PHYSICAL REVIEW D VOLUME 40, NUMBER 9 1 NOVEMBER 1989

Charge quantization in supersymmetric, technicolor, and composite models

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(Received 28 August 1989)

The possibility of quantizing the electric charge in supersymmetric, technicolor, and composite models is examined by studying some specific examples. By requiring an anomaly-free theory, the appropriate symmetry-breaking pattern of the gauge group, and mass generation, the electric charges may be determined.

It has been shown that electric charge quantization in the observable particle spectrum can be explained in terms of the standard model (SM). Charge quantization in this case is imposed by anomaly cancellation,[1] the spontaneous breaking of the gauge symmetry, and mass generation.[2,3] Although the SM has been extremely successful in describing nature up to energies presently accessible by experiment, there are many models which aspire to improve on the SM. Will charge quantization continue to be a consequence for models going beyond the SM? For the case of $U(1)_{em}$ embedded in a simple or semisimple group, it is well known that electric charge is quantized.[4] The $U(1)_{em}$ generator is a linear combination of the original simple or semisimple group generators which have discrete eigenvalues and are traceless. For gauge groups which have a U(1) factor, the quantization of electric charge is not as easily explained. The quantization of electric charge in such models has already been discussed in the context of gauge theories which rely on elementary Higgs scalars to break the symmetry and generate masses.[3] The electric charges can be quantized and fixed by imposing the following conditions: (i) the $U(1)_{em}$ gauge symmetry remains exact, (ii) the gauge[5] and mixed gauge-gravitational[6] anomalies cancel to ensure the renormalizability and the general covariance of the theory, respectively, and (iii) the masses of fermions are generated by the Higgs mechanism in the usual way,[7] including the possibility of Majorana mass terms so that the electric charge of the Majorana particle is zero.[8] The purpose of this paper is to extend the analysis of Refs. 2 and 3 to models in which fermions appear in the symmetry-breaking sector, and to composite models. Of course models with simple or semisimple gauge groups will automatically have charge quantization for the usual reasons. So only models based on gauge groups with U(1) factors will be considered in this paper.

To set the scene for the rest of the paper, charge quantization in the SM is reviewed. First, we note a general feature of the models to be considered. If we assume that the underlying dynamics of the model are based on the gauge principle, then the gauge bosons reside in the adjoint representation of the gauge group, and hence must have zero charge of the corresponding U(1) factor. Now consider the Higgs sector of the SM with the following transformation properties under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$:

$$\Phi = (\phi_1 \ \phi_2)^T \sim (1,2)(Y_\phi) . \tag{1}$$

If the gauge symmetry $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is broken to $SU(3)_c \otimes U(1)_{em}$ by the vacuum $|v\rangle$ of Φ via the Higgs mechanism, then the electric charges of the components of Φ are fixed. The $U(1)_{em}$ generator Q is unbroken so it must annihilate the vacuum. Working in the basis $|v\rangle = (0,v)^T$ and applying Q to the vacuum results in $Q = I_3 + \frac{1}{2}(Y/Y_\phi)$. By following convention we choose $Y_\phi = 1$. Therefore the electric charges of the Higgs bosons are determined from the symmetry breaking.

There remains the question of quantizing the charges of the SM fermions. Their transformation properties are listed as

$$\begin{aligned} &q_L \sim (3,2)(y_1),\ u_R \sim (3,1)(y_2),\ d_R \sim (3,1)(y_3),\\ &l_L \sim (1,2)(y_4),\ e_R \sim (1,1)(y_5). \end{aligned} \tag{2}$$

