and carrying out the traces, one then obtains

$$\mathcal{T}^{(Z)}_{\mu\nu}=\frac{4eg_Z^2}{M_Z^2}\int\frac{d^3p}{(2\pi)^3 2E}\left[a_Z(f_-+f_+)\left[\frac{2p_\mu p_\nu+(p_\mu q_\nu+q_\mu p_\nu)-g_{\mu\nu}p\cdot q}{q^2+2p\cdot q}+(q\to -q)\right]\right.$$
$$\left.-b_Z(f_--f_+)i\epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta\left[\frac{1}{q^2+2p\cdot q}+(q\to -q)\right]\right], \quad (2.11)$$

where

$$p^\mu=(E,\mathbf{p}),\quad E=\sqrt{\mathbf{p}^2+m^2}\ . \quad (2.12)$$

We have introduced the electron and positron distributions

$$f_\mp(p)=\frac{1}{e^{\beta(p\cdot v\mp\mu)}+1} \quad (2.13)$$

with p^μ defined in Eq. (2.12). It should be noticed that $p\cdot v$ is equal to the energy of the background particles in the rest frame of the medium and that, in this frame,

$$n_\mp=2\int\frac{d^3p}{(2\pi)^3}f_\mp(p) \quad (2.14)$$

give the number densities of electrons and positrons. The factor 2 in the last formula appears because of spin degeneracy.

The remainder of the calculation can be conveniently organized by making use of the following observations. Electromagnetic gauge invariance implies that $\mathcal{T}_{\mu\nu}$ must satisfy

$$q^\mu\mathcal{T}_{\mu\nu}=0\ . \quad (2.15)$$

From Eq. (2.11) it is easily verified that both $\mathcal{T}^{(W)}_{\mu\nu}$ and $\mathcal{T}^{(Z)}_{\mu\nu}$ separately satisfy this equation. However, from Eq. (2.11) we also see that both $\mathcal{T}^{(W)}_{\mu\nu}$ and $\mathcal{T}^{(Z)}_{\mu\nu}$ have the additional property that they are symmetric under the simultaneous interchange $\mu\to\nu, q\to -q$. Therefore, $\mathcal{T}_{\mu\nu}$ also satisfies[4]

$$q^\nu\mathcal{T}_{\mu\nu}=0\ . \quad (2.16)$$

Moreover, from Eq. (2.9) or (2.11), one can see that with our approximation of keeping only the leading-order terms in $1/M_W^2$, $\mathcal{T}_{\mu\nu}$ does not depend separately on k and k'; rather, it is a function of q only. The most general form of $\mathcal{T}_{\mu\nu}$ that is consistent with these properties is

$$\mathcal{T}_{\mu\nu}=\mathcal{T}_T R_{\mu\nu}+\mathcal{T}_L Q_{\mu\nu}+\mathcal{T}_P P_{\mu\nu}\ , \quad (2.17)$$

where

$$R_{\mu\nu}\equiv\tilde{g}_{\mu\nu}-Q_{\mu\nu}\ ,$$
$$Q_{\mu\nu}\equiv\frac{\tilde{v}_\mu\tilde{v}_\nu}{\tilde{v}^2}\ , \quad (2.18)$$
$$P_{\mu\nu}\equiv\frac{i}{\mathcal{Q}}\epsilon_{\mu\nu\alpha\beta}q^\alpha v^\beta\ .$$

We have defined

$$\tilde{g}_{\mu\nu}\equiv g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2} \quad (2.19)$$

and

$$\tilde{v}_\mu\equiv\tilde{g}_{\mu\nu}v^\nu\ . \quad (2.20)$$

Further, $\mathcal{T}_{T,L,P}$ are functions of the scalar variables

$$\Omega\equiv q\cdot v\ ,$$
$$\mathcal{Q}\equiv\sqrt{\Omega^2-q^2}\ , \quad (2.21)$$

the latter of which also appears in the definition of $P_{\mu\nu}$ in Eq. (2.18). We will indicate the dependence of $\mathcal{T}_{T,L,P}$ on these variables explicitly when needed.

The tensors $R_{\mu\nu}$, $Q_{\mu\nu}$, and $P_{\mu\nu}$ have the useful property that when one of them is contracted with the others it gives zero, while

$$R_{\mu\nu}R^{\mu\nu}=2\ ,$$
$$Q_{\mu\nu}Q^{\mu\nu}=1\ , \quad (2.22)$$
$$P_{\mu\nu}P^{\mu\nu}=-2\ .$$

The form factors $\mathcal{T}_{T,L,P}$ can thus be obtained by projecting as follows:

$$\mathcal{T}_T=\tfrac{1}{2}R_{\mu\nu}\mathcal{T}^{\mu\nu}=\frac{1}{2}\left[\mathcal{T}^\mu{}_\mu-\frac{v^\mu v^\nu}{\tilde{v}^2}\mathcal{T}_{\mu\nu}\right],$$
$$\mathcal{T}_L=Q_{\mu\nu}\mathcal{T}^{\mu\nu}=\frac{v^\mu v^\nu}{\tilde{v}^2}\mathcal{T}_{\mu\nu}\ , \quad (2.23)$$
$$\mathcal{T}_P=-\tfrac{1}{2}P_{\mu\nu}\mathcal{T}^{\mu\nu}\ .$$

