RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 40, NUMBER 9 1 NOVEMBER 1989

Spin-dependent cross sections of weakly interacting massive particles on nuclei

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(Received 30 May 1989)

We use measured nuclear magnetic moments together with some very general assumptions about nuclear wave functions to estimate the spin-dependent elastic cross sections of supersymmetric dark-matter particles on a variety of nuclei. Our cross sections are nearly always smaller than the corresponding single-particle estimates. In light nuclei with mirror partners, we make use of β-decay ft values in addition to magnetic moments and obtain even more accurate results.

Evidence from a variety of sources indicates that most matter in the Universe is not luminous.[1] Among the more popular dark-matter candidates is a class of supersymmetric "weakly interacting massive particles" (WIMP's) that includes photinos, Higgsinos, and Z-inos. The particular linear combination of these neutral objects that forms the lightest supersymmetric particle[2] (LSP) will be stable and (if it constitutes the bulk of dark matter) abundant. Attempts to detect the elastic scattering of LSP's are underway[3] and require calculations of expected cross sections so that conclusions about dark-matter flux or the absence thereof can be drawn.

LSP-nucleus cross sections depend on a variety of quantities at three or four scales. The precise composition of the LSP is a function of several unknowns, e.g., Higgs vacuum expectation values and supersymmetry-breaking parameters. The interaction of LSP's (no matter what their composition) with nucleons depends on the distribution of nucleonic spin among the quarks and gluons. Finally, because the scattering involves the entire nucleus, the cross section is strongly affected by *its* structure. The last dependence, which we take up here, is not totally beyond our ability to address and indeed has been estimated in the extreme single-particle model[4] and, for a few tractable nuclei, computed in the shell model[5] (also, see Ref. 6 for earlier, more general discussions). Unfortunately, in most heavy elements reliable shell-model calculations are difficult and single-particle arguments suspect. Here we approach the problem in a new way, using measured magnetic moments to estimate the elastic cross sections more accurately.

We will limit ourselves to the axial-vector part of the LSP-nucleus interaction; a scalar contribution also exists[7,8] but is proportional to the nuclear mass, and can be calculated without explicit nuclear wave functions. The spin-dependent (axial vector) amplitude for scattering of LSP's from nuclei (N) reduces in the nonrelativistic limit to the form[4,7]

$$M = A\langle N|a_p\mathbf{S}_p + a_n\mathbf{S}_n|N\rangle\cdot \mathbf{s}_{\mathrm{LSP}}\,, \tag{1}$$

where

$$\mathbf{S}_i = \sum_k \mathbf{s}_i(k),\quad i=p,n \tag{2}$$

is the total spin operator and a_p, a_n depend on the quark wave functions in the protons and neutrons and on the composition of the LSP. The quantity A in (1) lumps together coupling constants, mixing parameters for the LSP, and masses of bosons that are exchanged in the scattering. The Wigner-Eckart theorem implies that the matrix elements of both $\mathbf{S}_p$ and $\mathbf{S}_n$ are proportional to those of the total nuclear angular momentum $\mathbf{J}$, and the amplitude (1) is often presented in the form[9]

$$M = A\lambda\langle N|\mathbf{J}|N\rangle\cdot \mathbf{s}_{\mathrm{LSP}}\,, \tag{3}$$

where

$$\lambda = \frac{\langle N|(a_p\mathbf{S}_p + a_n\mathbf{S}_n)|N\rangle}{\langle N|\mathbf{J}|N\rangle} = \frac{\langle N|(a_p\mathbf{S}_p + a_n\mathbf{S}_n)\cdot\mathbf{J}|N\rangle}{J(J+1)}\,, \tag{4}$$

so that the cross section is proportional to $\lambda^2J(J+1)$ and the nuclear physics lumped together with a_p and a_n in the factor λ. We quote as an example the photino-nucleus cross section[4]

$$\sigma = \frac{4m_{\tilde\gamma}^2 m_N^2}{\pi(m_{\tilde\gamma}+m_N)^2}\left(\frac{e^2}{m_{\tilde q}^2}\right)^2\lambda^2 J(J+1)\,,$$

where $\tilde\gamma$ is a photino, $\tilde q$ is the exchanged squark, and in evaluating λ,

$$a_i = \sum_q Q_q^2\Delta_i q,\quad i=p,n\,.$$

Here Q_q is the charge of quark-type q, and $\Delta_i q$ is the fractional spin it carries in the proton or neutron. The latter quantity is either taken from a model or extracted from experiment.[4] The nonrelativistic quark model yields $a_p = 0.40$, $a_n = -0.01$ (note that the neutron contributes almost nothing), while a European Muon Collaboration measurement implies $a_p = 0.24$, $a_n = -0.17$. Extension of all of these results to Higgsinos, Z-inos, and linear combinations thereof is straightforward.[7]

