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Gauge model for the strong interactions

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A gauge theory is introduced with the gauge group $SU(4)\otimes SU(2)_L\otimes U(1)_{y'}$. In this theory the quarks transform under the fundamental four-dimensional representation of SU(4). Gauge anomaly cancellation is imposed on the known fermion generation structure (with four colors) to fix the $U(1)_{y'}$ charges. It is shown that the model requires the existence of only one extra Higgs multiplet to break the SU(4) group to $SU(3)_c$ and at the same time give the exotic $SU(3)_c$-singlet quarks mass. The model has the novel feature that the unbroken electric charge generator is embedded in all three simple factor groups of the semisimple gauge group.

The standard model (SM) is described by a Yang-Mills theory with the gauge group

$$SU(3)_c\otimes SU(2)_L\otimes U(1)_y\,. \tag{1}$$

This model has considerable experimental support as well as theoretical self-consistency. In this paper we would like to investigate the possibility that the strong force is a remnant of a larger group. We consider a gauge group G_{strong}, such that $G\supset SU(3)_c$. What should the group G be? We take the view that the known fermions with N colors (and top quark) should be anomaly-free. [This is certainly true for $G=SU(3)$.] Anomalies provide a powerful theoretical constraint on the possible quantum numbers of the fermions. For instance, it has been emphasized recently that the observed electric charges of the fermions can be derived almost uniquely from the cancellation of the anomalies alone.[1] This is a powerful result. From this point of view, one might expect that $G_{\text{strong}}=SU(3)$ simply from the fact that the hypercharge of the quark doublet is $-\frac{1}{3}$ that of the lepton doublets [for N colors, the cancellation of the $SU(2)_L^2\times U(1)_y$ anomaly implies that hypercharges of the left-handed quarks (y_q) and left-handed leptons (y_l) are related by $Ny_q+y_l=0$]. However, we will construct a simple model where the number of quark color degrees of freedom is larger than three. This model will preserve both the experimental consistency of the SM at low energies, as well as the successful SM prediction of the observed quark and lepton electric charges.

We consider the gauge theory defined by the gauge group

$$SU(4)\otimes SU(2)_L\otimes U(1)_{y'}\,. \tag{2}$$

This gauge group has been considered previously by Zee.[2] In Zee's model, additional fermions were required to cancel the gauge anomalies. We wish to *impose* anomaly cancellation on the known generation structure and to assume the existence of the usual Yukawa Lagrangian to derive the $U(1)_{y'}$ charges. These considerations imply that the SM generation has the form

$$f_L\sim(1,2,-1),\ e_R\sim(1,1,-2)\,,$$
$$Q_L\sim(4,2,\tfrac{1}{4}),\ u_R\sim(4,1,\tfrac{5}{4}),\ d_R\sim(4,1,-\tfrac{3}{4})\,. \tag{3}$$

To break the SU(4) group to the observed SU(3) low-energy theory, we introduce a colored Higgs multiplet χ. For economy, we would also like to use the colored Higgs multiplet to give the exotic $SU(3)_c$ neutral quarks a large mass. This will relate the mass scale of the extra quarks with the scale of SU(4) breaking. We now introduce two models. In model I, the following Yukawa Lagrangian is introduced (this Lagrangian is in addition to the usual Yukawa Lagrangian arising from the ordinary Higgs doublet):

$$L=\lambda_1\bar{u}_R\chi(d_R)^c+\lambda_2\bar{Q}_L\chi(Q_L)^c+\text{H.c.}\,, \tag{4}$$

where the superscript c indicates charge-conjugate field, and λ_1 and λ_2 are Yukawa couplings. Gauge invariance of this Lagrangian implies that χ transforms under the gauge group Eq. (2) as follows:

$$\chi\sim(10,1,\tfrac{1}{2})\,. \tag{5}$$

Note that this is the only extra Higgs multiplet required in our model [in addition to the usual doublet Higgs field $\phi\sim(1,2,1)$]. The **10** representation of SU(4) has the $SU(3)\otimes U(1)$ branching rule

$$\mathbf{10}=6(\tfrac{2}{3})+3(-\tfrac{2}{3})+1(-2)\,, \tag{6}$$

where the U(1) has been normalized so that the fundamental representation has the branching rule

$$\mathbf{4}=3(\tfrac{1}{3})+1(-1)\,. \tag{7}$$

Giving the $SU(3)_c$-singlet component of χ a vacuum expectation value (VEV) (of w) breaks $SU(4)\to SU(3)$. If the mass scale $g_sw\gg gu$ [where g_s, g are the SU(4) and $SU(2)_L$ coupling constants, respectively], then the breaking of the gauge group goes in two stages:

$$SU(4)\otimes SU(2)_L\otimes U(1)_{y'}\to SU(3)_c\otimes SU(2)_L\otimes U(1)_y$$
$$\to SU(3)_c\otimes U(1)_Q\,. \tag{8}$$

