Branching ratios for decays of light Higgs bosons

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We have carried out a coupled-channel ($\pi\pi, K\bar{K}$) analysis of the effects of final-state interactions on the branching ratios for the decay modes of a light Higgs boson. We find no large enhancement of $H\to\pi\pi$ relative to $H\to\mu\bar{\mu}$ for $m_H < 950$ MeV.

I. INTRODUCTION

A Higgs boson of mass less than m_B can be produced with large probability in the decays of a B meson.[1] The Higgs boson would be detected by observing its decay products. For $2m_\mu < m_H < 2m_\tau$ (3.6 GeV), the cleanest signal may be the $H\to\mu\bar{\mu}$ mode. But then one has to know that the branching ratio (BR) for $H\to\mu\bar{\mu}$ is not too small. At the most naive quark level, for $m_H < 2m_\tau$, the H will decay primarily into $s\bar{s}$ and $\mu\bar{\mu}$, with a ratio of about three to one (the color factor for $q\bar{q}$) or somewhat more, depending on what one takes for m_s. It was pointed out[2] that heavy quarks could also contribute substantially through a virtual-heavy-quark loop giving $H\to gg$ with the gluons then materializing as light hadrons—in particular, as $\pi\pi$. This observation was eventually refined into a low-energy theorem[3] (LET) for the $H\to\pi\pi$ amplitude. The crucial feature of this result is that the amplitude contains a term proportional to m_H^2 as well as one proportional to m_π^2. But even if one extrapolates this result well beyond its range of validity, it still leads to a BR for $H\to\mu\bar{\mu}$ which ranges from 40% at $m_H=0.3$ GeV to 7% at $m_H=2$ GeV. It was then observed by Raby and West[4] that strong final-state interactions could further enhance the rate for $H\to\pi\pi$, and hence decrease the BR for $H\to\mu\bar{\mu}$. They made an estimate and claimed very large enhancements for $H\to\pi\bar{\pi}$. We find that their estimate is a substantial overestimate for several reasons, the most important of which is that they treated the $f_0(975)$ (formerly, and in this paper, S^*) resonance as an elastic $\pi\pi$ resonance. But the S^* is strongly coupled to the $K\bar{K}$ channel and cannot be reasonably treated in a single-channel framework, even below the $K\bar{K}$ threshold (which is the range treated by Raby and West). In this paper, we provide a coupled-channel analysis which incorporates the constraints of unitarity on the S^* resonance parametrization both above and below the $K\bar{K}$ threshold. We also find a large enhancement of the rate for $H\to\pi\pi$ very close to the $K\bar{K}$ threshold, but not as large as the estimate of Ref. 4; and, more importantly, there is no large enhancement for m_H below 0.95 GeV.

II. FORMALISM

The amplitude for the decay $H\to\pi\pi$ defines a pion scalar form factor[5]

$$M(H\to\pi\pi)=\langle\pi\pi_{\rm out}|\mathcal{L}'_{\rm eff}(0)|0\rangle\equiv-(G\sqrt{2})^{1/2}F_\pi(q^2)\ , \tag{2.1}$$

$$\mathcal{L}'_{\rm eff}=\frac{\partial\mathcal{L}_{\rm eff}}{\partial H}\ ,$$

where H is the Higgs field. A similar definition holds for $F_K(q^2)$ with $\pi\pi$ replaced by $K\bar{K}$. The scalar form factor $F_\pi(q^2)$ satisfies a dispersion relation

$$F(q^2)=F(0)+\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}dq'^2\frac{\sigma(q'^2)}{q'^2(q'^2-q^2-i\epsilon)}\ , \tag{2.2}$$

$$\sigma(q^2)={\rm Im}F(q^2)\ . \tag{2.3}$$

The function $\sigma(q^2)$ has contributions from $\pi\pi$, $K\bar{K},\ldots$ real intermediate states.

