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Brief Reports

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Critique of parton-model calculations of azimuthal dependence in leptoproduction

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In leptoproduction the outgoing hadron jet has an asymmetry about the direction of the momentum transfer of the leptons, measured relative to the plane containing the lepton momenta. In the naive parton model the asymmetry arises from the transverse momentum of the constituent quarks. In calculating the cross section, including the asymmetry, one must not simply take the sum of the cross sections on the individual quarks, but instead follow a slightly different procedure. Failure to do so produces not only the wrong answer, but an answer inconsistent with Lorentz invariance.

Feynman and Bjorken and co-workers used the parton model to explain the then-emerging data on electroproduction. Treating the proton as if it were made of free quarks explained the observed Bjorken scaling: the structure functions depended nearly entirely on $x=Q^2/2M\nu$ rather than on Q^2 and ν separately. In succeeding years the parton model was justified and refined by asymptotically free quantum chromodynamics. More detailed experiments with electrons, muons, and neutrinos confirmed the anticipated variation of the structure function on Q^2 at fixed x. These effects typically have logarithmic dependence on Q^2. Effects that vanish as M^2/Q^2 are important for moderate values of Q^2 and such effects can arise in the Feynman-Bjorken model. The best known of these is the ratio of the longitudinal to transverse cross section:[1]

$$R=\frac{\sigma_L}{\sigma_T}=\frac{4(m^2+\langle p_\perp^2\rangle)}{Q^2}\ , \tag{1}$$

where $\langle p_\perp^2\rangle$ is the mean of the transverse momentum squared of the partons and m is the parton mass. The ratio is zero in the standard limit of $Q^2\to\infty$, but cannot be ignored for moderate Q^2. While corrections of order m^2/Q^2 are generally suspect in parton-model calculation since often the four-momentum of a parton is simply taken to be $p=xP$, where P is the nucleon momentum, relations such as that for R are more credible because they are for quantities that otherwise vanish.

Recent experimental results[2–4] have renewed interest in the azimuthal asymmetry in leptoproduction. In a naive parton model with quarks collinear with the proton, the azimuthal asymmetry vanishes. However, there are contributions from the transverse momenta of these partons[5,6] as well as from gluon-induced QCD corrections.[7,8]

The azimuthal asymmetry is defined by working in a frame in which the proton is collinear with the momentum transfer from the lepton. See Fig. 1. Thus for a charged-current weak interaction the proton and virtual W collide head on. The collision knocks out a parton, which produces a hadronic jet. In the plane transverse to the axis established by the proton–virtual-W direction an azimuthal angle ϕ is defined between the incident-lepton direction and the final-hadron-jet direction. It is the dependence of the differential cross section ϕ of that is of interest here.

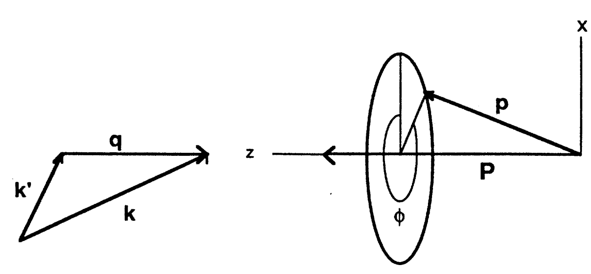

FIG. 1. Electron-parton scattering in the center of mass of the virtual photon (or W or Z) and the target proton. The incident electron momentum **k** and the final electron momentum **k**′ lie in the x-z plane. The momentum transfer **q**=**k**−**k**′ defines the negative z axis. The parton initially has momentum **p** with z component equal to a fraction, x of the proton momentum **P**. The transverse component of the parton momentum makes an azimuthal angle ϕ with the x axis in the x-y plane. The probability of scattering depends on ϕ since the invariants $\hat{s}$ and $\hat{u}$ are ϕ dependent.

