PHYSICAL REVIEW D VOLUME 40, NUMBER 11 1 DECEMBER 1989

Vector-pseudoscalar mass splitting and Υ-η_b mass difference

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(Received 8 June 1989; revised manuscript received 7 August 1989)

For the vector-pseudoscalar mass difference, we propose a new unified empirical formula which works very well for almost all known mesons whether or not they contain one light quark. For heavy mesons, we predict $\Upsilon-\eta_b=55.0\pm5.0$ MeV and in particular $\psi'-\eta_c'=96.5\pm5.0$ MeV without any additional parameters. This new relation seems also valid for the radially excited states of mesons. The well-known approximate relation $M_V^2-M_P^2\simeq0.56$ GeV^2 can be reproduced as a special case for a narrow region of the reduced mass. A problem concerning the difference in the hyperfine splittings of self-conjugate mesons with zero isospin and in other mesons is solved.

I. INTRODUCTION

The constituent-quark model[1,2] with a reasonable QCD-motivated potential has been fairly successful in describing the spectroscopy and decay of mesons.[3] The static, spin-averaged potential has been partly determined by fitting the experimental data and by perturbative and lattice QCD computations. The spin-dependent potentials, however, are supposed to be small corrections to the static potential and have to be determined by fitting the data of fine and hyperfine splittings. In the Breit-Fermi approximation, the mass difference between the spin-triplet and spin-singlet states of mesons comes from the hyperfine interaction which depends mainly on the short-range part of quark-antiquark potential. Therefore, a more detailed investigation of the regularity of hyperfine splittings may provide some information about the spin-dependent potentials.

It has been pointed out by Martin[4] and then discussed by several authors[5] that the difference between the mass squared of the vector meson and the mass squared of the corresponding pseudoscalar meson, i.e., the singlet partner of the spin-triplet S-wave meson, is approximately a flavor-independent constant (see Table I):

$$[M_V(^3S_1)]^2-[M_P(^1S_0)]^2\simeq0.56\ \text{GeV}^2\ . \tag{1}$$

This relation seems valid for the mesons containing at least one light quark which are not self-conjugate with zero isospin. However, this relation is clearly not valid for $J/\psi,\eta_c$ and quite probably also not valid for Υ,η_b.

On the other hand, for the light self-conjugate mesons of isospin 0, the mass-squared difference between ϕ and η' and that between ω and η is sometimes considered. Combining the situation of $J/\psi,\eta_c$, some previous works[6,7] guess that $M_V^2-M_P^2$ might increase very rapidly from 0.12 GeV^2 for ϕ,η' and 0.31 GeV^2 for ω,η to 0.71 GeV^2 for $J/\psi,\eta_c$ (see Table I).

Thus we are faced with the problem that $M_V^2-M_P^2$ is a flavor-independent constant for some mesons and is a strongly flavor-dependent quantity for other mesons.

In this paper we will investigate the evidence for the existence of a unified regularity in the hyperfine mass splitting of all mesons and suggest a new unified relation for M_V-M_P. Using this relation the problem mentioned above can be resolved. Some predictions and implications are also given.

TABLE I. The data of mass squared difference and mass difference between the vector meson (V) and corresponding pseudoscalar meson (P).

V,P	$M_V^2-M_P^2$ (GeV^2)	M_V-M_P (MeV)	$\overline{M}$ (MeV)	$R=\dfrac{M_V-M_P}{\overline{M}}$
ρ,π	0.5746 ± 0.0046	630.4 ± 3.0	612.5	$\sim100\%$
(ω,η_u)	(0.5746)	(633)	(625)	$(\sim100\%)$
K^*,K	0.5521 ± 0.0005	394.3 ± 0.3	792.8	$\sim50\%$
(ϕ,η_s)	(0.5470 ± 0.0254)	(300)	(945)	$(\sim30\%)$
D^*,D	0.5462 ± 0.0050	142.6 ± 2.0	1974.8	7.2%
D_s^*,D_s	0.5692 ± 0.0135	139.4 ± 3.3	2097.6	6.6%
$(J/\psi,\eta_c)$	0.7128 ± 0.0108	117.3 ± 1.8	3067.9	3.8%
B^*,B	0.5379 ± 0.0201	52 ± 6	5310	$\leq1\%$
ϕ,η'	0.1224 ± 0.0009			
ω,η	0.3113 ± 0.0010			

II. A UNIFIED REGULARITY IN THE HYPERFINE MASS SPLITTING OF MESONS

An interesting question is the following: Does there exist an essential difference in hyperfine mass splitting between the self-conjugate mesons of zero isospin and other mesons?