Above the cuts on the real axis we can write

$$F(q^2+i\epsilon)=e^{i\alpha(q^2)}|F(q^2)|\ . \tag{2.4}$$

Below the inelastic threshold, time-reversal invariance applied[5] to the $H\to\pi\pi$ amplitude determines that $\alpha(q^2)$ is the elastic $\pi\pi$ ($J=0$) phase shift $\delta_{\pi\pi}(q^2)$:

$$\alpha(q^2)=\delta_{\pi\pi}(q^2)\quad(q^2<4m_K^2) \tag{2.5}$$

(we ignore small 4π inelasticity). In this elastic range the $J=0$ partial-wave unitarity relations are

$${\rm Im}F=F^*\rho t\ , \tag{2.6}$$

$${\rm Im}t=\rho|t|^2\ , \tag{2.7}$$

$$\rho=k/W,\quad k=\left[\frac{W^2}{4}-m_\pi^2\right]^{1/2} \tag{2.8}$$

so t may be parametrized by the elastic s-wave $\pi\pi$ phase shift

$$t=\frac{1}{\rho}e^{i\delta}\sin\delta \ . \tag{2.9}$$

Here W is the c.m. energy; we use the notations

$$W^2=s=q^2(=m_H^2) \ . \tag{2.10}$$

We extend the consideration to two coupled channels; then the s-wave t_{ij} are the elements of a 2×2 (symmetric by time-reversal invariance) matrix, which satisfy the coupled-channel unitarity relations

$$\mathrm{Im}t_{ji}=t_{jk}^*\rho_k t_{ki} \ . \tag{2.11}$$

Including the $K\bar{K}$ contribution to σ_π (2.3), and the $\pi\pi$ contribution to σ_K, leads to the coupled unitarity equations for F_π,F_K:

$$\mathrm{Im}F_i=F_j^*\rho_j t_{ji} \ . \tag{2.12}$$

The solutions for the coupled integral equations for $F\pi$ and f_K are known in the literature.[6] They are in general fairly complicated if t_{ij} contained the left-hand cut singularity. We are interested in a simple parametrization of the t_{ij} such that they can be fitted to the experimental data and do not contain the left-hand cut singularity. In this case the unitarity equations are satisfied by the construction

$$F_i(s)=C_j(s)t_{ji}(W)/W \ , \tag{2.13}$$

where the right-hand side (RHS) of this equation is divided by W to remove any kinematical singularity which could be present in t_{ij} but which should not exist in F_i (see below), and where the $C_j(s)$ are real polynomials to be determined by certain "initial conditions" (a low-energy theorem). Note that as $k\rightarrow0$, $t_{11}(W)/W\rightarrow a$, the $\pi\pi$ s-wave scattering length.

An alternative approach[5,4] to satisfying the form-factor unitarity equations (2.6) and (2.12), is to construct an Omnes-Muskeleshvili (OM) representation. In their paper, Raby and West give the formalism for the two-channel OM representation, but conclude that it is intractable for actual application and use only the single-channel two-resonance approximation. We will see below that it is not difficult to give a two-pole parametrization of Eqs. (2.11)–(2.13) which gives a good representation of the known $\pi\pi$ and $K\bar{K}$ data and manifestly satisfies all the constraints of elastic and inelastic (two-channel) unitarity.

A standard starting point is the observation that t^{-1} satisfies a simple matrix unitarity equation

$$\mathrm{Im}t^{-1}=-\rho \ . \tag{2.14}$$

A simple parametrization of t^{-1} which satisfies (2.14) is

$$t^{-1}=\begin{bmatrix} \dfrac{M_1^2-s-2ik_i\Gamma_1}{2W\Gamma_1} & \dfrac{\lambda}{W} \\ \dfrac{\lambda}{W} & \dfrac{M_2^2-s-2ik_2\Gamma_2}{2W\Gamma_2} \end{bmatrix} . \tag{2.15}$$

Then

$$\begin{aligned} t_{11}&=\frac{2W\Gamma_1 D_2(s)}{D(s)} \ , \\ t_{12}=t_{21}&=\frac{-4W\lambda\Gamma_1\Gamma_2}{D(s)} \ , \\ t_{22}&=\frac{2W\Gamma_2 D_1(s)}{D(s)} \ ; \end{aligned} \tag{2.16}$$

$$\begin{aligned} D_1(s)&=M_1^2-s-2ik_1\Gamma_1 \ , \\ D_2(s)&=M_2^2-s-2ik_2\Gamma_2 \ , \\ D(s)&=D_1(s)D_2(s)-4\lambda^2\Gamma_1\Gamma_2 \ ; \end{aligned} \tag{2.17}$$

$$k_1=\left[\frac{s}{4}-m_1^2\right]^{1/2} , \quad k_2=\left[\frac{s}{4}-m_2^2\right]^{1/2} . \tag{2.18a}$$

