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Electromagnetic cascade showers in lead with the Landau-Pomeranchuk-Migdal effect included: Average behavior of the one-dimensional LPM shower in lead

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The numerical method developed by Fujimaki and Misaki can obtain solutions for an electromagnetic cascade shower, in which no restrictions on the forms of the differential cross sections for both bremsstrahlung and pair-production processes are made. By utilizing this method, the LPM showers (electromagnetic cascade showers in the presence of the Landau Pomeranchuk-Migdal effect) are calculated for primary energies in the range $10^{14}-10^{17}$ eV in lead. Physical properties of the LPM showers, such as electron transition curves, track lengths, and fractional dissipated energies, are obtained in the cases with and without the inclusion of ionization losses. In order to compare LPM showers with ordinary electromagnetic cascade showers in the absence of the LPM effect [hereafter defined as Bethe-Heitler (BH) showers] and to clarify the characteristics of the LPM showers, all physical quantities which have been obtained in the LPM showers have also been obtained in the BH showers. We discuss the characteristics of the LPM showers, especially with reference to the BH showers. The results obtained by other authors are compared with the results obtained by the author.

I. INTRODUCTION

Landau and Pomeranchuk[1] showed, using a semiclassical treatment, that in dense media, the bremsstrahlung and pair-production cross sections should decrease due to the multiple scattering by adjacent atoms when the incident energy becomes sufficiently large. This is in contrast with the Bethe-Heitler (BH) cross sections, which are essentially independent of initial energy in the usual cosmic-ray energy region. Later, developing the idea of Landau and Pomeranchuk, Migdal[2] gave cross sections for bremsstrahlung and pair-production processes based on quantum-electrodynamical calculations. The effect is nowadays generally called the Landau-Pomeranchuk-Migdal (LPM) effect.

Owing to the large decrease of LPM integral cross sections when compared to BH integral cross sections, and the strong deviation of the form of LPM differential cross sections from BH differential cross sections for the bremsstrahlung and pair-production processes at higher energy, electromagnetic cascade showers in the presence of the LPM effect (LPM showers hereafter) are expected to behave very differently from electromagnetic cascade showers in the absence of the LPM effect (usual cascade shower, BH showers, hereafter). We pointed out that the average behavior of LPM showers is much different from those of BH showers.[3] Also, we clarified that individual LPM showers at higher energies might behave quite differently from the average LPM shower at higher energy.[4,5]

Particularly, the LPM effect is expected to play an important role in emulsion-chamber experiments.[6] In emulsion chambers which consist mainly of lead and emulsion plates, the discrimination between LPM showers (electromagnetic cascade showers) and nuclear cascade showers (aggregate of electromagnetic cascade showers) might influence decisively the interpretations of elementary particle interactions at superhigh energies, for example, related to the interpretation of the "Centauro" type of the events.[7] For this reason, it is necessary to study the detailed structure of LPM showers in lead not only in the average aspects but also particularly in individual ones.

In the present paper, we limit our concern to the average aspects of LPM showers in one dimension.

The calculations are made by utilizing the matrix method, which was given by Fujimaki and Misaki.[8] We have calculated LPM showers in lead over the energy range $10^{14}-10^{17}$ eV. Only the adoption of this method makes it possible to carry out such systematic and extensive calculations. We have obtained track lengths, fractional dissipated energies, and several other quantities which characterize cascade showers which have not been obtained by other authors, in addition to the electron transition curves. In order to extract characteristics of the LPM showers, corresponding quantities in the BH shower are also given. Physical quantities of BH showers should be regarded as standard ones around which we can discuss the curious characteristics of LPM showers.

II. THE NUMERICAL METHOD FOR THE CALCULATIONS

A systematic description of the matrix method and its related problems will be presented in an independent paper.[9] Therefore, we briefly reintroduce this method.

Let $\pi(t)dt$ and $\gamma(t)dt$ be the average number of electrons and photons with energy at a depth t (in units of radiation lengths) in a cascade shower. Then, the fundamental equations which govern the average behavior of the cascade shower are given as

$$\begin{bmatrix} \pi(T+t) \\ \gamma(T+t) \end{bmatrix} = \begin{bmatrix} A(t) & B(t) \\ C(t) & D(t) \end{bmatrix} \begin{bmatrix} \pi(T) \\ \gamma(T) \end{bmatrix}, \tag{1}$$

where $A(t)$ is a transform operator which generates $\pi(T+t)$ from $\pi(T)$, $B(t)$ is a transform operator which generates $\pi(T+t)$ from $\gamma(T)$, $C(t)$ is a transform operator which generates $\gamma(T+t)$ from $\pi(T)$, and $D(t)$ is a transform operator which generates $\gamma(T+t)$ from $\gamma(T)$, respectively. For the time being, t in (1) may be defined as some positive value and no further restrictions on it are made. Then, these operators may be regarded as the black-box-type operators which are defined at the entrances and exits only, because of the definiteness of t, though these operators are closely connected with the bremsstrahlung and pair-production processes.

From (1), we obtain, generally,[9]

$$\begin{bmatrix} A(nt) & B(nt) \\ C(nt) & D(nt) \end{bmatrix} = \begin{bmatrix} A(t) & B(t) \\ C(t) & D(t) \end{bmatrix}^n . \tag{2}$$

Let us introduce a fundamental unit t_0, which may be one radiation length, for the convenience of the calculation. Then, from (2), the numbers of electrons and photons at the depth mt_0 are given as

$$\begin{bmatrix} \pi(mt_0) \\ \gamma(mt_0) \end{bmatrix} = \begin{bmatrix} A(t_0) & B(t_0) \\ C(t_0) & D(t_0) \end{bmatrix}^m \begin{bmatrix} \pi(0) \\ \gamma(0) \end{bmatrix}, \tag{3}$$

where m is an arbitrary integer, and $\pi(0)$ and $\gamma(0)$ are the number of electrons and photons at depth zero, respectively.