(1) The data show that the ratio of the hyperfine mass splitting and spin-averaged mass $\overline{M}[=(3M_V+M_P)/4]$, $R=(M_V-M_P)/\overline{M}$, is quite small for all known ground-state mesons except for ρ,π and K^*,K (see Table I). Hence, for most mesons, it should be a good approximation to treat the hyperfine interaction as a small perturbation to the static effective potential which determines the spin-averaged energy levels of the meson system. It is hardly likely that there is a big difference between $J/\psi,\eta_c$ ($R=3.8\%$) and D^*,D ($R=7.2\%$) or D_s^*,D_s ($R=6.6\%$).

(2) Another argument comes from the QCD calculation[8] of the effect of virtual gluon annihilation on the hyperfine splitting, which shows that this effect is quite small for $J/\psi,\eta_c$ and Υ,η_b. Hence this small correction does not seem to be able to cause two different regularities between the heavy self-conjugate mesons of zero isospin and other mesons.

(3) For the light self-conjugate mesons of isospin zero, since the η and η' have different quark contents from the ϕ and ω, respectively, and

$$\phi\simeq {}^3S_1(s\bar{s}),\quad \omega\simeq {}^3S_1\left[\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right], \tag{2}$$

the meson η' is *not* the corresponding spin singlet of ϕ, and the η is also *not* the corresponding spin singlet of ω. Hence we should not take the differences: $\phi^2-\eta'^2$ and $\omega^2-\eta^2$. The reasonable combinations should be $\phi^2-\eta_s^2$ and $\omega^2-\eta_u^2$, where $\eta_s={}^1S_0(s\bar{s})$ and $\eta_u={}^1S_0[(u\bar{u}+d\bar{d})/\sqrt{2}]$. The physical mesons η and η' are the linear combinations of η_s and η_u with a mixing angle θ_P:

$$\eta=X_\eta\eta_u+Y_\eta\eta_s,\quad \eta'=-Y_\eta\eta_u+X_\eta\eta_s\ , \tag{3}$$

where

$$\begin{aligned} X_\eta&=(\tfrac{1}{3})^{1/2}\cos\theta_P-(\tfrac{2}{3})^{1/2}\sin\theta_P\ ,\\ Y_\eta&=-(\tfrac{2}{3})^{1/2}\cos\theta_P-(\tfrac{1}{3})^{1/2}\sin\theta_P\ . \end{aligned} \tag{4}$$

In order to estimate the masses of η_s and η_u we investigate the mass matrix

$$M=\begin{bmatrix} X_\eta^2 m_\eta+Y_\eta^2 m_{\eta'} & -X_\eta Y_\eta(m_{\eta'}-m_\eta)\\ -X_\eta Y_\eta(m_{\eta'}-m_\eta) & X_\eta^2 m_{\eta'}+Y_\eta^2 m_\eta \end{bmatrix} \tag{5}$$

and parametrize it with the form[9]

$$M=\begin{bmatrix} m_{\eta_u}+2a^2 & \sqrt{2}\,ab\\ \sqrt{2}\,ab & m_{\eta_s}+b^2 \end{bmatrix}. \tag{6}$$

With this form of the mass matrix, $m_{\eta_s}+m_{\eta_u}\neq m_\eta+m_{\eta'}$, because there are some annihilation terms a and b, which may be interpreted in terms of a pseudoscalar quark-antiquark state passing through an intermediate two-gluon state to another pseudoscalar quark-antiquark state. Using the physical η and η' masses and the recent data of mixing angle $\theta_P=-15°\pm5°$, we can obtain $M[{}^1S_0(s\bar{s})]=720\pm36$ MeV with the assumption that $M({}^1S_0[(u\bar{u}+d\bar{d})/\sqrt{2}])=140$ MeV. It gives $\phi-\eta_s=300\pm36$ MeV and $\omega-\eta_u=633\pm10$ MeV. The corresponding mass-squared differences are $\phi^2-\eta_s^2=0.5470$ GeV^2 and $\omega^2-\eta_u^2=0.5746$ GeV^2. This result shows that the mass-squared difference of light self-conjugate mesons of zero isospin is almost the *same* as other non-self-conjugate mesons having similar reduced masses (see Table I) provided that the quark content of the pseudoscalar meson is the same as that of the vector meson.