Below the relevant threshold, $s_a=4m_a^2$,

$$k_a=i\left[m_a^2-\frac{s}{4}\right]^{1/2}\equiv i\kappa_a(s) \ . \tag{2.18b}$$

The conventional elasticity parameter η is introduced as

$$t_{11}=\frac{\eta e^{2i\delta_1}-1}{2i\rho_1}, \quad t_{12}=t_{21}=\frac{i\sqrt{(1-\eta^2)}e^{i(\delta_1+\delta_2)}}{2\sqrt{\rho_1\rho_2}}$$

$$(\text{above } K\bar{K} \text{ threshold}) \ . \tag{2.19}$$

In particular, a measure of the inelasticity above the $K\bar{K}$ threshold is

$$\iota\equiv\frac{1-\eta^2}{4}=k_1k_2\left|\frac{t_{12}}{W}\right|^2, \quad 0\le\iota\le0.25 \ . \tag{2.20}$$

We have found a set of values for the parameters in (2.15) which provides a good representation of the known elastic $\pi\pi$ and inelastic $\pi\pi\rightarrow K\bar{K}$ data. We use (all in GeV units)

$$m_\pi=0.138, \quad m_K=0.496 \ , \tag{2.21}$$

$$\begin{aligned} &M_1=0.87, \quad \Gamma_1=0.7, \quad M_2=0.92 \ , \\ &\Gamma_2=1.0, \quad \lambda=0.1 \ . \end{aligned} \tag{2.22}$$

Note that the output parameters ("observed resonances") can be substantially different from the input m_i, Γ_i. The roots of $D(s)$ determine the positions of the output resonances, and the widths are determined from the rates of variation at the resonance positions:

$$D(s_r)=0 \ ,$$

$$\Gamma_{(1)}=\Gamma_1 D_2(s_1)\Big/\left[-\frac{d\,\mathrm{Re}D(s)}{ds}\right]\Bigg|_{s_1} .$$

With the values (2.22) we find an ϵ with $M_\epsilon=0.85$, $\Gamma_\epsilon=0.63$ and a very narrow S^* just below the $K\bar{K}$ threshold; the effective S^* width is a very sensitive function of just how far below the threshold. These parameters produce an inelasticity ι (2.20) which rises sharply from the $K\bar{K}$ threshold to a peak value about 0.10 at $W=1.00$ and then falls back slowly. (η falls sharply from 1 at the $K\bar{K}$ threshold, then comes back up.) Experimentally, η is not well determined, but this result is at least consistent with the results from large-scale fits[7,8] to

$\pi\pi, K\bar{K}$ data: see, for example, Fig. 7 of Ref. 8. For the elastic $\pi\pi$ $I=0$ s-wave scattering, the parameters (2.22) in t_{11} of (2.16) give the scattering length equal to 0.26 m_π^{-1} and a phase shift which rises smoothly from $\pi\pi$ threshold (roughly linearly in W), passes through 90° at $W=0.85$, and then begins a rapid rise just below the $K\bar{K}$ threshold, all in good accord with detailed phase-shift analyses[7–9] of the data. We conclude that Eq. (2.16) with the parameter values (2.22) provides a reasonable account of the coupled $\pi\pi, K\bar{K}$ system, both below the $K\bar{K}$ threshold, and somewhat above it.