Applying this to the expression for $\mathcal{T}_{\mu\nu}$ given in Eq. (2.11), we obtain

$$\mathcal{T}^{(Z)}_T=\frac{2eg_Z^2}{M_Z^2}a_Z\left[A-\frac{B}{\tilde{v}^2}\right],$$
$$\mathcal{T}^{(Z)}_L=\frac{4eg_Z^2}{M_Z^2}a_Z\frac{B}{\tilde{v}^2}\ , \quad (2.24)$$
$$\mathcal{T}^{(Z)}_P=-\frac{4eg_Z^2}{M_Z^2}b_Z\mathcal{Q}C\ ,$$

where

$$A=\int\frac{d^3p}{(2\pi)^3 2E}(f_-+f_+)\left[\frac{2m^2-2p\cdot q}{q^2+2p\cdot q}+(q\to -q)\right],$$
$$B=\int\frac{d^3p}{(2\pi)^3 2E}(f_-+f_+)\times\left[\frac{2(p\cdot v)^2+2(p\cdot v)(q\cdot v)-p\cdot q}{q^2+2p\cdot q}+(q\to -q)\right], \quad (2.25)$$
$$C=\int\frac{d^3p}{(2\pi)^3 2E}(f_--f_+)\frac{p\cdot\tilde{v}}{\tilde{v}^2}\left[\frac{1}{q^2+2p\cdot q}+(q\to -q)\right].$$

The expressions for $\mathcal{T}^{(W)}_{T,L,P}$ can be obtained from Eq. (2.24) by using the substitutions indicated in Eq. (2.10).

The integrals in Eq. (2.25) must be interpreted as their principal values. The convenience of presenting the form factors $\mathcal{T}_{T,L,P}$ in this form lies in the fact that A, B, and C are scalars, so that the integrations can be performed in any reference frame, in particular in the rest frame of the medium which is defined by setting $v^\mu=(1,\mathbf{0})$. The details of this calculation have been summarized in the Appendix. To collect together some of the results for future reference, we introduce some notation. In the rest frame of the medium, we denote the components of the four-vectors q^μ and p^μ by

$$q^\mu=(\Omega,\mathcal{Q}), \quad p^\mu=(\mathcal{E},\mathcal{P}) \tag{2.26}$$

in the rest frame. Thus, $\mathcal{Q}=|\mathcal{Q}|$ and Ω are the invariants defined in Eq. (2.21), and $\mathcal{P}$ and $\mathcal{E}$ can also be written in manifestly covariant forms similar to the expressions in Eq. (2.21). In this notation, then

$$A-\frac{B}{\tilde{v}^2}=\begin{cases}-2\omega_0^2+O(\Omega^2) & (\text{for } \mathcal{Q}\to 0),\\ O(\mathcal{Q}^2) & (\text{for } \Omega\to 0),\end{cases} \tag{2.27}$$

$$\frac{B}{\tilde{v}^2}=\begin{cases}\dfrac{\mathcal{Q}^2\omega_0^2}{\Omega^2} & (\text{for } \mathcal{Q}\to 0),\\ \dfrac{1}{2}\displaystyle\int\frac{d^3\mathcal{P}}{(2\pi)^3}\frac{d}{d\mathcal{E}}(f_-+f_+)+O(\mathcal{Q}^2) & (\text{for } \Omega\to 0),\end{cases} \tag{2.28}$$

$$C=\begin{cases}-\dfrac{1}{2}\displaystyle\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}\frac{f_--f_+}{\mathcal{E}}\left[1-\frac{2\mathcal{P}^2}{3\mathcal{E}^2}\right]+O(\Omega^2) & (\text{for } \mathcal{Q}\to 0),\\ \dfrac{1}{2}\displaystyle\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}\frac{d}{d\mathcal{E}}(f_-+f_+)+O(\mathcal{Q}^2) & (\text{for } \Omega\to 0),\end{cases} \tag{2.29}$$

where ω_0^2 is defined in Eq. (A19).

We reiterate that, while these expressions are valid only for the limiting values of q indicated, no assumption has been made regarding the condition of the electron gas. Thus, they hold for a degenerate or a nondegenerate gas, whether or not it is relativistic.

III. PHYSICAL INTERPRETATION OF THE FORM FACTORS

Although the separation of $\mathcal{T}_{\mu\nu}$ into its $\mathcal{T}_{T,L,P}$ components is very convenient for carrying out the explicit calculations, it is not immediately obvious what is the physical interpretation of these form factors. For this purpose it is more convenient to rewrite Γ'_μ in the form

$$\Gamma'_\mu=[F_1\tilde{g}_{\mu\nu}\gamma^\nu+F_2\tilde{v}_\mu\not{v}+iF_3(\gamma_\mu v_\nu-\gamma_\nu v_\mu)q^\nu + iF_4\epsilon_{\mu\nu\alpha\beta}\gamma^\nu q^\alpha v^\beta]L\ , \tag{3.1}$$

where

$$\begin{aligned}F_1&=\mathcal{T}_T+\frac{\Omega^2}{\mathcal{Q}^2}(\mathcal{T}_L-\mathcal{T}_T)\ ,\\ F_2&=\frac{1}{\tilde{v}^2}(\mathcal{T}_L-\mathcal{T}_T)\ ,\\ iF_3&=-\frac{\Omega}{\mathcal{Q}^2}(\mathcal{T}_L-\mathcal{T}_T)\ ,\\ F_4&=\frac{\mathcal{T}_P}{\mathcal{Q}}\ .\end{aligned} \tag{3.2}$$

