PHYSICAL REVIEW D VOLUME 40, NUMBER 10 15 NOVEMBER 1989

Cosmic evolution of nontopological solitons

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(Received 12 June 1989)

Nontopological solitons are stable field configurations which may be formed in a primordial phase transition. We study their cosmic evolution and examine the possibility that such objects could contribute significantly to the energy density of the Universe. As the Universe cools, initially all but the largest lumps evaporate into free particles; those which survive may subsequently enter a brief accretion phase before they "freeze out" at a final size. Although the minimum critical charges which survive depend on particle masses and couplings, we develop an analysis which applies to a wide class of models. In most cases, solitons of moderate size survive the evaporation process only if there is a significant charge asymmetry or if they form at a temperature well below their binding energy per charge.

I. INTRODUCTION

Nontopological solitons (NTS's) are solutions of classical field theories which are stable by virtue of a conserved Noether charge carried by fields confined to a finite region of space. The prototype for such structures is the phenomenological bag model for hadrons, in which light quarks are confined in a false-vacuum region by Yukawa coupling to a scalar field.[1] In recent years, the bag model has genetically mutated into a rich variety of species, which include Q balls,[2] abnormal nuclei,[3] strange-quark matter nuggets,[4] cosmic neutrino balls,[5] as well as scalar[6-8] and gauge field[9] nontopological solitons. Despite the proliferation of models, they all share a number of family resemblances which allow them to be classified and discussed in general terms. The key feature of all models is that, for particle number Q greater than some minimum $Q_{\rm min}$, the confined soliton state has lower energy than the free particle state.

Recently, it has been suggested that nontopological solitons may be abundantly produced in a phase transition in the early Universe.[4,7,8,10] However, as pointed out by Alcock and Farhi[11] and Alcock and Olinto[12] in the context of strange matter, at sufficiently high temperature, such structures are in general unstable to evaporation into free (unconfined) particles. Non-topological solitons are states of low energy but high order. At temperatures $T \geq I_Q$, the NTS binding energy per particle, free particles have a lower Helmholtz free energy, $F=E-TS$, than the NTS state, due to the entropy term. (This holds as long as the temperature is not very near the critical temperature for the phase transition.[10]) At lower temperatures, the energy term dominates, and NTS's are the preferred states.

If we focus on the evolution of a single soliton of initial charge (or particle number) Q, as the Universe cools the lump begins to evaporate. If it is large enough to survive down to temperatures $T \leq I_Q$, it may then enter an accretion phase. Eventually, the accretion or evaporation rate becomes negligible compared to the expansion rate, and the system "freezes out" with a fixed charge. Thus, all solitons with initial charge less than a critical number Q_s die before they reach the accretion or freeze-out phase. Those with charges $Q \gg Q_s$ are born frozen; i.e., their evaporation and accretion rates are always negligible. However, evaporation is so efficient that Q_s is often larger than the charge inside the particle horizon at the time of formation; in these cases, it is unlikely that any solitons formed in the phase transition survive to the present. At the other extreme, if NTS's form at a temperature well below I_Q, then we expect $Q_s \sim Q_{\rm min}$. In addition, it has recently been suggested that,[13] if the particles are sufficiently strongly interacting to maintain chemical equilibrium at temperatures significantly below I_Q, then a new population of solitons may be built up via accretion and fusion processes at low temperature. However, this latter possibility is only feasible if $Q_{\rm min}$ is very small, of order unity (and only if the mass scale is less than about 10^4 GeV). Otherwise the formation of new solitons by free particle collisions is statistically suppressed at low temperature and the system cannot reach complete chemical equilibrium. Since, in almost all models considered in the literature, $Q_{\rm min} \gg 1$, we have relegated discussion of this possibility to an appendix.

In this paper, we follow the cosmological evolution of nontopological solitons from birth, through the adolescent struggle for survival, to placid old age. In the next section, we outline the basic properties of NTS's which we will need; a wide variety of models are encapsulated in a small number of parameters. For completeness, in Sec. III, we review several of the NTS formation mechanisms which have been proposed. In Sec. IV, we set up the formalism for treating NTS evolution. We discuss the evolution of single NTS lumps in Sec. V; this treatment is relevant if $Q_{\min}$ is not close to 1, in which case the system generally does not reach chemical equilibrium. In Sec. VI we discuss NTS destruction by "boiling," i.e., bubble nucleation, rather than surface evaporation. Our conclusions follow in Sec. VII.

Throughout most of the paper, we shall assume the Universe carries a net charge asymmetry, and that the particles involved are long lived. In a forthcoming paper we will drop the assumption of asymmetry and discuss how NTS's evolve as random charge fluctuations in a universe with zero net charge. In that paper we also will consider models with unstable particles and discuss cosmological constraints on particle-physics models with NTS's.

II. PROPERTIES OF NONTOPOLOGICAL SOLITONS

In all NTS models, a partially "confined" Bose or Fermi field ϕ, carrying a conserved additive quantum number, couples to a scalar field σ; the vacuum expectation value σ_0 of the "confining" field generates part of the ϕ mass, e.g., $m_\phi = m_0 + m_\phi(\sigma_0)$. (In the case of Q balls,[2] a single complex scalar field performs the roles of both ϕ and σ.) For appropriate couplings and sufficiently large charge, $Q > Q_{\min}$, the lowest-energy configuration consists not of free ϕ particles roaming the vacuum, but of a spherical "false vacuum" region in which $\sigma \simeq \sigma'$, surrounded by a domain wall where σ rapidly approaches its ground-state value. In the NTS interior, $m_\phi(\sigma') < m_\phi(\sigma_0)$, and the effectively light ϕ particles are trapped by the mass gap at the domain wall. Since, classically, a nonzero charge implies a time-dependent field, the confined particles carry kinetic energy, which varies as an inverse power of the volume. The potential (volume) and gradient (surface) energies vary with positive power of the volume. As a consequence, for fixed charge Q, the total energy is minimized at a finite radius R, and the resulting configuration is in hydrostatic equilibrium. We will consider solutions which carry no currents, in which case the lowest-energy configurations are spherically symmetric.

For large charges $Q \gg Q_{\min}$, the masses of all NTS models follow a simple scaling law:

$$M(Q) = \alpha m_\phi Q^p \ . \quad (1)$$

For NTS's with nonzero volume energy (bag constant), the radius scales as

$$R(Q) = \beta m_\phi^{-1} Q^{p/3} \quad \text{(volume)} \ . \quad (2)$$

On the other hand, models in which the interior potential energy density is degenerate with the vacuum, $U(\sigma') = U(\sigma_0)$, are confined by surface tension alone;[5,14] in this case,

$$R(Q) = \beta' m_\phi^{-1} Q^{p/2} \quad \text{(surface)} \ . \quad (3)$$

In this paper, we will focus almost exclusively on models with nonzero volume energy; i.e., we will assume the radius obeys Eq. (2). The dimensionless parameters p, α, and β depend on coupling constants in the theory. We shall only assume that they satisfy the criteria of NTS stability. The stability conditions are

$$M(Q) < m_\phi Q \quad (4)$$

and

$$\frac{d^2 M}{dQ^2} \le 0 \ . \quad (5)$$

Equation (4) ensures stability against decay into free ϕ particles, while Eq. (5) expresses stability against fission into smaller soliton fragments. Equations (1), (4), and (5) imply the upper bound $p \le 1$, and that the minimum stable charge is approximately $Q_{\min} \simeq \alpha^{1/(1-p)}$ (for $p < 1$). The binding energy per unit charge is defined as the energy required to remove a ϕ particle from a soliton:

$$I_Q = m_\phi + M(Q) - M(Q+1) \ . \quad (6)$$

Using Eq. (1), we can express this in terms of the mass of the free ϕ particles as

$$f(Q) \equiv I_Q / m_\phi = 1 - \alpha[(Q+1)^p - Q^p] \simeq 1 - \alpha p Q^{p-1} \ . \quad (7)$$

Note that for p not very close to 1, $f(Q) \simeq 1 - p(Q/Q_{\min})^{p-1}$, so $f(Q) \to 1$ as $Q/Q_{\min}$ grows large.

