Parametrization of the semileptonic baryon matrix elements

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We present a general parametrization of the semileptonic decays of the baryons obtained, as for the magnetic moments, performing a unitary transformation that connects the exact states to the model states. We show that again the nonrelativistic-quark-model (NRQM) description amounts to selecting certain terms among those of the general parametrization; in the no-flavor-breaking approximation the parametrization contains two parameters (a and b) linearly related to the Cabibbo's D,F and associated, in the NRQM language, to the one-quark and two-quark terms. The NRQM amounts to keeping the largest (one-quark) term with a coefficient a ($\simeq 0.75$); b is 3–10 % of a. Finally, we clarify the variety of possible terms that appear on going beyond the flavor-symmetric approximation.

INTRODUCTION

In a previous paper[1] we have introduced a unitary transformation connecting the exact states of a hadron to its model states. We have shown that, using this transformation, the baryon magnetic moments or the masses calculated with a relativistic field theory (e.g., QCD) can be parametrized as a sum of many terms with different spin-flavor structures, each multiplied by some coefficient. We have also shown that the nonrelativistic quark model[2] (NRQM) can be interpreted as providing a selection of certain terms in the above complete parametrization and that for the baryon masses and magnetic moments the terms considered in the NRQM are the dominant ones.

Here we apply the method of Ref. 1 to the semileptonic matrix elements of the baryons and compare the general parametrization with the NRQM calculation. We will see that (1) in the approximation of no flavor breaking the general parametrization is of course equivalent to the Cabibbo parametrization; it contains only two parameters (equivalent to D and F); (2) in the NRQM treatment the smaller of these two parameters is absent, leading to $D/F=3/2$; (3) as to the terms breaking flavor, their number, already at the first order, is too large to make the parametrization useful; and (4) a by-product of the treatment is to clarify (once more) why the frequent statement that the NRQM produces a value $\frac{5}{3}$ of g_A/g_V for the nucleon is wrong.

The weak current containing the quark fields is, of course,

$$j_\mu^+(x)=\cos\theta\,\bar\psi(x)\tau^+\gamma_\mu(1+\gamma_5)\psi(x) + \sin\theta\,\bar\psi(x)\lambda^+\gamma_\mu(1+\gamma_5)\psi(x)\ , \tag{1}$$

where θ is the Cabibbo angle and $\tau^+\equiv\lambda_1+i\lambda_2$ and $\lambda^+\equiv\lambda_4+i\lambda_5$ (the λ_i's are the Gell-Mann matrices; λ_8, to be used later, is simply Diag$[1,1,-2]$ without the $1/\sqrt{3}$ in front). In (1) the quark fields

$$\psi\equiv\begin{bmatrix}\mathcal{P}\\ \mathcal{N}\\ \lambda\end{bmatrix}$$

are defined (see Ref. 1) as the renormalized fields; but the values of the quark renormalized masses will not intervene except for establishing the flavor-breaking parameter $\Delta m/m_\lambda$.

We now parametrize the matrix elements of $\int j_\mu^+(\mathbf{x},0)\exp(i\Delta\mathbf{P}\cdot\mathbf{x})d^3\mathbf{x}$ in the limit in which the four-momentum transfer $\Delta P_\mu=0$ and, in particular, the initial and final baryons A and B are both at rest; the parametrization of the semileptonic matrix elements to be given below (starting from an underlying relativistic field theory) will be general and relativistic, although noncovariant, exactly as that[1] of the magnetic moments.

