Higgs mechanism for Kalb-Ramond gauge field

Soo-Jong Rey*
Department of Physics, University of California, Santa Barbara, California 93106
(Received 12 June 1989)

Kalb-Ramond gauge field theory, coupled to manifestly gauge-invariant and reparametrization-invariant second-quantized string fields, is studied. String condensation into the vacuum is shown to develop masses to the Kalb-Ramond gauge field which we call the string Higgs mechanism. In case the Kalb-Ramond field couples to the open strings which have a massless vector field in the spectrum, our field theory also exhibits the well-known Stueckelberg mechanism. To strengthen our conclusion, we construct a lattice version of the above string field theory and demonstrate that the mean-field approximation indicates the existence of the Higgs phase.

I. MOTIVATION

The Kalb-Ramond gauge field[1] appears in various field theories. Notable examples are the fundamental (super)string theories,[2,3] phenomenological QCD strings and color confinement models,[4] and axionic cosmic strings.[5]

The present paper discusses certain interesting aspects of the Kalb-Ramond gauge field theory. (Even though there are several proposals of non-Abelian extension of Kalb-Ramond gauge theory, none of them are satisfactory since the path ordering of the Wilson surfaces is ill defined.[6]) Specifically we are interested in the Higgs mechanism of the Kalb-Ramond gauge field. How would we achieve this? Let us recall that in the Abelian Higgs model the photon becomes massive (Meissner effect) once the Higgs field, to whose world line the photon couples, undergoes a charged Bose-Einstein condensation. Similarly, since the Kalb-Ramond gauge field couples to the world sheet of strings (our terminology such as "electric" and "magnetic" is in analogy with Maxwell's electromagnetism; we also normalize the action of the Kalb-Ramond gauge field so that the coupling constant appears in front of the Kalb-Ramond field minimal coupling term), bosonic string condensation into the vacuum may realize the Higgs mechanism to the Kalb-Ramond gauge field. (To the best knowledge of the author, Nambu[4] first suggested the Higgs mechanism for the Kalb-Ramond gauge field similar to the present paper even though he did not show how it works explicitly. See also Ref. 7.)

We demonstrate, in the present paper, that the above argument is indeed true: the condensation of the closed strings exhibits a Higgs mechanism to the Kalb-Ramond gauge fields (string Abelian Higgs model). We also find that, if coupled to open strings whose physical spectrum includes a massless Abelian vector gauge field,[8] the Kalb-Ramond gauge field becomes a massive vector field through the well-known Stueckelberg mechanism.[9] Therefore, both the Higgs mechanism and the Stueckelberg mechanism are two different ways that the Kalb-Ramond gauge field acquires mass through its coupling to the two different types of strings. We also study a lattice version of the string Abelian Higgs model. The phase structure is examined within the mean-field approximation. The existence of the Higgs phase is demonstrated, in which the Kalb-Ramond gauge field becomes massive.

II. PLAQUETTE FORMULATION OF THE STRING QUANTIZATION

We consider a relativistic closed string immersed in D-dimensional ($D \geq 3$) spacetime. An infinitesimal string plaquette $\Sigma^{\mu\nu}(X(\sigma,\tau))$ is taken as a dynamical variable which is related to the string coordinate $X^\mu(\sigma,\tau)$ by

$$d\Sigma^{\mu\nu}(X(\sigma,\tau)) \equiv \Sigma^{\mu\nu}(X(\sigma,\tau))d\sigma \wedge d\tau = \frac{\partial(X^\mu,X^\nu)}{\partial(\sigma,\tau)} d\sigma \wedge d\tau \; . \tag{1}$$

The world-sheet parametrization is described by $\sigma=[0,2\pi]$ and $\tau=[-\infty,\infty]$. In the presence of the background Kalb-Ramond gauge field,

$$H_{\mu\nu\lambda}(X) \equiv \frac{1}{3!}\nabla_{[\mu}B_{\nu\lambda]}(X) \; ,$$

the first-quantized string action[10] is

$$\mathcal{S}_{\text{closed}} = \int d\sigma\, d\tau \{ T(\Sigma^{\mu\nu}\Sigma_{\mu\nu})^{1/2}[X(\sigma,\tau)] + q\Sigma^{\mu\nu}(X(\sigma,\tau))B_{\mu\nu}(X(\sigma,\tau))\} \; . \tag{2}$$

Here, T is a "bare" string tension and q is a coupling constant of the string to the Kalb-Ramond gauge field (i.e., the "electric" charge). Using Eq. (1), one can verify that the action equation (2) is manifestly reparametrization invariant. If we vary the action equation (2) with respect to the variable X^μ using the implicit relation of $\Sigma^{\mu\nu}(X)$ given in Eq. (1), we get a string equation of motion in the background of the Kalb-Ramond gauge field:

$$T\frac{\partial(\Sigma_{\mu\nu}/\sqrt{\Sigma^2},X^\nu)}{\partial(\sigma,\tau)} + qH_{\mu\nu\lambda}(X)\Sigma^{\nu\lambda}=0 \; . \tag{3}$$

This is indeed a correct equation of motion, being a string analog of the Lorentz equation for a charged particle in an external electromagnetic background field.