Two notes are in order about the scaling laws, Eqs. (1) and (2). First, although they are strictly valid only in the limit $Q \gg Q_{\min}$, we will use them as approximate formulas for all Q larger than $Q_{\min}$. Second, these scaling laws break down at very large charge, $Q_{\max} \sim [(\beta/\alpha)(m_{\rm Pl}/m_\phi)^2]^{3/2p}$, when gravity becomes important. Since we are interested in only moderately sized lumps, $Q \ll Q_{\max}$, we can ignore this complication; for discussions of soliton stars, we refer the reader to Ref. 15.

Although the analysis we develop holds for general p, we will often focus on two particular cases, $p=1$ and $\frac{3}{4}$, to demonstrate how the results vary with p. In the literature, many models have p essentially equal to 1, e.g., Q balls, fermion NTS's, interacting scalar NTS's, bag models, and strange matter. In this case, the requirement of stability is just $\alpha < 1$, and the binding energy per charge is independent of Q, i.e., $f(Q) = 1 - \alpha$. (Actually, in these models, due to the surface energy correction, p is slightly smaller than one; this ensures stability against fission.) For example, for strange matter, $f(Q) \simeq 0.05$ and $m_\phi = m_n \simeq 1$ GeV. Configurations with $p = \frac{3}{4}$ arise in a scalar model in which the confined field has little or no self-interaction.[6-8] We shall see that the qualitative evolution of NTS's in the early Universe is sensitive to both p and $Q_{\min}$, as well as the mass scale m_ϕ.

III. PRIMORDIAL FORMATION OF NTS's

One can envision several mechanisms by which nontopological solitons may be formed in the early Universe.

In principle, such objects arise in a symmetry-breaking phase transition if suitably large regions carrying a net charge $Q > Q_{\rm min}$ become trapped in the false-vacuum state. In the simplest scenario, the NTS interior $\sigma = \sigma'$ corresponds to a metastable local minimum of the potential $U(\sigma)$ (dashed curve, Fig. 1), and (i) may or (ii) may not be a phase of restored symmetry. In case (i), the universe at high temperature is initially in the "NTS vacuum" and undergoes a first-order phase transition to the true vacuum at a critical temperature $T_c \sim \sigma_0$. Charge prefers to live in the false vacuum, where the ϕ mass is smallest. Thus, as bubbles of the true-vacuum phase nucleate, grow, and eventually percolate, the charge may become highly concentrated in the shrinking "NTS vacuum" regions. Such a mechanism was proposed by Witten for the formation of strange-matter nuggets in the QCD phase transition.[4]

If the false vacuum is not a phase of restored symmetry, i.e., case (ii) above, then NTS's may form in a second-order phase transition.[7] In this instance, during the transition, a fraction of space evolves to the metastable minimum σ' instead of the true vacuum σ_0. The relative probability of ending up in the NTS phase is given roughly by the Boltzmann formula $p_{\rm false}/p_{\rm true} = \exp(-8\xi_G^3 \Delta f / T_G)$. Here, T_G is the Ginzburg temperature, the temperature at which thermal fluctuations between the true and false vacua freeze out, ξ_G is the correlation length at the Ginzburg temperature, and Δf is the difference in free energy density between the true and false vacua. We may interpret T_G as the temperature at which NTS's are first formed; typically T_G is just below T_c, the critical temperature for the phase transition. The size distribution of the proto-NTS false-vacuum regions is calculable from percolation theory, while the scale size is set by the correlation length ξ_G. Generally, false-vacuum regions with sizes $R \gg \xi_G$ are exponentially suppressed; NTS's with very large charges are correspondingly rare. This holds particularly if the probability $p_{\rm false} \lesssim \frac{1}{3}$, when only finite NTS regions form. On the other hand, if $p_{\rm false} \gtrsim \frac{1}{3}$, both the true vacuum and the NTS phase initially percolate, forming a web of infinite interconnecting segments. In this percolating-NTS case, the infinite false-vacuum regions eventually pinch off into finite blobs when the pressure squeezing them from the true vacuum becomes dynamically important. If the vacuum asymmetry is small, i.e., $\Delta f \ll T_G^4$, pinch-off may occur well after the phase transition is completed; the finite NTS's formed in this way are thus born at a temperature $T_{\rm pinch} \ll T_G$, and have a greater chance of surviving.

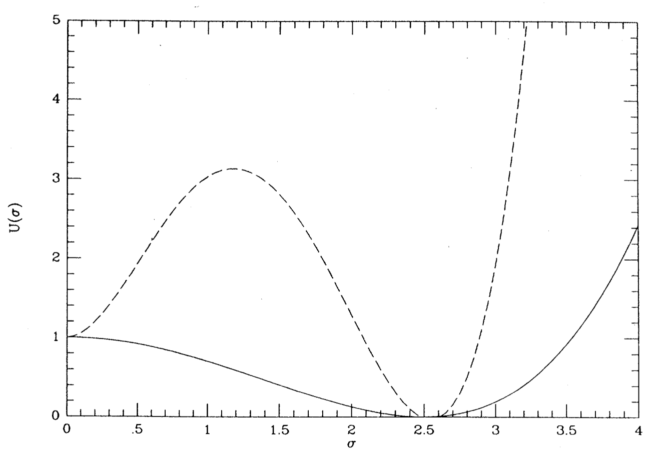

FIG. 1. Schematic σ (confining field) potentials for theories with NTS solutions, with a presumed NTS phase at $\sigma = 0$ and true vacuum $\sigma_0 = 2.5$ (arbitrary units). For the dashed curve, the "NTS vacuum" is metastable in (the absence of charge), while for the solid curve it is unstable.

In a large class of models, the false-vacuum NTS interior is not a metastable minimum of the potential, but is instead an unstable, symmetry-preserving local maximum (solid curve, Fig. 1). In this case, solitons cannot form simply by charge piling up in a metastable phase. To see how formation occurs in this case,[10] consider the structure of an NTS (of fixed charge) as a function of increasing temperature. Because of finite-temperature quantum effects, when the temperature reaches the range near T_G, the NTS radius expands enormously. If we now imagine starting with a dilute gas of solitons at $T \sim 0$ and similarly dialing up the temperature, then at $T \sim T_G$, the gas percolates. Reversing the film, as the temperature drops below T_G, the Universe, initially in the "NTS vacuum," shatters into soliton fragments, which subsequently shrink as they cool. (We note that this process may happen at a modest temperature compared to σ_0: a large charge asymmetry can delay the transition, since charge prefers the NTS vacuum.) The essential idea is that, very close to T_G, the free energy of the NTS state becomes small compared to the state of a gas of free ϕ's. As a result, solitons can be formed at little cost. Thus, at temperatures $T \sim T_G$, we expect a population of NTS's in thermal equilibrium, with an exponential size distribution. (Although the initial NTS geometries are expected to be nonspherical, relaxation due to surface tension occurs rapidly.)

From this discussion, several general features emerge. In the scenarios outlined above, charge concentration into solitons may arise either due to a net cosmological charge asymmetry (like the baryon asymmetry), or due to random fluctuations in a Universe containing zero net charge. We will treat the first case in this paper. In addition, if thermal equilibrium is approximately maintained down to a temperature of order T_G, as above, we expect a quasiexponential size distribution of solitons produced in a phase transition.

Finally, independent of what may happen in a phase transition, NTS's may form at temperatures below T_G by several mechanisms. First, as mentioned in the Introduction, in models with small $Q_{\rm min}$, solitons may form via accretion and fusion after the temperature falls below the binding energy I_Q. Second, some field theory models give rise to both topological and nontopological solitons. For example, if ϕ and σ are complex scalar fields in a theory with the symmetry breaking $U(1)\times U(1)' \to U(1)$, the spectrum contains both cosmic strings and nontopological solitons. For appropriate couplings, the strings carry trapped charged zero modes which, when excited, act as a supercurrent.[16] In this case, NTS's can be spawned in the decay of charged superconducting strings, when the charge condensate separates from the underlying topological vortex.[8,17] As a final speculative possibility, in some models nontopological solitons might form in the gravitational collapse of density perturbations, due to unstable spatial fluctuations in the classical σ field.