The Yukawa couplings which connect left- and right-handed fermions and give them masses through spontaneous symmetry breaking are given by

$$\mathcal{L} = \Gamma_u \bar{q}_L \tilde{\Phi} u_R + \Gamma_d \bar{q}_L \Phi d_R + \Gamma_e \bar{l}_L \Phi e_R + \text{H.c.} , \tag{3}$$

where $\tilde{\Phi} = i\tau_2 \Phi^*$. This imposes some relations between the hypercharges: i.e.,

$$y_2 = y_1 + 1,\ y_3 = y_1 - 1,\ \text{and}\ y_5 = y_4 - 1 . \tag{4}$$

So far the hypercharges are not completely determined. To proceed further, the constraint of anomaly cancellation is imposed. There are three nontrivial gauge anomalies and one mixed gauge-gravitational anomaly in this case:

$$\begin{aligned} &[SU(3)_c]^2 U(1)_Y:\ y_2 + y_3 = 2y_1 ,\\ &[SU(2)_L]^2 U(1)_Y:\ 3y_1 + y_4 = 0 ,\\ &[U(1)_Y]^3:\ 6y_1^3 - 3y_2^3 - 3y_3^3 + 2y_4^3 - y_5^3 = 0 ,\\ &\text{Tr}[Y]:\ 6y_1 - 3y_2 - 3y_3 + 2y_4 - y_5 = 0 . \end{aligned} \tag{5}$$

Solving Eqs. (4) and (5) results in[2]

$$y_1 = \tfrac{1}{3},\ y_2 = \tfrac{4}{3},\ y_3 = -\tfrac{2}{3},\ y_4 = -1,\ y_5 = -2 . \tag{6}$$

This corresponds to the usual set of charge assignments for the SM fermions.

It has been noted that if a right-handed neutrino ν_R (with hypercharge y_6) is introduced to the SM, the situation changes dramatically.[2] The analogous set of equations to those in Eqs. (4) and (5) no longer fixes the charge, but leaves a free parameter. In this case, if the right-handed neutrino is required to have a Majorana mass, then the SM fermion charges are recovered.[8]

In order to derive the above result, Eq. (4) was crucial. Since these relations are due to the symmetry-breaking and mass-generation sector of the theory, one can see that charge quantization and symmetry breaking are intimately related. Other than Eq. (4) anomaly cancellation was the major constraint. It therefore seems reasonable to investigate cases where the symmetry-breaking sector of the theory also contributes to the gauge anomalies. There are two obvious classes of theories where this can occur: supersymmetric models and (extended) technicolor theories. In the first case the Higgs bosons have fermionic partners which contribute to the anomalies. In (extended) technicolor models the Higgs sector is replaced by a fermionic system.

Let us first consider the spectrum of the supersymmetric standard model.[9] It consists of left- and right-handed chiral superfields having the quantum numbers of Eq. (2) together with two types of left-handed Higgs superfields:

$$\Phi_{1L}\sim(1,2)(-1),\ \Phi_{2L}\sim(1,2)(\phi)\,. \tag{7}$$

(The arbitrary normalization factor has been defined so that Φ_{1L} has hypercharge -1.) Let n_G, n_1, and n_2 be the number of generations of quarks and leptons, Φ_{1L} and Φ_{2L}, respectively. Assuming that the standard superpotential term

$$W=\lambda_1 l_L(e^c)_L\Phi_{1L}+\lambda_2 q_L(u^c)_L\Phi_{2L}+\lambda_3 q_L(d^c)_L\Phi_{1L} \tag{8}$$

is invariant under $U(1)_Y$ leads to the relations

$$y_5=y_4-1,\ y_2=y_1+\phi,\ y_3=y_1-1\,. \tag{9}$$

Imposing gauge and mixed anomaly cancellation then yields three solutions:

$$\text{(A)}\ y_1=0,\ y_2=1,\ y_3=-1,\ y_4=1,\ y_5=0\,,\quad \phi=1,\ n_2-n_1=n_G\,; \tag{10a}$$

$$\text{(B)}\ y_1=\tfrac{1}{3},\ y_2=\tfrac{4}{3},\ y_3=-\tfrac{2}{3},\ y_4=-1,\ y_5=-2\,,\quad \phi=1,\ n_2=n_1,\ n_G\ \text{unconstrained}\,; \tag{10b}$$

$$\text{(C)}\ y_1=\tfrac{2}{3},\ y_2=\tfrac{5}{3},\ y_3=-\tfrac{1}{3},\ y_4=-3,\ y_5=-4\,,\quad \phi=1,\ n_1-n_2=n_G\,. \tag{10c}$$