Only nuclei with an odd number of either protons or neutrons can have nonzero spin. Recently, full-scale nuclear shell-model calculations of λ were performed in certain of the lighter odd-even isotopes.[5] Thus far, however, attempts to evaluate λ in heavier nuclei have relied on the single-particle model, in which only the last odd nucleon contributes to $\mathbf{S}$ and $\mathbf{J}$. In this limit,

$$\lambda_{\mathrm{s.p.}} = \tfrac{1}{2}a_{\mathrm{odd}}\left(1+\frac{\frac{3}{4}-l(l+1)}{j(j+1)}\right) = a_{\mathrm{odd}}\frac{s_{\mathrm{odd}}}{j}\,, \tag{5}$$

where j and l are the single-particle total and orbital angular momenta and the subscript "odd" refers to whichever kind of nucleon is unpaired. The quantity $s_{\rm odd}$ is the expectation value of the single-particle spin of the last odd nucleon. In accordance with convention and everywhere in what follows, we will evaluate the z components of vector operators in the maximum m_j state; in the equation above, for example, $s_{\rm odd}=\langle j,j\,|\,s_{z_{\rm odd}}\,|\,j,j\rangle$.

While (5) is often an adequate lowest-order estimate, an examination of nuclear magnetic moments indicates that in many nuclei λ will be quite different. Ignoring for the moment the role of mesons in the nucleus, we can write the magnetic dipole operator in the form

$$\boldsymbol{\mu}=\boldsymbol{\mu}_p+\boldsymbol{\mu}_n=(g_p^l\mathbf{L}_p+g_p^s\mathbf{S}_p)+(g_n^l\mathbf{L}_n+g_n^s\mathbf{S}_n)\,, \qquad (6)$$

where the free-nucleon g factors are

$$g_p^l=1,\ g_n^l=0,\ g_p^s=5.586,\ g_n^s=-3.826\,. \qquad (7)$$

The abundant data on magnetic moments provide a test of the single-particle estimates of $\mathbf{S}_p$ and $\mathbf{S}_n$ that enter λ. Near closed shells, the model predictions are usually (though not always) fairly accurate, but further away they tend to overestimate the spin contribution to magnetic moments. In an open shell, the last odd particle will polarize the others of the sample type in the direction opposite its own spin, resulting in a spin-quenching effect that is entirely absent from the single-particle picture.

Suppose, however, that some of the assumptions underlying the single-particle model are relaxed. If, for instance, we abandon the requirement that only the *last* odd nucleon is important, while still retaining the idea that the even system of nucleons (protons in odd-neutron nuclei and vice versa) carries little angular momentum, we are on much firmer ground; a variety of arguments and data support this less inclusive claim.[10,11] Furthermore, a direct relation between the total spin of the odd system and the nuclear magnetic moment will then obtain. Introducing the subscripts "odd" and "even," each referring either to protons or neutrons in a given odd-even nucleus, we have[12]

$$\mu=g_{\rm odd}^lJ+(g_{\rm odd}^s-g_{\rm odd}^l)S_{\rm odd}+(g_{\rm even}^l-g_{\rm odd}^l)J_{\rm even}+(g_{\rm even}^s-g_{\rm even}^l)S_{\rm even}\,, \qquad (8)$$

where now all vectors [like $s_{\rm odd}$ in Eq. (5)] are represented by the expectation values of their z components in the maximum m_j state of the nucleus. If, in accordance with our assumption, $J_{\rm even}$ and $S_{\rm even}$ above are both small, then

$$S_{\rm odd}\approx\frac{\mu-g_{\rm odd}^lJ}{g_{\rm odd}^s-g_{\rm odd}^l}\,. \qquad (9)$$

Thus, in this variant of the "odd-group" model,[13] the measured magnetic moment directly determines $S_{\rm odd}$ and λ, which is now given by

$$\lambda=\frac{a_pS_p+a_nS_n}{J}\approx\frac{a_{\rm odd}S_{\rm odd}}{J}\,. \qquad (10)$$

The results of Eq. (9) for a variety of candidate nuclei are displayed in Table I, alongside the single-particle predictions. In some cases, the differences are substantial. The

TABLE I. Estimates for the odd-system spin in candidate detector nuclei (the even-system spin is taken to be 0). The third column is the odd-group result discussed in the text. The fourth column is the single-particle estimate. The top and bottom parts of the table contain odd-proton and odd-neutron nuclei, respectively.