Thus, F_1 is of the form of the standard charge-radius form factor. The physical interpretation of the other terms is obtained by considering, as usual, the interaction with an external, static field. Thus, taking the external field of the form $A^\mu=(\phi,\mathbf{0})$ in the rest frame of the medium, we see that F_2 yields an additional contribution to the charge radius. To obtain the interpretation of $F_{3,4}$ let us recall some of the results of Ref. 2. That work analyzed the implications of the terms in Γ'_μ of the form

$$iD'_E(\gamma_\mu v_\nu-\gamma_\nu v_\mu)q^\nu\gamma_5+iD'_M\epsilon_{\mu\nu\alpha\beta}\gamma^\nu\gamma_5 q^\alpha v^\beta\ . \tag{3.3}$$

It was shown there that, for massive neutrinos, D'_E and D'_M can be regarded as additional contributions to the electric and magnetic dipole form factors, in the nonrelativistic limit. Comparing Eq. (3.3) with Eq. (3.1) and using Eq. (3.2), we then identify

$$\begin{aligned}D'_E&=-\frac{i\Omega}{2\mathcal{Q}^2}(\mathcal{T}_L-\mathcal{T}_T)\ ,\\ D'_M&=-\frac{\mathcal{T}_P}{2\mathcal{Q}}\ .\end{aligned} \tag{3.4}$$

The discussion so far has implicitly assumed that we are treating the case of Dirac neutrinos. In particular, Eq. (3.1) holds only if the neutrinos are Dirac particles. For Majorana neutrinos we have to add the contribution from diagrams similar to those of Fig. 1, but with the external neutrino lines crossed. Such diagrams would contribute to the process $\bar{\nu}\to\bar{\nu}\gamma$ for Dirac neutrinos, but for Majorana neutrinos they contribute to the same process. The net result is that

$$\Gamma^{\text{Majorana}}_\mu(k,k',v)=\Gamma_\mu(k,k',v)+\Gamma^C_\mu(-k',-k,v)\ , \tag{3.5}$$

where Γ_μ is the quantity that we have already calculated and Γ^C_μ is obtained from Γ_μ by multiplying every quantity that appears in Γ_μ by its charge-conjugation phase η_C. The values of η_C for the various quantities are given in Table II of Ref. 2. For the present purpose, it suffices to know that for the particular form of Γ'_μ given in Eq. (3.1), Γ^C_μ is obtained simply by making the replacement $\gamma_\alpha L\to-\gamma_\alpha R$. Noting that, as a consequence of the local limit $M_W\to\infty$, the form factors F_i depend only on q and not on k,k' separately, it is trivial to see that

$$F_i(-k',-k,v)=F_i(k,k',v)\ , \tag{3.6}$$

so that Eq. (3.5) yields

$$\Gamma^{\text{Majorana}}_\mu=-[F_1\tilde{g}_{\mu\nu}\gamma^\nu+F_2\tilde{v}_\mu\not{v}+iF_3(\gamma_\mu v_\nu-\gamma_\nu v_\mu)q^\nu + iF_4\epsilon_{\mu\nu\alpha\beta}\gamma^\nu q^\alpha v^\beta]\gamma_5\ . \tag{3.7}$$

Therefore, in particular,

$$D_E'^{\text{Majorana}} = -\frac{i\Omega}{Q^2}(\mathcal{T}_L - \mathcal{T}_T) ,$$
$$D_M'^{\text{Majorana}} = \frac{\mathcal{T}_P}{Q} . \tag{3.8}$$

These results illustrate very simply some of the conclusions of Ref. 2. For example, if the chemical potential μ of the electron background is zero, then $f_- = f_+$ and therefore, from Eq. (2.25), $\mathcal{T}_P = 0$, which in turn implies that $D_M' = 0$ for both Dirac and Majorana neutrinos. This result is a consequence of CP invariance, as can be deduced by referring to Eq. (11d) of Ref. 2. Since the Lagrangian is CP invariant and, for $\mu = 0$, the background is CP symmetric, D_M' must satisfy

$$D_M'(-k', -k, v) = -D_M'(k, k', v) , \tag{3.9}$$

independently of whether the neutrino is of the Dirac or Majorana type. Since in our case D_M' is a function only of $q = k - k'$, Eq. (3.9) implies that it is zero. However, when $f_- \neq f_+$, we have CP as well as CPT asymmetries in the medium, and therefore D_M' is nonzero in general.

As another example, consider the static limit ($\Omega \to 0$) of D_E'. It will be shown in the Appendix that $\mathcal{T}_{T,L}$ have well-defined limits as $\Omega \to 0$. Thus, Eqs. (3.4) and (3.8) imply that $D_E' = 0$ as $\Omega = 0$ for both Dirac and Majorana neutrinos. This result can be understood in terms of time-reversal invariance as follows.