Here, let us introduce δt such that $\delta t = t_0/n$. Here, let n be a sufficiently large integer and consequently δt be so small that either the bremsstrahlung or pair-production process may occur at most once in δt. Then, we have

$$\begin{bmatrix} A(t_0) & B(t_0) \\ C(t_0) & D(t_0) \end{bmatrix} = \begin{bmatrix} A(\delta t) & B(\delta t) \\ C(\delta t) & D(\delta t) \end{bmatrix}^n . \tag{4}$$

Further, in order to examine the change in the number of shower particles in a cascade in a definite energy range, let us divide the whole energy range into a large number of smaller energy cells. Further, we denote by π_i and γ_i the expected numbers of electrons and photons in the ith cell, respectively, and $A_{i,j}$, $B_{i,j}$, $C_{i,j}$, and $D_{i,j}$ are the mean transfer probabilities from the jth cell to the ith cell in energy.

Then, $\pi(t)$ and $\gamma(t)$ can be regarded as a column matrix whose components are π_i and γ_i, respectively, and the transform operators $A(\delta t)$, etc., can be regarded as square matrices whose components are $A_{i,j}$, etc., and whose dimensions are equal to two times of the numbers of energy divisions. Hence, the fundamental equation which describes the mean behavior of the cascade shower is expressed finally as[9]

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \cdots \\ \cdots \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \cdots \\ \cdots \end{bmatrix}_{t+\delta t} = \begin{bmatrix} A_{11} A_{12} \cdots & B_{11} B_{12} \cdots \\ A_{21} A_{22} \cdots & B_{21} B_{22} \cdots \\ A_{31} A_{32} \cdots & B_{31} B_{31} \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ C_{11} C_{12} \cdots & D_{11} D_{12} \cdots \\ C_{21} C_{22} \cdots & D_{21} D_{22} \cdots \\ C_{31} C_{32} \cdots & D_{31} D_{32} \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}_{\delta t} \times \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \cdots \\ \cdots \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \cdots \\ \cdots \end{bmatrix}_{t} . \tag{5}$$

From (3)–(5), we obtain the number of shower particles at the depth mt_0 under the boundary condition corresponding to $\pi(0)$ and $\gamma(0)$ which are given in an appropriate way.

In our calculation, we adopt $\delta t = \frac{1}{128}$, and use 32 logarithmically and equally divided energy division per decade for primary energies from 10^{14} to 10^{17} eV in lead.

In order to provide the validity of the method both with and without ionization losses, we have solved the same problems by both this method and the analytical method which is methodologically independent.

We calculated electron numbers and track lengths under approximation A (Ref. 10) and the total number of electrons under approximation B (Ref. 11) for the comparison between the method and the analytical one. The agreement between the results was excellent. In the case with ionization loss, we cannot treat zero energy exactly. So, in this case, we adopt 10^3 eV as "zero" energy. The agreement between results obtained by independent methods guarantees automatically the validity of the results for LPM showers obtained by the method, because the forms of the cross sections in the cascade showers are essentially independent on the accuracy of the calculation.

By utilizing the confirmed validity of the matrix method, we compare LPM showers without ionization loss by this method with the corresponding ones obtained by Konishi, Misaki, and Fujimaki[3] in Fig. 1. The agreement is excellent. The latter results are obtained using an exact Monte Carlo method. The validity of the Monte Carlo method had been proved from the fact that it applied to the case under approximation A, whose results

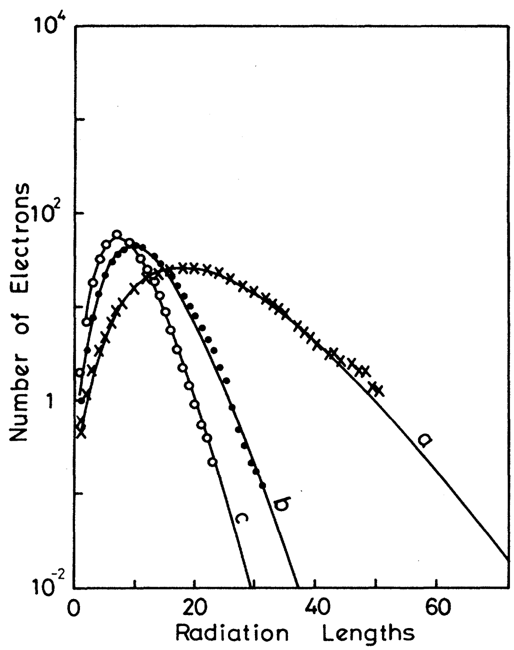

FIG. 1. Comparison between the present results and the results obtained by Konishi, Misaki, and Fujimaki. The comparison is made for LPM showers without ionization loss in lead, keeping E_0/E_{th} fixed and values of $E_0=10^{11}$, 10^{13}, and 10^{15} eV. The cross ($\times$), closed circles (●), and open circles (○) denote $E_0=10^{15}$, 10^{13}, and 10^{11} eV, respectively.

agreed excellently with analytical solutions.[12,13] Further, the results obtained by Konishi, Misaki, and Fujimaki had been confirmed by other independent calculations.[14]

III. CALCULATIONAL RESULTS

A. Above what energy can we neglect the effect of ionization loss in LPM showers

There are two different physical concerns in which we can and cannot neglect the effect of ionization loss, respectively.