In obvious notation the only nonvanishing matrix elements (with $\Delta P_\mu=0$) are

$$\langle 1\rangle_{\Delta S=0}=\left\langle B\uparrow\left|\int\bar\psi(\mathbf{x},0)\tau^+\gamma_4\psi(\mathbf{x},0)d^3\mathbf{x}\right|a\uparrow\right\rangle \equiv\langle B\uparrow|T^+|A\uparrow\rangle\ , \tag{2}$$

$$\langle 1\rangle_{\Delta S=1}=\left\langle B\uparrow\left|\int\bar\psi(\mathbf{x},0)\lambda^+\gamma_4\psi(\mathbf{x},0)d^3\mathbf{x}\right|A\uparrow\right\rangle \equiv\langle B\uparrow|L^+|A\uparrow\rangle\ , \tag{3}$$

and

$$\langle\sigma_z\rangle_{\Delta S=0}=\left\langle B\uparrow\left|\int\bar\psi(\mathbf{x},0)\tau^+\gamma_4\sigma_z\psi(\mathbf{x},0)d^3\mathbf{x}\right|A\uparrow\right\rangle\ , \tag{4}$$

$$\langle\sigma_z\rangle_{\Delta S=1}=\left\langle B\uparrow\left|\int\bar\psi(\mathbf{x},0)\lambda^+\gamma_4\sigma_z\psi(\mathbf{x},0)d^3\mathbf{x}\right|A\uparrow\right\rangle\ . \tag{5}$$

We now express the exact states $|A\rangle$ and $|B\rangle$ (each of which can be thought as a superposition of an infinite number of Fock states with quarks, antiquarks, and gluons) as

$$|A\rangle=V|\phi_A\rangle,\quad |B\rangle=V|\phi_B\rangle\ , \tag{6}$$

where V is the unitary operator transforming the three

quark-model states $|\phi_A\rangle, |\phi_B\rangle$ into the exact states $|A\rangle, |B\rangle$; both the model states and the operator V are defined as in (Ref. 1); recall that ϕ_A and ϕ_B are the usual $L=0$ wave functions of the octet baryons [Eq. (10) of Ref. 1]. Inserting (6) in (2) and (3) we have, for the Fermi matrix elements,

$$\langle 1\rangle_{\Delta S=0}=\langle \phi_B\uparrow|V^\dagger T^+ V|\phi_A\uparrow\rangle$$

$$=\left\langle \phi_B\uparrow\left|\sum_{i=1}^{3}\tau_i^+\right|\phi_A\uparrow\right\rangle , \tag{7}$$

where the last equality is due to the fact that V commutes with T^+ (because of isospin conservation) and $V^\dagger V=1$; for $\langle 1\rangle_{\Delta S=1}$ we have

$$\langle 1\rangle_{\Delta S=1}=\langle \phi_B\uparrow|V^\dagger L^+ V|\phi_A\uparrow\rangle$$

$$=\left\langle \phi_B\uparrow\left|\sum_{i=1}^{3}\lambda_i^+\right|\phi_A\uparrow\right\rangle+O((\Delta m/m_\lambda)^2) , \tag{8}$$

where the last passage is due to the Ademollo-Gatto theorem[3] (Fermi matrix elements receive only second-order corrections by flavor breaking due to the fact that L^+ is a generator of flavor symmetry).

We turn to the parametrization of the Gamow-Teller $\langle \sigma_z\rangle$ matrix elements; from (4)–(6) we have

$$\langle \sigma_z\rangle_{\Delta S=0}=\left\langle \phi_B\uparrow\left|V^\dagger\int \bar\psi(\mathbf{x},0)\tau^+\gamma_4\sigma_z\psi(\mathbf{x},0)d^3\mathbf{x}\,V\right|\phi_A\uparrow\right\rangle\equiv\langle \phi_B\uparrow|\Gamma_z^+|\phi_A\uparrow\rangle , \tag{9}$$

$$\langle \sigma_z\rangle_{\Delta S=1}=\left\langle \phi_B\uparrow\left|V^\dagger\int \bar\psi(\mathbf{x},0)\lambda^+\gamma_4\sigma_z\psi(\mathbf{x},0)d^3\mathbf{x}\,V\right|\phi_A\uparrow\right\rangle\equiv\langle \phi_B\uparrow|\Omega_z^+|\phi_A\uparrow\rangle . \tag{10}$$

We now summarize the main points of the procedure.

