# Hyperfine-interaction meson spectroscopy and the linkage between contituent-, dynamical-, and current-quark masses

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$SU(3)_f$ current-quark masses and the dynamical-quark mass characterizing the chiral noninvariance of the QCD vacuum are related to hyperfine-interaction constituent-quark mass expressions for $SU(3)_f$ mesons.

Since quarks are nonasymptotic states confined to live within colorless hadrons, there is a great deal of ambiguity associated with assigning them masses. In this paper we endeavor to develop the phenomenological connection between the constituent-quark masses associated with hadron spectroscopy and static hadron properties, the dynamical-quark mass characterizing the chiral noninvariance of the QCD vacuum, and the current-quark masses associated with current divergences and higher-momentum-transfer physics.

The nonrelativistic (valence-) quark model (NRQM) approach to baryon spectroscopy,[1,2] in which the ground-state baryon masses are the sum of all constituent-quark masses and quark-quark hyperfine-interaction terms,

$$m_{\text{baryon}} = m_1 + m_2 + m_3 + B\left[\frac{\mathbf{s}_1\cdot\mathbf{s}_2}{m_1 m_2} + \frac{\mathbf{s}_1\cdot\mathbf{s}_3}{m_1 m_3} + \frac{\mathbf{s}_2\cdot\mathbf{s}_3}{m_2 m_3}\right], \tag{1}$$

succeeds in fitting the eight SU(2) baryon-multiplet masses $(N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega)$ within 5% provided[3]

$$m_u = m_d = 363 \text{ MeV},\quad m_s = 538 \text{ MeV},\quad B = (298 \text{ MeV})^3. \tag{2}$$

The corresponding NRQM approach to $SU(3)_f$-nonsinglet mesons is astonishingly successful, despite the fact that this approach is in apparent contradiction to a chiral-symmetry realization of the pion. As in Eq. (1), the meson mass is assumed to include a hyperfine interaction term in addition to the sum of constituent-quark masses:

$$m_{\text{meson}} = m_q + m_{\bar q} + M\left[\frac{\mathbf{s}_q\cdot\mathbf{s}_{\bar q}}{m_q m_{\bar q}}\right]. \tag{3}$$

Since $\mathbf{s}_q\cdot\mathbf{s}_{\bar q} = -\frac{3}{4}$ for spin-0 mesons and $+\frac{1}{4}$ for spin-1 mesons, the hyperfine interaction provides a means for obtaining a pion mass substantially lighter than its $u$ and $d$ constituents. If we assume equal constituent values for $m_u$ and $m_d$, a least-squares fit of (3) to $\pi$, $K$, $\rho$, and $K^*$ isospin-multiplet masses is obtained using

$$\hat m \equiv m_{u,d} = 306.2 \text{ MeV},\quad m_s = 487.9 \text{ MeV},\quad M = 5.932\times 10^7 \text{ MeV}^3 = (390.0 \text{ MeV})^3. \tag{4}$$

These values succeed extremely well (Table I) in fitting the masses of the (nonisoscalar) SU(3)-meson isospin multiplets. Moreover, application of (3) and (4) to the $\omega$-$\phi$ system in the ideal mixing limit $(\phi = s\bar s)$ yields $m_\omega(=m_\rho) = 771$ MeV and $m_\phi = 1038$ MeV, values within 2% of physical. The hyperfine-interaction term in (3) is also useful in charmed-meson spectroscopy, serving to correlate the sizes of $(F^* - F)$ and $(D^* - D)$ relative to $(K^* - K)$ and $(\rho - \pi)$ mass differences.[4] The phenomenological consistency of (3) is further supported by the approximate agreement between the constituent masses in the baryonic (2) and mesonic (4) fits, as well as the rough agreement with the $M/B = 2$ prediction of the color SU(3) for the relative strengths of $q\bar q$ and $qq$ hyperfine-interaction couplings.