Returning to (2.13), to compute F_π, F_K we have to determine $C_{1,2}(s)$. These are determined from a low-energy theorem. From the amplitudes of Fig. 1, one obtains the effective Lagrangian for (2.1):

$$\mathcal{L}_{\text{eff}}=(G\sqrt{2})^{1/2}H\left[-\sum_{\text{light }q} m_q\bar{q}q+N_h\frac{\alpha_s}{12\pi}G^A_{\mu\nu}G^{\mu\nu}_A\right] . \tag{2.23}$$

Here N_h is the number of quarks with mass $\geq m_H$. The $\pi\pi$ and $K\bar{K}$ matrix elements of the first term in (2.23) are evaluated by standard PCAC (partial conservation of axial-vector current) and SU(3) current-algebra techniques. The second term is related through the trace anomaly of broken scale invariance to the trace of the energy-momentum tensor, and the $\pi\pi$ and $K\bar{K}$ matrix elements of θ^μ_μ are evaluated in the chiral limit.[3] The result is a LET (Ref. 10):

$$\frac{1}{(G\sqrt{2})^{1/2}}\langle M_aM_b|\mathcal{L}'_{\text{eff}}|0\rangle=\delta_{ab}\left(\tfrac{11}{9}m_a^2+\tfrac{2}{9}q^2+\cdots\right) . \tag{2.24}$$

The $I=0$ $\pi\pi$ state is $\sum_{a=1}^{3}|M_aM_a\rangle/\sqrt{3}$ and the $I=0$ $K\bar{K}$ state is $\sum_{a=4}^{7}|M_aM_a\rangle/2$. In this Cartesian basis both $\pi_a\pi_a$ and $K_a\bar{K}_a$ are normalized as identical particles. The result is that the *rate* for $H\to\pi\pi$ $(I=0)$ is three and a half times the rate for $H\to\pi^+\pi^-$ and the rate for $H\to K\bar{K}$ $(I=0)$ is twice the rate for $H\to K^+K^-$. So the "initial conditions" for (2.13) are

$$F_1(0)=\sqrt{3}\tfrac{11}{9}m_\pi^2=0.040 ,\qquad F_1'(0)=\sqrt{3}\tfrac{2}{9}=0.385 , \tag{2.25a}$$

$$F_2(0)=2\tfrac{11}{9}m_K^2=0.601 ,\qquad F_2'(0)=2\tfrac{2}{9}=0.444 . \tag{2.25b}$$

Then, using the values of the t_{ij}/W and their derivatives at $s=0$, we determine

$$C_1(s)=0.0872+0.24s ,\qquad C_2(s)=0.5563-0.0050s . \tag{2.26}$$

Note that only products of a C with a t/W are physically significant (2.13). And these products are fixed by the LET (for small s); hence, they are not so sensitive to small variations in the parameters of the fit as the individual factors might be, in particular the small coefficient of s in $C_2(s)$.

With the form factors $F_{1,2}(s)$ computed (2.13), we can compute the rates for $H\to\pi\pi$ and $H\to K\bar{K}$, and their ratios relative to $H\to\mu\bar{\mu}$:

$$\Gamma(H\to\mu\bar{\mu})=\frac{G\sqrt{2}}{8\pi}m_Hm_\mu^2\left[1-\frac{4m_\mu^2}{m_H^2}\right]^{3/2} , \tag{2.27}$$

$$\Gamma(H\to\pi\pi,K\bar{K})=\frac{G\sqrt{2}}{32\pi}\frac{|F_{1,2}|^2}{m_H}\left[1-\frac{4m_{1,2}^2}{m_H^2}\right]^{1/2} \quad (s=m_H^2) . \tag{2.28}$$

Then define the ratio of hadronic-to-muonic decay rates. Below the $K\bar{K}$ threshold the hadronic decay is just $\pi\pi$:

$$f_\pi=\frac{\Gamma(H\to\pi\pi)}{\Gamma(H\to\mu\bar{\mu})}=\frac{1}{4}\frac{|F_1|^2}{m_H^2m_\mu^2}\frac{\left[1-\frac{4m_\pi^2}{m_H^2}\right]^{1/2}}{\left[1-\frac{4m_\mu^2}{m_\mu^2}\right]^{3/2}} . \tag{2.29}$$

Above the $K\bar{K}$ threshold we have

$$f=\frac{\Gamma(H\to\pi\pi)+\Gamma(H\to K\bar{K})}{\Gamma(H\to\mu\bar{\mu})}=f_\pi+f_K . \tag{2.30}$$