The Kalb-Ramond gauge field has a U(1) gauge invariance

$$\delta_\Lambda B_{\mu\nu}(X)=\partial_{[\mu}\Lambda_{\nu]}(X) . \tag{4}$$

Under this transformation, the second term of Eq. (2) is changed into (the boundary terms at $\tau=\pm\infty$ vanish since the gauge function Λ_μ should be the same at these boundaries[6])

$$q\int d\sigma\, d\tau\, \Sigma^{\mu\nu}\nabla_{[\mu}\Lambda_{\nu]}= -q\int_{\sigma=2\pi} d\tau \frac{\partial X^\nu}{\partial\tau}\Lambda_\nu + q\int_{\sigma=0} d\tau\frac{\partial X^\nu}{\partial \tau}\Lambda_\nu . \tag{5}$$

These terms vanish only for a closed string, having periodic boundary condition in the σ direction. If the string is open, one needs to extend the gauge invariance by including the massless photon background field coupling to $+e$ and $-e$ electric charges at both ends of the open string. Thus the open-string action in the background of the Kalb-Ramond gauge field and the photon field reads

$$\mathcal{S}_{\text{open}}=\mathcal{S}_{\text{closed}}+e\int d\sigma\, d\tau\, \Sigma^{\mu\nu}F_{\mu\nu}(X(\sigma,\tau)) . \tag{6}$$

The extended gauge transformation now reads

$$\delta_\Lambda B_{\mu\nu}(X)=\partial_{[\mu}\Lambda_{\nu]}(X)$$

and (7)

$$\delta_{\eta,\Lambda}A_\mu(X)=\partial_\mu\eta(X)+\frac{q}{e}\Lambda_\mu(X) .$$

Note that the gauge transformation involves a singular transformation of the Maxwell field.

The "momentum" conjugate to the string plaquette $\Sigma^{\mu\nu}$ is

$$P_{\mu\nu}(X)\equiv\frac{\delta\mathcal{S}_{\text{closed}}}{\delta\Sigma^{\mu\nu}(X)}=T\frac{\Sigma_{\mu\nu}}{\sqrt{\Sigma^2}}(X)-qB_{\mu\nu}(X) . \tag{8}$$

This is the generator of string loop deformation within $(D-1)$-dimensional subspace normal to the string. From this, we find the "physical on-shell" condition [Nambu[11] showed how the Hamilton-Jacobi formulation can be achieved for the free action (i.e., no external Kalb-Ramond gauge field)]

$$(P_{\mu\nu}+qB_{\mu\nu})^2[X(\sigma,\tau)]=T^2 . \tag{9}$$

Upon quantization of the strings, we have a wave functional of the strings $\Psi[\Omega]$. Here, $\Omega(\cdot)$ denotes a parametrized string loop $\Omega\equiv\{X^\mu(\sigma)=X^\mu(\sigma+2\pi):\sigma\in[0,2\pi]\}$. The wave functional should be invariant under a string reparametrization $\Omega(\sigma)\to\tilde{\Omega}(\tilde{\sigma})$:

$$\Psi[\Omega(\sigma)]=\Psi[\tilde{\Omega}(\tilde{\sigma})] . \tag{10}$$

Thus, from Eq. (9), the quantum-mechanical wave equation may be written as

$$\left[-i\frac{\delta}{\delta\Sigma^{\mu\nu}(X(\cdot))}+qB_{\mu\nu}(X(\cdot))\right]^2\Psi[\Omega]=T^2\Psi[\Omega] . \tag{11}$$

Here, we used the fact that the wave functional $\Psi[\Omega]$ is manifestly reparametrization invariant.

The definition of the plaquette derivative on the string functional $\Psi[\Omega]$ is as follows. For a given loop configuration Ω, we add an infinitesimal plaquette $\Sigma^{\mu\nu}(X(\sigma,\tau))$ at a point $X(\sigma)\in\Omega$. This induces a normal direction deformation of the string loop $\Omega(\cdot)$ into $\Omega(\cdot)+\Delta\Omega(\cdot)$ such that $\partial\Sigma=\Delta\Omega$. The procedure is depicted in Fig. 1.