Of course, the empirical successes of (3) in relating the pion mass to the masses of heavier mesons are somewhat embarrassing. The nonrelativistic addition of constituent masses in (3) would appear inappropriate for generating a light $(m_\pi \sim 138$ MeV) bound state from much heavier constituents $(\hat m \sim m_N/3)$. Moreover, (3) contains no information about chiral symmetry; a naive application of (3) would have the pion mass grow as $\hat m$ decreases, in apparent contradiction to our understanding of the pion as the Nambu-Goldstone particle of broken $SU(2)_f$ chiral symmetry. Such breakdown (more carefully considered) may be characterized as explicit, through the introduction of small quark masses in the QCD Lagrangian, or dynamical, through the chiral noninvariance of the QCD

TABLE I. Comparison of physical, isomultiplet-averaged masses of $SU(3)_f$-nonsinglet mesons with masses predicted from Eq. (3) using the three parameter values of Eq. (4).

| | Physical | Eq. (3) |
|---|---|---|
| $\pi^0$ | 138 | 137.8 |
| $K$ | 496 | 496.3 |
| $\rho$ | 770 | 770.6 |
| $K^*$ | 894 | 893.4 |

 

vacuum. The quark masses characterizing chiral-symmetry breakdown in the former case are known to be quite small compared to those in (2) or (4). By contrast, chiral noninvariance of the QCD vacuum allows the occurrence of a chiral-noninvariant vacuum expectation value $\langle 0|{:}q(x)\bar{q}(y){:}|0\rangle$, whose contribution to the fermion two-point function leads to a "dynamical mass"[5–8] quite comparable to $\hat{m}$ in (2) and (4). In the absence of explicit chiral-symmetry breaking (i.e., mass terms in the QCD Lagrangian), dynamical-symmetry breaking through the chiral-noninvariant QCD vacuum necessarily would lead to a zero mass pion. The only way to reconcile (3) to this boundary condition is to identify the dynamical quark mass with the zero-pion-mass limit of the constituent mass $\hat{m}$:

$$\lim_{\hat{m}\to m_{\rm dyn}} (m_\pi) = 2m_{\rm dyn} - 3M/4m_{\rm dyn}^2 = 0 \ , \tag{5}$$

in which case

$$m_{\rm dyn} = (3M/8)^{1/3} \approx 281 \text{ MeV} \ . \tag{6}$$

This phenomenological estimate is quite comparable to the theoretical value[6–8] generated through $\langle 0|{:}q(x)\bar{q}(y){:}|0\rangle$:

$$\begin{aligned} m_{\rm dyn} &= \left[ \frac{4\pi\alpha_s(1\text{ GeV})}{3} \left| \langle \bar{q}q \rangle_{1\text{ GeV}} \right| \right]^{1/3} \\ &= 290\text{–}320 \text{ MeV} \end{aligned} \tag{7}$$

in which $\alpha_s$ (1 GeV) is taken to be 0.5, constituent with the relative widths of strangeonium $\phi$ and charmonium $\psi$ (Refs. 1 and 9) and,

$$|\langle \bar{q}q \rangle_\mu| \equiv |\langle 0|\bar{q}(0)q(0){:}|0\rangle_\mu| \xrightarrow[\mu\to 1\text{ GeV}]{} (225\text{–}250 \text{ MeV})^3 \ ,$$

consistent with QCD sum-rule phenomenology.[10]

The passage from a dynamical-quark mass (6) to the meson isodoublet constituent-quark mass (4) is linked to the existence of nonzero quark mass terms in the QCD Lagrangian. Such terms, which necessarily break the chiral symmetry of the QCD Lagrangian, prevent the pion from being a true Nambu-Goldstone state, except in the limit that such terms vanish. Since the pion is substantially lighter than any other hadron, such terms are clearly quite small compared to constituent-quark masses; lattice simulations of QCD obtain up and down Lagrangian quark masses of order 10 MeV in the limit in which the strong interactions are turned off entirely.[11]

Similarly, the $SU(3)_f$ current-quark masses, as computed via Heisenberg equations of motion, satisfy the current-divergence quark operator equations

$$\begin{aligned} i\partial\cdot A^3 &= -\hat{m}_{\rm cur}\bar{q}\gamma_5\lambda^3 q, \\ i\partial\cdot A^{6+i7} &= -(m_s+\hat{m})_{\rm cur}\bar{d}\gamma_5 s \ . \end{aligned} \tag{8}$$