Case (B) corresponds to the supersymmetric extension of the SM. If one imposes the requirement that the Higgs multiplets give each other masses, then $n_1=n_2$ necessarily and case (B) becomes the only viable solution. If one arranges for the vacuum to leave the putative electromagnetic gauge group unbroken, then all three solutions imply charge quantization. It is just that cases (A) and (C) yield the wrong charges.

We now examine technicolor theories, in which the elementary Higgs scalar is replaced by a bound state of fermions. The SM gauge group is extended to[10]

$$G_{TC}\otimes SU(3)_c\otimes SU(2)_L\otimes U(1)_Y, \tag{11}$$

where G_{TC} is the gauge group of the new strong technicolor (TC) interaction. The usual quarks and leptons are TC singlets. The new fermions on which G_{TC} acts are called technifermions. In analogy with QCD, G_{TC} is an asymptotically free unbroken gauge group and it is assumed to be confining, with all physical states being TC singlets.

To investigate charge quantization in TC models, we choose to examine a simple but typical example. For simplicity, we will assume that the technifermions belong to the fundamental representation of G_{TC} [where for example G_{TC} may be $SU(N)_{TC}$ or $SO(N)_{TC}$]. The technifermions transform as

$$\begin{aligned}&Q_{aL}\sim(N,3,2)(Y_1),\ \Psi_L\sim(N,1,2)(Y_4)\,,\\&U_{aR}\sim(N,3,1)(Y_2),\ E_R\sim(N,1,1)(Y_5)\,,\\&D_{aR}\sim(N,3,1)(Y_3),\ N_R\sim(N,1,1)(Y_6)\,,\end{aligned} \tag{12}$$

and the SM fermions transform as

$$\begin{aligned}&q_{aL}\sim(1,3,2)(y_1),\ l_L\sim(1,1,2)(y_4)\,,\\&u_{aR}\sim(1,3,1)(y_2),\ e_R\sim(1,1,1)(y_5)\,,\\&d_{aR}\sim(1,3,1)(y_3)\,,\end{aligned} \tag{13}$$

where a is the color index.

When TC becomes strong, the chiral symmetry of the techniworld (in the limit of zero SM gauge couplings) is spontaneously broken at a scale Λ_{TC} by the condensates

$$\langle\bar U_a U_a\rangle=\langle\bar D_a D_a\rangle=3\langle\bar E E\rangle=3\langle\bar N N\rangle\neq 0\,. \tag{14}$$

To fix the values of the otherwise *a priori* arbitrary hypercharges, we proceed as we did for the SM. By assumption, the condensates of Eq. (14) will break the symmetry of $SU(2)_L\otimes U(1)_Y$ to $U(1)_{em}$. To arrive at this, the quantum numbers of the fermion bilinears which condense must be adjusted so that they transform as $(1,1,2)(1)$, the same as the Higgs doublet in the SM. To see how this can give information about the hypercharges of the technifermions consider, for example, the condensate

$$\langle\bar U_a U_a\rangle=\langle\bar U_{aR}U_{aL}+\bar U_{aL}U_{aR}\rangle\,. \tag{15}$$

The first term on the right-hand side of Eq. (15) transforms with $I_3=\frac{1}{2}$ under $SU(2)_L$ and Y_1-Y_2 under $U(1)_Y$. This condensate must be invariant under $U(1)_{em}$ and so

$$Y_1-Y_2=-1\,. \tag{16a}$$

Similarly, from $\langle\bar D_a D_a\rangle$, $\langle\bar E E\rangle$, and $\langle\bar N N\rangle$, one obtains

$$Y_1-Y_3=1,\ Y_4-Y_5=1,\ \text{and}\ Y_4-Y_6=1\,. \tag{16b}$$

The condensates are also assumed to generate the quark and lepton masses due to an effective interaction of the form