When we are interested in the calorimetric-type experiments and air-shower-type experiments, then we must consider the effect of ionization loss, because, in those experiments, we treat shower particles whose energies fall to zero energy. Further, we are not interested in cascade showers with small numbers of shower particles in these experiments.

On the contrary, in the emulsion-chamber-type experiments, we are only interested in the shower particles near the core of the cascade showers whose energies are so high that we can neglect the effect of ionization loss. Further, we are interested in small numbers of shower particles, even less than one particle, because we make probabilistic arguments in the analysis of the experiments.

For the examination of the effect of ionization loss, we calculate LPM showers both with and without ionization losses for the same threshold energies. The results, which are given in Fig. 2, indicate that we have to consider the effect of ionization loss below a threshold energy of 10^9 eV. This holds exactly in the BH showers, because it is quite independent of the form of the cross sections which generate the cascade shower.

B. Cascade showers without ionization loss: Comparison of LPM showers with BH showers

1. Transition curves of electron numbers in the LPM showers

In Figs. 3(a)–3(d), the transition curves of electron numbers for primary energies from 10^{14} to 10^{17} eV in lead are presented. Comparing these figures with corresponding ones under approximation A (Ref. 12), it is easily understood that the LPM showers have the following characteristics when compared with the BH showers: the LPM showers develop more slowly than the BH showers do and reach shower maximum at greater depths, and electron numbers at the shower maximum in the LPM showers are smaller than the BH shower's. The LPM showers attenuate more slowly than the BH showers. These factors, which distinguish an LPM shower from a

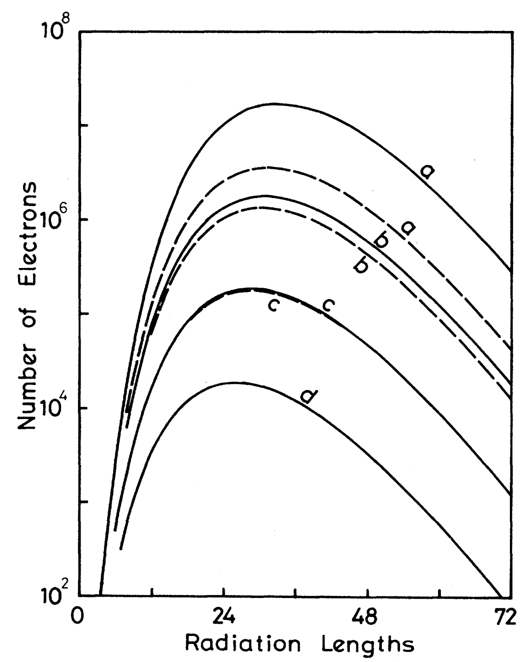

FIG. 2. Comparison of electron numbers in lead for the same threshold energies between LPM showers in the cases with and without ionization losses. The solid lines denote the electron transition curves in the cases without ionization loss, and the dotted lines denote the corresponding curves in the cases with ionization loss. The primary energies are 10^{15} eV. The letter attached to each curve denotes the threshold energy: $a=10^6$ eV, $b=10^7$ eV, $c=10^8$ eV, and $d=10^9$ eV.