(a) After normal ordering and contraction of all the quark and gluon creation and destruction operators, $\mathbf{\Gamma}^+$ and $\mathbf{\Omega}^+$ defined in (9) and (10) are three-body operators, to be constructed in terms of the space-spin-flavor coordintaes of the three quarks that appear in the wave functions ϕ_A and ϕ_B.

(b) Because of the choice of the model states ϕ_A and ϕ_B (wave functions factorizable into a space-color antisymmetric factor times a symmetric spin-flavor factor) the most general structure of the axial-vector operators $\mathbf{\Gamma}^+$ and $\mathbf{\Omega}^+$, after integration on the space-color coordinates, is very simple, as shown in detail[1] for the analogous case of the magnetic moments.

We assume, as in Ref. 1, that the underlying field theory breaks flavor symmetry only via the mass difference between the quarks λ and $\mathcal{N},\mathcal{P}$ so that the only flavor dependence is proportional to the projection operator P^λ on the λ quark: $P^\lambda=(1-\lambda_8)/3$.

To construct all the $\mathbf{\Gamma}^+$'s and $\mathbf{\Omega}^+$'s in (9) and (10) we must write the most general expression of a three-quark axial vector, linear in the τ_i^+'s [Eq. (9)] or in the λ_i^+'s [Eq. (10)], containing a product of P_i^λ's, up to a maximum of three. Here we start with the case of no flavor breaking, where no P^λ appears in $\mathbf{\Gamma}^+$ or $\mathbf{\Omega}^+$. Later we consider first-order flavor breaking (one P^λ).

THE FLAVOR-SYMMETRICAL CASE

In this case the only axial vectors, linear in τ_i^+ or λ_i^+, that can be used to construct $\mathbf{\Gamma}^+$ or $\mathbf{\Omega}^+$ are

$$\mathbf{\Gamma}^+:\ \sum_{i=1}^{3}\tau_i^+\boldsymbol{\sigma}_i,\quad \sum_{\substack{i,k=1\\ i\neq k}}^{3}\tau_i^+\boldsymbol{\sigma}_k,\quad \sum_{\substack{i,k=1\\ i\neq k}}^{3}(\tau_i^+-\tau_k^+)(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k) , \tag{11}$$

$$\mathbf{\Omega}^+:\ \sum_{i=1}^{3}\lambda_i^+\boldsymbol{\sigma}_i,\quad \sum_{\substack{i,k=1\\ i\neq k}}^{3}\lambda_i^+\boldsymbol{\sigma}_k,\quad \sum_{\substack{i,k=1\\ i\neq k}}^{3}(\lambda_i^+-\lambda_k^+)(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k) . \tag{12}$$

In principle $\mathbf{\Gamma}^+$ can be a linear combination of the three operators (11) and $\mathbf{\Omega}^+$ a linear combination of the three operators (12). Note that each operator in (11) and (12) is symmetrical in the three quarks, as it must be because ϕ_A and ϕ_B are symmetrical in the spin-flavor variables. This is the reason why in (11) and (12) we have not included

$$\sum_{\substack{i,k,j=1\\ i\neq k\neq j}}^{3}\tau_j^+(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k)$$

or

$$\sum_{\substack{i,k,j=1\\ i\neq k\neq j}}^{3}\lambda_j^+(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k) .$$

We will now show that the terms in (11) and (12) containing $(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k)$ have vanishing matrix elements. In fact, with $2\mathbf{J}=\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3$, it is

$$\sum_{\substack{i,k=1\\ i\neq k}}^{3}(\tau_i^+-\tau_k^+)(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k)=\sum_{i=1}^{3}\tau_i^+\boldsymbol{\sigma}_i\times(2\mathbf{J})-(2\mathbf{J})\sum_{i=1}^{3}\tau_i^+\boldsymbol{\sigma}_i \tag{13}$$

and it is straightforward to check that the matrix element of the right-hand side of (13) between the two $J=\frac{1}{2}$ states ϕ_A and ϕ_B vanishes for any A,B. A similar conclusion holds, of course, for

$$\sum_{\substack{i,k=1\\ i\neq k}}^{3}(\lambda_i^+-\lambda_k^+)(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k) .$$