	J	$S_{\rm odd}$	$s_{\rm odd}$ (s.p.)
^{7}Li	$\frac{3}{2}$	0.38	0.50
^{11}B	$\frac{3}{2}$	0.26	0.50
^{15}N	$\frac{1}{2}$	−0.17	−0.17
^{19}F	$\frac{1}{2}$	0.46	0.50
^{27}Al	$\frac{5}{2}$	0.25	0.50
^{35}Cl	$\frac{3}{2}$	−0.15	−0.30
^{51}V	$\frac{7}{2}$	0.36	0.50
^{69}Ga	$\frac{3}{2}$	0.11	0.50
^{71}Ga	$\frac{3}{2}$	0.23	0.50
^{75}As	$\frac{3}{2}$	−0.01	0.50
^{79}Br	$\frac{3}{2}$	0.13	0.50
^{81}Br	$\frac{3}{2}$	0.17	0.50
^{93}Nb	$\frac{9}{2}$	0.36	0.50
^{107}Ag	$\frac{1}{2}$	−0.13	−0.17
^{109}Ag	$\frac{1}{2}$	−0.14	−0.17
^{127}I	$\frac{5}{2}$	0.07	0.50
^{133}Cs	$\frac{7}{2}$	−0.20	−0.39
^{139}La	$\frac{7}{2}$	−0.16	−0.39
^{203}Tl	$\frac{1}{2}$	0.24	0.50
^{205}Tl	$\frac{1}{2}$	0.25	0.50
^{3}He	$\frac{1}{2}$	0.56	0.50
^{9}Be	$\frac{3}{2}$	0.31	0.50
^{17}O	$\frac{5}{2}$	0.49	0.50
^{29}Si	$\frac{1}{2}$	0.15	0.50
^{47}Ti	$\frac{5}{2}$	0.21	0.50
^{49}Ti	$\frac{7}{2}$	0.29	0.50
^{67}Zn	$\frac{5}{2}$	−0.23	−0.36
^{73}Ge	$\frac{9}{2}$	0.23	0.50
^{91}Zr	$\frac{5}{2}$	0.34	0.50
^{111}Cd	$\frac{1}{2}$	0.16	0.50
^{113}Cd	$\frac{1}{2}$	0.16	0.50
^{115}Sn	$\frac{1}{2}$	0.24	0.50
^{117}Sn	$\frac{1}{2}$	0.26	0.50
^{199}Hg	$\frac{1}{2}$	−0.13	−0.17
^{207}Pb	$\frac{1}{2}$	−0.15	−0.17

changes in the WIMP-nucleus cross section scale like the square of the ratio of the two columns (a_{odd} cancels out), so the effects of tying the nuclear spin to measured quantities are clearly not negligible.

At this point, it is important to note that while it improves substantially on single-particle estimates, the odd-group picture is not perfect. The even-system angular momenta S_{even} and J_{even}, though small, do not in fact vanish identically. Furthermore, meson-exchange currents can renormalize the g factors entering (8) and (9). Completely reliable calculations of all these effects are in general difficult to carry out.[14] Fortunately, the existence of "mirror pairs" in the mass range $A<50$ allows the use of two experimentally measured magnetic moments and a Gamow-Teller transition to isolate corrections to the odd-group results in the β-stable partner.[12,15] For these nuclei, the quantity $S_{odd}-S_{even}$ is directly related to the β-decay ft value:

$$R^2(S_{odd}-S_{even})^2=\left[\frac{6170}{ft}-1\right]\frac{J}{J+1}, \quad (11)$$

where $R=g_A/g_V$ is the ratio of axial-vector–to–vector coupling constants. In addition, there is a close relation between the sum of the two mirror magnetic moments (the isoscalar moment) and $S_{odd}+S_{even}$. Pion exchange currents are isovector in character, and so we need consider only the small Mayer-Jensen correction μ_x to μ_p induced by heavy meson exchange.[16] Because J_{even} in (8) cancels in the isoscalar sum, we have

$$\begin{aligned}\mu_{IS}&=(g^l_{odd}+g^l_{even})J\\&\quad+(g^s_{odd}-g^l_{odd}+g^s_{even}-g^l_{even})(S_{odd}+S_{even})+\mu_x\\&=J+0.76(S_{odd}+S_{even})+\mu_x\,, \quad (12)\end{aligned}$$

where IS means isoscalar. Equations (11) and (12) contain four unknowns: S_{odd}, S_{even}, R, and μ_x. The last of these is quite small, and can be estimated from theory.[15,16] Attempts to isolate the quantity R, in effect the renormalization of the axial-vector coupling in nuclei, have a long history.[15] Despite all the data in mirror nuclei, model-dependent assumptions must at some point still enter the analysis. Here we follow the work of Ref. 12, in which a fit to a large number of mirror pairs led to an average value $R=1.00\pm0.02$. This result, applied uniformly to all the mirror nuclei and combined with theoretical values for μ_x (which we take from Ref. 15), allows us to extract S_{odd} and S_{even}; we display them, where the data allow their determination, in Table II. Note that S_{even} is in fact quite small, in accordance with our original more naive assumption reflected in Table I. Since the precise value of R is the chief uncertainty in our procedure, we devote an additional portion of Table II to results obtained with $R=1.25$, the free-neutron value; the relatively small changes in most of the numbers validate our approach. Between the two columns we have reproduced where available the shell-model results of Ref. 5. By