Consider the restrictions on D_E' due to the Hermiticity of the Lagrangian and time-reversal (T) symmetry, both of which hold in our case. Equations (11b) and (13) of Ref. 2 imply the following relations for D_E':

$$\text{Hermiticity:}\quad D_E'^*(k', k, v) = D_E'(k, k', v) ,$$
$$T\ \text{symmetry:}\quad D_E'^*(k, k', v) = -D_E'(k, k', v) . \tag{3.10}$$

Again, these relations should hold for Dirac as well as Majorana neutrinos. The second equation in (3.10) implies that D_E' is purely imaginary while the first equation then implies that D_E' must change sign under the substitution $q \to -q$, or equivalently, that D_E' is an odd function of Ω and therefore must vanish when $\Omega = 0$. Since the zero-momentum limit of D_E' gives the matter-induced electric dipole moment,[2] it is zero within our approximations.

IV. APPLICATIONS

Apart from illustrating some of the general aspects considered in Ref. 2, the results of the present work can be useful in various contexts. In this section we consider two possible applications.

A. Plasmon decay

The process $\gamma_{\text{pl}} \to \nu\bar{\nu}$, where γ_{pl} represents a photon mode propagating through a plasma, is an important process in the cooling mechanism of white dwarfs.[5] The rate for this process was calculated many years ago in the context of a local $V - A$ theory[6] and was later corrected to include the effects of the neutral currents of the standard model.[7] Here we show how the rate for this process can be deduced straightforwardly from our results.

The amplitude for the process

$$\gamma_{\text{pl}}(q) \to \nu(k')\bar{\nu}(k) \tag{4.1}$$

is given by

$$iM = -i\sqrt{N_{q\lambda}}\, e^{\mu}(q\lambda)\bar{u}(k')\Lambda_\mu v(k) , \tag{4.2}$$

where

$$\Lambda_\mu = \Gamma_\mu'(-k, k', v) \tag{4.3}$$

and $\sqrt{N_{q\lambda}}$ is the normalization factor that is associated with photons in external lines[8] and e^μ is the polarization vector. Since, in the local limit ($M_W \to \infty$), Γ_μ' depends only on q and not on k and k' separately, we can write

$$\Lambda_\mu = (\mathcal{T}_T R_{\mu\nu} + \mathcal{T}_L Q_{\mu\nu} - \mathcal{T}_P P_{\mu\nu})\gamma^\nu L , \tag{4.4}$$

where the tensors $R_{\mu\nu}$, $Q_{\mu\nu}$, and $P_{\mu\nu}$ are defined in Eq. (2.18). In writing Eq. (4.4), it was also necessary to use the facts that $\mathcal{T}_{T,L,P}$ are even functions of q, as Eqs. (2.24) and (2.25) show. In Eq. (4.3) we are dropping the background-independent part since it is of order $(q^2)^2/M_W^2 m^2$ and hence a factor $\sim q^2/m^2$ smaller than the background-dependent term, m being the electron mass.[9]

To proceed we borrow the following result from Ref. 8. It was shown there that the tensors $R_{\mu\nu}$, $Q_{\mu\nu}$, and $P_{\mu\nu}$ have the following representations in terms of the polarization vectors with definite helicity:

$$R_{\mu\nu} = -[e_\mu^*(q+)e_\nu(q+) + e_\mu^*(q-)e_\nu(q-)] ,$$
$$Q_{\mu\nu} = -e_\mu(q3)e_\nu(q3) , \tag{4.5}$$
$$P_{\mu\nu} = e_\mu^*(q+)e_\nu(q+) - e_\mu^*(q-)e_\nu(q-) .$$

From this we obtain

$$e^\mu(q\lambda)\Lambda_\mu = (\mathcal{T}_T - \lambda\mathcal{T}_P)e_\nu(q\lambda)\gamma^\nu L \quad \text{for } \lambda = + \text{ or } - ,$$
$$e^\mu(q3)\Lambda_\mu = \mathcal{T}_L e_\nu(q3)\gamma^\nu L , \tag{4.6}$$

which simplify the calculations enormously.

The decay rate, in the rest frame of the medium, is given by

$$d\Gamma = \frac{1}{2\Omega_{Q\lambda}}(2\pi)^4\delta^4(q - k - k')|M|^2 \frac{d^3k}{(2\pi)^3 2\omega}\frac{d^3k'}{(2\pi)^3 2\omega'} , \tag{4.7}$$

where $q^\mu = (\Omega_{Q\lambda}, \mathbf{Q})$ with $\Omega_{Q\lambda}$ representing the photon energy in the rest frame of the medium. Below we give the details only for the case of longitudinally polarized plasmons, since the calculation is very similar for the other polarization states.

Substituting Eq. (4.6) in Eq. (4.2) and carrying out the traces involved in $|M|^2$, we obtain

$$\Gamma_L = \frac{N_{Q3}|\mathcal{T}_L|^2}{(2\pi)^2\Omega_{Q3}} J_{\mu\nu} Q^{\mu\nu} . \tag{4.8}$$

Here $J_{\mu\nu}$ is the kinematic integral

$$J_{\mu\nu}\equiv-\int\frac{d^3k}{2\omega}\int\frac{d^3k'}{2\omega'}\delta^4(q-k-k')\times(k_\mu k'_\nu+k_\nu k'_\mu-k\cdot k' g_{\mu\nu})=\frac{\pi}{6}(q^2 g_{\mu\nu}-q_\mu q_\nu)\ , \tag{4.9}$$

which gives

$$\Gamma_L=\frac{\Omega_{Q3}|\mathcal{T}_L|^2}{24\pi}\left[1-\frac{Q^2}{\Omega_{Q3}^2}\right]N_{Q3}\ . \tag{4.10}$$

Then, recalling from Ref. 8, the formula

$$N_{Q3}=\frac{2\Omega_{Q3}}{\left[\frac{\partial}{\partial\Omega}(q^2\epsilon_l)\right]_{\Omega=\Omega_{Q3}}} \tag{4.11}$$

we finally obtain

$$\Gamma_L=\left[\frac{|\mathcal{T}_L|^2}{12\pi\left[\frac{\partial\epsilon_l}{\partial\Omega}\right]}\right]_{\Omega=\Omega_{Q3}}\ , \tag{4.12}$$

where we have used the fact that Ω_{Q3} is the solution to $\epsilon_l(\Omega_{Q3},Q)=0$.