This will induce a deformation of the string wave functional $\Psi[\Omega]$. The rate of deformation of the string wave functional is defined as the plaquette derivative

$$\Psi[\Omega+\Delta\Omega_X]-\Psi[\Omega] \equiv\int_0^{2\pi}\sqrt{h}\, d\sigma\frac{\delta\Psi}{\delta\Sigma^{\mu\nu}(X(\cdot))}\Delta\Sigma^{\mu\nu}(X(\cdot)) . \tag{12}$$

The loop space metric is denoted by

$$h(\sigma)=\left[\frac{\partial X^\mu(\sigma)}{2\pi\partial\sigma}\right]^2 .$$

III. THE SECOND QUANTIZATION OF STRING ABELIAN HIGGS MODEL

We now consider the second-quantized closed-string field theory interacting with the Kalb-Ramond gauge field, the string Abelian Higgs model. The theory may be described by an action

$$\mathcal{S}=\int d^DX\frac{1}{2\times 3!}H^2_{\mu\nu\lambda}(X) +\int DX(\sigma)\int_0^{2\pi}\sqrt{h}\, d\sigma\left[\left|\frac{D\Phi[\Omega]}{D\Sigma^{\mu\nu}(X)}\right|^2 -M^2|\Phi[\Omega]|^2\right]+\mathcal{S}_{\text{int}} . \tag{13}$$

Here, the dynamical string tension is denoted by M. The covariant plaquette derivative is defined by

$$\frac{D}{D\Sigma^{\mu\nu}(X(\cdot))}=\frac{\delta}{\delta\Sigma^{\mu\nu}(X(\cdot))}+iqB_{\mu\nu}(X(\cdot)) .$$

The string field $\Phi[\Omega(\cdot)]$ is a complex scalar functional which is reparametrization invariant. The term $\mathcal{S}_{\text{int}}$ gives rise to the usual joining and splitting interactions among

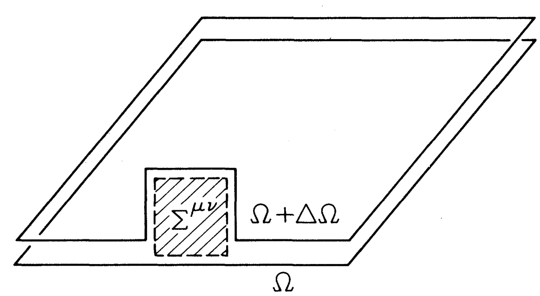

FIG. 1. Addition of an infinitesimal plaquette at $X(\sigma)$ on a loop Ω.

strings and thus has cubic, quartic, or more contact terms:

$$S_{\rm int}=g\int DX_1DX_2\Phi^*[\Omega_1+\Omega_2]\Phi[\Omega_1]\Phi[\Omega_2]+g^2\int DX_1DX_2DX_3DX_4\Phi^*[\Omega_1+\Omega_2]\times\Phi^*[\Omega_3+\Omega_4]\Phi[\Omega_1+\Omega_3]\Phi[\Omega_2+\Omega_4]+\cdots\ . \quad (14)$$

If there were no external background Kalb-Ramond field, the above action (13) is invariant under a global U(1) gauge transformation $\Phi[\Omega]\to e^{i\omega}\Phi[\Omega]$ and the string reparametrization $\Phi[\Omega]\to\tilde{\Phi}[\tilde{\Omega}]$.

In fact, interactions of strings with the Kalb-Ramond gauge field or with other modes of the string spectrum can be deduced from the local U(1) gauge invariance. (This formalism is very analogous to the string field theory proposed by Marshall and Ramond.[12] See also the formulation by Hosotani.[13]) Local U(1) gauge transformation of the complex string functional is

$$\Phi[\Omega(\cdot)]\to\Phi'[\Omega(\cdot)]=\exp\{iq\omega[\Omega(\cdot)]\}\Phi[\Omega(\cdot)]\ . \quad (15)$$

Note that the interaction terms in Eq. (14) are invariant under the loop space U(1) gauge transformation Eq. (15).