It is important to observe that the unbroken electric-charge gauge group $U(1)_Q$ couples to the generator

$$Q=I_3+Y'/2+T/8\,, \tag{9}$$

where T is the SU(4) generator, with fundamental representation

$$T=\begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{10}$$

It is straightforward to verify that the electric charges of the ordinary quarks and leptons have their experimentally observed values. In fact, comparing Eq. (9) with the usual formula for the electric charge $Q=I_3+Y/2$, we see that the familiar hypercharge generator can be expressed as $Y=Y'+T/4$.

In model II, we will add the following gauge anomaly-free multiplet of fermions to each generation:

$$F_L\sim(1,2,0),\ E_R\sim(1,1,-1),\ V_R\sim(1,1,1) . \tag{11}$$

We then introduce the Lagrangian

$$L=\lambda_1\bar{Q}_L\chi(F_L)^c+\lambda_2\bar{u}_R\chi(E_R)^c+\lambda_3\bar{d}_R\chi(V_R)^c . \tag{12}$$

In this model the extra Higgs field χ transforms as

$$\chi\sim(4,1,\tfrac{1}{4}) . \tag{13}$$

The Lagrangian Eq. (12) will give the $SU(3)_c$-singlet quarks and the fermions in Eq. (11) a mass. Model II simplifies the Higgs sector at the expense of the introduction of the new fermion fields in Eq. (11). Also note that the SU(2) global anomaly[3] will imply an even number of generations for model I, with no new restrictions for model II. We will now discuss some issues relevant to both models.

How do the exotic heavy gluons and quarks transform under the unbroken $SU(3)_c\otimes U(1)_Q$ group? There are 15 fields transforming in the adjoint representation of SU(4) which mediate the force. Using Eq. (9) we find that, under $SU(3)_c\otimes U(1)_Q$,

$$15=8(0)+3(\tfrac{1}{6})+\bar{3}(-\tfrac{1}{6})+1(0) . \tag{14}$$

Thus, we have the familiar chargeless and massless eight gluons of the unbroken $SU(3)_c$ gauge group. In addition we have a heavy color triplet of charge-$\frac{1}{6}$ gluons and their antiparticles, and one $SU(3)_c\otimes U(1)_Q$ neutral heavy gluon field. (This field will mix with the Z boson as we will show later.) In addition to the exotic gluons we have exotic $SU(3)_c$-singlet quarks with electric charges $+\frac{1}{2}$ and $-\frac{1}{2}$. These quarks are not expected to form bound states due to the massive nature of the exotic gluons. We emphasize that since the mass scale of the exotic quarks is coupled with the scale of SU(4)-symmetry breaking, it is not surprising that they have not been observed experimentally. Let us now discuss the neutral gauge sector of the theory. The covariant derivative can be expressed as

$$D^\mu=\partial^\mu+igW_0^\mu I_3+ig'B^\mu Y'/2+ig_sC^\mu(\tfrac{3}{8})^{1/2}T$$
$$+ig_sG_a^\mu\lambda^a/2+(\text{charged-gauge-boson terms}) , \tag{15}$$

where g, g', and g_s are the $SU(2)_L$, $U(1)_{y'}$, and SU(4) coupling constants, respectively. Recall that the generator T is normalized in Eq. (10). [The factor $(\frac{3}{8})^{1/2}$ in Eq. (15) is there so that the strong coupling constant g_s coincides with its usual definition.] The neutral-gauge-boson mass matrix arises from the Higgs-boson kinetic terms

$$L=(D_\mu\phi)^\dagger D^\mu\phi+(D_\mu\chi)^\dagger D^\mu\chi . \tag{16}$$

The $SU(3)_c$ gluons are massless as expected and decouple from the mass matrix. The remaining three neutral gauge bosons are W_0, B, and C. The mass matrix has the form