To compare this with the old result[6] of the $V-A$ theory, we use the approximate result of Eq. (A21) for $\mathcal{T}_L$ and, on the other hand, recall that ϵ_l is given simply by

$$\epsilon_l(\Omega,Q)=1-\frac{4e^2\omega_0^2}{\Omega^2}\ , \tag{4.13}$$

where ω_0^2 is defined in Eq. (A19). Thus

$$\Omega_{Q3}^2=4e^2\omega_0^2\ , \tag{4.14}$$

and therefore

$$\Gamma_L=\frac{(\Omega_{Q3}^2-Q^2)^2G_F^2}{24\pi^2\alpha\left[\frac{\partial\epsilon_l}{\partial\Omega}\right]_{\Omega=\Omega_{Q3}}}\ , \tag{4.15}$$

which is the known result.

For completeness, we give the formula for the decay rate of the transverse modes as well. Following steps analogous to those leading to Eq. (4.10), we obtain, for the transverse modes,

$$\Gamma_\lambda=\frac{\Omega_{Q\lambda}|\mathcal{T}_T-\lambda\mathcal{T}_P|^2}{24\pi}\left[1-\frac{Q^2}{\Omega_{Q\lambda}^2}\right]N_{Q\lambda}\quad\text{for }\lambda=+\text{ or}-\ . \tag{4.16}$$

The $\mathcal{T}_P$ term in this equation comes from the axial-vector current, and is usually neglected in the standard calculations.[6] This is justified by the fact that $\mathcal{T}_P$ is a factor of Q/m smaller than $\mathcal{T}_T$, as can easily be deduced from Eq. (2.24) with the help of the formulas in Eqs. (2.27) and (2.29). Nevertheless, as a by-product of our calculation, we obtain the correction from the axial-vector current when the factor Q/m is not necessarily small, represented by the presence of $\mathcal{T}_P$ in Eq. (4.16).

Apart from its ability to reproduce known results for $\gamma_{\rm pl}$ decay, our model of calculation provides an efficient way to include corrections to these results, such as the contribution from more diagrams involving the exchange of other particles that are predicted by extensions of the standard model.

B. Neutrino index of refraction in matter in the presence of a magnetic field

As first shown by Wolfenstein,[10] neutrinos propagating through matter have an index of refraction. Thus, the dispersion relation for neutrinos in a medium differ from the one in vacuum, viz., $E=|\mathbf{p}|$, for massless neutrinos. Recently, it has been shown how the dispersion relation, and equivalently the index of refraction, can be obtained from the calculation of the neutrino self-energy using covariant techniques of finite-temperature field theory.[11,12] The method used in Refs. 11 and 12, coupled with the results of the present work, provide a simple and efficient way to calculate the correction to the index of refraction due to an external magnetic field, as we will show now.

However, before presenting the details of the calculation we should stress the following. The possibility that neutrinos may have electromagnetic dipole moment interactions, which change left-handed neutrino into right-handed ones, can have important consequences in the context of the solar-neutrino puzzle.[13,14] Moreover, the combined effect of matter density and magnetic fields on neutrino flavor oscillations and spin precession has been studied.[14,15] However, the existence of such chirality flipping interactions require either (a) neutrino masses, (b) right-handed neutrinos, or (c) lepton-number violation so that, for example, transitions of the form $\nu_{eL}\to\nu_{\mu R}^c$ can occur. Further, none of these ingredients is contained in the standard model with just one Higgs doublet, and therefore require that it be enlarged.

On the contrary, the effect that we discuss here is quite different. In particular, it does not require any of the ingredients mentioned above. The new form factors that we calculate preserve flavor and chirality, and are present even in the standard model with massless left-handed neutrinos. These new terms arise because of the interaction of the neutrino with the particles in the background. In the presence of an external magnetic field the new terms, instead of inducing flavor or spin transitions, contribute to the neutrino index of refraction and can modify the Wolfenstein resonance condition[10] for neutrino oscillation in matter.

Let us consider the scattering of a neutrino by an external electromagnetic field. If we define the Fourier transform of the external potential by

$$A_\mu^{\rm ext}(x)=\int\frac{d^4q}{(2\pi)^4}A_\mu(q)e^{-iq\cdot x} \tag{4.17}$$

then the contribution to the off-shell ν-ν S-matrix element is

$$S_{(\nu\nu)}=-i\Gamma_\mu(k,k',v)A^\mu(k'-k)\ . \tag{4.18}$$

We now restrict ourselves to a static field. In addition, we assume that the field is uniform in space. The justification for the latter assumption is that we are working in a region of space that is microscopically large but macroscopically small, and therefore the external field is approximately constant over the region. The macroscopic dependence of the external field on **x** can be restored at the end of our calculation.