The masses in (8) necessarily vanish in the $\partial\cdot A=0$ limit of Lagrangian chiral symmetry, thereby suggesting the identification of current-algebra masses in (8) with Lagrangian quark masses. However, there are phenomenological reasons[12] for believing that the scale of the current-algebra isodoublet quark masses is of order $m_\pi/2$, in contrast with the much smaller values for Lagrangian quark masses obtained from lattice simulations. To reconcile these scales, it has been argued elsewhere[13] that chiral-symmetric condensates, such as the dimension-4 gluon condensate $\langle F^{\mu\nu}F_{\mu\nu}\rangle$ can enhance the apparent scale of any explicit chiral-symmetry-breaking terms entering the Lagrangian.

For example, the $O(\alpha_s)$ contribution to the pole $(m)$ of the quark propagator in the presence of both QCD condensates and a renormalized Lagrangian quark mass $(m_L)$ has been calculated explicitly to be[8,14]

$$\begin{aligned} m &= m_L \left[ 1 + \frac{\pi\alpha_s\langle F^{\mu\nu}F_{\mu\nu}\rangle m}{3(m-m_L)^2(m+m_L)^3} \right] + \frac{4\pi\alpha_s|\langle\bar{q}q\rangle|}{3m^2} \\ &= m_L^{\rm eff} + \frac{4\pi\alpha_s|\langle\bar{q}q\rangle|}{3m^2} \ . \end{aligned} \tag{9}$$

In the limit of explicit Lagrangian chiral symmetry $[m_L=0]$, the contribution of the chiral-invariant condensate $\langle F^{\mu\nu}F_{\mu\nu}\rangle$ necessarily vanishes, and the identification of the propagator pole with (7), the dynamical-quark mass generated entirely through chiral noninvariance of $\langle\bar{q}q\rangle$, is recovered. However, if $m_L \neq 0$, the contribution of an explicit Lagrangian mass term to the propagator pole (9) is substantially enhanced by its $\langle F^{\mu\nu}F_{\mu\nu}\rangle$-dependent coefficient.[13] If we choose to identify (1) the current-quark mass with $m_L^{\rm eff}$, the gluon-condensate enhanced contribution from explicit chiral-symmetry breaking, (2) the constituent-quark mass with the propagator pole $m$, and (3) the dynamical-quark mass with the propagator pole in the $m_L\to 0$ limit, we find that (9) yields a simple relationship between current-, constituent-, and dynamical-quark masses:[6,12]

$$m_{\rm con} = m_{\rm cur} + m_{\rm dyn}^3/m_{\rm con}^2 \ . \tag{10}$$

Equation (10) can also be obtained from the asymptotic momentum dependence of the two Bethe-Salpeter solutions to the Schwinger-Dyson equations for the quark propagator.[15] The irregular solution only depends weakly (i.e., logarithmically) on external momentum, corresponding to the renormalization-group behavior of the Lagrangian quark mass. The regular solution exhibits inverse proportionality to the square of external momentum; its suppression at high off-shell momenta is suggestive of a dynamical mass contribution to static hadron properties that disappears at momenta characteristic of scaling in deep-inelastic scattering. The sum of these two contributions can be identified with the constituent-quark mass of Eq. (10), provided the (on-shell) momentum is self-consistently constrained to the constituent-quark mass shell.

A spectroscopic assessment of the isodoublet current-quark mass can be obtained from utilization of (3) to relate the constituent isodoublet quark mass $\hat{m}_{\rm con}$ to the physical pion mass:

$$m_\pi = 2\hat{m}_{\rm con} - 3M/4\hat{m}_{\rm con}^2 \ . \tag{11a}$$

If we use (10) and (6) to replace $1/\hat{m}_{\rm con}^2$ in (11a) with

$(\hat{m}_{\rm con}-\hat{m}_{\rm cur})/m_{\rm dyn}^3=8(\hat{m}_{\rm con}-\hat{m}_{\rm cur})/3M$, we find that the remaining factors of $\hat{m}_{\rm con}$ cancel, and that