Thus in the flavor-symmetric case the most general $\mathbf{\Gamma}^+$ and $\mathbf{\Omega}^+$ are

$$\mathbf{\Gamma}^+=a\sum_{i=1}^{3}\tau_i^+\boldsymbol{\sigma}_i+b\sum_{\substack{i,k=1\\ i\neq k}}^{3}\tau_i^+\boldsymbol{\sigma}_k , \tag{14}$$

$$\Omega^+ = a\sum_{i=1}^{3}\lambda_i^+\sigma_i + b\sum_{\substack{i,k=1\\ i\neq k}}^{3}\lambda_i^+\sigma_k \ , \tag{15}$$

where a and b are two parameters. Note the following: The coefficients a and b in (14) and (15) *must be the same* because in the flavor-symmetrical approximation the operator V appearing in the (middle) expressions (9) and (10) is the same and is flavor independent; no matter how complicated, V acts in the same way on $\bar{\psi}(\mathbf{x},0)\tau^+\gamma_4\sigma\psi(\mathbf{x},0)$ and on $\bar{\psi}(\mathbf{x},0)\lambda^+\gamma_4\sigma\psi(\mathbf{x},0)$.

The above can be thus summarized by the statement that in the flavor-symmetric approximation the most general expressions of $\langle 1\rangle$ and $\langle\sigma_z\rangle$ are

$$\langle 1\rangle_{\Delta S=0} = \left\langle \phi_B\uparrow \left| \sum_{i=1}^{3}\tau_i^+ \right| \phi_A\uparrow \right\rangle \ ,$$
$$\langle 1\rangle_{\Delta S=1} = \left\langle \phi_B\uparrow \left| \sum_{i=1}^{3}\lambda_i^+ \right| \phi_A\uparrow \right\rangle \ , \tag{16}$$

$$\langle\sigma_z\rangle_{\Delta S=0} = \left\langle \phi_B\uparrow \left| a\sum_{i=1}^{3}\tau_i^+\sigma_{zi} + b\sum_{\substack{i,k=1\\ i\neq k}}^{3}\tau_i^+\sigma_{zk} \right| \phi_A\uparrow \right\rangle$$
$$\equiv \left\langle \phi_B\uparrow \left| (a-b)\sum_{i=1}^{3}\tau_i^+\sigma_{zi} + b\sum_{i=1}^{3}\tau_i^+(2J_z) \right| \phi_A\uparrow \right\rangle, \tag{17}$$

$$\langle\sigma_z\rangle_{\Delta S=1} = \left\langle \phi_B\uparrow \left| a\sum_{i=1}^{3}\lambda_i^+\sigma_{zi} + b\sum_{\substack{i,k=1\\ i\neq k}}^{3}\lambda_i^+\sigma_{zk} \right| \phi_A\uparrow \right\rangle$$
$$\equiv \left\langle \phi_B\uparrow \left| (a-b)\sum_{i=1}^{3}\lambda_i^+\sigma_{zi} + b\sum_{i=1}^{3}\lambda_i^+(2J_z) \right| \phi_A\uparrow \right\rangle. \tag{18}$$

Note that $(2J_z)$ in the second form of (17) and (18) can be replaced by 1 (since the functions $\phi_A\uparrow$ and $\phi_B\uparrow$ have $J_z=\frac{1}{2}$); we have left it as $(2J_z)$ only to make clear the passage from the first to the second form.

It should be stressed that in spite of the noncovariant appearance of (16)–(18), they are fully relativistic (they have been derived from a relativistic field theory in the limit $\Delta P_\mu=0$) and general (in the flavor-symmetric approximation) because, as shown, no other term can be present.