For a static and uniform field, only the F_4 term in Eq. (3.1) contributes for Γ'_μ. This can be seen as follows. In the static limit $F_3=0$ as Eq. (3.2) shows, since $\mathcal{T}_{T,L}$ are finite in that limit. The F_2 term in the static limit contains a factor $v\cdot A$, which is zero for an external *magnetic* field. On the other hand, by taking the $\Omega\to 0$ limit of F_1 in Eq. (3.2) and using the result that $\mathcal{T}_T(0,\mathcal{Q})=O(\mathcal{Q}^2)$, it is easy to see that the F_1 term has a factor $\mathcal{Q}^2 A_\mu$, which is zero for a uniform field as well. To rewrite the F_4 term in a convenient way, we can define

$$B_\mu \equiv \tilde{F}_{\mu\nu}v^\nu = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}v^\nu F^{\alpha\beta} \tag{4.19}$$

so that in the rest frame of the medium, $B^\mu=(0,\mathbf{B})$. Thus, Eq. (4.18) gives

$$S_{(\nu\nu)} = -iF_4\gamma^\mu L B_\mu(k'-k) \ . \tag{4.20}$$

Taking a uniform magnetic field implies that

$$B_\mu(k'-k)=(2\pi)^4\delta^4(k'-k)B_\mu^{\text{ext}} \ , \tag{4.21}$$

where B_μ^{ext} is the magnetic field in coordinate space. Therefore, we finally arrive at

$$S_{(\nu\nu)} = -iF_4(2\pi)^4\delta^4(k'-k)\gamma^\mu L B_\mu^{\text{ext}} \ . \tag{4.22}$$

Thus, in the presence of the external magnetic field, the neutrino acquires an additional contribution to its self-energy that depends on the field. Identifying the self-energy as usual by Σ, defined by

$$S_{(\nu\nu)} = -iR\Sigma L \ , \tag{4.23}$$

the complete self-energy in the medium in the presence of an external magnetic field can be written as

$$\Sigma = a\not{k} + b\not{v} + c\not{B}^{\text{ext}} \ , \tag{4.24}$$

where the coefficients a and b have been calculated previously[11,12] and

$$c=F_4(\Omega=0,\ \mathcal{Q}\to 0) \ . \tag{4.25}$$

In Refs. 11 and 12, it has been shown that, with $B_\mu^{\text{ext}}=0$, Eq. (4.24) can be used to determine the dispersion relation satisfied by neutrinos propagating through the medium. Further, it was shown that this method reproduces the Wolfenstein formula for the index of refraction. Borrowing from Refs. 11 and 12 the general procedure and the result that $a=0$, the dispersion relation for the present case is found to be

$$\omega_k = |\mathbf{k} - c\mathbf{B}^{\text{ext}}| + b \ , \tag{4.26}$$

where now all the quantities refer to the rest frame of the medium. Here,

$$b=(n_- - n_+)\times\begin{cases}\dfrac{g^2}{4M_W^2}+\dfrac{g_Z^2}{M_Z^2}a_Z & \text{for } \nu_e \ ,\\[2mm] \dfrac{g_Z^2}{M_Z^2}a_Z & \text{for } \nu\neq\nu_e \ .\end{cases} \tag{4.27}$$

As for c, remembering Eq. (3.2) and using Eqs. (2.24) and (2.29), we obtain

$$c=2e\int\frac{d^3p}{(2\pi)^3 2E}\frac{d}{dE}(f_- - f_+)\times\begin{cases}\dfrac{g^2}{4M_W^2}-\dfrac{g_Z^2}{M_Z^2}b_Z & \text{for } \nu_e \ ,\\[2mm] -\dfrac{g_Z^2}{M_Z^2}b_Z & \text{for } \nu\neq\nu_e \ .\end{cases} \tag{4.28}$$

The most significant feature of Eq. (4.26) is that, for neutrinos moving in different directions in space, one obtains different dispersion relations and therefore different speeds. Equivalently, the index of refraction, defined by

$$n=\frac{|\mathbf{k}|}{\omega_k} \ , \tag{4.29}$$

is not isotropic. This is caused by the direction of the external magnetic field.

For weak fields, Eq. (4.26) yields

$$\omega_k = |\mathbf{k}| - c\hat{\mathbf{k}}\cdot\mathbf{B}^{\text{ext}} + b \tag{4.30}$$

which in turn implies

$$n \simeq 1 + \frac{c\hat{\mathbf{k}}\cdot\mathbf{B}^{\text{ext}} - b}{|\mathbf{k}|} \ . \tag{4.31}$$