The BR for $H\to\mu\bar{\mu}$ is

$$B(H\to\mu\bar{\mu})=\frac{1}{f+1} . \tag{2.31}$$

III. RESULTS AND DISCUSSION

In Table I we give the values of F_1, $|F_1|^2$, and $|F_2|^2$. [From the real and imaginary parts of F_1, below the $K\bar{K}$ threshold, one can immediately extract the elastic $\pi\pi$ $I=0$ s-wave phase shifts produced by our parametrization (2.16) and (2.22).] In Table II and Fig. 2, we give the values of f_π, f_K as functions of the Higgs-boson mass ($W=m_H$). Note the dip in f_π just below the S^* resonance, which is the result of destructive interference between the two terms in (2.13). Note also that f_π never exceeds 4 until m_H is within 50 MeV of the S^* resonance and $K\bar{K}$ threshold, i.e., the BR for $H\to\mu\bar{\mu}$ is never less the 20% for $m_H<950$ MeV.

In the text following Eq. (2.22) we described qualitatively the goodness of our fit to the s wave $I=0$, $\pi\pi, K\bar{K}$ data. (See also F_1 in Table II.) Here we emphasize that the parameters in (2.22) are overdetermined by the quantitative and qualitative features described there. We list seven such features: the scattering length, the energy at

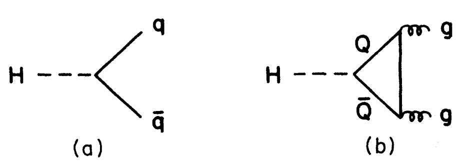

FIG. 1. Feynman diagrams for Higgs-boson decay into (a) light quarks (b) gluons through virtual intermediate heavy quarks.

TABLE I. Values of F_1 and F_2 in the range $0.3 \le m_H \le 1.5$ GeV. F_1 and F_2 are in units of GeV^2.

m_H	F_1	$\lvert F_1\rvert^2$	$\lvert F_2\rvert^2$
0.3	$0.092+0.012i$	0.009	
0.4	$0.121+0.042i$	0.016	
0.5	$0.150+0.090i$	0.031	
0.6	$0.163+0.163i$	0.053	
0.7	$0.130+0.244i$	0.076	
0.8	$0.041+0.265i$	0.072	
0.9	$-0.017+0.088i$	0.008	
1.0	$0.390+1.458i$	2.279	28.712
1.1	$-0.166+0.787i$	0.648	3.658
1.2	$-0.322+0.605i$	0.468	1.690
1.3	$-0.383+0.478i$	0.375	0.976
1.4	$-0.405+0.389i$	0.315	0.426
1.5	$-0.411+0.325i$	0.275	0.416

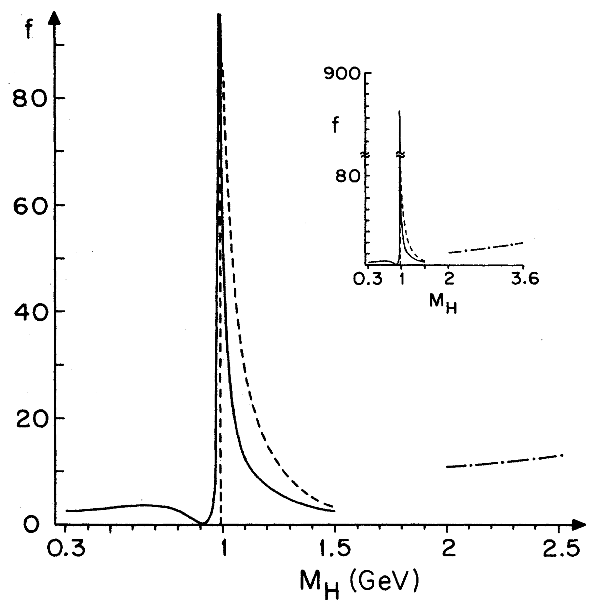

FIG. 2. The ratios f_π (solid line), f_k (dashed line), and f_{QCD} (dashed-dotted line).

which δ rises through $\pi/2$, the slope there, the position and slope of the rapid rise of δ near the $K\bar{K}$ threshold, and the rapid rise and then turnover of the inelasticity. We therefore expect that anyone who does a five-parameter, two-pole, two-channel fit will arrive at parameters very similar to (2.22).