Thus, there are a variety of ways in which nontopological solitons may be born in the early Universe. However, in the temperature range $T_G \geq T \geq I_Q$, they are not the lowest free energy states of given charge, and are vulnerable to evaporation. To study this process, we review properties of NTS's in equilibrium at finite temperature in the next section. We shall confine our attention to temperatures at least slightly below the Ginzburg temperature T_G, so that finite-temperature corrections to the *structure* of NTS's (Ref. 10) may be neglected.

IV. NONTOPOLOGICAL SOLITONS IN EQUILIBRIUM

We are interested in the question of whether NTS's formed in a primordial phase transition could have survived to contribute significantly to the energy density of the Universe. First consider NTS's in thermal equilibrium, where a distribution of lumps of all charges Q coexist with free ϕ particles. In kinetic equilibrium, the number density of NTS's of charge Q at temperature T is

$$n_Q(T) = g_Q \left[\frac{M(Q)T}{2\pi} \right]^{3/2} \exp\left[\frac{\mu(Q)-M(Q)}{T} \right] , \quad (8)$$

where g_Q is the internal partition function for a NTS of charge Q, $\mu(Q)$ is the NTS chemical potential, and $M(Q)$, defined in Sec. II, is the NTS mass. In Eq. (8) we have made the reasonable assumption that NTS's are nonrelativistic, $M(Q) \gg T$, and nondegenerate, $\mu(Q) \ll T$. At temperatures $T \ll m_\phi$, the distribution of free ϕ particles obeys an expression analogous to Eq. (8).

Chemical equilibrium between the relative number densities of ϕ's and Q's is established if the accretion and evaporation reactions, $(Q+1) \leftrightarrow (Q) + \phi$, occur rapidly compared to the expansion rate. In this case, the chemical potentials obey

$$\mu(Q+1) = \mu(Q) + \mu(\phi) . \quad (9)$$

Inverting Eq. (8) and substituting in Eq. (9), we obtain the familiar Saha equation

$$\frac{n_{Q+1}}{n_Q n_\phi} = \frac{g_{Q+1}}{g_Q g_\phi} \left[\frac{M(Q+1)}{M(Q)} \right]^{3/2} \left[\frac{2\pi}{m_\phi T} \right]^{3/2} e^{I_Q/T} , \quad (10)$$

where I_Q is the NTS binding energy per charge [Eq. (6)]. Unless the minimum stable NTS charge is very small, $Q_{\min} \simeq 1$, the reactions above are in fact not sufficient to establish chemical equilibrium. If, as an initial condition, the Universe contains only ϕ particles, accretion alone obviously cannot generate solitons. The missing ingredient is the fusion reaction, $\phi + \phi + \cdots + \phi \to Q_{\min}$.

Typically, one expects chemical equilibrium to be maintained until either the fusion or the accretion and/or evaporation rates become slower than the expansion rate. In principle, the proper way to study this freeze-out process is to integrate the (infinite set of) coupled rate equations for the number densities of all charges and free particles, i.e., numerically solve the Boltzmann equations for the system. We do not follow this approach. Instead, in Appendix A, we estimate freeze-out number densities using the "poor cosmologist's" approximation: the freeze-out densities are roughly the equilibrium densities at the epoch when the reaction rates drop below the expansion rate.

Such an approach, however, assumes that chemical equilibrium is initially established and then lost. For most NTS models, this is unjustified. In particular, when $Q_{\min}$ is not very small (as is usually the case) and/or there is a potential-energy barrier between the true and false vacua (the NTS vacuum is a metastable minimum), then, as we show in Appendix A, the fusion rate is strongly suppressed and can never approach equilibrium. (Essentially, this is analogous to what would happen in big-bang nucleosynthesis if the lightest stable element had a large baryon number.) In the limit that fusion can be neglected, the total number of NTS's is a nonincreasing function of time. Since new lumps are not being produced, it makes sense instead to follow the evolution of individual lumps of initial charge Q separately. (We shall also assume that accretion of lumps by lumps is rare.) We develop this approach in the next section.

V. EVOLUTION OF SINGLE LUMPS

To follow the evolution of individual solitons, we need to calculate the emission and absorption rates of massive ϕ particles by NTS's of charge Q. To do this, we apply detailed balance arguments in the usual way.[11] First, we massage the Saha equation into a more convenient form. Since ϕ is a charged field, we assume it has $g_\phi = 2$ degrees of freedom. In addition, up to corrections of order T/m_ϕ, we expect $g_{Q+1}/g_Q \simeq 1$, and we can further approximate $M(Q+1)/M(Q) = 1 + O(p/Q)$. Substituting into Eq. (10), we find

$$\frac{n_{Q+1}}{n_Q n_\phi} = \frac{1}{2} \left[\frac{2\pi}{m_\phi T} \right]^{3/2} \exp(I_Q/T) . \quad (11)$$

In chemical equilibrium, the rate at which NTS's emit ϕ's is equal to the absorption rate. Since the binding energy grows with an increase in charge, absorption is energetically favored. Thus, if the particles are sufficiently strongly interacting, we expect a geometric cross section for absorption:

$$\sigma = 4\pi f_\phi R(Q)^2 = 4\pi f_\phi \left[\frac{\beta}{m_\phi} \right]^2 Q^{2p/3} , \quad (12)$$

where the NTS radius $R(Q)$ is given by Eq. (2). Here, $f_\phi(\leq 1)$ is the absorption efficiency, which depends on coupling constants in the theory; we discuss it in more detail in Appendix B. The accretion rate per unit volume is simply

$$R[(Q)+\phi\to(Q+1)]=n_\phi n_Q \sigma v_\phi \ , \tag{13}$$

where $v_\phi=(T/2\pi m_\phi)^{1/2}$ is the mean velocity of free nonrelativistic ϕ particles. The emission rate from a lump of charge $Q+1$ is proportional to the number density of lumps times the evaporation rate $r_{\rm evap}$:

$$R[(Q+1)\to(Q)+\phi]=n_{Q+1}r_{\rm evap} \ . \tag{14}$$

The two rates must be equal in chemical equilibrium; using Eqs. (11)–(14), the evaporation rate from a NTS is

$$r_{\rm evap}=\frac{2}{\pi}f_\phi \beta^2 \frac{T^2}{m_\phi}Q^{2p/3}\exp(-I_Q/T) \ . \tag{15}$$

Although the evaporation rate was derived from the assumption of chemical equilibrium, Eq. (15) applies more generally; in particular, it is valid whenever the ϕ particles are kept in kinetic nondegenerate equilibrium [Eq. (8)], e.g., by collisions with the ambient plasma. (The power of detailed balance is that it applies to situations outside of *chemical* equilibrium.) Thus, the accretion and evaporation rate from a lump of charge Q is

$$\frac{dQ}{dt}=r_{\rm abs}-r_{\rm evap}=\frac{2}{\pi}f_\phi\beta^2\frac{T^2}{m_\phi}Q^{2p/3}\left[\sqrt{2}\frac{n_\phi}{T^3}\left[\frac{\pi T}{m_\phi}\right]^{3/2}-\exp(-I_Q/T)\right] . \tag{16}$$

Equation (16) describes the evolution of an individual soliton in a thermal bath of ϕ particles. Before applying it, we must expose two additional hidden assumptions.[11] (i) As it evaporates or accretes, an NTS is cooled or heated with respect to the environment; we assume, however, that it is kept in good thermal contact with the surrounding medium. (ii) We assume the exterior ϕ gas is dilute, so that the flow of particles into or out of the lump is not inhibited.