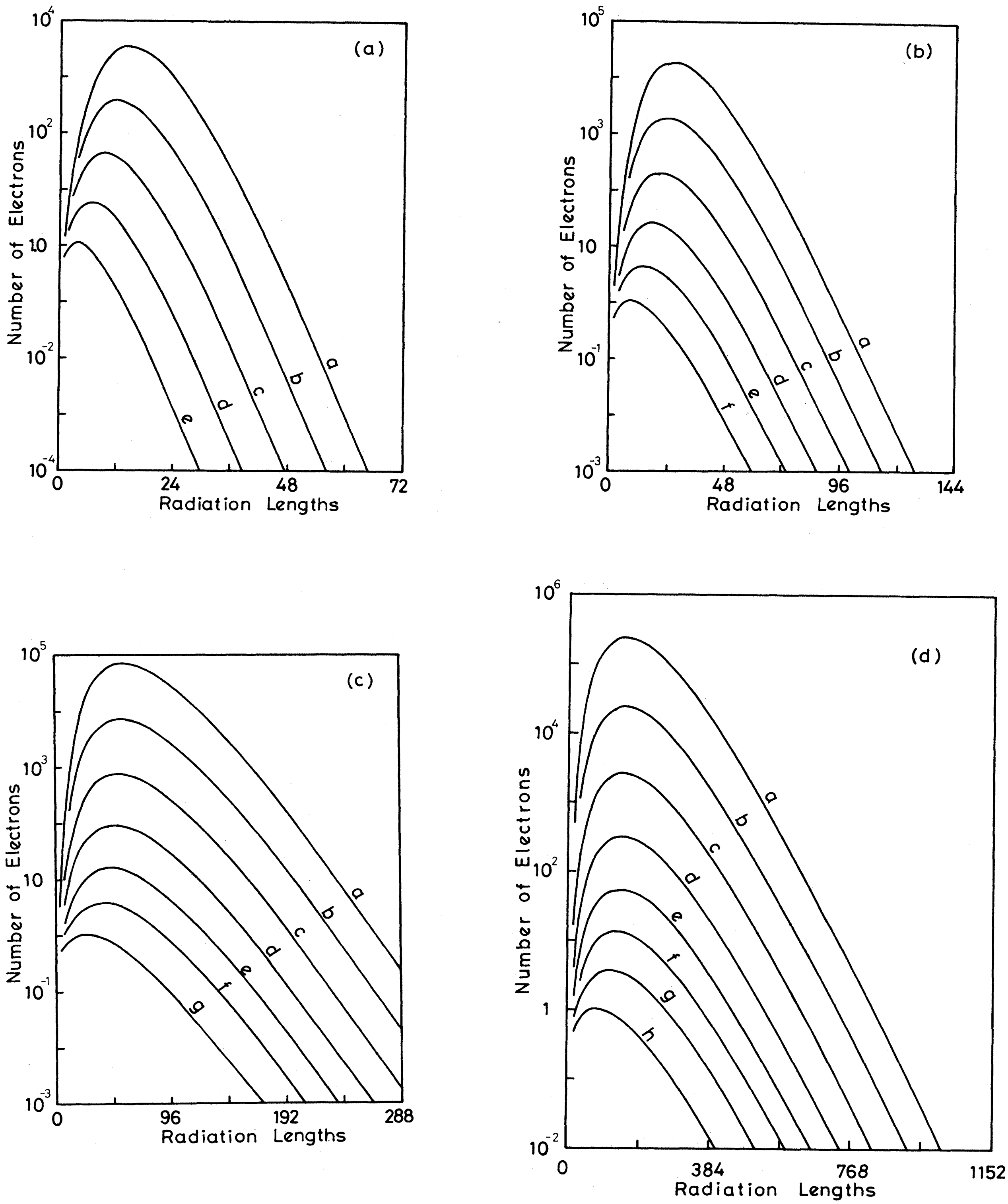

FIG. 3. Transition curves of electron numbers in lead in the presence of the LPM effect without ionization loss. (a) The primary energy is 10^{14} eV and the letter attached to each curve denotes the threshold energy: $a=10^9$ eV, $b=10^{10}$ eV, $c=10^{11}$ eV, $d=10^{12}$ eV, and $e=10^{13}$ eV. (b) The primary energy is 10^{15} eV and the letter attached to each curve denotes the threshold energy: $a=10^9$ eV, $b=10^{10}$ eV, $c=10^{11}$ eV, $d=10^{12}$ eV, $e=10^{13}$ eV, and $f=10^{14}$ eV. (c) The primary energy is 10^{16} eV and the letter attached to each curve denotes the threshold energy: $a=10^9$ eV, $b=10^{10}$ eV, $c=10^{11}$ eV, $d=10^{12}$ eV, $e=10^{13}$ eV, $f=10^{14}$ eV, and $g=10^{15}$ eV. (d) The primary energy is 10^{17} eV and the letter attached to each curve denotes the threshold energy: $a=10^9$ eV, $b=10^{10}$ eV, $c=10^{11}$ eV, $d=10^{12}$ eV, $e=10^{13}$ eV, $f=10^{14}$ eV, $g=10^{15}$ eV, and $h=10^{16}$ eV.

TABLE I. Numerical values for electron numbers at shower maximum $N_{\max}$, depth of shower maximum $T_{\max}$, and full width at half maximum (FWHM) in both BH and LPM showers for the case of no ionization loss, in lead. The numerical values are due to a LPM shower, while the corresponding ones in parentheses are due to a BH shower.

E_0 (eV)	10^{14}	10^{15}	10^{16}	10^{17}
$N_{\max}$	3.44×10^3	1.84×10^4	7.00×10^4	2.37×10^5
	(4.13×10^3)	(3.74×10^4)	(3.47×10^5)	(3.24×10^6)
$T_{\max}$ (r.l.)	15	26	58	152
	(11)	(13)	(16)	(18)
FWHM	12	23	59	176
(r.l.)	(10)	(11)	(12)	(13)

BH shower, become remarkably more significant as the incident energy of the shower increases.

In order to elucidate the identified characteristics of the LPM showers compared to the BH showers, $N_{\max}$ (the electron number at the shower maximum), $T_{\max}$ (the depth of the shower maximum), and FWHM (full width at half maximum, the distance between the depth at half maximum of the electron number before and after the shower maximum) are given in Table I in both cases, for primary energies from 10^{14} to 10^{17} eV in lead. The threshold energy is fixed as 10^9 eV.

2. *The absorption rate of the LPM shower at greater depths*

One of the features which characterizes the cascade shower is its absorption rate at great depth. According to the analytical theory of showers under approximation A (Refs. 10 and 13), the shower at depth t which is far from zero can be described as

$$\Pi(E_0,E,t)\propto\exp[\lambda_1(s)t] , \tag{6}$$

where s is called the shower age. Therefore, the absorption rate of the shower at great depth is expressed as $\lambda_1(s)$. If we are now interested in the behavior of the shower at infinite depth, then the analytical theory[10,13] provides

$$\lim_{\substack{s\to\infty\\(t\to\infty)}} \lambda_1(s)=-\sigma_0 , \tag{7}$$

where σ_0 is the total cross section for pair production in the complete screening, which is expressed as $1/\lambda_p^{\mathrm{BH}}$, where λ_p^{BH} denotes the mean free path for the pair-production process in complete screening. That is, the tail of the shower under approximation A is governed by the total cross section for pair production, and is expressed as $\exp(-\sigma_0 t)$.

It was already suggested by Ivanenko, Kirilov, and Lyutov,[15] based on their analytical theory, that the tail of an LPM shower without ionization loss would also be governed by the total pair-production cross section, but they did not show the validity of their assertion quantitatively. In Table II we show the absorption rates of our LPM showers at great depth in lead for 10^{15} eV, at different threshold energies. In this case, the absorption rates of electron numbers at great depths approach $1/\sigma_{\mathrm{LPM}}(E_0)$, the inverse of the total cross section for pair production with primary energy of E_0 in the presence of the LPM effect. The results verify the validity of the indication by Ivanenko, Kirilov, and Lyutov. This is the theoretical reason why the Nishimura-Kamata conjecture[11] that LPM showers behave essentially the same as BH showers does not hold true.