Here we concentrate on the gauge invariance associated with the Kalb-Ramond gauge field. Since we should preserve the reparametrization invariance, the functional $\omega[\Omega]$ must be reparametrization invariant by itself. Let us take[12]

$$\omega[\Omega(\cdot)]=\oint_\Omega dX^\mu\Lambda_\mu(X)\ . \quad (16)$$

One now considers a one-parameter trajectory of the strings Ω with coordinates $X^\mu(\sigma,\tau)$ where $\sigma=[0,2\pi]$ and $\tau=[0,1]$ denotes the closed-string parametrization and the evolution parametrization, respectively. Then we can rewrite Eq. (15) as

$$\omega[\Omega(\cdot)]=\oint_{C=\partial\mathcal{A}}d\sigma\frac{\partial X^\mu(\sigma)}{\partial\sigma}\Lambda_\mu(X(\sigma))=\int_{\mathcal{A}}d\Sigma^{\mu\nu}(X(\sigma,\tau))\partial_{[\mu}\Lambda_{\nu]}(X(\sigma,\tau))\ . \quad (17)$$

The plaquette integration is over the string world sheet $\mathcal{A}$ whose boundary is C. Now, we look at

$$\left[\frac{\delta}{\delta\Sigma^{\mu\nu}(X(\cdot))}+iqB'_{\mu\nu}(X(\cdot))\right]\Phi'[\Omega(\cdot)]=\exp\{iq\omega[\Omega(\cdot)]\}\left[\frac{\delta}{\delta\Sigma^{\mu\nu}(X(\cdot))}+iq\frac{\delta\omega[\Omega]}{\delta\Sigma^{\mu\nu}(X(\cdot))}+iqB'_{\mu\nu}(X(\cdot))\right]\Phi[\Omega]\ . \quad (18)$$

Using Eq. (17) and the basis relation

$$\delta\Sigma^{\mu\nu}\frac{\delta}{\delta\Sigma^{\alpha\beta}}=\tfrac{1}{2}(\delta^\mu_\alpha\delta^\nu_\beta-\delta^\mu_\beta\delta^\nu_\alpha)\ ,$$

we rewrite Eq. (18) as

$$\exp(iq\omega[\Omega])\left[\frac{\delta}{\delta\Sigma^{\mu\nu}(X)}+iq[B'_{\mu\nu}(X)-\partial_{[\mu}\Lambda_{\nu]}(X)]\right]\Phi[\Omega]\ .$$

Thus if the gauge transformation of the Kalb-Ramond gauge field is given by Eq. (15), we find that the covariant derivative transforms homogeneously and the action (13) remains gauge invariant.

The spontaneous symmetry breakdown of the local U(1) gauge invariance associated with the Kalb-Ramond gauge field is achieved by condensations of the closed strings into the vacuum. Namely, when the "physical" string tension M becomes complex, $M^2\le 0$, we have

$$\langle 0|\Phi[\Omega(\cdot)]|0\rangle=\phi_0[X]\ . \quad (19)$$

(The string condensation criterion can be determined assuming that the strings are topological defects of a local field theory. This analysis was undertaken in Ref. 14.) Note that the vacuum expectation value of string field Φ can be an arbitrary functional of string coordinates subject to the Poincaré invariance constraint. This means that, infinitely many string particle states on the Regge trajectory become tachyonic and condense into the vacuum. The simplest choice corresponds to $\phi_0[X]=\phi_0/2q=$const and we get

$$\mathcal{L}=\tfrac{1}{12}H^2_{\mu\nu\lambda}+\frac{\phi_0^2}{4}B^2_{\mu\nu}\ . \quad (20)$$

This exhibits a genuine Higgs mechanism for the Kalb-Ramond gauge field theory. The mass of the Kalb-Ramond gauge field is generated by the "charged" condensation of the closed-string loops into the vacuum. A unique feature of string condensation compared to the usual point-particle condensation is the possibility of string-coordinate-dependent functional of the vacuum expectation value. As mentioned above, however, only Poincaré-invariant condensates are allowed. Thus the functional $\omega[\Omega]$ must only involve nonzero modes of $X^\mu(\sigma)$ and must be Lorentz scalar. In addition, to have a meaningful loop space formulation of string field theory, the reparametrization invariance must be retained.