For the remainder of this paper, we suppose there is a primordial asymmetry of ϕ's over anti-ϕ's, $\eta_\phi\equiv(n_\phi-n_{\bar\phi})/n_\gamma$, where $n_\gamma=2.4T^3/\pi^2$ is the photon number density at temperature T. We shall further assume that ϕ and $\bar\phi$ are stable, but that they can annihilate into a pair of *massless* particles, say, neutrinos or photons. (These assumptions will be relaxed in our forthcoming paper.) The latter assumption guarantees that annihilation takes place inside as well as outside solitons. For stable ϕ's the Q asymmetry can be written as

$$\eta_\phi=2.5\times10^{-8}\Omega_\phi h^2 \ {\rm GeV}/m_\phi \ , \tag{17}$$

where h is the Hubble constant in units of 100 km/sec Mpc, and Ω_ϕ is the ϕ energy density in units of the critical density. Requiring that ϕ particles and charges do not overclose the Universe, $\Omega_\phi h^2\le 1$, yields an upper bound on η_ϕ as a function of m_ϕ. We define j as the fraction of ϕ particles in the NTS phase, so the number density of *free* ϕ particles is $n_\phi(T)=\eta_\phi(1-j)n_\gamma$. The initial value of j depends on the efficiency with which NTS's are formed; e.g., in case (ii) of Sec. III, we expect $j_i\simeq p_{\rm false}\le 0.5$, while for case (i), Witten envisioned $j_i\sim 0.9$. In general, however, j is a temperature-dependent function which increases (decreases) during the accretion (evaporation) epoch. With these definitions, the expression for the evolution of a lump becomes

$$\frac{dQ}{dt}=\frac{2}{\pi}f_\phi\beta^2\frac{T^2}{m_\phi}Q^{2p/3}\left[1.92\eta_\phi(1-j)\left[\frac{T}{m_\phi}\right]^{3/2}-e^{-f(Q)m_\phi/T}\right] , \tag{18}$$

where $f(Q)$ is the binding energy per ϕ mass [Eq. (7)].

The evolution of a nontopological soliton is determined by three temperature scales. The evolution begins at the formation temperature T_i, which is of order the Ginzburg temperature T_G or smaller, depending on the formation mechanism. Once formed, the competition between the two terms in the brackets of Eq. (18) determines the fate of the soliton. These terms are equal at the "turnaround" temperature T_t, which marks the transition between evaporation and accretion phases. Finally, the freeze-out temperature T_F is defined as the temperature at which both the emission and absorption rates fall below the expansion rate, $(|\dot Q|/QH)_{T_F}=1$. Consider a model with $T_i\sim m_\phi$. From Eqs. (17) and (18), at $T\sim m_\phi$ evaporation dominates over accretion due to the small asymmetry term. Thus, only those charges which are large enough to survive down to T_t or T_F are present today. There are two exceptions to this general picture. (1) If ϕ is an unstable particle with a lifetime shorter than the age of the Universe, the asymmetry is not constrained by Eq. (17), and may be large. (2) If ϕ is stable, but only annihilates into particles more massive than $f(Q)m_\phi$, then annihilation inside NTS's is suppressed at low temperature. In this case, NTS's can grow by accreting *both* ϕ's and $\bar\phi$'s, and the absorption rate is independent of η_ϕ. We discuss both of these scenarios in our forthcoming work.

To study lump evolution, it is convenient to introduce the dimensionless temperature variable

$$x\equiv m_\phi/T \ . \tag{19}$$

During the radiation-dominated early Universe, $t=0.3g_*^{-1/2}(m_{\rm Pl}/m_\phi^2)x^2$ (where g_* is the effective number of relativistic degrees of freedom), so the evolution equation can be rewritten

$$\frac{dQ}{dx}=\frac{1.2}{\pi g_*^{1/2}}f_\phi\beta^2\left[\frac{m_{\rm Pl}}{m_\phi}\right]x^{-1}Q^{2p/3}\times[1.92\eta_\phi(1-j)x^{-3/2}-e^{-f(Q)x}] \ . \tag{20}$$

From the discussion above, there are several possible fates that may befall a NTS with stable ϕ's: (a) death by evaporation; (b) evaporation followed by freeze-out, $x_i<x_F<x_t$; (c) evaporation followed by accretion and then freeze-out, $x_i<x_t<x_F$; (d) accretion followed by freeze-out, $x_t<x_i<x_F$; and (e) frozen at birth, $x_i>x_F$. Which of these fates we obtain depends on the mass scale, the NTS charge, the asymmetry parameter, and the formation temperature.

The turnaround temperature T_t, at which accretion takes over from evaporation, is given implicitly by

$$\ln 1.92\eta_\phi(1-j)=1.5\ln x_t - f(Q)x_t \ . \tag{21}$$

Equation (21) may be derived either by equating the accretion and evaporation terms in Eq. (20) or by setting $n_Q = n_{Q+1}$ in Eq. (11). In Fig. 2 we show $x_t = m_\phi/T_t$ as a function of $\eta_\phi(1-j)$, for several values of $f(Q)$. Two trends are obvious. For fixed binding energy $f(Q)$, turnaround occurs "later," i.e., at larger x_t, as η_ϕ decreases, since the supply of free ϕ particles is dropping. Thus, from Eq. (17), for a large mass scale m_ϕ, the evaporation phase is relatively long in units of the mass scale. Second, NTS's with lower binding energy per unit mass turn around later, as expected. For models with $p<1$, $f(Q)\to 1$ as $Q\to\infty$, so lumps with large charge spend less time in the evaporation phase than their smaller brethren. (The structure in Fig. 2 at very large asymmetry, $\eta>1$, is only relevant if ϕ is unstable, so we postpone discussion of it to our forthcoming paper.)

Now consider the freeze-out temperature T_F. During the radiation-dominated epoch, the expansion rate is

$$H=1.67g_*^{1/2}T^2/m_{\rm Pl}=1.67g_*^{1/2}(m_\phi^2/m_{\rm Pl})x^{-2} \ .$$

To obtain an analytic estimate of x_F, we consider case (b) separately from (c) and (d). For case (b), freeze-out occurs during the evaporation phase, when the absorption term can be neglected; using Eq. (20), we find

$$x_F=\frac{1}{f(Q)}\left[\ln\left[\frac{f_\phi\beta^2}{2.62g_*^{1/2}}\right]+43.9-\ln\left[\frac{m_\phi}{\rm GeV}\right] - \left[1-\frac{2p}{3}\right]\ln Q\right] \quad (x_F<x_t) \ . \tag{22}$$

Note that as the mass scale m_ϕ or the charge Q increases, freeze-out occurs earlier, i.e., at smaller x. Also, for large relative binding energy $f(Q)$, NTS's freeze out at early times since more strongly bound objects evaporate less efficiently.

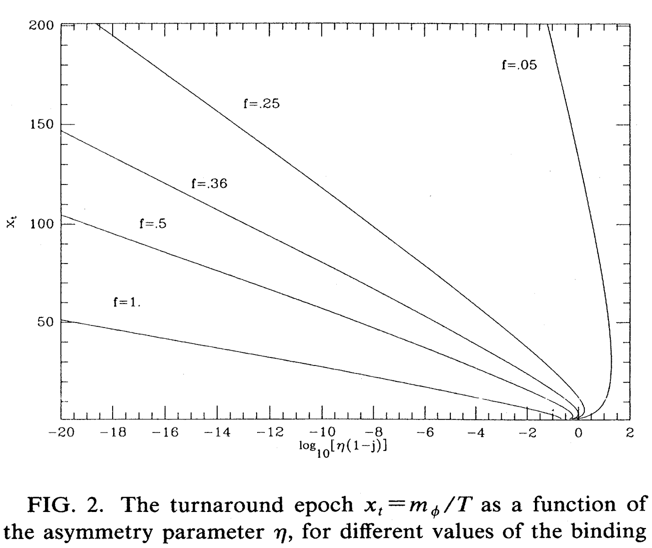

FIG. 2. The turnaround epoch $x_t = m_\phi/T$ as a function of the asymmetry parameter η, for different values of the binding energy per unit mass, $f(Q)=0.05$, 0.25, 0.36, 0.5, and 1.