All these arguments motivated us to look for a unified regularity in the hyperfine mass splitting for all mesons.

From the recent data of $M_V^2-M_P^2$ (see Fig. 1), one can see that $M_V^2-M_P^2$ is likely a flavor-dependent quantity and is rather unlikely a constant. If there exists a unified regularity, one formula should be able to demonstrate this flavor dependence. On the other hand, the data of M_V-M_P have relatively small error (see Table II) and have the regularity that the hyperfine mass splitting decreases almost monotonically with increasing m_i+m_j (see Fig. 2), where m_i and m_j are the constituent masses of quarks contained in the meson.

Based upon these investigations we suggest a new unified empirical relation for the hyperfine splitting as

$$M_V-M_P=\frac{\delta(\mu)}{\overline{M}} \tag{7a}$$

with

$$\delta(\mu)=p\alpha_s(\mu)\mu^q\ , \tag{7b}$$

where $p=1.950$ GeV$^{1.45}$ and $q=0.55$ are flavor-

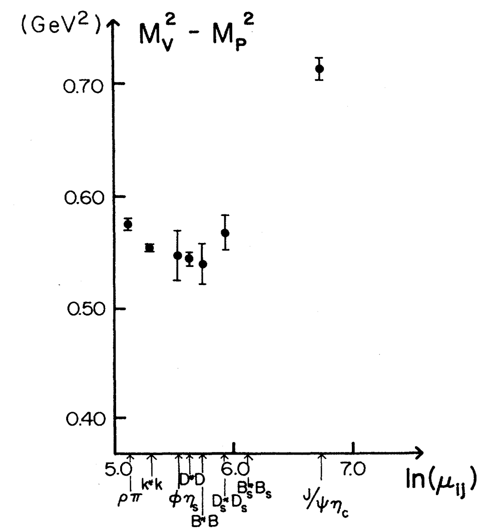

FIG. 1. The mass-squared difference between the vector meson and the corresponding pseudoscalar meson vs $\ln(\mu)$, where μ is in MeV.

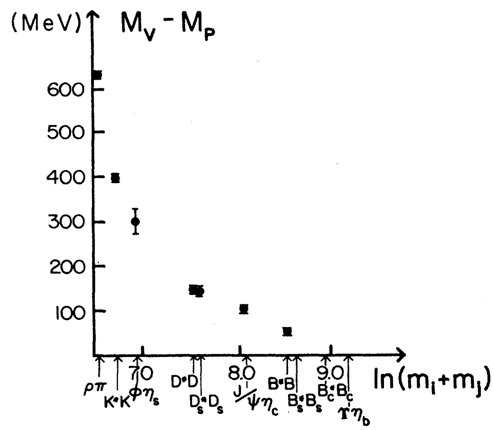

FIG. 2. The mass difference between the vector meson and the corresponding pseudoscalar meson vs $\ln(m_i+m_j)$, where m_i+m_j is in MeV.

independent constants, and $\alpha_s(Q)$ is the QCD running coupling constant with $Q=2\mu$ and $\Lambda=100$ MeV:

$$\alpha_s(Q)=\frac{4\pi}{\beta_0 \ln\left[\frac{Q^2}{\Lambda^2}\right]+\frac{\beta_1}{\beta_0}\ln\left[\ln\left[\frac{Q^2}{\Lambda^2}\right]\right]}, \tag{8}$$

where $\beta_0=11-2n_f/3$, and $\beta_1=102-38/3n_f$, and n_f is the number of quark flavors.

We note that although Eq. (7) is an empirical relation, some motivation can be given as follows. In the framework of a potential model, a static spin-averaged potential consists of two parts: a short-range part $V(r)$ which transforms as a time component of a Lorentz four-vector and a long-range part $S(r)$ which transforms as a Lorentz scalar. In the Breit-Fermi approximation the hyperfine splitting is

$$M_V-M_P=\frac{2}{3m_i m_j}\langle \nabla^2 V(r)\rangle_S , \tag{9}$$

where the subscript S denotes that the expectation value is taken over the S-wave ($l=0$) state.