Next, suppose that the string field $\Phi[l(\cdot)]$ describes an open string denoted by $l\equiv\{X^\mu(\sigma):\sigma\in[0,2\pi]\}$. In this case, the massless photon couples to the ends of open string, which carries equal and opposite electric charges as we discussed in the previous section [see Eq. (6)]. Thus the string field with the photon coupling may be written as a path-ordered Wilson line

$$\Phi_0[l(\cdot)]=P\exp\left[ie\int_l A_\mu(X)\frac{dX^\mu}{d\tau}d\tau\right]\frac{\phi_0}{2q}\ . \quad (21)$$

Inserting this configuration into the action equation (13), we get

$$\mathcal{L}_{\rm open}=\tfrac{1}{12}H^2_{\mu\nu\lambda}(X)+\frac{\phi_0^2}{4}\left|B_{\mu\nu}(X)+\frac{e}{q}F_{\mu\nu}(X)\right|^2\ . \quad (22)$$

Thus the vector field of the open string couples to the external Kalb-Ramond gauge field through the $B_{\mu\nu}F^{\mu\nu}$ term.[1]

The Lagrangian equation (22) is nothing but the Stueckelberg formalism[9] of a massive pseudovector field in terms of Kalb-Ramond tensor field. Namely, we may redefine the Kalb-Ramond field

$$\bar{B}_{\mu\nu}=B_{\mu\nu}+\frac{e}{q}\nabla_{[\mu}A_{\nu]} \ . \tag{23}$$

The kinetic term of the Kalb-Ramond gauge field does not change after this field redefinition so the new Lagrangian is

$$\mathcal{L}_{\text{open}}=\tfrac{1}{12}H^2_{\mu\nu\lambda}(X)+\left[\frac{q}{e}\right]^2\bar{B}^{\,2}_{\mu\nu} \ . \tag{24}$$

Since we redefined the Kalb-Ramond field, the coupling of the open string to the external field $\bar{B}_{\mu\nu}$ now reads

$$\mathcal{S}=\int d\sigma\, d\tau\left[[\Sigma^{\mu\nu}\Sigma_{\mu\nu}]^{1/2}+q\Sigma^{\mu\nu}\left[B_{\mu\nu}(X)+\frac{e}{q}\nabla_{[\mu}A_{\nu]}(X)\right]\right] \ . \tag{25}$$

This is precisely the first-quantized string action equation (9) we deduced from the extended gauge invariance equation (7). We thus find that the Kalb-Ramond field combines with the massless Abelian vector field to become a massive pseudovector field and interact through the minimal coupling described by Eq. (25).

IV. LATTICE FORMULATION OF THE STRING ABELIAN HIGGS MODEL

Let us now turn to a (spacetime) lattice formulation of the closed-string Abelian Higgs model Eq. (13). Using this formulation, we will analyze the phase structure within the mean-field approximation. Consider a Euclidean D-dimensional hypercube lattice of lattice spacing a. The Kalb-Ramond gauge field is defined on each plaquette. Thus we consider a plaquette operator

$$U_p[B_{\mu\nu}]=\exp[iqa^2B_{\mu\nu}(p)] \ . \tag{26}$$

The kinetic term of the Kalb-Ramond gauge field is

$$\mathcal{S}_H=\beta\sum_{(\text{cube})}\text{Re}\left[\prod_{p\in\text{cube}}U_p\right] \ . \tag{27}$$

The lattice coupling constant of the Kalb-Ramond gauge field is denoted by β. The gauge transformation of the plaquette operator under the transformation equation (4) turns out to be

$$U_p\to\left[\prod_{l\in\partial p}\Lambda_l\right]U_p \quad\text{where}\quad \Lambda_l=\exp[iqa\Lambda_\mu(l)] \ . \tag{28}$$

Namely, for a given plaquette p on which we assign the plaquette operator U_p, the gauge transformation is to put the link operators Λ_l on every perimeter link $l\in\partial p$ of the plaquette p.

Similarly, we can write down the gauge-invariant kinetic term of string loop fields $\Omega(C)$ defined for each closed link as

$$\mathcal{S}_\Phi=\sum_{(C)}\left[\sum_{p\in\partial C}[\bar{\Omega}(C+p)\cdot U_p\cdot\Omega(C)+\bar{\Omega}(C-p)\cdot\bar{U}_p\cdot\Omega(C)]+[M^2a^4+2(D-1)]\bar{\Omega}(C)\Omega(C)\right] \ . \tag{29}$$

Here $C+p$ denotes the addition of a plaquette p to the closed link C and the summation over the plaquette means all possible plaquette additions to the given link C. The dynamical string tension M is related to the bare string tension of T as[14]

$$M^2=a^{-4}[\exp(Ta^2)-2(D-1)] \ . \tag{30}$$

The second term that depends upon the spacetime dimensionality is a contribution from the entropy of the string configuration.

Thus the total action is written as

$$\mathcal{S}_0[U_p,\Omega(C),\beta,M^2]=\mathcal{S}_H+\mathcal{S}_\Phi \ .$$

We now evaluate the partition function

$$Z(\beta,M^2)=\int\prod_p dU_p\prod_C d\Omega(C)\times\exp[-\mathcal{S}_0(U_p,\Omega_C,\beta,M^2)] \ , \tag{31}$$

where we assume proper Haar measures defined both on the Abelian U(1) group and on the reparametrization group of strings.