$$m_\pi=2\hat{m}_{\rm cur}\ . \tag{11b}$$

This result is a startling confirmation of the current-quark mass additivity in mesons predicted in Ref. 12 from the $m_\pi/2$ scale of isodoublet current-quark masses necessary to account for the observed 5–6% Goldberger-Treiman discrepancy. Indeed, one could have begun with the nonrelativistic picture of the pion suggested by (11b) of two (virtually) unbound current quarks, each carrying half the pion mass, which are then separately related via (10) to the (constituent) masses dressed by the $1/p^2$ dynamical contributions in order to yield (11a). It is important to note that our use of (10) in (11a) yields an expression for $m_\pi(m_{\rm cur})$ that is completely independent of a dynamical mass scale, as would appear appropriate for a Goldstone pion. By contrast, had we used the naive additivity relation $\hat{m}_{\rm con}=\hat{m}_{\rm cur}+m_{\rm dyn}$ instead of (10) to eliminate the constituent mass $\hat{m}_{\rm con}$ from (11a), we would have obtained $m_\pi=6m_{\rm cur}-6m_{\rm cur}^2/m_{\rm dyn}+O(m_{\rm cur}^3/m_{\rm dyn}^2)$, an expression that is difficult to interpret physically.

The distinction between current and Lagrangian masses is crucial in attempting to reconcile the relationship (11b) to the $m_\pi\sim m_{u,d}^{1/2}$ behavior obtained from lattice simulations of QCD (Ref. 11). The lattice-generated relationship between $m_\pi$ and $m_{u,d}$ puts the physical value for $m_\pi$ in correspondence with a very small ($\lesssim 10$ MeV) value for $m_{u,d}$, suggesting that the lattice-simulation mass be identified with the Lagrangian mass $\hat{m}_L$ rather than the corresponding current mass $\hat{m}_L^{\rm eff}$ of Eq. (9). Indeed, if we utilize the $O(\alpha_s)$ relationship (9) between $m_L$ and $m_L^{\rm eff}$, we find from (11b) that

$$\begin{aligned} m_\pi&=2m_L^{\rm eff}\\ &=2\hat{m}_L\left[1+\frac{\pi\alpha_s\langle F^{\mu\nu}F_{\mu\nu}\rangle\hat{m}_{\rm con}}{3(\hat{m}_{\rm con}-\hat{m}_L)^2(\hat{m}_{\rm con}+\hat{m}_L)^3}\right]\ . \end{aligned} \tag{12a}$$

Moreover, we see from (10) and (11b) that $\hat{m}_{\rm con}$ is implicitly a function of $m_\pi$,

$$\hat{m}_{\rm con}=(m_\pi/2)+m_{\rm dyn}^3/\hat{m}_{\rm con}^2\ , \tag{12b}$$

where the constant $m_{\rm dyn}$ is given by (6). Equations (12a) and (12b) taken together suffice to define $m_\pi$ as dependent on a single variable $\hat{m}_L$, in that (12a) is now a constraint of the form

$$m_\pi=F[\hat{m}_L,\hat{m}_{\rm con}(m_\pi)]\ . \tag{12c}$$

In other words, the constraint (12c) between $m_\pi$ and $\hat{m}_L$ serves to define $m_\pi$ implicitly as a function of $\hat{m}_L$. We see from Fig. 1 that the relationship between $m_\pi$ and $\sqrt{m_L}$ obtained through (12) is very nearly linear, consistent with the lattice result.[11,16]

Equation (10) may be utilized in conjunction with (3) to express all SU(2)$_f$-nonsinglet meson masses as power series in the appropriate current masses. To do this, it will prove useful to regard (10) as implicitly defining $m_{\rm con}$ as a function of $m_{\rm cur}$. Through successive differentiations of both sides of (10) with respect to $m_{\rm cur}$, one can easily obtain Taylor-series coefficients for $m_{\rm con}(m_{\rm cur})$ expanded about $m_{\rm cur}=0$:

$$m_{\rm con}=m_{\rm dyn}+\frac{m_{\rm cur}}{3}+\frac{m_{\rm cur}^2}{9m_{\rm dyn}}+\frac{2m_{\rm cur}^3}{81m_{\rm dyn}^2}+O(m_{\rm curr}^5)\ . \tag{13}$$

($m_{\rm cur}^4$ has a coefficient of zero.)