It is now straightfoward to relate the parameters a and b to D and F of the Cabibbo parametrization. It is sufficient to consider two independent matrix elements and express them in terms of a,b or D,F. For the $N\to P$ matrix elements it is

$$\langle 1\rangle_{NP}=1 \ ,$$
$$\langle\sigma_z\rangle_{NP}=\tfrac{5}{3}(a-b)+b=\tfrac{5}{3}a-\tfrac{2}{3}b=D+F \ . \tag{19}$$

For $\Sigma^-\to\Lambda$ we have

$$\langle 1\rangle_{\Sigma^-\Lambda}=0, \quad \langle\sigma_z\rangle_{\Sigma^-\Lambda}=\sqrt{2/3}(a-b)=\sqrt{2/3}D \tag{20}$$

and thus

$$a=\tfrac{1}{3}D+F, \quad b=F-\tfrac{2}{3}D \tag{21a}$$

or

$$D=a-b, \quad F=\tfrac{2}{3}a+\tfrac{1}{3}b \ . \tag{21b}$$

The physical meaning of a and b is very direct: a is the coefficient of an expression additive in the quarks, whereas b is the coefficient of an expression containing at the same time the variables of two quarks. As already stated in connection with the magnetic moments, the NRQM, in its simplest form, amounts to selecting, among all the possible terms, only those additive in the coordinates of the individual quarks; here this implies to put $b=0$ in (17) and (18); indeed, $b=0$ corresponds to the well-known NRQM result $D/F=3/2$. Note also that the NRQM approximation ($b=0$) implies $(g_A/g_V)_{NP}=\frac{5}{3}a$; the frequent assertion that the NRQM predicts $(g_A/g_V)_{NP}=\frac{5}{3}$ is incorrect; as remarked long ago[4] $\frac{5}{3}$ is a result of abstract SU(6), not of the NRQM.

The value of D/F depends to some extent on how the experimental data are analyzed, in particular on whether the Cabibbo angle is fixed by the same hyperon semileptonic decays[5] or is determined differently (e.g., as in Ref. 6, from the $0^+\to0^+$ β transitions). In the first case[5] ($\sin\theta_C=0.231\pm0.003$) one gets $D=0.756\pm0.011$, $F=0.477\pm0.012$ and $D/F=1.58\pm0.04$; one has a fit to the hyperon decays that does not show appreciable SU(3) violations; in the second case[6] (with $\sin\theta_C=0.225\pm0.002$) the overall fit requires $D/F=1.74\pm0.04$ and an appreciable amount of flavor breaking is needed to have a reasonably good fit to all the data. We shall discuss below the question of the flavor breaking; here we note only the values of the ratio b/a corresponding to the values of D/F stated above. It is $b/a=[1-(2D/3F)]/[1+(D/3F)]$ and therefore for $D/F=1.58$ one has $b/a=-0.034$, whereas for $D/F=1.74$ it is $b/a=-0.10$. In both cases b is much smaller than a, from $\approx3\%$ to $\approx10\%$. Thus the "two-quark term" (that with coefficient b) in (17) and (18) is depressed with respect to the "additive" one-quark term (with coefficient a) as it happens for the magnetic moments.[1] This is another confirmation, additional to those in Ref. 1, that the additive terms prevail over the two-quark ones.

FIRST-ORDER FLAVOR BREAKING

We now extend the above treatment [Eqs. (16)–(18)] to take into account flavor breaking to first order. Nothing changes, to first order, for the vector current ($\langle 1\rangle$ matrix elements). On the other hand, for the axial-vector current ($\langle\sigma_z\rangle$ matrix elements) a variety of new possible terms appear. We list them, noting that for first-order flavor breaking only one P^λ operator can appear.

$\Delta S=0$: in addition to $\sum_i\tau_i^+\sigma_i, \sum_{k\neq i}\tau_i^+\sigma_k$, the matrix elements of the following operators appear in the parametrization of $\langle\sigma_z\rangle_{\Delta S=0}$ to first order:

$$(1)\ \sum_i \tau_i^+ \sigma_i P_k^\lambda,\quad (2)\ \sum_{i\neq k} \tau_i^+ \sigma_k P_k^\lambda,$$
$$(3)\ \sum_{i\neq k\neq j} \tau_i^+ \sigma_k P_j^\lambda . \tag{22}$$