In Ref. 6 it is shown that azimuthal dependence arises in a very simple way. The matrix element for neutrino-quark charged-current scattering is proportional to $\hat{s}$ while that for antineutrino-quark scattering is proportional to $-\hat{u}$ where

$$\hat{s}=(k+p)^2,\quad \hat{u}=(k'-p)^2\ . \tag{2}$$

Here the initial- and final-lepton momenta are k and k', and the initial- and final-quark momenta are p and p'. In the target rest frame with the z axis along the momentum-transfer direction, in the limit $Q \ll \nu$ ($\nu = E - E'$ is the energy lost by the lepton in the target rest frame),

$$\begin{aligned} k &\approx \left[E, \frac{Q}{\nu}\sqrt{EE'}, 0, E - \frac{Q^2E'}{2\nu^2}\right], \\ k' &\approx \left[E', \frac{Q}{\nu}\sqrt{EE'}, 0, E' - \frac{Q^2E}{2\nu^2}\right]. \end{aligned} \tag{3}$$

This is not a good frame for doing the calculation. If we boost along the z axis until the proton has a large momentum P, the parton's momentum can be written to sufficient accuracy as

$$p = xP + p_\perp\ , \tag{4}$$

with

$$p_\perp = (0, p_\perp\cos\phi, p_\perp\sin\phi, 0)\ . \tag{5}$$

The azimuthal angle ϕ of the initial parton will be the azimuthal angle of the outgoing parton since the momentum transfer is along the z axis. Now it is easy to see that[9]

$$\begin{aligned} \hat{s} &= 2k\cdot p = 2xk\cdot P + 2k\cdot p_\perp \\ &= 2MEx\left[1 - \frac{2p_\perp}{Q}\sqrt{1-y}\,\cos\phi\right], \\ -\hat{u} &= 2k'\cdot p = 2xk'\cdot P + 2k'\cdot p_\perp \\ &= 2ME'x\left[1 - \frac{2p_\perp}{Q\sqrt{1-y}}\cos\phi\right], \end{aligned} \tag{6}$$

where $y = \nu/E$. It follows that

$$\begin{aligned} \sigma_\nu &\propto \left[1 - \frac{2p_\perp}{Q}\sqrt{1-y}\,\cos\phi\right]^2, \\ \sigma_{\bar\nu} &\propto \left[1 - \frac{2p_\perp}{Q\sqrt{1-y}}\cos\phi\right]^2, \\ \sigma_{ep} &\propto \hat{s}^2 + \hat{u}^2 \propto \left[1 - \frac{2p_\perp}{Q}\sqrt{1-y}\,\cos\phi\right]^2 \\ &\qquad + (1-y)^2\left[1 - \frac{2p_\perp}{Q\sqrt{1-y}}\cos\phi\right]^2. \end{aligned} \tag{7}$$

We have assumed that only quarks, not antiquarks participate in the neutrino scattering. For the asymmetries then

$$\begin{aligned} \langle\cos\phi\rangle_{\nu p} &= -\frac{2p_\perp}{Q}\sqrt{1-y}\ , \\ \langle\cos\phi\rangle_{\bar\nu p} &= -\frac{2p_\perp}{Q\sqrt{1-y}}\ , \\ \langle\cos2\phi\rangle_{\nu p} &= \frac{p_\perp^2}{Q^2}(1-y)\ , \\ \langle\cos2\phi\rangle_{\bar\nu p} &= \frac{p_\perp^2}{Q^2(1-y)}\ , \\ \langle\cos\phi\rangle_{ep} &= -\left[\frac{2p_\perp}{Q}\right]\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}\ , \\ \langle\cos2\phi\rangle_{ep} &= \frac{2p_\perp^2}{Q^2}\frac{1-y}{1+(1-y)^2}\ . \end{aligned} \tag{8}$$

This simple analysis has been challenged by the assertion that the cross section for scattering on the proton is given by the sum partonic cross sections. The partonic cross sections involve not just the matrix element squared, but the flux factor as well. The flux factor is simply $2\hat{s}$. According to this reasoning

$$\sigma_\nu \propto \hat{s}^2/\hat{s},\quad \sigma_{\bar\nu} \propto \hat{u}^2/\hat{s}\ . \tag{9}$$

Since $\hat{s}$ depends on $\cos\phi$, this would dramatically affect the results.