In (13), $m_{\rm dyn}$ is related via (6) to the hyperfine-interaction coefficient $M$ appearing in (3) ($m_{\rm dyn}^3=3M/8$). Upon substitution of (13) into (3) we obtain the following power-series expressions for masses of the spin-0 and spin-1 mesons [$m_\pi=2\hat{m}_{\rm cur}$ as in (11b)]:

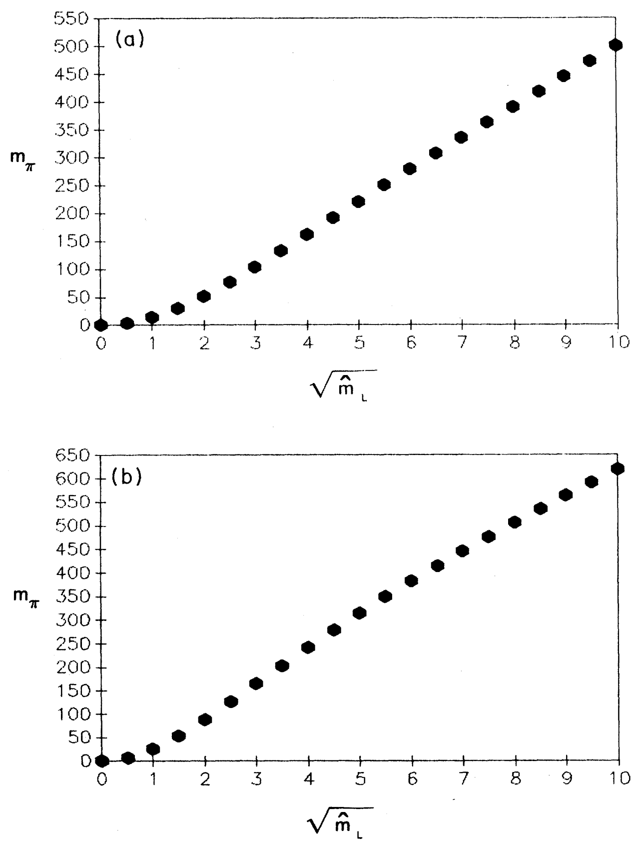


FIG. 1. The dependence of the pion mass on the Lagrangian mass is determined numerically from Eqs. (12). In (a) we use the "standard" value for the gluon condensate $\alpha_s\langle F_{\mu\nu}F^{\mu\nu}\rangle=(441\ {\rm MeV})^4$. In (b) we use double this value, as discussed in Ref. 13. The breakdown of proportionality between $m_\pi$ and $\hat{m}_L^{1/2}$ at very small $\hat{m}_L$ can be understood by noting that $m_\pi(\hat{m}_L)$, implicitly defined through (12c), is analytic at $\hat{m}_L=0$. Thus one can construct a Taylor series for $m_\pi(\hat{m}_L)$ expanded about $\hat{m}_L=0$ [$K\equiv\pi\langle\alpha_sF_{\mu\nu}F^{\mu\nu}/3\rangle$]: $m_\pi=\hat{m}_L(2+2Km_{\rm dyn}^4)-\hat{m}_L(14K/3m_{\rm dyn}^4+8K^2/3m_{\rm dyn}^9)+O(\hat{m}_L)$. We see from this series that $M_\pi$ is proportional to $\hat{m}_L$ (rather than $\hat{m}_L^{1/2}$) for sufficiently small $\hat{m}_L$.