As an example, consider the nonrelativistic and nondegenerate limit of the electron gas. In this case, $n_+=0$ in the expression for b in Eq. (4.27). Then, using Eqs. (A4) and (A11), we can simplify Eq. (4.28) as

$$c=-\mu_B\beta n_-\times\begin{cases}\dfrac{g^2}{4M_W^2}-\dfrac{g_Z^2}{M_Z^2}b_Z & \text{for } \nu_e \ ,\\[2mm] -\dfrac{g_Z^2}{M_Z^2}b_Z & \text{for } \nu\neq\nu_e \ ,\end{cases} \tag{4.32}$$

where μ_B is the Bohr magneton, $e/(2m)$. Thus, Eq. (4.30) yields in this case

$$\omega_k = |\mathbf{k}| + b(1+\mu_B\beta r\hat{\mathbf{k}}\cdot\mathbf{B}^{\text{ext}}) \ , \tag{4.33}$$

where

$$r=\begin{cases}\dfrac{1-(4g_Z^2M_W^2)b_Z/(g^2M_Z^2)}{1+(4g_Z^2M_W^2)a_z/(g^2M_Z^2)} & \text{for } \nu_e \ ,\\[2mm] -\dfrac{b_Z}{a_Z} & \text{for } \nu\neq\nu_e \ .\end{cases} \tag{4.34}$$

The relative importance of the matter-density effects and the magnetic-field effects thus depend on the magnitude of $\mu_B \beta r|\mathbf{B}|$. Since the magnetic field in the Sun is at most a few tenths of a tesla and the core temperature is of order 1 keV, we obtain $\mu_B \beta r|\mathbf{B}| \sim 10^{-8}$, using $r \sim 1$ from Eq. (4.34) and $\mu_B = 5.8\times 10^{-11}$ MeV/T.

Although the application of these results to the solar neutrinos appears to be of no consequences, their possible application in other physical contexts remains an open question and their implications should be kept in mind.

V. CONCLUSIONS

In the present work we have carried out an explicit calculation of the neutrino electromagnetic vertex in a background of electrons. We have been motivated in part by our desire to provide examples of some of the results obtained in Ref. 2. However, our calculations also have some practical applications. We have illustrated this by rederiving in a simple manner the standard formulas for the decay rate of plasmons into neutrinos. Those calculations serve to show the way how to calculate other, similar decay processes in a medium. We have also determined the index of refraction of neutrinos propagating in matter in the presence of a magnetic field. Although the application of this to the solar-neutrino puzzle seems to be uninteresting, their possible implication in other contexts deserve further attention. Some of these issues are currently under study.

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APPENDIX: CALCULATIONAL DETAILS

Here we give expressions for the functions A, B, and C, defined in Eq. (2.25), in various limiting cases. Since these functions are scalars, we can choose to do the integrals in the rest frame of the medium, defined by $v^\mu=(1,\mathbf{0})$. In this frame, the components of the four-vectors q^μ and p^μ are denoted as in Eq. (2.26).

1. Static limit

We consider first the static limit, i.e.,

$$\Omega = 0 \ . \tag{A1}$$

In this limit, the expression for C in Eq. (2.25) reduces to

$$C(0,\mathcal{Q}) = \frac{1}{2}\int \frac{d^3\mathcal{P}}{(2\pi)^3}(f_- - f_+) \times \left[\frac{1}{2\mathcal{P}\cdot\mathcal{Q} - \mathcal{Q}^2} - \frac{1}{2\mathcal{P}\cdot\mathcal{Q} + \mathcal{Q}^2}\right] . \tag{A2}$$

For small $\mathcal{Q}$, one can easily expand the function within the large parentheses as a power series in $\mathcal{Q}$. This yields

$$C(0,\mathcal{Q}) = \frac{1}{16\pi^2}\sum_{N=0}^{\infty}(\tfrac{1}{2}\mathcal{Q})^{2N}\int_0^\infty \frac{d\mathcal{P}}{\mathcal{P}^{2N}}(f_- - f_+) \times \int_{-1}^{1} d(\cos\theta)\frac{1}{\cos^{2N+2}\theta}$$
$$= -\frac{1}{8\pi^2}\sum_{N=0}^{\infty}\frac{(\frac{1}{2}\mathcal{Q})^{2N}}{2N+1}\int_0^\infty \frac{d\mathcal{P}}{\mathcal{P}^{2N}}(f_- - f_+) , \tag{A3}$$

where θ is the angle between the three-vectors $\mathcal{P}$ and $\mathcal{Q}$. In the nonrelativistic and classical limit, we can put

$$f_+ = 0 \quad \text{and} \quad f_- \approx e^{-\beta(\mathcal{E}-\mu)} , \tag{A4}$$

where $\mathcal{E} \approx m + \mathcal{P}^2/2m$. The integrals in Eq. (A3) can then be evaluated to give

$$C(0,\mathcal{Q}) = -\frac{1}{8\pi^2}\sum_{N=0}^{\infty}\frac{(\frac{1}{2}\mathcal{Q})^{2N}}{2(2N+1)}e^{\beta(\mu-m)} \times \left[\frac{2m}{\beta}\right]^{1/2-N}\Gamma(\tfrac{1}{2}-N) . \tag{A5}$$

We can easily eliminate the explicit dependence on the chemical potential by invoking the expression for the number density of electrons. Using Eq. (2.14) with the nonrelativistic limit in Eq. (A4), we obtain

$$n_- = \frac{1}{2\pi^2}e^{\beta(\mu-m)}\left[\frac{2m}{\beta}\right]^{3/2}\Gamma(\tfrac{3}{2}) , \tag{A6}$$

so that finally

$$C(0,\mathcal{Q}) = -\frac{\beta n_-}{8m}\sum_{N=0}^{\infty}\frac{1}{(2N+1)!!}\left[-\frac{\beta\mathcal{Q}^2}{4m}\right]^N . \tag{A7}$$