We have already remarked that Raby and West[4] (RW) have overestimated the enhancement of $H\to\pi\pi$ by the S^* by treating it as an elastic $\pi\pi$ resonance, ignoring the constraint of coupled-channel unitarity. In addition, even in the context of a single-channel resonance parametrization, we have found two other errors in the treatment of RW, each of which also goes in the direction of overestimating the (dominant) S^* enhancement. RW use a resonance parametrization of a single-channel Omnes-Muskeleshvili representation [RW Eq. (64)]

$$F(s)=P(s)\Omega(s) .$$

[In the single-channel approximation this is entirely equivalent to the single-channel version of (2.13).] For $P(s)$ they take the LET result (2.24) [RW Eq. (95)]. [They put in the i-spin factor of 3 in the calculation of the rate, see RW Eqs. (97) and (98).] However this does not satisfy the LET because $\Omega'(0)\neq 1$. When the contribution of $\Omega'(0)$ is taken out to satisfy the LET, the value of F at the S^* is decreased by almost $\frac{1}{2}$ (and F is squared in the rate). A second error in RW is that for the denominators in their resonance forms they use $M^2-s-ik\Gamma$ [RW Eqs. (67) and (96)]. The conventional form is $M^2-s-2ik\Gamma\approx 2M(M-W-i\Gamma/2)$. [See, for example, the Particle Data Group (PDG) review.] Thus when they take a (total) S^* width of 33 GeV from the PDG review and use it in their formulas, they are affectively using a width of 16.5 GeV. Thus they overestimate the peak height by another factor of 2 in amplitude and 4 in rate. (This also explains why they obtain $\Gamma_\epsilon=1.3$ GeV from fitting to elastic $\pi\pi$ phase shift, while we obtain 0.63 GeV, fitting to the *same* phase shift, with the conventional resonance form, or its equivalent $\Gamma=2/[d\delta/dW]$.) Compounding all of these factors leads to a large overestimate of the S^* contribution.

TABLE II. Values of the $\pi\pi/\mu\bar{\mu}$ and $K\bar{K}/\mu\bar{\mu}$ ratios.

m_H	f_π	f_K
0.3	2.46	
0.4	2.65	
0.5	3.11	
0.6	3.57	
0.7	3.68	
0.8	2.63	
0.9	0.229	
1.0	52.5	86.9
1.1	12.3	30.9
1.2	7.42	15.5
1.3	5.05	8.70
1.4	3.65	5.21
1.5	2.77	3.28

Somewhere above the $K\bar{K}$ threshold, additional channels $(\eta\eta, 4\pi, \rho\rho, \ldots)$ become important and our two-channel formalism is no longer adequate. At still higher energies, where many channels are open, for the inclusive hadronic ratio $f_{\text{had}}=\Gamma(H\to\text{hadrons})/\Gamma(H\to\mu\bar{\mu})$, one can use the quark-gluon QCD description[11] [see (2.23)]

$$\Gamma_{\mu\bar{\mu}}:\Gamma_{s\bar{s}}:\Gamma_{gg}\approx m_\mu^2:3m_s^2:\left[\frac{\alpha_s}{\pi}\right]^2\left[\frac{N_h^2}{9}\right]m_H^2 . \tag{3.1}$$

For m_H in the range 2–3.6 GeV we take $\alpha_s/\pi\approx 0.1$ and $N_h=3$ (or 2). Then roughly

$$f_{\text{had}}\approx\frac{\Gamma_{ss}+\Gamma_{gg}}{\Gamma_{\mu\bar{\mu}}}\approx 7+m_H^2 . \tag{3.2}$$

This is also plotted in Fig. 2.

In the range above 1.2 or 1.3 GeV where our two-channel description breaks down, but below some several GeV where the inclusive QCD description becomes accurate; it is very difficult to make any detailed calculation. But we remark that the large enhancement associated with the S^* is a very special case. The S^* is a very narrow resonance, and, more importantly, it is located very close to a threshold to which it is strongly coupled. This special set of circumstances is not repeated at higher masses. s-wave resonances at higher energy are broad, and there are no sharply defined thresholds. The most reasonable guess is just to extrapolate $f_{\rm QCD}$ [Eq. (3.2)] back to smaller q^2 with some smooth broad bumps [$f_0(1400)$, $f_0(1590)$,?] superposed on it. A conservative estimate is that $B_{\mu\mu}$ is not less than 1% for m_H in the range from 1.1 to 3.6 GeV.