$\Delta S=1$: In addition to $\sum_i \lambda_i^+ \sigma_i, \sum_{k\neq i} \lambda_i^+ \sigma_k$ the matrix elements of the following operators appear:

$$(1)\ \sum_i \lambda_i^+ \sigma_i P_i^\lambda,\quad (2)\ \sum_{i\neq k} \lambda_i^+ \sigma_k P_i^\lambda,$$
$$(3)\ \sum_{i\neq k} \lambda_i^+ \sigma_i P_k^\lambda,\quad (4)\ \sum_{i\neq k} \lambda_i^+ \sigma_k P_k^\lambda, \tag{23}$$
$$(5)\ \sum_{i\neq k\neq j} \lambda_i^+ \sigma_k P_j^\lambda$$

plus terms containing $(\sigma_i \times \sigma_k)$.

Clearly the number of parameters, if we take into account all the above terms (without knowing their coefficients from, say, QCD), is too large to make a general parametrization with first-order flavor breaking of interest; the no-flavor-breaking approximation is the only case where a parametrization exists with few enough parameters to have predictive power (Cabibbo parametrization). One might try to reduce the number of flavor-breaking terms by assuming that the contributions of the "additive" terms among (22) and (23) are more important than the remaining ones, and neglecting the two-quark terms; then only the term $\sum_i \lambda_i^+ \sigma_i P_i^\lambda$ in the $\Delta S=1$ matrix element would remain and just one more parameter would intervene: namely, that (call it Δa) multiplying $\sum_i \lambda_i^+ \sigma_i P_i^\lambda \equiv \sum_i \lambda_i^+ \sigma_i$. In this case (16) and (17) do not change, whereas (18) changes into

$$\langle \sigma_z \rangle_{\Delta S=1} = \left\langle \phi_B \uparrow \left| (a+\Delta a) \sum_{i=1}^{3} \lambda_i^+ \sigma_{zi} + b \sum_{\substack{i,k=1 \\ i\neq k}}^{3} \lambda_i^+ \sigma_{zk} \right| \phi_A \uparrow \right\rangle , \tag{24}$$

where we expect Δa to be of the order $\approx a(\Delta m / m_\lambda)$. However, if the flavor-breaking effect that fits the data amounts, as suggested in Ref. 6, to a difference between a value of $(D/F)_{\Delta S=0}$ ($\simeq 1.34$) and $(D/F)_{\Delta S=1}$ ($\simeq 1.82$), then the introduction of Δa along cannot produce such a difference. The reason is that if $b=0$, the value of D/F is always $\frac{3}{2}$, independently of the value of a; now b is not zero, but is very small as seen above; thus a comparatively small change in a (by Δa) cannot produce the necessary change in D/F. Then the flavor breaking must be due also to a difference in b between the $\Delta S=0$ and $\Delta S=1$ transitions and possibly also to the presence (with coefficients some percent of a) of many other terms listed in (23). At this stage we do agree with Ref. 5 that the possibilities are too many to make a more detailed discussion fruitful, although we agree with Ref. 6 that the data seem to indicate effects of flavor breaking (and that there is no reason why such effects should be absent or totally negligible in this case).

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[4]Among the many references where I stressed this point, compare G. Morpurgo, in *Theory and Phenomenology in Particle Physics,* proceedings of the International School of Physics "Ettore Majorana," Erice, Italy, 1968, edited by A. Zichichi (Academic, New York, 1969), pp. 83–217, in particular, p. 126.

[5]J. M. Gaillard and G. Sauvage, Annu. Rev. Nucl. Particle Sci. **34**, 351 (1984).

[6]J. F. Donoghue and B. R. Holstein, Phys. Rev. D **25**, 2015 (1982) [we have preferred to give the values of D/F rather than of $\alpha_D = D/(D+F)$ as done in the paper by Donoghue and Holstein]. Compare also A. Garcia and P. Kielanowsky, Phys. Lett. **110B**, 498 (1982). The numbers quoted in the text are from the first of these references, but the analysis in the second reference (particularly the fit B) is similar.