Assuming the short-range part $V(r)$ is a QCD Coulomb-type potential $V(r)=-k\alpha_s(\mu)/r$, and using $m_i+m_j \simeq \overline{M}$ we obtain

$$M_V-M_P \propto \frac{\alpha_s(\mu)}{\overline{M}}\left[\frac{|\psi_s(0)|^2}{\mu}\right]. \tag{10}$$

Furthermore, using the data of the leptonic widths of vector mesons[10] and the QCD-corrected Van Royen-Weisskopf formula[11]

$$|\psi_s(0)|^2=\frac{\Gamma(V\to e+e-)M_V^2}{16\pi\alpha^2 e_q^2(1-16\alpha_s/3\pi)}, \tag{11}$$

we have $|\psi_s(0)|^2 \propto \mu^{1+q}$, where $q=0.60\pm0.20$ (in Refs. 12 and 14, the potential model calculations gave $q=0.50-0.60$). Taking $q=0.55$ we obtain $M_V-M_P \propto [\alpha_s(\mu)/M]\mu^{0.55}$; this is just Eq. (7).

To calculate the hyperfine splittings we use three different parameter sets of quark masses:

Set (1): $m_u=m_d=270$ MeV, $m_s=600$ MeV, $m_c=1700$ MeV, $m_b=5000$ MeV ;

Set (2): $m_u=m_d=336$ MeV, $m_s=485$ MeV, $m_c=1670$ MeV, $m_b=4980$ MeV ;

Set (3): $m_u=m_d=300$ MeV, $m_s=450$ MeV, $m_c=1700$ MeV, $m_b=5000$ MeV .

TABLE II. Comparison of the hyperfine splitting (in MeV) predicted from relation (7) and other potential models. Sets (1), (2), and (3) refer to different parameter sets given in Sec. II.

M_V-M_P	Data	Set (1)	Set (2)	Set (3)	Ref. 13	Ref. 14
$\rho-\pi$	630.43±3	539.6	492.8	486.7	620	608
$\rho'-\pi'$	300±100	214.7	202.6	192.5	150	332
$(\omega-\eta_u)$	(633±10)	533.1	483.7	475.7		
K^*-K	394.3±0.3	370.4	364.8	363.7	430	399
$K^{*\prime}-K'$	~200	183.4	180.4	175.7	130	212
$(\phi-\eta_s)$	(300±36)	298.2	300.0	298.1		318
D^*-D	142.6±2.1	143.0	143.1	143.0	160	162
$D_s^*-D_s$	139.4±3.3	139.1	137.7	137.5	150	148
$J/\psi-\eta_c$	117.3±2	115.2	114.7	115.3	140	77
$\psi'-\eta_c'$	92±6	96.5	96.2	96.5	60	40
B^*-B	52±6	52.8	52.7	53.2	60	56
$B_s^*-B_s$		52.3	51.2	53.0	60	48
$B_c^*-B_c$		58.2	58.2	58.3	70	42
$\Upsilon-\eta_b$		55.0	55.0	54.9	60	34
$\Theta-\eta_t$		13.5	13.5	13.5	30	7

The comparison of the computed values from Eq. (7) with the data and other potential models is shown in Table II. It should be noted that in the cases of (ϕ,η_s) and (Υ,η_b) which the masses of spin triplet are known, Eq. (7) may be reshaped into

$$M_V-M_P=2[M_V-\sqrt{M_V^2-\delta(\mu)}] \tag{12}$$

which allows us to calculate M_P, the mass of the spin singlet, from M_V and $\delta(\mu)$. On the other hand, if the mass of spin singlet is known, another relation

$$M_V-M_P=\tfrac{1}{3}[\sqrt{4M_P^2+12\delta(\mu)}-2M_P] \tag{13}$$

can be used to calculate the mass of spin triplet.

From Table II we can make several observations.

(i) The new relation (7) gives a very good and unified description of almost all known ground-state mesons; the errors are less than 1%. Even for ρ,π (error=15%) and K^*,K (error=7%), our relation still gives reasonably good agreement with the data. Hence we have a unified regularity in the hyperfine mass splitting and the problem discussed previously has been resolved.