We briefly consider the application of these results to the experimental searches for the H in B decays. In the standard model with a single physical H and just three generations of fermions, the BR for the decay of a B meson into H plus anything is[1]

$$\frac{B(B\to HX)}{B(B\to l\nu X)} = \frac{27\sqrt{2}}{64\pi^2}G_F m_b^2 \frac{|V_{tb}V_{ts}^*|^2}{|V_{cb}|^2}\left[\frac{m_t}{m_b}\right]^4 \frac{\left[1-\frac{m_H^2}{m_b^2}\right]^2}{f(m_c^2/m_b^2)} . \qquad (3.3)$$

In order to evade the Linde-Weinberg bound[12] and have an H with mass less than m_B, in the minimal standard model, it is required that the top mass be ≥ 80 GeV. Then, with the $b\to cl\nu$ phase-space factor $f(m_c^2/m_b^2)=0.5$ and $B(B\to l\nu X)=0.12$, Eq. (3.3) gives the theoretical value

$$B(B\to HX)\geq 0.26\left[1-\frac{m_H^2}{m_b^2}\right]^2 \quad (m_t\geq 80\ \text{GeV},\ m_H<m_B) . \qquad (3.4)$$

There are a series of experiments[13] which give

$$B(B\to\mu\bar{\mu}X)_{\rm expt}\leq 0.008 \quad (\text{for } m_H>0.3\ \text{GeV}) . \qquad (3.5)$$

In the range $0.3\leq m_H\leq 0.95$ our calculation gives $B(H\to\mu\bar{\mu})>0.20$, hence

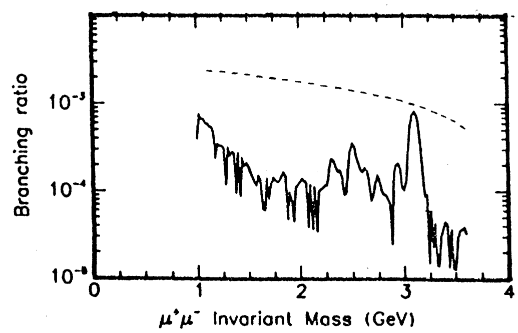

FIG. 3. The CLEO (Ref. 14) upper limit on $B(B\to\mu\bar{\mu}X)$ (solid line) and the theoretical lower limit on $B(B\to HX)B(H\to\mu\bar{\mu})$ under the conditions stated in the text (dashed line).

$$B(B\to HX)B(H\to\mu\bar{\mu})>0.05 \quad (0.3\leq m_H\leq 0.95) , \qquad (3.6)$$

which exceeds (3.5) by a factor of 6. In the range 0.95–1.1 GeV the strong S^* enhancement of the $\pi\pi$ and $K\bar{K}$ decay modes of the H depresses the BR for $H\to\mu\bar{\mu}$ so much that no conclusion can be drawn from this mode. For $m_H>1.1$ GeV, up to 3.6 GeV, the recent CLEO experiment[14] provides stronger bounds. In Fig. 3 we have reproduced the CLEO limits for $B(B\to\mu\bar{\mu}X)$ (Fig. 6 of Ref. 14) and superimposed the theoretical lower limit following from (3.4) (with $m_b=4.9$ GeV) and our conservative estimate $B(H\to\mu\bar{\mu})>0.01$ in this range:

$$B(B\to HX)B(H\to\mu\bar{\mu})>26\times 10^{-4}\left[1-\frac{m_H^2}{m_{b^2}}\right]^2 \quad (1.1\leq m_H\leq 3.6) \qquad (3.7)$$

we see that the theoretical lower limit substantially exceeds the experimental upper limit everywhere in this range, except just at the position of the J/ψ resonance, where the experiment observes $B\to\psi X$ followed by $\psi\to\mu\bar{\mu}$ at the level 8×10^{-4}.

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