The partition function may be evaluated by the mean-field method.[15] First we decompose the string loop field $\Omega(C)$ as a product of each individual component excitations of strings:

$$\Omega(C)=\prod_{l\in\partial C}\prod_m U_l(m) \ . \tag{32}$$

The mth string excited state link operator is denoted schematically by $U_l(m)$. When the physical string tension takes $M^2\le 0$ [see Eq. (30)], the string field $\Omega(C)$ is expected to fluctuate around a nonzero value. Thus we decompose the string loop operator as

$$\Omega(C)=\Omega_l[A_\mu]\omega(C),\quad |\omega(C)|\approx\left[\frac{-M^2}{g}\right] \ . \tag{33}$$

All the "heavy" mode excitations other than the massless vectorlike spectrum are denoted by $\omega(C)$. Its magnitude is proportional to the physical string tension. The massless vector field spectrum may be written as

$$\Omega_l[A_\mu]=\prod_{l\in\partial C}\exp[iqaA_\mu(l)] \ .$$

Inserting this into Eq. (31) we get

$$Z(\beta,g_R^2)=\int\prod_p dU_p\prod_C d\Omega_l(A)\times\exp[-\mathcal{S}_{\text{eff}}(U_p,\Omega_l,\beta,g_R^2)] \ ,$$

where

$$\mathcal{S}_{\rm eff}=-\ln\int\prod_{C}d\omega(C)\exp\{-\mathcal{S}_0[U_p,\Omega_l\omega(C),\beta,M^2]\} \;. \tag{34}$$

We now approximate the effective action $\mathcal{S}_{\rm eff}$ as

$$\tfrac{1}{2}g_R^2\sum_{(C)}\left\{\sum_{p\in\partial C}\left[\left[\prod_{l\in C+p}\overline{\Omega}_l\right]\cdot U_p\cdot\left[\prod_{l\in C}\Omega_l\right]+\text{H.c.}\right]\right.$$
$$\left.+[M^2a^4+2(D-1)]\right\}$$
$$=g_R^2\sum_p \text{Re}\left[U_p\prod_{l\in\partial p}\Omega_l\right] \tag{35}$$

with the lattice coupling constant $g_R^2=2|\omega|^2$ for the string field. Thus we finally get

$$Z(\beta,g_R^2)=\int\prod_p dU_p\prod_l d\Omega_l\exp(-\overline{\mathcal{S}}_{\rm eff}) \;,$$

where

$$\overline{\mathcal{S}}_{\rm eff}=\beta\sum_{(\text{cube})}\text{Re}\left[\prod_{p\in\text{cube}}U_p\right]+g_R^2\sum_p\text{Re}\left[U_p\prod_{l\in\partial p}\Omega_l\right] \tag{36}$$

modulo irrelevant numerical constant normalization. This is precisely the partition function of the Kalb-Ramond gauge field coupled to the "massless phantom photon" field A_μ. Note that the gauge invariance of Eq. (28) associated with the plaquette operator [U(1) Kalb-Ramond tensor field gauge invariance] appears lost in this mean-field theory. However, the original gauge invariance associated with $\Omega(C)$ [whose continuum version is Eq. (15)] shows that there are residual gauge invariances

$$U_p\to\left[\prod_{l\in\partial p}\Lambda_l\right]U_p \quad\text{and}\quad \Omega_l\to\Lambda_l\Omega_l \;. \tag{37}$$

With the ansatz $\text{Re}\langle U_p\rangle=V_p$, $\text{Re}\langle U_l\rangle=V_l$, and $\text{Im}\langle U_p\rangle=\text{Im}\langle U_l\rangle=0$, we get, from Eq. (36), the mean-field solutions[16]

$$V_p=\frac{I_1(\alpha_p)}{I_0(\alpha_p)} \quad\text{and}\quad V_l=\frac{I_1(\alpha_l)}{I_0(\alpha_l)} \;. \tag{38}$$

Here I_0, I_1 are modified Bessel functions while

$$\alpha_p=2(D-2)\beta V_p^5+g_R^2V_l^4$$

and (39)

$$\alpha_l=2(D-1)g_R^2V_pV_l^3 \;.$$

From these, one can determine phase boundary in (β,g_R^2) space, extended from the two critical points $(\beta_c,g_R^2=0)$ and $(\beta=\infty,g_R^{*2})$, at which the first-order phase transition takes place. Analysis of Eqs. (38) and (39) shows that there are three phases of the theory as drawn in Fig. 2.