(ii) For heavier quarkonium $b\bar{b}$ and $t\bar{t}$ our relation gives a very clear prediction: $\Upsilon-\eta_b=55.0\pm5.0$ MeV if $m_b=5.0\pm0.1$ GeV and $\Theta-\eta_t=13.5\pm2.0$ MeV if $m_t=50$ GeV and $\Theta({}^3S_1(t\bar{t}))=96\pm2$ GeV.

(iii) It is surprising that Eq. (7) also gives a very good agreement with the data for the first radially excited state of charmonium. In fact, it is obvious from Eq. (7) that for a meson $(q_i,\bar{q}_j)$ with fixed $\mu=m_im_j/(m_i+m_j)$, the $\delta(\mu)$ value is the same for different radially excited states. Hence we predict

$$\psi'-\eta_c'=96.5\pm5.0 \text{ MeV}, \tag{14}$$

which agrees well with the latest data:[10] $(\psi'-\eta_c')_{\text{expt}}=92\pm6$ MeV.

Assuming that Eq. (7) also holds for the first radially excited state of light mesons, we can predict $\pi(2\,{}^1S_0)=1400$ MeV [taking $\rho(2\,{}^1S_0)=1600$ MeV (Ref. 10)], which also agrees well with the data: $\pi(2\,{}^1S_0)_{\text{expt}}=1300\pm100$ MeV (Ref. 10).

(iv) For charmonium and bottomonium, a number of ${}^3S_1$ states are well known ($1S$-$4S$ for $c\bar{c}$ and $1S$-$6S$ for $b\bar{b}$). Hence we can use Eq. (12) to determine all corresponding spin-singlet levels, which can be tested by future experiments. We also can determine the spin-averaged levels which can be used as basic "data" to compare with the prediction from the potential models.

(v) If we use $(M_V+M_P)/2$ instead of $\bar{M}$, from Eq. (7) we obtain

$$M_V^2-M_P^2\simeq 2\delta(\mu). \tag{15}$$

For a narrow region of μ: 0.20 GeV$\leq\mu\leq$0.45 GeV (which corresponds to the region from K^*,K,ϕ,η_s, D^*,D,D_s^*,D_s to B^*,B), it is easy to show that $\delta(\mu)\simeq0.28$ GeV^2 is approximately a constant and then the well-known relation (1) is reproduced. The result on hyperfine splittings of baryons given by Lipkin[15] is obviously still valid under the new relation.

III. DISCUSSION

(i) All results given above are model independent and the predictions will be tested by the future experiments.

(ii) Our prediction is not *too* sensitive to the constituent-quark mass parameters, which we listed in the last section. They are, of course, commonly used in the potential models. One can see from Table II that for three different sets of parameters, all predicted values are almost unchanged except for ρ,π. A 66-MeV change of u-quark mass and a 115-MeV change of s-quark mass only lead to a change of 40 MeV for $\rho-\pi$, 6 MeV for K^*-K, and less than 1 MeV for all other mesons.

(iii) The fact that the new relation (7) works very well for all known mesons presumably means that the source of hyperfine splitting for the lightest mesons ρ,π probably is essentially the same as that for the heavier mesons, i.e. Eq. (7) roughly holds for $\rho-\pi$. A 100-MeV difference between the theoretical value and the data of $\rho-\pi$ is not unexpected, because the pion system is a special and an ultrarelativistic $q,\bar{q}$ bound system and has been hard to deal with for a long time.[16,17] In the past few years, many works have been done within the bound-state quark model with the symmetry broken spontaneously in the Nambu-Goldstone mode.[18] An interesting result given by Ref. 19 is that the vector-scalar mass splitting ρ-π comes from an effect of relativistic kinematics in the presence of a long-range central force without appealing to any short-range spin-spin interaction. This mechanism is different from the perturbation result in the potential model. We hope to study the question of which of these mechanisms is preferable.

ACKNOWLEDGMENTS

I would like to thank J. McCarthy and P. K. Kabir for their warm hospitality at INPP, University of Virginia, where this work was finally completed. The author would also like to thank V. de Alfaro, V. Dixit, R. Furlong, W. Kwong, I-Hsiu Lee, H. Lipkin, W. Marciano, A. Martin, M. Peshkin, and E. Predazzi for various helpful discussions. This work was supported by the INPP, University of Virginia, the National Natural Science Fund, and Zhejiang Provincial Natural Science Fund, P. R. China.

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