3. *The track lengths of the electrons in the LPM shower*

According to the analytical theory under approximation A, the track length is given as[10,13]

$$P_0^{(\pi)}(E_0,E_{\mathrm{th}})=0.437E_0/E_{\mathrm{th}} , \tag{8}$$

where E_0 and E_{th} denote the primary energy and the threshold energy, respectively. The track length under approximation A is not conserved from shower to shower, but fluctuates less than other observable quantities of cascade showers under this approximation.

In Table III we give the track lengths of electrons for primary energies from 10^{14} to 10^{17} eV in lead and for different threshold energies. It is clear from this table that LPM showers provide longer track lengths at higher threshold energies, owing to the higher content of energetic particles, compared to BH showers; but the track length of electrons for lower threshold energies in the LPM showers approaches the corresponding one for BH showers. As the track length denotes the area covered by the corresponding transition curve, we may conclude that both the transition curves and the area they enclose are different at higher threshold energies for LPM showers

TABLE II. Absorption rate (in r.l.$^{-1}$) of LPM shower in lead initiated by photon of energy 10^{15} eV at great depths. The total cross section for pair production in the presence of the LPM effect is 2.23×10^{-1} r.l.$^{-1}$ for the primary energy of 10^{15} eV.

t (r.l.) \ E_{th} (eV)	10^9	10^{10}	10^{11}	10^{12}	10^{13}	10^{14}
100	2.07×10^{-1}	2.03×10^{-1}	2.05×10^{-1}	2.07×10^{-1}	2.09×10^{-1}	2.12×10^{-1}
120	2.10×10^{-1}	2.11×10^{-1}	2.13×10^{-1}	2.14×10^{-1}	2.15×10^{-1}	2.18×10^{-1}
140	2.17×10^{-1}	2.17×10^{-1}	2.18×10^{-1}	2.19×10^{-1}	2.20×10^{-1}	2.21×10^{-1}
160	2.20×10^{-1}	2.21×10^{-1}	2.22×10^{-1}	2.22×10^{-1}	2.23×10^{-1}	2.24×10^{-1}
180	2.22×10^{-1}	2.23×10^{-1}	2.24×10^{-1}	2.25×10^{-1}	2.25×10^{-1}	2.26×10^{-1}

TABLE III. Track length (in r.l.) of electrons in the LPM shower in lead without ionization loss for primary energies from 10^{14} to 10^{17} eV and the various threshold energies.

E_{th} (eV) \ E_0 (eV)	10^{14}	10^{15}	10^{16}	10^{17}
10^9	4.47×10^4	4.44×10^5	4.46×10^6	4.45×10^7
10^{10}	4.56×10^3	4.55×10^4	4.55×10^5	4.54×10^6
10^{11}	4.85×10^2	4.86×10^3	4.86×10^4	4.85×10^5
10^{12}	5.89×10^1	6.04×10^2	6.07×10^3	6.07×10^4
10^{13}	8.63	1.02×10^2	1.06×10^3	1.07×10^4
10^{14}		2.04×10^1	2.47×10^2	2.59×10^3
10^{15}			5.81×10^1	7.05×10^2
10^{16}				1.73×10^2

and BH showers, while in the lower-energy region the transition curves are different but the area enclosed by the curves approach each other. It is easily understood that, whether the shower is a LPM shower or a BH shower, almost all particles in the shower have lower energies where the LPM effect is no longer effective.

When we are interested in a low threshold energy, therefore, the track length in the LPM shower approaches the corresponding one of the BH shower.

4. *The development of the LPM shower with primary energy of E_{LPM}*

Stanev *et al.*[14] introduced the concept of the $E_{\rm LPM}$ which characterizes the energy above which the LPM effect is significant. The numerical value of $E_{\rm LPM}$ is 3.5×10^{13} eV in the case of lead. $E_{\rm LPM}$ is introduced in relation to the differential probabilities for bremsstrahlung and pair-production processes. Consequently, we are interested in how, at a primary energy of $E_{\rm LPM}$, the development of the cascade shower totally is affected. In Fig. 4, a comparison between a BH shower with the primary energy of $E_{\rm LPM}$ and a LPM shower with the same primary energy are made for lead. From the figure, we can adopt the $E_{\rm LPM}$ as the energy which characterizes the primary energy above which the discrimination between BH and LPM showers is possible. The concept of $E_{\rm LPM}$ mentioned above holds exactly in the case with ionization loss.

C. Cascade showers with ionization loss: Comparison of LPM showers with BH showers

1. *The transition curves of electron number in the BH showers*

In order to compare LPM showers with BH showers and extract characteristics of LPM showers, the transition curves of electron numbers with different threshold energies, the electron transition curves under approximation B, are given for primary energies of $10^{14}-10^{17}$ eV in Figs. 5(a)–5(d). As expected, the electron number for a definite energy threshold converges towards the electron number above zero energy as the threshold energy tends to zero.

2. *The transition curves of electron number in the LPM showers*

The transition curves of electron numbers with different threshold energies for primary energies of $10^{14}-10^{17}$ eV are given in Figs. 6(a)–6(d) for the comparison with BH showers. In Table IV, several quantities which characterize features of the cascade showers, that is, $N_{\rm max}$, $T_{\rm max}$, and FWHM for both the BH and the LPM showers analogous to the case without ionization loss given in Table I, are given.