When $\beta\le\beta_c$ and $g_R^2\le g_R^{*2}$, the Kalb-Ramond gauge field and the photon field are confined due to strong disorder. If we increase $\beta\to\infty$ beyond the critical value β_c, the Kalb-Ramond field is weakly coupled and becomes massless while the photon field remains confined still.

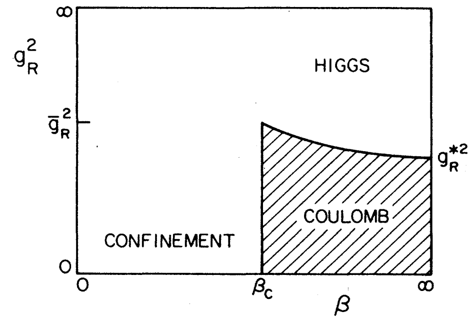

FIG. 2. The phase diagram of the string Abelian Higgs model.

Thus, this is a phase in which the Kalb-Ramond field exhibits Coulombic behavior. On the other hand, if we increase g_R^2 also beyond the critical value g_R^{*2}, a weakly interacting, massless photon couples to the Kalb-Ramond gauge field. They combine to result in a massive Kalb-Ramond gauge field. This is precisely the parameter domain that the string Higgs mechanism shows up. In four dimensions, however, there exists a nonperturbative disorder[17] for all values of β due to the "magnetic" instantons of the Kalb-Ramond gauge field. Thus the Coulomb phase in the figure is not present in four dimensions. The physical consequence is string electric confinement[18] due to the Kalb-Ramond "magnetic" instantons.

V. DISCUSSION

In this paper we mainly concentrated on the Kalb-Ramond field coupled to the string fields. However, it is straightforward to generalize this to the higher- $(p+1)$-rank antisymmetric tensor gauge fields (and only Abelian fields due to the reason discussed earlier) by coupling to the appropriate extended objects (p-branes). The p-branes, once condensed, exhibit p-form gauge field as a Goldstone excitation mode. This Goldstone field is absorbed by the $(p+1)$-rank antisymmetric tensor gauge field and turn into a massive gauge field.

Can we also describe the graviton and the dilaton couplings to the string in terms of the plaquette variables as we did for the Kalb-Ramond gauge field? It is easy to see that appropriate couplings are

$$\mathcal{S}=\int d\sigma\,d\tau\{T\,D(X)[\Sigma^{\mu\nu}G_{\mu\alpha}(X)G_{\nu\beta}(X)\Sigma^{\alpha\beta}]^{1/2}$$
$$+q\Sigma^{\mu\nu}B_{\mu\nu}(X)\} \;. \tag{40}$$

If the fluctuation of these massless modes is small, the linearized interaction term reads $[G_{\mu\nu}(X)=\delta_{\mu\nu}+h_{\mu\nu}(X)]$

$$\int d\sigma\,d\tau\left[T\sqrt{\Sigma^2}D(X)+T\frac{\Sigma^\mu_\alpha\Sigma^{\nu\alpha}}{\sqrt{\Sigma^2}}h_{\mu\nu}(X)+q\Sigma^{\mu\nu}B_{\mu\nu}(X)\right] \;. \tag{41}$$

Thus we derive their currents to be

$$\mathcal{T}_{\mu\nu}(X) = \frac{\delta \mathcal{S}}{\delta G^{\mu\nu}(X)} = T \int d\sigma\, d\tau \left[\frac{\Sigma_{\mu\alpha}\Sigma_\nu^\alpha}{\sqrt{\Sigma^2}} \right] (Y) \delta^{(D)}[X - Y(\sigma,\tau)]$$

and

$$\mathcal{T}(X) = \frac{\delta \mathcal{S}}{\delta D(X)} = T \int d\sigma\, d\tau \sqrt{\Sigma^2} \delta^{(D)}[X - Y(\sigma,\tau)] \; . \tag{42}$$

We note that these are nothing but the energy-momentum tensor and its trace part of a single string, coupled to the linearized graviton and the dilaton. The string plaquette couples to the gravity and the dilaton fields nonlinearly (in fact, this difficulty has been noticed by Kalb and Ramond[1]), unlike the Kalb-Ramond gauge field. This nonlinear structure makes it hard to formulate graviton and dilaton coupling of strings in terms of the world-sheet plaquette as a dynamical variable. An early attempt has been made by Marshall and Ramond[11] based upon the local U(1) gauge invariance of the loop space [see Eq. (15)]. This interesting approach is yet to be investigated thoroughly.