For fixed mass scale, the freeze-out epoch x_F grows as Q drops (unless $f_\phi \ll 1$, see Appendix B). Therefore, for small enough charge freeze-out will occur *after* turnaround. We denote the charge at the transition point, $x_F = x_t$, by $Q_{\rm eq}$; for $Q<Q_{\rm eq}$, we have $x_F > x_t$, i.e., case (c) or (d) above. From Eqs. (17), (21), and (22),

$$Q_{\rm eq}=(2.2\times10^{11})^{3/(3-2p)}\left[\frac{m_\phi}{\rm GeV}\right]^{-6/(3-2p)} \times\left[\frac{f_\phi\beta^2(1-j)\Omega_\phi h^2}{x_t^3/g_*^{1/2}}\right]^{3/(3-2p)} \ . \tag{23}$$

For $p=\frac{3}{4}$ and 1, $Q_{\rm eq}$ scales as m_ϕ^{-4} and m_ϕ^{-6}, respectively. The dominant dependence in Eq. (23) resides in the first two terms. Thus, approximately independently of p, for $m_\phi \gtrsim 10^4$ GeV, we have $Q_{\rm eq}<1$. Since $Q_{\rm eq}$ is the largest charge which can have an accretion phase, this mass scale marks an important boundary.

Given that NTS's are generally born in the evaporation phase [except for case (d)], we would like to know how large the initial charge must be in order to survive to the accretion or freeze-out epoch. Approximately integrating Eq. (20) during the evaporation phase, the smallest charge which survives is

$$Q_s(x_i)\simeq(4.6\times10^{18})^{3/(3-2p)}\left[\frac{m_\phi}{\rm GeV}\right]^{-3/(3-2p)} \times\left[\frac{(1-2p/3)f_\phi\beta^2 e^{-f(Q)x_i}}{[1+f(Q)x_i]g_*^{1/2}}\right]^{3/(3-2p)} , \tag{24}$$

where we have assumed $x_F, x_t \gg x_i$ and have used a standard approximation for the exponential integral. In arriving at Eq. (24), we have also made use of the fact that, for $Q\gg1$, $f(Q)$ is approximately independent of Q. For models with $p=\frac{3}{4}$, the surviving charge scales as m_ϕ^{-2}, while for solitons with $p=1$, Q_s scales with m_ϕ^{-3}.

From Fig. 2, we see typically $x_t \sim 10-100$. Thus, comparing Eqs. (23) and (24), we find $Q_s(x_i)>Q_{\rm eq}$ if $f(Q)x_i<17+\ln(m_\phi/{\rm GeV})$. That is, unless NTS's form at a temperature well below $f(Q)m_\phi$, cases (c) and (d) are eliminated: all those that *would* accrete before freeze-out in fact do not survive. If NTS's are created in a phase transition, we generally expect them to form at a temperature within an order of magnitude of their mass scale, $0.1<x_i<10$, for which we find $Q_s>Q_{\rm eq}$. This can be circumvented, on the other hand, if NTS's form well after the phase transition is completed, for example, by pinch-off in the asymmetric vacuum model, or by the decay of cosmic strings. In these cases, $x_i\gg1$, $Q_s(x_i)$ can be substantially reduced, and an accretion phase is possible. For example, with $p=\frac{3}{4}$ and $f(Q)=1$ we obtain, for $x_i=1$ and $x_i=10$, respectively, $Q_s(1)\sim10^{33}({\rm GeV}/m_\phi)^2$ and $Q_s(10)\sim10^{24}({\rm GeV}/m_\phi)^2$.

For NTS's formed at relative small values of x_i, then, the only hope of survival is to start with sufficiently large charge, $Q>Q_s$. However, this runs into two potential difficulties. First, for most of the formation mechanisms discussed in Sec. III, large charges are exponentially suppressed and thus may be rather rare. As a result, their contribution to the present energy density of the Universe is likely to be small. Second, it is difficult to imagine forming lumps with a charge larger than that contained in the particle horizon at the time of formation. The total charge inside the horizon is

$$Q_H(x)=1.2\times10^{48}\frac{\Omega_\phi h^2 x^3}{g_*^{3/2}}\left[\frac{\text{GeV}}{m_\phi}\right]^4 . \qquad (25)$$

For example, for $p=1$, with $f(Q)\simeq x_i\simeq 1$, we have $Q_s(1)>Q_H(1)$ for $m_\phi>10^{-4}$ GeV; that is, solitons formed at early epochs ($x_i\sim 1$) in these models are never large enough to survive ($Q>Q_s$) if their formation is limited by causality. Only if they form at a sufficiently late epoch, since Q_s depends exponentially on $f(Q)x_i$, can this problem be overcome. For models with $p=\frac{3}{4}$, on the other hand, at $x_i=1$, we find $Q_s(1)<Q_H(1)$ if $m_\phi<10^6$ GeV. In this case, models with sufficiently small mass scale can have surviving NTS's which form early. Again, at later formation epochs, the situation can change substantially; e.g., for $x_i=10$ and $p=\frac{3}{4}$, we find $Q_s(10)<Q_H(10)$ for $m_\phi<10^{12}$ GeV.

In Fig. 3 we summarize our results for the case in which there is a primordial charge asymmetry. Here we have chosen $f_\phi\beta^2=1$, $g_*=100$, $\Omega_\phi h^2=1$, $f(Q)=1$ and the two cases $p=\frac{3}{4}$ and 1. As a function of mass scale, we compare the charge inside the horizon at formation, $Q_H(x_i)$, with the smallest charge that survives evaporation, $Q_s(x_i)$, for $x_i=1$ and 10, obtained by numerically integrating the rate equation, Eq. (20). For models with $Q_s(x_i)>Q_H(x_i)$, even horizon-sized solitons disappear by evaporation. For $f(Q)x_i=10$, both models have a window of survival if $m_\phi<10^{12}$ GeV. We also show the charge $Q_{\rm eq}$ at which freeze-out and turnaround coincide, $x_F=x_t$. Recall that if $Q>Q_{\rm eq}$, the soliton will freeze out before turnaround; in the opposite case, there may be accretion before freeze-out. However, for all models and mass scales, Fig. 3 shows that $Q_{\rm eq}\ll Q_s$ so that these charges die before they have a chance to grow. In Fig. 4, we show results of the numerical evolution of $Q(x)$ for a sample model with $p=\frac{3}{4}$ and $m_\phi=10^4$ GeV, in the neighborhood of $Q_s(1)\simeq10^{26}$. For a very narrow range of Q_i, the value of the charge at freeze-out depends sensitively on the initial charge. Outside this range, the final charge is either zero (for $Q<Q_s$) or $\simeq Q_i$ (for $Q_i>Q_s$).

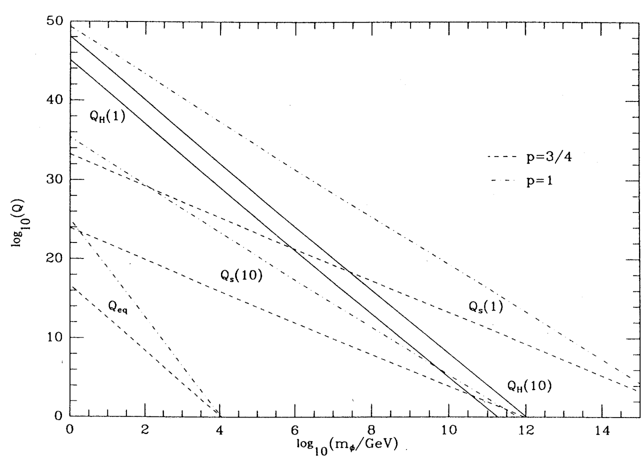

FIG. 3. The minimum charge which survives evaporation, $Q_s(x_i)$, is shown as a function of the mass scale m_ϕ for the two cases $p=0.75$, 1, and for $x_i=1$ and 10. Also shown are the charge inside the horizon, $Q_H(x_i)$ and the charge below which an accretion phase is possible before freeze-out, $Q_{\rm eq}$.