3. *The fractional dissipated energies*

The fractional dissipated energy is an important concept in the cascade shower with ionization loss. This

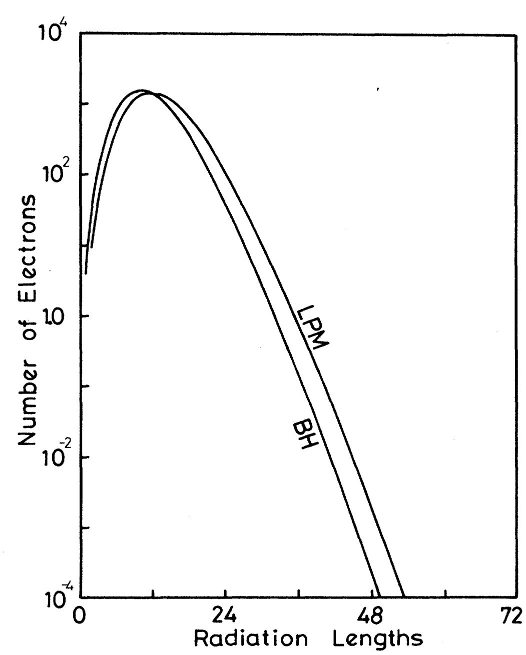

FIG. 4. Comparison between the LPM shower with incident energy $E_{\rm LPM}$ and the BH shower with the same energy in lead. In both showers, the ionization loss is neglected. $E_{\rm LPM}$ in lead is 3.5×10^{13} eV.

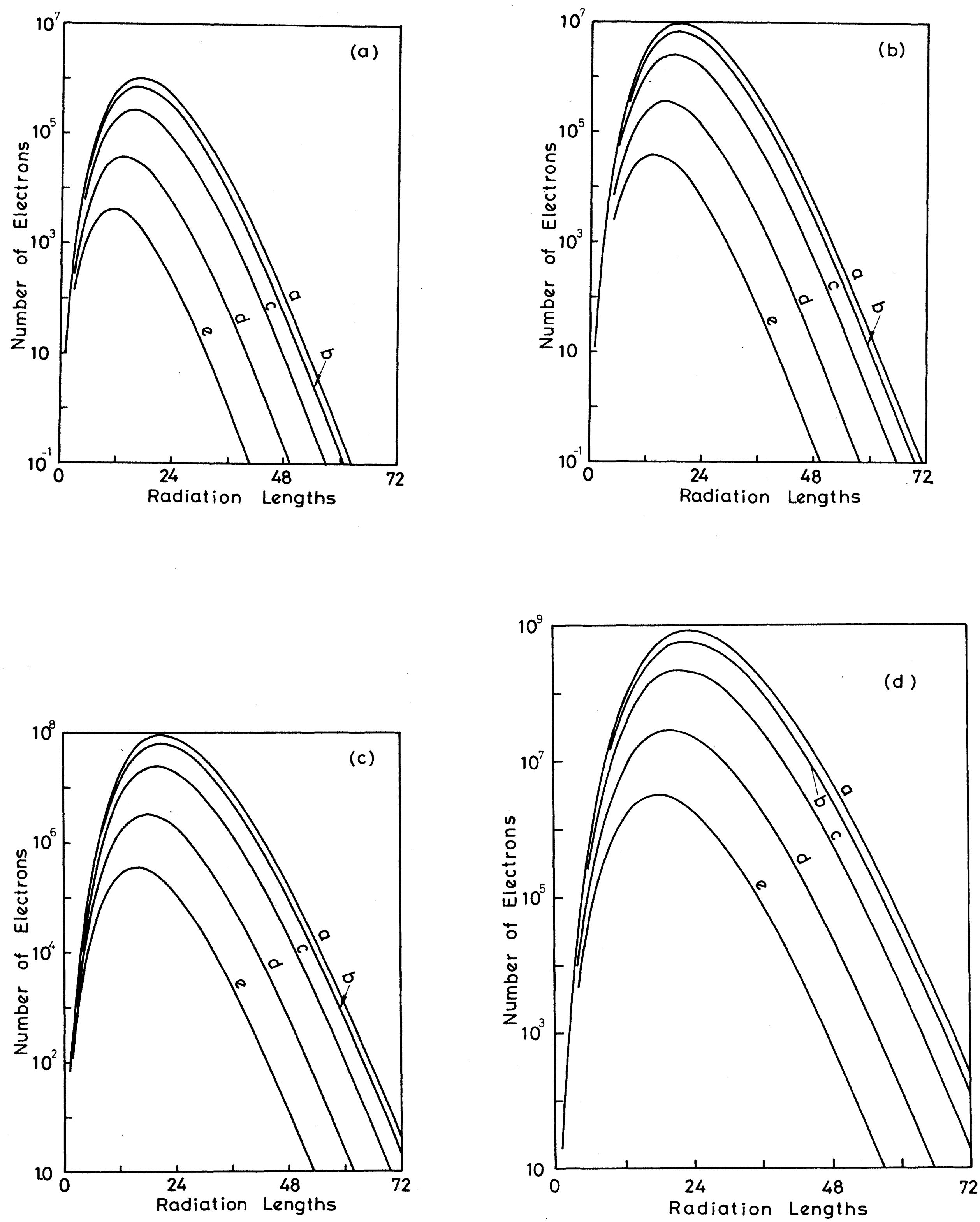

FIG. 5. (a) Transition curves of electron numbers in lead under approximation B initiated by photons of 10^{14} eV for various threshold energies. The letter attached to each curve denotes the threshold energy: $a = 10^3$ eV, $b = 10^6$ eV, $c = 10^7$ eV, $d = 10^8$ eV, and $e = 10^9$ eV. (b) Transition curves for photons of 10^{15} eV. (c) Transition curves for photons of 10^{16} eV. (d) Transition curves for photons of 10^{17} eV.