$$L_{\text{mass}}=\tfrac{1}{2}V^TM^2V , \tag{17}$$

where

$$V^T=(W_0,B,C) , \tag{18}$$

and

$$M^2=\begin{pmatrix} g^2u^2/2 & -g'gu^2/2 & 0 \\ -g'gu^2/2 & g'^2u^2/2+\rho g'^2w^2/8 & -\rho g'g_sw^2(\frac{3}{8})^{1/2} \\ 0 & -\rho g'g_sw^2(\frac{3}{8})^{1/2} & 3\rho g_s^2w^2 \end{pmatrix} , \tag{19}$$

where $\rho=1$ for model I and $\frac{1}{4}$ for model II. Evaluating the mass eigenvalues in the limit $g_s^2\gg g^2$ we obtain

$$M_\gamma^2=0 ,$$
$$M_Z^2=\tfrac{1}{2}(g^2+g'^2)u^2-\frac{g'^4u^2}{48g_s^2}+O\left[\frac{g'^6u^2}{g_s^4}+\frac{g'^6u^4}{g_s^4w^2}\right] ,$$
$$M_{Z'}^2=3\rho g_s^2w^2 , \tag{20}$$

where M_γ, M_Z, and $M_{Z'}$ denote the masses of the photon, Z boson, and the new Z' boson. Observe that the change in the Z-boson mass due to the mixing is expressed in terms of known parameters (and clearly, naturally small). In particular, note that to order (g'^4/g_s^2), M_Z^2 is independent of w. One can also check that the numerical coefficients of the higher-order terms are very small. Calculating δM_Z we obtain

$$\delta M_Z=\frac{-\alpha_{\text{em}}M_Z\tan^2\theta_w}{48\alpha_s} , \tag{21}$$

where θ_w is defined by $\tan\theta_w=g'/g$. Note that the value for g_s will depend on the scale of the symmetry breaking $SU(4)\rightarrow SU(3)$ (i.e., at $s=M_{Z'}^2$). For illustration, here we will assume that $M_{Z'}$ is of the same order of magnitude as M_Z. Recall that since the coupling constants scale very slowly with energy scale, g_s should not be very sensitive to the precise value for $M_{Z'}$. Hence, for the ratio of the finestructure constant and the strong structure constant we take

$$\frac{\alpha_{em}}{\alpha_s}=\frac{1}{10}\,. \tag{22}$$

Using Eq. (22) we find that $\delta M_Z = -60$ MeV.

In the limit $g'^2 \ll g_s^2$, one can also check that the gauge couplings of the mass eigenstate Z boson reduces to the SM couplings. Indeed we find that Z_μ couples to the generator

$$g\cos\theta_w \frac{\tau_3}{2} - g'\sin\theta_w\left[\frac{Y'}{2}+\frac{T}{8}\right]+\Delta\,, \tag{23}$$

where Δ corresponds to a small correction term of order g'^3/g_s^2. To leading order we calculate Δ:

$$\Delta = g\cos\theta_w\left[\frac{g'^2\sin^2\theta_w}{48g_s^2}\right]\frac{\tau_3}{2} - g'\sin\theta_w\left[\frac{\sin^2\theta_w g'^2}{48g_s^2}-\frac{g'^2}{24g_s^2}\right]\frac{Y'}{2} - g'\sin\theta_w\left[\frac{g'^2u^2}{6g_s^2\sin^2\theta_w \rho w^2}-\frac{g'^2}{24g_s^2}+\frac{g'^2\sin^2\theta_w}{48g_s^2}\right]\frac{T}{8}\,. \tag{24}$$

Thus Δ is very small, giving a change in Z-boson gauge couplings of less than one percent (unless $w<u$). The new neutral gauge boson Z' couples dominantly to the quarks, and weakly to the leptons through small gauge-boson mixing angles. Experimental limits on the Z' boson are therefore expected to be very weak. Of course, the extra charged gluons can couple to both the quarks and leptons through the photon coupling. The limits on these particles are expected to be of order 100 GeV.