$$m_K=(m_s+\hat{m})_{\rm cur}+\frac{(m_s-\hat{m})^2_{\rm cur}}{9m_{\rm dyn}}\left[1-2\frac{(m_s^2+m_s\hat{m}+\hat{m}^2)_{\rm cur}}{27m^2_{\rm dyn}}\right]+O((m_s)^5_{\rm cur})\ , \tag{14}$$

$$m_\rho=\frac{8m_{\rm dyn}}{3}+\frac{2\hat{m}_{\rm cur}}{9}+\frac{8\hat{m}^2_{\rm cur}}{27m_{\rm dyn}}+\frac{16\hat{m}^3_{\rm cur}}{243m^2_{\rm dyn}}+O(\hat{m}^5_{\rm cur})\ , \tag{15}$$

$$m_{K^*}=\frac{8m_{\rm dyn}}{3}+\frac{(\hat{m}+m_s)_{\rm cur}}{9}+\frac{(3\hat{m}^2+3m_s^2+2m_s\hat{m})_{\rm cur}}{27m_{\rm dyn}}+\frac{8(\hat{m}^3+m_s^3)_{\rm cur}}{243m^2_{\rm dyn}}+\tfrac{2}{729}(m_s^4+\hat{m}^4-m_s^3\hat{m}-m_s\hat{m}^3)_{\rm cur}+O((m_s)^5_{\rm cur})\ , \tag{16}$$

$$m_\phi=\frac{8m_{\rm dyn}}{3}+\frac{2(m_s)_{\rm cur}}{9}+\frac{8(m_s)^2_{\rm cur}}{27m_{\rm dyn}}+\frac{16(m_s)^3_{\rm cur}}{243m^2_{\rm dyn}}+O((m_s)^5_{\rm cur})\ . \tag{17}$$

Ideal mixing is assumed in (17); correspondingly (15) is the ideal mixing expression for $m_\omega$. Thus we have obtained a model for corrections to the meson masses that is quadratic-and-higher order in the current mass. Of course, such terms would be modified by including additional nonhyperfine interaction terms; the higher order terms in (14)–(17) are listed primarily to show that they are calculable and manifestly smaller in magnitude than the leading terms. The strange current-quark mass can be ascertained either through a direct fit of (14)–(17), or alternatively, from (10) through use of the dynamical mass (6) and the spectroscopically fitted constituent masses (4). We find that

$$(m_s)_{\rm cur}=(m_s)_{\rm con}-m^3_{\rm dyn}/(m_s)^2_{\rm con}=394\ {\rm MeV}\ . \tag{18}$$

This large value, which succeeds in accounting for the $KN\Lambda$ Goldberger-Treiman discrepancy,[12] can be reconciled with a Lagrangian mass of order 200 MeV once gluon condensate enhancement effects are properly taken into account.[13] The large $SU(3)_f$ current-quark masses of (11b) and (18) can be shown to be consistent, in the infinite-momentum frame, with quadratic mass formulas induced entirely by kinematic considerations.[17] Moreover, similarly large current masses are obtained[18] when current-divergence expressions for quark masses, such as (7), are sandwiched between all $SU(3)_f$ ground-state hadrons ($0^-$, $1^-$, $\frac{1}{2}^+$, and $\frac{3}{2}^+$).

We now consider briefly how the dynamical quark mass can be related to hyperfine-interaction ground-state baryon spectroscopy. One approach toward obtaining the dynamical mass appropriate for baryon spectroscopy is to rearrange (10) in order to find the value for $m_{\rm dyn}$ consistent with both the baryon-spectroscopic value for the isodoublet constituent-quark mass $\hat{m}=363$ MeV (2) and the isodoublet current-quark mass $(\hat{m})_{\rm cur}=m_\pi/2$:

$$m_{\rm dyn}=\{\hat{m}^2_{\rm con}[\hat{m}_{\rm con}-(\hat{m})_{\rm cur}]\}^{1/3}=339\ {\rm MeV}\ . \tag{19}$$

Further phenomenological support for the 58-MeV difference between (19) and the meson-spectroscopic dynamical mass (6) may be obtained by using (1) to relate the baryonic dynamical mass to the chiral-limiting nucleon mass $m_N^{\rm CL}$:

$$m_N^{\rm CL}=3m_{\rm dyn}-3B/4m^2_{\rm dyn}\ . \tag{20}$$

A direct estimate of $m_N^{\rm CL}$ can be obtained from the $\sigma_N\simeq 60$-MeV nucleon $\sigma$ term extracted from low-energy $\pi N$ scattering:

$$m_N^{\rm CL}=m_N-\sigma_N\approx 881\ {\rm MeV}\ . \tag{21}$$

Substitution of (21) into (20) yields 348 MeV for the dynamical mass, a value close to (19) but 67 MeV larger than that generated through (5) (Ref. 19).