In fact, this is incorrect. In the proper calculation the matrix element squared for scattering from the proton is obtained by taking the Lorentz-invariant matrix element squared for the partonic process and dividing by x, the ratio of the parton's energy E_p to the proton's energy E. This is so because the relation between the S-matrix element and the Lorentz-invariant amplitude induces a factor $(2E)^{-1/2}$ for each incident particle. The resulting quantity $|\mathcal{M}_{\text{parton}}|^2/x$ should then be divided by the flux factor $2s$ for the proton. Now xs is very nearly $\hat{s}$ so there is little difference in the predicted total rates, but the consequences are important for the azimuthal dependence.

Alternatively, one recalls that the cross section is obtained by dividing the observed reaction rate by the incident flux. The rate is determined by the rate at which the partonic processes take place. This in turn is given by the partonic cross section times the partonic flux, the product being just the partonic matrix element squared.

These arguments can be summarized by the equation

$$|\mathcal{M}|^2_{\text{proton}} = \int dx\, d^2p_\perp f(x,p_\perp)\frac{|\mathcal{M}|^2_{\text{parton}}}{x} \quad \text{(correct)}\ , \tag{10}$$

where $f(x,p_\perp)$ gives the distribution of partons in the proton. The alternative

$$\sigma_{\text{proton}} = \int dx\, d^2p_\perp f(x,p_\perp)\sigma_{\text{parton}} \quad \text{(incorrect)} \tag{11}$$

is adequate only if we can ignore the transverse momenta of the partons.

We gain additional insight by examining the general form of the cross section for $\nu p \to \mu h X$ where h is some observed hadron. The matrix element for a particular choice of X is

$$\mathcal{M} \propto \bar{\mu}(k')\gamma_\mu \tfrac{1}{2}(1-\gamma_5)\nu(k)\langle h,X|J^\mu|p\rangle \tag{12}$$

and

$$d\sigma \propto \delta^4\left[p+q-h-\sum p_i'\right]\frac{d^3h}{2E_h}\prod_i \frac{d^3p_i'}{2E_i'}\frac{d^3k'}{2k'}l_{\mu\nu}\langle p|J^{\nu\dagger}|hX\rangle\langle hX|J^\mu|p\rangle\ . \tag{13}$$

Here h represents the momentum of the observed hadron (or hadron jet), p_i' are the unobserved hadrons, and

$$l_{\mu\nu}=\mathrm{Tr}\,\not{k}\gamma_\mu\not{k}'\gamma_\nu\tfrac{1}{2}(1-\gamma_5)\ . \tag{14}$$

The semi-inclusive cross section is obtained by summing over possible choices of X. This yields a structure function analogous to those familiar in fully inclusive lepton production:

$$W^{\mu\nu}(P,q,h)\propto \sum_n \delta^4\left[P+q-h-\sum p_i'\right]\langle P|J^{\dagger\nu}|h,X_n\rangle\langle h,X_n|J^\mu|P\rangle\ . \tag{15}$$

The tensor $W^{\mu\nu}$ must be constructed from the vectors p, q, and h, and the scalars P^2, q^2, h^2, $P\cdot q$, $P\cdot h$, and $q\cdot h$. Let us take k and k' in the $x-z$ plane with $\mathbf{q}=\mathbf{k}-\mathbf{k}'$ along the z axis in the laboratory frame, as before:

$$\begin{aligned} k&=k(1,\sin\alpha,0,\cos\alpha)\ ,\\ k'&=k'(1,\sin\beta,0,\cos\beta)\ ,\\ q&=(\nu,0,0,\sqrt{\nu^2+Q^2})\ ,\\ P&=(M,0,0,0)\ ,\end{aligned} \tag{16}$$

so

$$k\sin\alpha=k'\sin\beta\ .$$

The laboratory lepton scattering angle is $\theta=\beta-\alpha$, and

$$\begin{aligned}\sin\alpha&=\left[\frac{Q^2}{\nu^2+Q^2}\frac{k'}{k}\right]^{1/2}\cos(\theta/2)\ ,\\ \sin\beta&=\left[\frac{Q^2}{\nu^2+Q^2}\frac{k}{k'}\right]^{1/2}\cos(\theta/2)\ .\end{aligned} \tag{18}$$