TABLE II. Improved estimates for the proton and neutron spin in all stable mirror nuclei for which data exist. The results of columns 3 and 6 were obtained by setting $R=1.00$, the preferred value in the nuclear medium. Columns 5 and 8 were obtained with $R=1.25$, the free-neutron value, to illustrate the low degree of sensitivity to changes in R. Columns 4 and 7 contain the results of Ref. 5 where full-scale shell-model calculations were performed. The top and bottom parts of the table contain odd-proton and odd-neutron nuclei, respectively.

			S_p			S_n	
	J	$R=1.00$	Ref. 5	$R=1.25$	$R=1.00$	Ref. 5	$R=1.25$
^{11}B	$\frac{3}{2}$	0.292	0.292	0.264	0.006	0.008	0.034
^{15}N	$\frac{1}{2}$	−0.145		−0.127	0.037		0.019
^{19}F	$\frac{1}{2}$	0.415	0.441	0.368	−0.047	−0.109	−0.001
^{27}Al	$\frac{5}{2}$	0.333		0.304	0.043		0.072
^{31}P	$\frac{1}{2}$	0.181		0.166	0.032		0.047
^{35}Cl	$\frac{3}{2}$	−0.094	−0.059	−0.083	0.014	−0.011	0.004
^{39}K	$\frac{3}{2}$	−0.196		−0.171	0.055		0.030
^{3}He	$\frac{1}{2}$	−0.081		−0.021	0.522		0.462
^{13}C	$\frac{1}{2}$	−0.009		−0.026	−0.172		−0.155
^{17}O	$\frac{5}{2}$	−0.036	0.000	0.019	0.508	0.500	0.453
^{21}Ne	$\frac{3}{2}$	0.020		0.047	0.294		0.266
^{25}Mg	$\frac{5}{2}$	0.040		0.073	0.376		0.343
^{29}Si	$\frac{1}{2}$	0.054		0.069	0.204		0.189

nuclear-physics standards the agreement between these numbers and the $R = 1.00$ column is excellent.

Shell-model calculations are difficult in the heavier mirror nuclei because of the size of the matrices involved. In ^{29}Si, an isotope of obvious practical importance, there is no shell-model result. Our analysis there, however, should be very reliable; the contribution of μ_x is small, and the result is relatively insensitive to changes in R. Because ^{29}Si is an open-shell nucleus, we are not surprised to find a large deviation from the single-particle estimate. This new result significantly alters calculated LSP-nucleus event rates in silicon detectors.

We have not evaluated λ because [see Eq. (10)] it contains the model-dependent coefficients a_p and a_n. It is worth pointing out, though, that photino scattering in the nonrelativistic quark model[4] will cause problems because of the large value of a_p relative to a_n. In odd-neutron nuclei S_p is small and, as a result, difficult to determine with any precision. The inevitable uncertainty will be amplified when multiplied by a_p.

In the mass region $A > 50$ there are no mirror pairs, and for now we must rely on the estimates in Table I, where the roles of even-system spin and meson-exchange currents are neglected. For relatively light WIMP's, the kinematics of recoil make heavier nuclei less desirable candidates, but detector technology may require their use. A reliable estimate in ^{73}Ge is therefore of clear importance. Surely, the odd-group results in Table I can be improved on, but this will necessarily involve complicated details of nuclear structure and meson exchange. In the meantime, these estimates stand as clear improvements on calculations that assume but one active nucleon.

The chief result of this paper is that spin-dependent cross sections are uniformly suppressed from single-particle estimates. Spin-independent scattering, explicated in Refs. 7 and 8, therefore becomes correspondingly more important. The confidence level of our results is related to the sensitivity presented in Table II, but is difficult to specify precisely. But in any case, uncertainties from unknown supersymmetry parameters and quark-gluon wave functions now dwarf those remaining in nuclear structure. Accelerator experiments can help us unravel supersymmetry. For the theorist, the nucleonic quantities a_p and a_n may now be the best subject for investigation.

This work was supported in part by the U.S. Department of Energy under Contract No. DE-F603-88ER-40397.

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