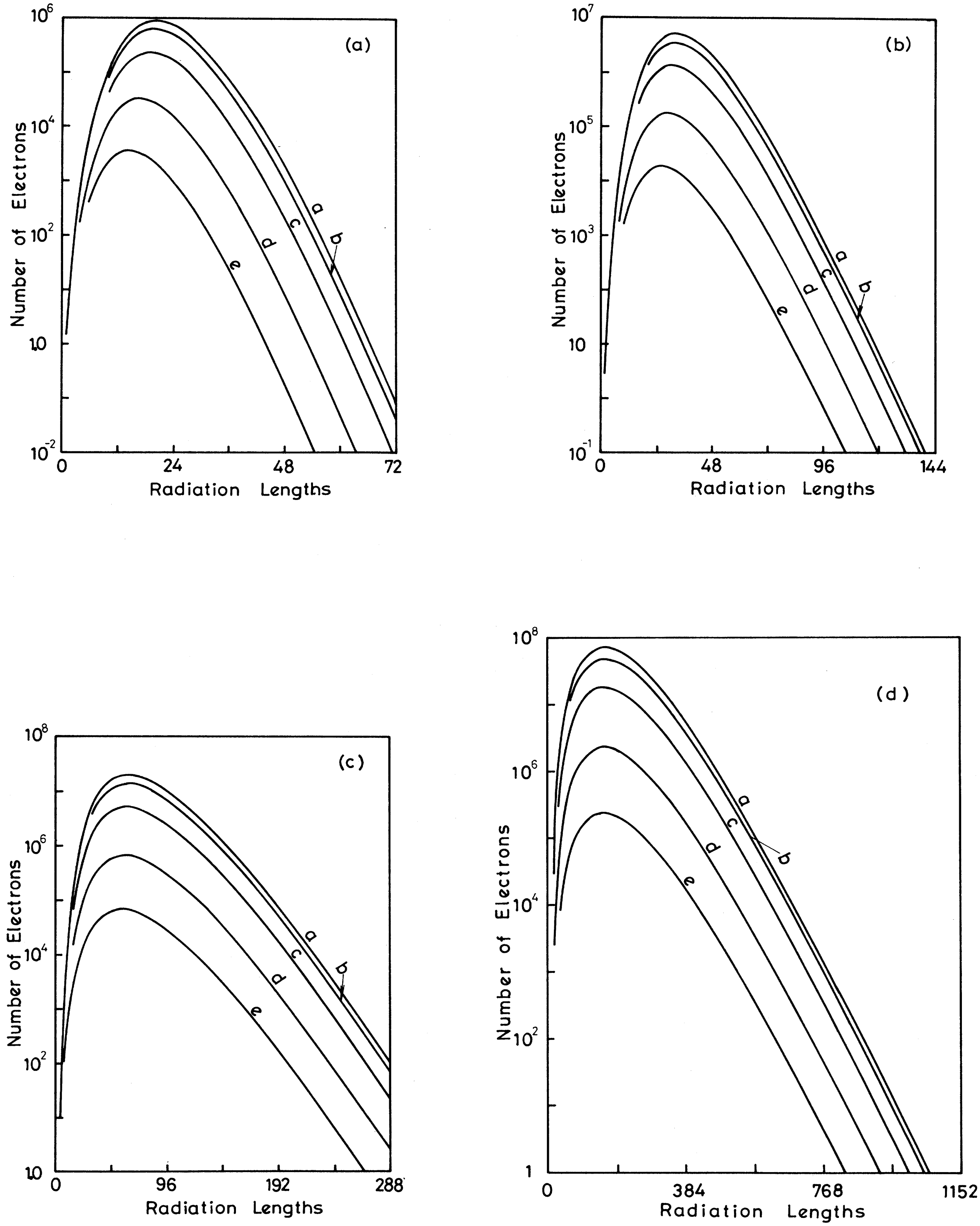

FIG. 6. Transition curves of electron numbers in lead in the presence of the LPM effect with ionization loss. (a) The primary energy is 10^{14} eV and the letter attached to each curve denotes the threshold energy: $a = 10^3$ eV, $b = 10^6$ eV, $c = 10^7$ eV, $d = 10^8$ eV, and $e = 10^9$ eV. (b) The primary energy is 10^{15} eV. (c) The primary energy is 10^{16} eV. (d) The primary energy is 10^{17} eV.

TABLE IV. Numerical values for electron numbers at shower maximum $N_{\max}$, depth for shower maximum $T_{\max}$ and full width half maximum (FWHM), in both BH and LPM showers in the case with ionization loss in lead. The numerical values are due to a LPM shower, while the corresponding ones in parentheses are due to a BH shower.

E_0 (eV)	10^{14}	10^{15}	10^{16}	10^{17}
$N_{\max}$	8.79×10^5	5.19×10^6	2.06×10^7	7.01×10^7
	(1.02×10^6)	(9.53×10^6)	(8.98×10^7)	(8.56×10^8)
$T_{\max}$ (r.l.)	20	32	64	159
	(17)	(19)	(21)	(24)
FWHM	14	24	60	176
	(12)	(13)	(14)	(15)

concept is available for the design of shower detectors. It is defined as

$$F_{\mathrm{LPM(BH)}}(E_0/\epsilon,t)=(\epsilon/E_0)\int_0^t \Pi_{\mathrm{LPM(BH)}}(E_0,0,t)dt \ , \quad (9)$$

where $\Pi_{\mathrm{LPM(BH)}}(E_0,0,t)$ denotes the number of electrons above zero energy in a LPM shower and a BH shower with ionization loss ϵ (7.6 MeV/radiation length) and with a depth t, respectively. From the law of energy conservation, we obtain

$$F_{\mathrm{LPM(BH)}}(E_0/\epsilon,t=\infty)=1 \ . \quad (10)$$

In Table V the fractional dissipated energies are given for primary energies from 10^{14} to 10^{17} eV in a LPM shower and a BH shower, respectively. Comparing LPM showers with BH showers, it is revealed that we need much more material for the design of a calorimeter in which the LPM showers are absorbed than assumed usually at higher energy. At a primary of 10^{17} eV, we need ten times as much material for the absorption of the cascade shower in the presence of the LPM effect than in its absence of the effect.