$$\mathcal{L}_{\rm eff}=\frac{\langle\bar F_R F_L\rangle}{\Lambda^2}(\bar f_L f_R+\text{H.c.})\,, \tag{17}$$

where f and F are the generic SM fermions and technifermions, respectively. For example, to give the u-quark mass, one can have

$$\mathcal{L}^u_{\rm eff}=\frac{1}{\Lambda^2}\bar u_L u_R(\langle\bar U_{aR}U_{aL}\rangle+\cdots)\,. \tag{18}$$

One then has

$$y_1 - y_2 = 1 . \tag{19a}$$

A similar analysis for the other SM fermions gives

$$y_1 - y_3 = 1 \text{ and } y_4 - y_5 = 1 . \tag{19b}$$

By applying the anomaly cancellation conditions we have

$$\begin{aligned}
&[SU(2)_L]^2U(1)_Y\text{: } 3NY_1 + NY_4 + 3y_1 + y_4 = 0 ,\\
&[SU(3)_c]^2U(1)_Y\text{: } 2NY_1 - NY_2 - NY_3 + 2y_1 - y_2 - y_3 = 0 ,\\
&[SU(N)_{TC}]^2U(1)_Y\text{: } 6Y_1 - 3Y_2 - 3Y_3 + 2Y_4 - Y_5 - Y_6 = 0 ,\\
&[U(1)_Y]^3\text{: } 6NY_1^3 - 3NY_2^3 - 3NY_3^3 + 2NY_4^3 - NY_5^3 - NY_6^3 + 6y_1^3 + 2y_4^3 - 3y_2^3 - 3y_3^3 - y_5^3 = 0 ,\\
&\mathrm{Tr}[Y]\text{: } 6NY_1 - 3NY_2 - 3NY_3 + 2NY_4 - NY_5 - NY_6 + 6y_1 - 3y_2 - 3y_3 + 2y_4 - y_5 = 0 .
\end{aligned} \tag{20}$$

Combining Eqs. (16), (19), and (20), we are able to fix $y_4 = -1$ and $y_5 = -2$, but not the other hypercharges. There are two special cases where the hypercharges are completely determined. (i) If $Y_i = y_i$, then by solving the above equations we obtain the standard hypercharges for the technifermions and the SM fermions,

$$\begin{aligned}
&y_1 = Y_1 = \tfrac{1}{3},\ y_2 = Y_2 = \tfrac{4}{3},\ y_3 = Y_3 = -\tfrac{2}{3} ,\\
&y_4 = Y_4 = -1,\ y_5 = Y_5 = -2,\ Y_6 = 0 .
\end{aligned} \tag{21}$$

(ii) The anomalies due to the technifermions and SM fermions cancel separately. In this case, the hypercharges of the SM fermion, y_i, are fixed as in the SM. For the hypercharges of the technifermions the situation is the same as in the SM with the inclusion of a right-handed neutrino.[2] Notice that if the TC group G_{TC} [e.g., $G_{TC} = SU(N)$] has a $(G_{TC})^3$ anomaly, then the technifermion N_R plays an important role in the cancellation of this anomaly. However, if G_{TC} is automatically anomaly free for all representations [e.g., $G_{TC} = SO(N)$ with $N \neq 6$], then the existence of N_R is unnecessary and Y_6 can be set to zero.