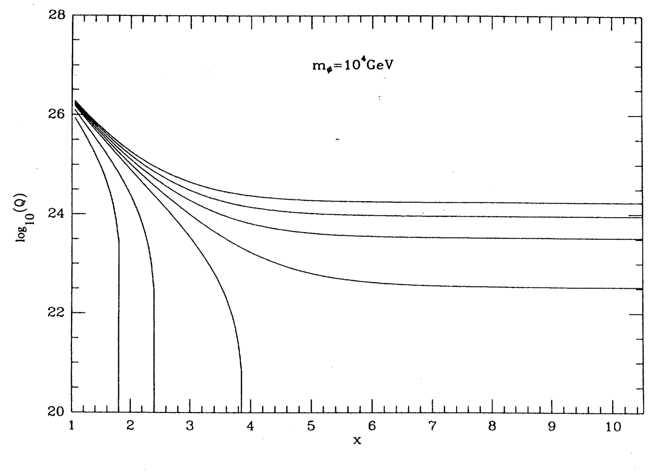

FIG. 4. A sample numerical integration of $Q(x)$, for the parameters $f(Q)=1$, $m_\phi=10^4$ GeV and $p=\frac{3}{4}$. In this example, $Q_s\simeq10^{26}$.

As noted above, for $m_\phi\le10^4$ GeV, it is possible to have $x_F>x_t$, so that an accretion phase is possible before freeze-out occurs. Thus, even if all solitons formed at $x_i\sim1$ evaporate away, a later period of accretion starting from individual charges can build up new solitons before freeze-out occurs.[13] As noted in Sec. IV, this is only feasible for very small values of the minimum charge, $Q_{\min}\sim1$, so that free ϕ particles can act as seeds for new lumps. We return to this possibility in Appendix A.

VI. NTS BOILING

Although NTS's may be formed at a phase transition at T_c, they are not the lowest free energy phase until the Universe cools below T_t. In the last section we studied the struggle for survival of NTS's against surface evaporation. In this section we discuss the boiling of NTS's, i.e., bubble nucleation of the lowest free energy phase (massive ϕ particles in the true vacuum) throughout the volume of a NTS.

Imagine heating up a NTS from zero temperature. Surface evaporation starts at temperatures just above T_t (or T_F, if $T_t<T_F$). Bubble formation is negligible for temperatures just above T_t, due to the energy cost associated with the bubble surface. However, at higher temperatures ($T\sim T_G$), bubble nucleation throughout the volume may dominate the evaporation process. The probability of bubble nucleation is very sensitive to the

surface tension of the bubble wall, σ_b, which is determined by the mass scale and coupling constants. NTS's can survive the boiling phase if σ_b is greater than a critical value derived below.

Following Ref. 12, we calculate the thermodynamic work expended to create a bubble of free ϕ's inside a NTS:

$$W=-\frac{4\pi}{3}r^3(P_b-P_{\rm NTS})+4\pi\sigma_b r^2 , \tag{26}$$

where r is the radius of the bubble, P_b is the pressure inside the bubble, and $P_{\rm NTS}$ is the pressure inside the NTS. W is maximized at the critical radius $r_c=2\sigma_b/(P_b-P_{\rm NTS})$. Bubbles with radius $r<r_c$ shrink away while those with $r \geq r_c$ grow. The nucleation rate Γ is determined by the abundance of critical size bubbles:

$$\Gamma \sim T^4 e^{-W_c/T} , \tag{27}$$

where

$$W_c=\frac{16\pi}{3}\frac{\sigma_b^3}{(P_b-P_{\rm NTS})^2} . \tag{28}$$

The soliton is assumed to be in pressure equilibrium with its surroundings. Therefore, $P_{\rm NTS}$ is just equal to the exterior thermal radiation pressure. The pressure inside the bubble, P_b, has the same thermal component, plus the pressure due to the massive ϕ gas, P_ϕ. Hence,

$$P_b-P_{\rm NTS}=P_\phi=(2/\pi^3)^{1/2}m_\phi^{3/2}T^{5/2}\exp(-I_Q/T) \tag{29}$$

and

$$\frac{W_c}{T}=\frac{8\pi^4}{3}\frac{\sigma_b^3}{m_\phi^3 T^6}e^{2I_Q/T} . \tag{30}$$

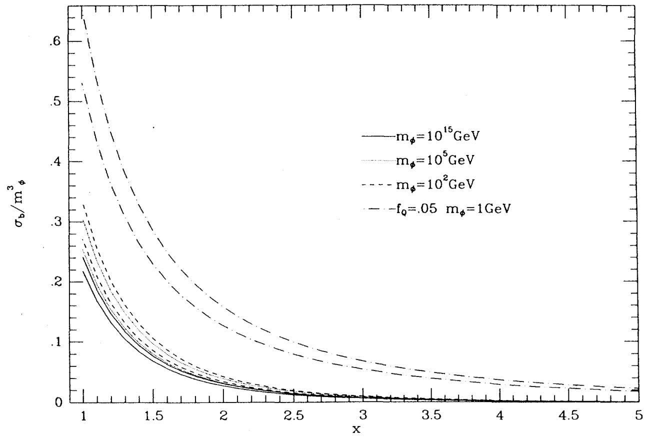

FIG. 5. The surface tension $\sigma_{\rm min}$ and $\sigma_{\rm max}$ as a function of x, for mass scales $m_\phi=10^2$, 10^5, 10^{15} GeV, for $f(Q)=1$. Also shown is the strange-matter case, $f(Q)=0.05$, $m_\phi=m_n \simeq 1$ GeV. Boiling dominates evaporation for $\sigma_b<\sigma_{\rm max}$.

At temperature T, the characteristic number density of bubbles, n_b, is of order Γt, where t is the expansion time scale, so $n_b \simeq 0.3 g_*^{-1/2} m_{\rm PL} T^2 \exp(-W_c/T)$. If $n_b \gg n_{\rm NTS}$, the charge density inside the soliton, the NTS phase is so far out of thermodynamic equilibrium that the probability of remaining in such a phase is negligible. (The NTS would likely not have been formed, let alone survived.) The condition $n_b<n_{\rm NTS}$ translates into a constraint on the surface tension $\sigma_b>\sigma_{\rm min}$, where

$$\sigma_{\rm min}(x,Q,m_\phi)=0.55m_\phi^3\frac{\exp[-2f(Q)x/3]}{x^2}\left[1-0.023\ln\left[\frac{m_\phi}{1\ {\rm GeV}}Q^{1-p}x^2 g_*^{1/2}\beta^{-3}\right]\right]^{1/3} . \tag{31}$$

Here we have assumed a spherical NTS lump of charge Q and radius R_Q given by Eq. (2).

On the other hand, if σ_b is too large, boiling may never occur. Boiling will only dominate surface evaporation if the surface area of the NTS lump is smaller than the surface area of all the bubbles inside the lump. Boiling is more effective than surface evaporation if $\sigma_b<\sigma_{\rm max}$, where $\sigma_{\rm max}$ is given implicitly by

$$\sigma_{\rm max}(x,Q,m_\phi)=0.57m_\phi^3\frac{\exp[-2f(Q)x/3]}{x^2} \times\left[1+0.021\left\{2f(Q)x+\ln\left[x^3 g_*^{-1/2}\beta Q^{p/3}\left[\frac{1\ {\rm GeV}}{m_\phi}\right]\left[\frac{\sigma_{\rm max}}{m_\phi^3}\right]^3\right]\right\}\right]^{1/3} . \tag{32}$$

Both $\sigma_{\rm min}$ and $\sigma_{\rm max}$ depend on the specific NTS model. In Fig. 5, we plot $\sigma_{\rm min}(x)$ (lower curve) and $\sigma_{\rm max}(x)$ (upper curve) for different values of m_ϕ, where we set $Q=Q_H(m_\phi, x=1)$ [Eq. (25)]; we have chosen $f(Q)=1$, except for the case $m_\phi=1$ GeV, where we set $f(Q)=0.05$, characteristic of strange matter. We have also set $p=\beta=1$, since the dependence on β (~ 1) is only logarithmic, and the difference between $p=1$ and $p=0.75$ is unnoticeable. For $\sigma_b<\sigma_{\rm min}$, there is copious boiling and NTS's disintegrate. Between the two curves, $\sigma_{\rm min}<\sigma_b<\sigma_{\rm max}$, boiling dominates over surface evaporation, and for $\sigma_b>\sigma_{\rm max}$ boiling can be neglected.