For the purpose of expressing $W^{\mu\nu}$ we may use P, q, and $h_\perp$ where $h_\perp$ is the portion of h perpendicular to q and P:

$$h_\perp=(0,h_\perp\cos\phi,h_\perp\sin\phi,0)\ . \tag{19}$$

The most general symmetric Hermitian form for $W^{\mu\nu}$ is

$$\begin{aligned}W^{\mu\nu}_{\text{symm}}&=Ag^{\mu\nu}+BP^\mu P^\nu+Cq^\mu q^\nu+D(P^\mu q^\nu+P^\nu q^\mu)\\ &\quad+Eh_\perp^\mu h_\perp^\nu+F(h_\perp^\mu P^\nu+h_\perp^\nu P^\mu)\\ &\quad+G(h_\perp^\mu q^\nu+h_\perp^\nu q^\mu)\ .\end{aligned} \tag{20}$$

The most general antisymmetric form for $W^{\mu\nu}$ that is T invariant is

$$W^{\mu\nu}_{\text{antisymm}}=i(HP_\alpha q_\beta+IP_\alpha h_{\perp\beta}+Jq_\alpha h_{\perp\beta})\epsilon^{\mu\nu\alpha\beta}\ . \tag{21}$$

The functions A through J depend only on P^2, q^2, $h_\perp^2$, $P\cdot h_\perp$, $q\cdot h_\perp$, and $P\cdot q$, all of which are independent of ϕ.

Contracting with $l_{\mu\nu}$ gives the most general possible form for the cross section $d\sigma/d^3k'd^3h$. Because of current conservation the contributions from C, D, and G vanish. The A, B, and H terms have no $\cos\phi$ dependence. The F, I, and J terms are proportional to $\cos\phi$ while the E term includes a portion proportional to $\cos 2\phi$. Thus,

$$\frac{d\sigma}{d^3k'd^3h}\propto A+B\cos\phi+C\cos 2\phi\ . \tag{22}$$

A term such as $\hat{u}^2/\hat{s}$ would give

$$\sigma\propto\frac{\left[1-\dfrac{2p_\perp}{Q\sqrt{1-y}}\cos\phi\right]^2}{1-\dfrac{2p_\perp\sqrt{1-y}}{Q}\cos\phi} \tag{23}$$

in violation of the general result just derived.

Because the naive parton model puts the quarks on shell both before and after scattering we have the conditions $h^2=0=(h-q)^2$. These result in an overall δ function $\delta(q^2-2q\cdot k)$ multiplying Eq. (22). Now the cross section of interest is measured against the fixed lepton directions so the δ function must be eliminated by integrating over d^3h, not d^3k'. Since $\mathbf{q}$ is along the z axis $q\cdot h$ has no azimuthal dependence and thus none is introduced in evaluating the integral over d^3h.

It might be argued that the parton model is not reliable beyond terms of order $p_\perp/Q$ or $(p_\perp/Q)^2$ and thus the denominator should be expanded. This would result in a $\cos\phi$ dependence allowed by the above analysis. However, the question is one of principle. Independent of the dynamical reliability of the model the result ought to be consistent with the Lorentz-invariance arguments.

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[9]Notice Eqs. (11) and (12) in Ref. 6 are missing factors of 2 in front of $p_\perp$. Equations (13)–(15) in Ref. 6 are correct as they appear there. I regret the confusion caused by my communication with the authors of Ref. 2, which is reflected in their footnote 1. Contrary to the assertion there, the equations for $\langle \cos\phi \rangle$ are correct in Ref. 6.