In the above discussion, the effective Lagrangian of Eq. (17) was assumed to generate the SM fermion masses. It is obvious that this four-Fermi interaction is not renormalizable. In order to have a renormalizable theory, it is assumed that Eq. (17) is due to the exchange of scalar or vector (V_μ) particles. Since TC theories already assume that there are no fundamental scalars the latter option is taken. This is, in fact, the extended technicolor (ETC) model.[11] In one such model the ETC group is assumed to be $SU(N+1)_{ETC}$. The technifermions and SM fermions are embedded into the fundamental representation of $SU(N+1)_{ETC}$. At some energy scale Λ_{ETC}, $SU(N+1)_{ETC}$ is spontaneously broken to $SU(N)_{TC}$ with the technifermions transforming as the fundamental representation and the SM fermions transforming as singlets of $SU(N)_{TC}$. In this case, it is essential to have the right-handed-neutrino ν_R with hypercharge y_6, so that the $[SU(N+1)_{ETC}]^3$ anomaly vanishes. In this model the vector bosons V_μ correspond to the broken generators of $SU(N+1)_{ETC}$ and induce an effective four-Fermi interaction between the f and F, such that

$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{\Lambda_{ETC}^2}(\bar{f}_L \gamma^\mu F_L \bar{F}_R \gamma_\mu f_R + \mathrm{H.c.}) . \tag{22}$$

After the formation of the technifermion condensates and a Fierz rearrangement, one obtains Eq. (17) with Λ changed to Λ_{ETC}. Notice that because f and F are embedded in the same representation of $SU(N+1)_{ETC}$, one then naturally has $Y_i = y_i$. Carrying out the same analysis as for the TC model, the hypercharges satisfy the same equations as those in the SM with a right-handed neutrino. Therefore the electric charges are not completely determined.

There are several ways in which the hypercharges may be completely fixed. One possibility is that the ETC group should be automatically anomaly-free for all representations. For example, if the ETC group is $SO(N+1)_{ETC}$ ($N+1 \neq 6$), then it is unnecessary for ν_R and N_R to appear in the theory. In this case, Y_6 does not appear in the theory, and hence the standard hypercharge assignments are recovered. It may also be possible, due to some mechanism, that ν_R and N_R develop Majorana masses. This gives $Y_6 = 0$ which in turn fixes all the other hypercharges, and hence the electric charges, to their standard values.[8]

Of course, extended technicolor theories in the past have encountered phenomenological problems with flavor-changing neutral currents[11] (FCNC's). The above examples of ETC models are therefore only indicative of the basic structure of a hypothetical realistic model. (One should note that recently progress has been made on the FCNC problem.[12]) However, we feel that this structure, though motivated by unrealistic models, should have more general relevance.

We now move on to consider another interesting facet of electromagnetic interactions. Hitherto we have studied models where $SU(2)_L \otimes U(1)_Y$ is gauged and the photon is a linear combination of the neutral $SU(2)_L$ and $U(1)_Y$ gauge bosons. This embroiled us in the issue of electroweak symmetry breaking. The other possibility, which seems viable only in composite models, is that the photon, although an elementary particle, is not the result of electroweak symmetry breaking. In these models the $W^\pm$

and Z^0 bosons are composite and $U(1)_{em}$ rather than $SU(2)_L \otimes U(1)_Y$ is gauged. This situation is distinct from that considered in the paper so far, and for completeness should be analyzed.

In order to say something definite about charge quantization one should have in mind a reasonably well-defined and phenomenologically successful model of composite electroweak bosons. A major example of such a model is that due to Abbott and Farhi.[13] We will consider gauge anomaly cancellation at the fundamental (preonic) level and show that charge quantization results.

The gauge symmetry of the Abbott and Farhi model is $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, in which the $SU(2)_L$ coupling constant becomes large and confining at a mass scale of the order of the known weak-interaction scale. Therefore all physical states must be $SU(2)_L$ singlets. In this model weak isospin SU(2) is not the $SU(2)_L$ local symmetry, but rather a certain global symmetry. The particle spectrum of the model is given as follows. The preons transforming under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ are

$$\begin{aligned} &Q_{aL} \sim (3,2)(Y_1)\,, \quad \Psi_L \sim (1,2)(Y_4)\,, \\ &u_{aR} \sim (3,1)(Y_2), \quad e_R \sim (1,1)(Y_5)\,, \\ &d_{aR} \sim (3,1)(Y_3), \quad \nu_R \sim (1,1)(Y_6)\,, \\ &\phi \sim (1,2)(Y_\phi)\,, \end{aligned} \tag{23}$$