Let us now examine the Higgs sector of the theory. In model I, the Higgs field χ transforms as the irreducible **10** representation of SU(4), and under the unbroken gauge group $SU(3)_c \otimes U(1)_Q$ it transforms as

$$6(\tfrac{1}{3})+3(\tfrac{1}{6})+1(0)\,. \tag{25}$$

The Higgs field χ can be expressed as a complex symmetric 4×4 matrix. The most general Higgs potential has the form

$$\begin{aligned} V(\phi,\chi) = {}& \lambda_1(\phi^\dagger\phi-u^2)^2+\lambda_2(\mathrm{Tr}\chi^\dagger\chi-w^2)^2 \\ &+\lambda_3[\mathrm{Tr}\chi^\dagger\chi\chi^\dagger\chi-(\mathrm{Tr}\chi^\dagger\chi)^2] \\ &+\lambda_4(\phi^\dagger\phi-u^2)(\mathrm{Tr}\chi^\dagger\chi-w^2)\,. \end{aligned} \tag{26}$$

The potential has a minimum provided $\lambda_1, \lambda_2 > 0$ and the minimum occurs when

$$\chi = \begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&w\end{bmatrix},\quad \phi=\begin{bmatrix}0\\u\end{bmatrix} \tag{27}$$

provided $\lambda_3 < 0$.[4] The real part of the $SU(3)_3 \otimes U(1)_Q$-singlet Higgs field and the $6(\frac{1}{3})$ Higgs field become massive and these are the physical Higgs fields. The remaining seven Higgs degrees of freedom [i.e., the imaginary part of the $1(0)$ field and the complex $3(\frac{1}{6})$ component] are massless and are absorbed by the seven exotic gluon fields, which have become massive. The massive neutral physical Higgs boson will mix with the ordinary SM physical Higgs particle via the λ_4 term in Eq. (26).

In Model II, the Higgs field χ transforms as the four-dimensional representation of SU(4). The Higgs field χ can be represented by a 4×1 column matrix. In this model the Higgs potential also has the form given in Eq. (26), except that the λ_3 term is not an independent invariant in this case, and it cancels out. Note that there are only eight real Higgs degrees of freedom in model II [since model II has χ transforming in the complex **4** representation of SU(4)]. Seven of these degrees of freedom are absorbed by the seven new gluons which have become massive. The remaining physical Higgs field gains mass from the Higgs potential and will mix with the SM physical Higgs boson.

In summary, we have extended the SM gauge group to $SU(4)\otimes SU(2)_L \otimes U(1)_{y'}$. Gauge anomaly cancellation is used to fix the $U(1)_{y'}$ charges of the fermion generation. An exotic Higgs field χ is added to the ordinary Higgs-doublet field ϕ. The exotic Higgs field plays the dual role of giving the exotic $SU(3)_c$ neutral quarks a large mass, as well as breaking the strong-interaction group down to $SU(3)_c$. The exotic Higgs field is analogous to the familiar SM Higgs doublet ϕ. This Higgs field also plays a dual role, by giving the ordinary quarks and leptons their masses, as well as breaking the electroweak gauge group down to $U(1)_Q$. The two Higgs fields ϕ and χ break the gauge group of the theory to $SU(3)_c \otimes U(1)_Q$ which remains unbroken. The model has the interesting feature that the electric charge generator is embedded in all three simple factor groups of the semisimple gauge group $SU(4)\otimes SU(2)_L \otimes U(1)_{y'}$.

The model has only a few additional parameters, the most important of these is the scale of SU(4)-symmetry breaking defined by the VEV of $\chi(=w)$. We expect that $w>u$, since no charge-one-half particles have been discovered. This is only an expectation, Yukawa couplings it seems, may be large (cf., M_t/u). The experimental limits on the additional exotic gluons (both charged and the uncharged Z' boson), are not expected to be very stringent. For instance the stringent experimental constraints on flavor-changing neutral currents will be satisfied because the model retains the Glashow-Iliopoulos-Maiani (GIM) mechanism of the SM. We also believe that the model contains a degree of uniqueness, much like that of the standard model.

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[1]C. Q. Geng and R. E. Marshak, Phys. Rev. D **39**, 693 (1989); R. Foot, G. C. Joshi, H. Lew, and R. R. Volkas, Mod. Phys. Lett. (to be published); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **63**, 938 (1989); J. A. Minahan, P. Ramond, and R. C. Warner, University of Florida Report No. UFIFT-HEP-89-16 (unpublished); X.-G. He, G. C. Joshi, and B. H. McKellar, University of Melbourne Report No. UM-P-89/68, 1989 (unpublished); R. Foot, University of Wisconsin Report No. MAD/TH/89-6 (unpublished).

[2]A. Zee, Phys. Lett. **84B**, 91 (1979).

[3]E. Witten, Phys. Lett. **117B**, 324 (1982).

[4]L.-F. Li, Phys. Rev. D **9**, 1723 (1974).