From Fig. 5, we see that, for models with $f(Q)\simeq 1$, boiling is important if $x_i \sim 1$. In these models, the surface tension σ_b must be larger than about $0.3m_\phi^3$ to avoid boiling. Alternatively, for smaller σ_b, NTS's can survive if they form later. More generally, the efficiency of boiling is larger for $f(Q)x_i \lesssim 1$. For example, for strange matter, $f(Q) \simeq 0.05$, $x_i \simeq 10$, and strange lumps can only survive if[12] $\sigma_b \geq 0.006 m_n^3$ (much larger than the theoretically expected value[4]).

VII. CONCLUSION

We have discussed the cosmic evolution of nontopological solitons, developing a general analysis which applies to virtually all classes of models, and thus separating general features from model-specific results. We have found that, for a wide range of parameters and models, primordially formed NTS's of moderate charge disappear quickly, either via surface evaporation or bubble nucleation, leaving behind a distribution of free massive particles. For a range of mass scales, depending on the scaling parameter p, there is a window of survival between the smallest charges which survive evaporation, Q_s, and the largest that can be causally formed, Q_H. This window, in turn, depends on the epoch of NTS formation; in scenarios with delayed NTS formation, the chances of survival are enhanced, and, if $m_\phi \lesssim 10^4$ GeV, sufficiently small charges may then undergo an accretion phase. Our results pertain to models constrained to have small charge asymmetry by the existence of stable ϕ particles. If this constraint is relaxed, the results can change considerably.

ACKNOWLEDGMENTS

This work originated at the Aspen Center for Physics 1988 summer workshop, and much of it was completed while two of us (J.F. and A.O.) were visitors at the Institute for Theoretical Physics, Santa Barbara. We thank both institutions for their hospitality. This research was supported in part by NASA Grant No. NAGW-1340 and the U.S. Department of Energy (DOE) at Fermilab, by NSF Grant No. PHY-82-17853, the DOE and NASA at the University of California at Santa Barbara, and by DOE Contract No. W-7405-ENG-48 at Livermore.

APPENDIX A

In this appendix, we consider a system of interacting ϕ particles and solitons which reaches chemical equilibrium for a time in the early Universe. Recall that this might occur in two ways. (1) If $Q_{\min} \sim 1$, chemical equilibrium can be established by the accretion and evaporation reactions $(Q+1) \leftrightarrow (Q) + \phi$. (This possibility was recently studied by Griest and Kolb[13] in the context of a particular toy NTS model with $p = \frac{3}{4}$.) (2) In most NTS models, unless the coupling constants are specially chosen, $Q_{\min} \gg 1$. In this case, the fusion and dissociation reactions $\phi + \phi + \cdots + \phi \leftrightarrow (Q)$ are also required to establish complete chemical equilibrium. However, at the temperatures at which fusion can be effective in generating new solitons, $T < T_t$, these reactions are likely to be strongly suppressed compared to the expansion rate, for reasons given at the end of this section.

In both cases, in chemical equilibrium the chemical potentials are related by

$$\mu(Q) = Q\mu(\phi) , \tag{A1}$$

where $\mu(Q)$, $\mu(\phi)$ are given implicitly by Eq. (8). We define the mass defect by

$$B_Q = Qm_\phi - M(Q) = Qm_\phi(1 - \alpha Q^{p-1}) \equiv Qm_\phi \tilde{f}(Q) , \tag{A2}$$

and note that $\tilde{f}(Q) \to f(Q)$ [Eq. (7)] as $p \to 1$ or as $Q \to \infty$, independent of p. Then, from Eqs. (1), (8), (A1), and (A2), the number density of solitons of charge Q in chemical equilibrium can be written

$$n_Q(T) = \frac{g_Q}{g_\phi^Q} n_\phi^Q (\alpha Q^p)^{3/2} \left[\frac{2\pi}{m_\phi T} \right]^{3(Q-1)/2} e^{B_Q/T} . \tag{A3}$$

It is convenient to introduce the *charge fraction* of solitons of charge Q, $Y_Q \equiv n_Q Q/N$, where the total charge density is given by

$$N = n_\phi + \sum_{Q_{\min}}^{\infty} Q n_Q = \eta_\phi n_\gamma . \tag{A4}$$

Note that, in general, the charge fraction is different from the mass fraction. From Eqs. (A3) and (A4), we have

$$Y_Q = \frac{g_Q}{g_\phi^Q} \alpha^{3/2} Q^{1+(3p/2)} \times \left[\frac{2.4\eta_\phi}{\pi^{1/2}} \left[\frac{2T}{m_\phi} \right]^{3/2} \right]^{Q-1} (1-j)^Q e^{B_Q/T} \tag{A5}$$

and the constraint

$$\sum_{Q_{\min}} Y_Q = j . \tag{A6}$$

The charge fraction is obtained by simultaneously solving Eqs. (A5) and (A6).

The qualitative behavior of the NTS population is relatively simple to understand. At high temperature, the relative abundance of NTS's with large Q is suppressed by the small asymmetry factor $\sim \eta_\phi^{Q-1}$. At lower temperature, the exponential and $Q^{1+(3p/2)}$ terms favor the formation of large Q objects. The trend from small to large Q shifts at the "turnaround" temperature defined in Sec. V: in the Saha equation, we find $n_Q = n_{Q+1}$ at $T = T_t$. Since $f(Q)$ is approximately constant for large Q, x_t is roughly Q independent. Thus the turnaround is rapid: slightly above T_t, large charges are strongly suppressed, while slightly below it, they are strongly favored. As the temperature drops through T_t, large charges quickly build up until the available "fuel" of free particles is depleted, thereby shutting the system out of chemical equilibrium.

At high temperature, $x < x_t$, Y_Q falls rapidly to zero with increasing Q. Consequently, in this regime, only the first few terms contribute significantly to the sum in Eq. (A6), and the equilibrium charge fraction may be easily calculated. Thus, if $x_F < x_t$, i.e., if freeze-out occurs before turnaround, the final abundance $Y_Q[x_F(Q)]$ can be estimated analytically. As discussed in Sec. V, this is always the case for $m_\phi > 10^4$ GeV. At $x < x_t$, we expect the total NTS charge fraction to be small, $j \ll 1$. To zeroth order of approximation, we can then estimate Y_Q by setting $j=0$ in Eq. (A5).

Alternatively, we can obtain a more transparent expression for $Y_Q(x_F)$ without approximation. Setting $g_\phi=2$ as in most cases of interest, the freeze-out abundance for $x_F < x_t$ may be written

$$Y_Q(x_F)=10^7(\alpha x_F)^{3/2}g_Q\left[\frac{m_\phi}{\text{GeV}}\right](\Omega_\phi h^2)^{-1}Q^{1+(3p/2)}(4.8\times10^{-8})^Q \times\left[\frac{1-j}{x_F^{3/2}}\left[\frac{4.4\times10^{18}f_\phi\beta^2Q^{(2p/3)-1}}{g_*^{1/2}}\right]^{\tilde{f}(Q)/f(Q)}\left[\frac{\text{GeV}}{m_\phi}\right]^{1+[\tilde{f}(Q)/f(Q)]}\Omega_\phi h^2\right]^Q , \quad \text{(A7)}$$

where $x_F(Q)$ is given by Eq. (22), $f(Q)$ by Eq. (7), and $\tilde{f}(Q)$ by Eq. (A2). If we approximate $\tilde{f}(Q)=f(Q)$, valid at large Q, this may be rewritten using Eq. (23):

$$Y_Q(x_F)=10^7\frac{(\alpha x_F)^{3/2}g_Q}{\Omega_\phi h^2}\left[\frac{m_\phi}{\text{GeV}}\right]Q^{1+(3p/2)} \times\left[\left[\frac{Q_{\text{eq}}}{Q}\right]^{1-(2p/3)}\left[\frac{x_t}{x_F}\right]^{3/2}\right]^Q \quad \text{(A8)}$$

This gives the freeze-out abundance for $Q \gg Q_{\text{eq}}$ and thus $x_F < x_t$. The case $x_F \sim x_t$ must in general be handled numerically.