and the composites which represent the known quarks and leptons are

$$\begin{aligned} &u_{aL} = Q_{aL}\phi \sim (3,1)(Y_1+Y_\phi), \quad \nu_L = \Psi_L\phi \sim (1,1)(Y_4+Y_\phi)\,, \\ &d_{aL} = Q_{aL}\tilde{\phi} \sim (3,1)(Y_1-Y_\phi), \quad e_L = \Psi_L\tilde{\phi} \sim (1,1)(Y_4-Y_\phi)\,, \\ &u_{aR} \sim (3,1)(Y_2), \quad e_R \sim (1,1)(Y_5)\,, \\ &d_{aR} \sim (3,1)(Y_3), \quad \nu_R \sim (1,1)(Y_6)\,. \end{aligned} \tag{24}$$

Notice that the right-handed fermions remain elementary at the composite level.

For three generations of SM fermions, the model has a global $SU(12) \otimes SU(2)$ symmetry for the preons when the color and electromagnetic interactions are turned off. The 't Hooft anomaly-matching conditions,[14] necessary for the generation of light composite fermions, are satisfied by the bound-state spectrum in Eq. (24). These conditions play no role in charge quantization.

The cancellation of the gauge and mixed gauge-gravitational anomalies yield the same equations as in the SM with a right-handed neutrino. Anomaly cancellation is not sufficient to fix the hypercharges of the model. However, the model allows fermion mass terms which are similar to the Yukawa terms of the SM:

$$\begin{aligned} \mathcal{L}_{\text{mass}} \sim \lambda_u \bar{u}_{aR}(Q_{aL}\Phi) + \lambda_d \bar{d}_{aR}(Q_{aL}\tilde{\Phi}) \\ + \lambda_e \bar{e}_R(l_L\tilde{\Phi}) + \lambda_\nu \bar{\nu}_R(l_L\phi) + \text{H.c.}\,. \end{aligned} \tag{25}$$

The constraints imposed by the mass terms and the gauge anomaly equations give

$$\begin{aligned} &Y_1,\ Y_2 \text{ arbitrary}, \quad Y_4 = -3Y_1\,, \\ &Y_3 = 2Y_1 - Y_2, \quad Y_5 = -2Y_1 - Y_2\,, \\ &Y_\phi = Y_2 - Y_1, \quad Y_6 = -4Y_1 + Y_2\,. \end{aligned} \tag{26}$$

We have included the right-handed neutrino ν_R in this model. Imposing a Majorana mass term for ν_R gives $Y_2 = 4Y_1$ and $Y_6 = 0$. We also get the same result if ν_R is excluded from the model.

Note in this model $U(1)_Y$ is identified as $U(1)_{em}$. No electroweak mixing is involved at the preonic level.[15] The electric charges are determined up to an overall normalization factor which is of no physical consequence. Therefore charge quantization is obtained from Eq. (26) provided either ν_R is omitted, or is given a Majorana mass. The mathematics is actually the same as for the SM, but the interpretation of the result is different, due to the fact that $SU(2)_L \otimes U(1)_Y$ play quite different roles in the two theories.

To summarize, we have examined charge quantization in supersymmetric, technicolor, and composite models by studying some simple examples. In the supersymmetric SM we discovered three solutions, all of which feature charge quantization. If one requires the Higgs superfields to pair up in mass terms then the phenomenologically realistic solution is the only viable one. By requiring an anomaly-free theory, the appropriate symmetry-breaking pattern, and mass generation, the charges in the technicolor models were determined. In the Abbott-Farhi model, where the photon does not result from $SU(2)_L \otimes U(1)_Y$-symmetry breaking, we also found anomaly cancellation and mass generation imply charge quantization, provided ν_R is either omitted or given a Majorana mass.

This work was supported by the Australian Research Council. One of us (H.L.) would like to acknowledge the assistance of the Commonwealth Postgraduate Award.

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