This difference between the baryon and meson dynamical masses in (6) and (19) may be understood as the underlying source of the $\sim 60$ MeV disparity[20,21] which seems to characterize the difference between baryon and meson constituent-quark masses, as evidenced by comparison of (2) and (4). Curiously, the averages of mesonic (4) and baryonic (2) constituent masses,

$$(\hat{m}^{\rm meson}+\hat{m}^{\rm baryon})_{\rm con}/2\approx 335\ {\rm MeV}\ ,\qquad (m_s^{\rm meson}+m_s^{\rm baryon})_{\rm con}/2\approx 514\ {\rm MeV}\ , \tag{22}$$

are quite close to the corresponding quark masses obtained from the NRQM's analysis of hadron magnetic moments:[1]

$$\hat{m}_{\rm con}=M_p/\mu_p\approx 336\ {\rm MeV}\ ,\qquad m_{s,\rm con}=-M_N/3\mu_\Lambda\approx 513\ {\rm MeV}\ . \tag{23}$$

Such values also characterize the masses entering quark triangle diagrams contributing to pion and kaon charge radii.[22] Moreover, the alternative relativistic chiral-symmetry picture with pion-quark coupling strength[6] $g_{\pi qq}\approx 2\pi/\sqrt{N_c}\approx 3.6$ leading to the quark-level Goldberger-Treiman relation

$$\hat{m}_{\rm con}=f_\pi g_{\pi qq}=338\ {\rm MeV} \tag{24}$$

can be extended[23] to the kaon ($g_{Kqq}\approx g_{\pi qq}$) for the observed decay constant ratio $f_K/f_\pi\approx 1.25$:

$$(m_s+\hat{m})_{\rm con}/2=f_K g_{Kqq}\to m_{s,\rm con}=[2(f_K/f_\pi)-1]\hat{m}\approx 507\ {\rm MeV}\ . \tag{25}$$

Once again, the results (24) and (25) are consistent with the averaged spectroscopic masses (22). A similar averaging of mesonic and baryonic determinations of the dynamical mass yields $m_{\rm dyn}\simeq 310$–$315$ MeV $\simeq m_N/3$, a value compatible with (7) and other model estimates,[24] such as the vector-dominance value of the pion radius $r_\pi=m^{-1}_{\rm dyn}\approx 0.63$ fm.

To conclude, we have examined naive NRQM constituent-quark meson spectroscopy in light of (10), a relation between current-, constituent-, and dynamical-quark masses that follows from the anticipated $1/p^2$

dependence of the quark mass's dynamical component. We find that the current quark mass contributes additively to the pion mass [and very nearly additively to the kaon mass (14) as well], providing a framework for explaining why the NRQM expression (3) is applicable to the pion. By considering NRQM constituent-quark baryon spectroscopy, we have also shown how the $\sim 60$-MeV NRQM difference between baryon and meson constituent-quark masses is a reflection of a corresponding difference between the NRQM baryon and meson dynamical masses. The SU(3) current-quark masses we obtain are much larger than the widely quoted range of values. We believe this discrepancy can be removed by careful treatment of how chiral-invariant QCD condensates (in particular, the dimension-4 gluon condensate) can magnify the renormalized Lagrangian quark mass $(m_L)$ devolving from the electroweak Yukawa sector.[13] Presumably, it is this Lagrangian mass that is accessible from lattice QCD calculations through analysis of the limit in which QCD interactions are turned off, an interpretation supported by the effectively linear relationship we obtain between $m_\pi$ and $\hat{m}_L^{1/2}$ (Fig. 1).

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