In the above, we showed how the closed-string condensation into the vacuum can generate spontaneously the mass to the Kalb-Ramond gauge field. Our classical string Abelian Higgs model is more heuristic than rigorous. The issue of reparametrization gauge invariance is very important to have a consistent, fully interacting *quantum* string field theory (we do not have any such theory at hand for the moment). We did not discuss the unitarity and quantum consistency issues since only classical physics is all we need. The Becchi-Rouet-Stora-Tyutin (BRST) formalism is expected to add only some technical complications to the basic picture we presented here. Still it would be nice to understand the precise way that BRST string field theories realize the Higgs mechanism we discussed in this paper. One possible approach may be through a background-independent string field theory[19] and considering a Kalb-Ramond field background.

Finally, we mention that the fermionic strings in the background of the Kalb-Ramond gauge field are a straightforward generalization of the present paper.

ACKNOWLEDGMENTS

The author is deeply gratful to F. Wilczek who provided the author with initial motivations and several important suggestions in the early stage of the present work. For suggestive comments, I thank G. Horowitz and P. Ramond, who also pointed out Ref. 7. This work was supported in part by the National Science Foundation under Grant No. PHY85-06686.

---

*Electronic address: SOO@SBITP.BITNET, SOO@UCSBAP.BITNET.

[1]M. Kalb and P. Ramond, Phys. Rev. D **9**, 2273 (1974); Y. Nambu, Phys. Rep. **23**, 250 (1976); see also E. Cremmer and J. Scherk, Nucl. Phys. **72**, 117 (1974).

[2]R. Rohm and E. Witten, Ann. Phys. (N.Y.) **170**, 454 (1986), and references therein.

[3]P. G. O. Freund and R. I. Nepomechie, Nucl. Phys. **B199**, 482 (1982).

[4]Y. Nambu, Phys. Rev. D **10**, 4262 (1974); G. Parisi, *ibid.* **11**, 970 (1975).

[5]A. Vilenkin, Phys. Rep. **121**, 263 (1985), and references therein.

[6]C. Teitelboim, Phys. Lett. **167B**, 63 (1986).

[7]F. Gliozzi, T. Regge, and M. A. Virasoro, Phys. Lett. **81B**, 178 (1979).

[8]M. Kalb, Phys. Rev. D **17**, 2713 (1978).

[9]E. C. G. Stueckelberg, Helv. Phys. Acta **30**, 209 (1957); D. Boulware and W. Gilbert, Phys. Rev. **126**, 1563 (1962).

[10]Y. Nambu, in *Proceedings of the International Conference on Symmetries and Quark Models,* Wayne State University, 1969, edited by Ramesh Chand (Gordon and Breach, New York, 1970), p. 269, and manuscript prepared for the Copenhagen Symposium, 1970 (unpublished); L. Susskind, Nuovo Cimento **A69**, 457 (1970); H. B. Nielsen, in *15th International Conference on High Energy Physics,* Kiev, 1970, edited by V. Shelest (Naukova Dumka, Kiev, U.S.S.R., 1972); O. Hara, Prog. Theor. Phys. **46**, 1549 (1971); T. Goto, *ibid.* **46**, 1560 (1971).

[11]Y. Nambu, Phys. Lett. **92B**, 327 (1980); **102B**, 149 (1981); T. Eguchi, Phys. Rev. Lett. **44**, 126 (1980); Y. Hosotani, *ibid.* **47**, 207 (1981).

[12]C. Marshall and P. Ramond, Nucl. Phys. **B85**, 375 (1975). See also T. Banks, Phys. Lett. **89B**, 369 (1980); W. Siegel, *ibid.* **151B**, 391 (1985); **151B**, 396 (1985).

[13]Y. Hosotani, Phys. Rev. Lett. **55**, 1719 (1985).

[14]D. Foerster, Phys. Lett. **77B**, 211 (1978); **78B**, 473 (1978); H. Kawai, Prog. Theor. Phys. **65**, 351 (1981).

[15]See, for example, J. Drouffe and J. Zuber, Phys. Rep. **102**, 1 (1983).

[16]M. Lehto, H. B. Nielsen, and M. Ninomiya, Phys. Lett. **115B**, 129 (1982).

[17]R. Pearson, Phys. Rev. D **26**, 2013 (1982); P. Orland, Nucl. Phys. **B205**, 107 (1982).

[18]S.-J. Rey, University of California at Santa Barbara report, 1989 (unpublished).

[19]G. Horowitz, J. Lykken, R. Rohm, and A. Strominger, Phys. Rev. Lett. **57**, 283 (1986).