There is another way to expand the integrand in Eq. (A2) which brings out the leading term of $C(0,\mathcal{Q})$ more elegantly. For this, change the integration variable $p \to p + \frac{1}{2}q$ in the first term of Eq. (A2) and $p \to p - \frac{1}{2}q$ in the other. This gives

$$C(0,\mathcal{Q}) = \frac{1}{2}\int \frac{d^3\mathcal{P}}{(2\pi)^3}\left[\frac{1}{2\mathcal{P}\cdot\mathcal{Q}}\right] \times [\Delta f(p+\tfrac{1}{2}q) - \Delta f(p-\tfrac{1}{2}q)] , \tag{A8}$$

where

$$\Delta f(p) = f_-(p) - f_+(p) . \tag{A9}$$

This time, while making the expansion of the integrand, we remember that $f_\mp$ depend on $\mathcal{P}$ only through $\mathcal{E}$, so that we obtain

$$C(0,\mathcal{Q}) = \frac{1}{2}\int \frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}\frac{d}{d\mathcal{E}}(f_- - f_+) + O(\mathcal{Q}^2) . \tag{A10}$$

Performing integration by parts, this result can easily be seen to coincide with that of Eq. (A3). Furthermore, for a classical and nonrelativistic electron gas, Eq. (A4) implies

$$\frac{df_-}{d\mathcal{E}} = -\beta f_- , \tag{A11}$$

so that we immediately obtain

$$C(0,\mathcal{Q})=-\frac{\beta n_-}{8m}+O(\mathcal{Q}^2)\ , \tag{A12}$$

which is the same as the leading term of Eq. (A7).

Formulas for the other functions, A and B, are obtained in similar fashion. Omitting the intermediate steps, the results are

$$A(0,\mathcal{Q})=B(0,\mathcal{Q})=\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3}\frac{d}{d\mathcal{E}}(f_-+f_+)+O(\mathcal{Q}^2)\ . \tag{A13}$$

It must be remarked that the $O(\mathcal{Q}^2)$ terms are different for A and B. In particular, this result implies that $\mathcal{T}_T(0,\mathcal{Q})$ vanishes as $\mathcal{Q}\to 0$.

Again, as an example, consider the classical and nonrelativistic gas. Then

$$A(0,\mathcal{Q})=B(0,\mathcal{Q})=-\frac{\beta n_-}{4}+O(\mathcal{Q}^2)\ . \tag{A14}$$

2. Long-wavelength limit

The long-wavelength limit is defined by taking $\mathcal{Q}\to 0$ keeping Ω fixed. In order to evaluate A, B, and C in this limit we first combine the denominators that appear in Eq. (2.25). Let us consider C to start with:

$$C(\Omega,\mathcal{Q})=\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}(f_--f_+)2q^2\left[\frac{\mathcal{E}-\Omega\mathcal{P}\cdot\mathcal{Q}/\mathcal{Q}^2}{q^4-4(p\cdot q)^2}\right]\ . \tag{A15}$$

The denominator in the large parentheses is now expanded for $\mathcal{Q}\to 0$. We will assume also that $\Omega\ll\mathcal{E}$. Omitting the terms linear in $\mathcal{P}$, which integrate to zero, we obtain

$$\left[\frac{\mathcal{E}-\Omega\mathcal{P}\cdot\mathcal{Q}/\mathcal{Q}^2}{q^4-4(p\cdot q)^2}\right]\to\frac{-1}{4\mathcal{E}\Omega^2}\left[1-\frac{2(\mathcal{P}\cdot\mathcal{Q})^2}{\mathcal{E}^2\mathcal{Q}^2}\right]\ . \tag{A16}$$

In the integral we can replace $\mathcal{P}^i\mathcal{P}^j\to\frac{1}{3}\mathcal{P}^2\delta^{ij}$, so that finally we get

$$C(\Omega,0)=-\frac{1}{2}\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}\left[\frac{f_--f_+}{\mathcal{E}}\right]\left[1-\frac{2\mathcal{P}^2}{3\mathcal{E}^2}\right]\quad(\text{for }\Omega\ll\mathcal{E})\ . \tag{A17}$$

Carrying out similar steps for A and B we obtain

$$B(\Omega,\mathcal{Q})|_{\mathcal{Q}\to 0}=\mathcal{Q}^2\omega_0^2/\Omega^2\ , \tag{A18}$$

where

$$\omega_0^2\equiv\int\frac{d^3\mathcal{P}}{(2\pi)^3 2\mathcal{E}}(f_-+f_+)\left[1-\frac{\mathcal{P}^2}{3\mathcal{E}^2}\right] \tag{A19}$$

and

$$A(\Omega,\mathcal{Q})|_{\mathcal{Q}\to 0}=B(\Omega,\mathcal{Q})|_{\mathcal{Q}\to 0}-3\omega_0^2\ . \tag{A20}$$

In particular, using $\tilde{v}^2=-\mathcal{Q}^2/q^2$ and substituting the result for B into Eq. (2.24), we obtain, with the substitutions of Eq. (2.10),

$$\mathcal{T}_L^{(W)}=-\frac{eg^2\omega_0^2}{M_W^2}\frac{q^2}{\Omega^2}=-8e\omega_0^2\frac{G_F}{\sqrt{2}}\frac{q^2}{\Omega^2}\ , \tag{A21}$$

which is the result used to obtain Eq. (4.15) from Eq. (4.12).

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