We finish this section by discussing the fusion of free ϕ's into a soliton for models in which $Q_{\min}$ is not small. This rate must be faster than the expression rate in order to maintain chemical equilibrium. We shall show that this is unlikely at temperatures below m_ϕ. We estimate the rate by assuming that a soliton is formed when $Q \geq Q_{\min}$ ϕ particles get together in a fluctuation with a charge density comparable to the NTS's charge density. This criterion can be made rigorous by considering the effective σ potential $U(\sigma,q)$ in a background of nonzero charge density q. At a charge density $q \sim q_{\text{NTS}}$, the unconfined phase $\sigma=\sigma_0$ becomes degenerate with the NTS phase and at higher densities the NTS phase is preferred. If the vacuum potential $U(\sigma,0)$ has no barrier (see Fig. 1), then a soliton forms whenever a sufficiently dense charge fluctuation occurs. However, if there is a barrier between the NTS state and the unconfined phase (dashed curve in Fig. 1), a charge fluctuation is not sufficient to form an NTS: in addition, the σ field must tunnel through the barrier, leading to an extra suppression of the fusion rate. We will ignore this factor, so the estimate below will yield an upper bound on the formation rate.

For simplicity we focus on the case $p=1$; the results may be easily generalized to other models. Consider a fluctuation region with fixed volume $V_{\min}=(4\pi/3)\beta^3 m_\phi^{-3}Q_{\min}$, the size of a NTS with charge $Q_{\min}$, containing an average charge $\bar{Q}=\bar{q}V_{\min}$, where the mean charge density $\bar{q}\simeq(2.4\eta_\phi/\pi^2)T^3$. The probability of finding a charge Q in a region with average charge $\bar{Q}$ is of order

$$P(Q,\bar{Q})\simeq(2\pi\bar{Q})^{-1/2}\exp\left[-\frac{(Q-\bar{Q})^2}{2\bar{Q}}\right]dQ . \quad \text{(A9)}$$

Here we have assumed $\bar{Q}$ is of order a few or more, so that the Gaussian approximation to the Poisson distribution is reasonably accurate, and have ignored antiparticles. We are interested in a fluctuation with charge $Q > Q_{\min}$. At the temperatures of interest, $T < m_\phi$, we can safely assume $\eta_\phi \ll (m_\phi/\beta T)^3$, so that $Q_{\min} \gg \bar{Q}$. Thus, the probability of a fluctuation with charge large enough to make a soliton is

$$P(Q > Q_{\min})\simeq\left[\frac{\bar{Q}}{2\pi}\right]^{1/2}Q_{\min}^{-1}e^{-Q_{\min}^2/2\bar{Q}} . \quad \text{(A10)}$$

The number density of minimum charge NTS's is $n_{\text{sol}}\simeq P(Q > Q_{\min})/V_{\min}$, and their abundance relative to free ϕ particles is approximately

$$\frac{n_{\text{sol}}}{n_\phi}\sim\left[\frac{x^3}{\beta^3 Q_{\min}^3\eta_\phi}\right]^{1/2}\exp\left[-\frac{Q_{\min}}{2}\left[\frac{x^3}{\beta^3\eta_\phi}\right]\right] . \quad \text{(A11)}$$

Thus, if $Q_{\min}$ is large and/or the asymmetry is small, the formation of solitons from thermal fluctuations is strongly suppressed at temperatures below m_ϕ ($x > 1$).

APPENDIX B

The factor f_ϕ measures the probability of absorption into a NTS of an incoming particle ϕ, relative to its geometric cross section. Since the mass term for ϕ generally drops abruptly (and monotonically) as the NTS surface is approached from the exterior, an incoming ϕ particle effective sees a square-well potential inside the NTS. Since there is no potential barrier, ϕ particles have no difficulty entering the NTS, but they can as easily escape at the other side. The incoming ϕ's (massive outside the NTS) have energies higher than the binding energy of the massless ϕ gas inside the NTS. Therefore, f_ϕ is determined by the probability of an incoming ϕ being trapped by scattering down to lower energies.

The probability of at least one collision inside a soliton of radius R_Q is given by $p_c=1-\exp(-R_Q/\lambda_\phi)$, where λ_ϕ is the mean free path of ϕ's inside the NTS. Thus, $\lambda_\phi=(n_s\sigma_\phi)^{-1}$, where n_s is the number density of scatterers and σ_ϕ is the cross section for ϕ scattering. If

$R_Q \gg \lambda_\phi$, the incident ϕ will have enough scatterings to be trapped, and $f_\phi = p_c \simeq 1$. On the other hand, if $R_Q < \lambda_\phi$, then $p_c \simeq R_Q/\lambda_\phi \ll 1$. In this case, there will typically be at most one scattering, and f_ϕ is given by the probability of scattering *and* losing enough energy in one collision to be trapped.

If a ϕ particle has initial kinetic energy E outside the NTS, it will have energy $E_i = E + m_\phi$ inside the NTS. (Here, we are assuming ϕ is massless inside an NTS.) The particle is captured if it scatters down to a final energy inside the NTS, $E_f < m_\phi$. In the case of multiple scattering, capture generally occurs, since the incident energy $E \sim T \lesssim m_\phi$. In the single-scattering regime $\lambda_\phi \gtrsim R_Q$ the capture probability is model dependent. If we consider isotropic cross sections, on average half of the incident energy is lost per collision. If the target particles responsible for capture are not ϕ particles, the condition for trapping in one scattering is roughly $E < m_\phi$. However, if the dominant scatterers are ϕ's the incident energy must be lower; otherwise, the initially bound ϕ will be scattered out of the NTS, with no net gain in charge. In this case, the condition for single scatter capture is approximately $E \lesssim I_Q$, and thus $T \lesssim I_Q$.

The mean free path λ_ϕ is also model dependent. In the simplest NTS models, the possible interactions are either ϕ-ϕ or ϕ-σ scattering. In some models, although the cross section for ϕ-σ scatterng is appreciable, the density of σ particles inside the NTS is small (e.g., if σ is very heavy). In these models, ϕ-ϕ scattering can occur either through a ϕ^4 term in the Lagrangian, or through the exchange of a σ particle.

For example, consider a model where ϕ-ϕ scattering through a $g\phi^4$ interaction dominates, with cross section $\sigma_s = g^2/32\pi E_i^2$. If the ϕ's inside the NTS are nondegenerate, the density of scatterers is $n_s = 3Q/(4\pi R_Q^3)$. Then

$$f_\phi = 1 - \exp\left[-\left[\frac{Q}{Q_*}\right]^{1-2p/3}\right], \tag{B1}$$

where

$$Q_* = \left[\frac{128\pi^2}{3}\frac{\beta^2}{g^2}\frac{(1+x)^2}{x^2}\right]^{3/(3-2p)}. \tag{B2}$$

In this case, $f_\phi \sim 1$ for $Q \gg Q_*$ (multiple scattering), and $f_\phi \simeq (Q/Q_*)^{1-2p/3}$ for $Q < Q_*$. At $x = 1$, for models with $p = \frac{3}{4}$, we find $Q_* \sim 10^6(\beta/g)^4$, while for $p = 1$, $Q_* \sim 10^9(\beta/g)^6$. Thus, the approximation $f_\phi = 1$ is valid for large charges, but for small charges the scattering probability is suppressed. Consequently, small charges freeze out earlier; for $Q < Q_*$,

$$x_F \simeq x_F(f_\phi = 1) - \left[1 - \frac{2p}{3}\right]\ln\left[\frac{Q_*}{Q}\right], \tag{B3}$$

and

$$Q_{\rm eq} \simeq \left[\frac{Q}{Q_*}\right] Q_{\rm eq}(f_\phi = 1), \tag{B4}$$

$$Q_s \simeq \left[\frac{Q}{Q_*}\right] Q_s(f_\phi = 1). \tag{B5}$$

In principle, the distribution of NTS's with charge $Q \ll Q_*$ may be frozen out from the beginning ($x_F \lesssim x_i$) and thus survive. For these objects, the mean free path is much larger than the NTS size, so the time scale to repopulate the evaporated tail of the interior ϕ distribution is longer than the expansion timescale. In this case, the initial distribution of small NTS's is preserved, and may contribute significantly to the energy density of the universe. On the other hand, if ϕ is kept in thermal equilibrium by scattering with *other* particles, the mean free path is much shorter and we expect $f_\phi \sim 1$ down to smaller values of Q.

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