TABLE V. Fractional dissipated energies in both a BH shower and a LPM shower for primary energies from 10^{14} to 10^{17} eV. The numerical values are due to a LPM shower, while the corresponding ones in parentheses are due to a BH shower.

F (r.l.) \ E_0 (eV)	10^{14}	10^{15}	10^{16}	10^{17}
0.1	14.3	22.4	42.6	99.1
	(11.6)	(13.5)	(15.4)	(17.3)
0.2	16.4	26.1	51.0	124
	(13.5)	(15.6)	(17.6)	(19.7)
0.3	18.2	28.9	57.9	144
	(15.0)	(17.2)	(19.4)	(21.5)
0.4	19.8	31.6	64.6	163
	(16.3)	(18.6)	(20.8)	(23.1)
0.5	21.3	34.1	70.8	182
	(17.6)	(19.9)	(22.3)	(24.6)
0.6	22.8	36.8	77.8	202
	(19.0)	(21.5)	(23.8)	(26.3)
0.7	24.7	39.8	85.7	226
	(20.4)	(23.1)	(25.6)	(28.0)
0.8	26.9	43.6	96.6	255
	(22.3)	(24.9)	(27.6)	(30.2)
0.85	28.2	45.8	102	275
	(23.6)	(26.7)	(28.9)	(31.5)
0.9	32.6	48.9	110	300
	(25.0)	(27.9)	(30.6)	(33.1)
0.95	33.0	53.6	123	340
	(27.2)	(30.5)	(32.9)	(35.7)
0.98	36.7	58.7	138	387
	(29.6)	(32.5)	(35.7)	(38.3)

4. Early stage of development of the LPM showers

In Fig. 7 the early stages of the development of the LPM shower are shown for primary energies of $10^{13}-10^{17}$ eV in lead for zero threshold energy. Comparing these figures with Figs. 5(a)–5(c), which give the corresponding results with ionization loss and in the absence of the LPM effect (under approximation B), we can find a distinct contrast between BH showers and LPM showers. LPM showers with higher primary energies develop more slowly than LPM showers with lower primary energies, while BH showers with higher primary energies always develop more rapidly than BH showers with lower primary energies, which comes from the prolonged mean free paths for bremsstrahlung and pair production due to the LPM effect. The situation is the same in the case without ionization loss.

IV. EXAMINATIONS OF OTHER CALCULATIONS

In Sec. II we verified the validity and accuracy of our method. In this section, we examine other calculations based on our accurate results and discuss the limitations

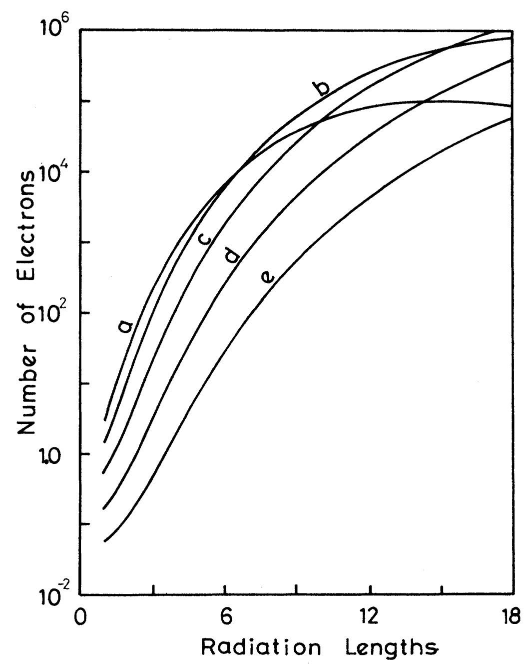

FIG. 7. Early stage of the development of the LPM shower with ionization loss in lead. The primary energies range from 10^{13} to 10^{17} eV, and the threshold energy is "zero." $a=10^{13}$ eV, $b=10^{14}$ eV, $c=10^{15}$ eV, $d=10^{16}$ eV, and $e=10^{17}$ eV.

of other calculations. Here, let us limit our concern to LPM showers with ionization loss.

A. Comparison with Pomansky's results

As early as 1967, Pomansky[16] gave transition curves of electron number for primary energies from 10^{13} to 10^{16} eV in lead obtained in a numerical way. We find big differences between his calculations and ours not only in absolute values of electron numbers but also in the rates of development and absorption of the transition curves. The main reason which produced such big errors in his calculations seems to be the propagation of the accumulated errors of his numerical procedure. In spite of the big differences with ours, the pioneering nature of Pomansky's work deserves recognition.

B. Comparison with the calculations of Stanev *et al.*

Stanev *et al.*[14] calculate LPM showers with ionization loss by a hybrid method, which combines a Monte Carlo simulation with analytical calculations. The hybrid method is frequently utilized in cases where it is impossible to utilize the full Monte Carlo method. Although the hybrid method is questionable for treating fluctuation problems in cascade showers, we may treat average quantities by this method. In Fig. 8 we compare the results of Stanev *et al.* with our results. The agreement between them is fairly good, which confirms the validity of the hybrid method as far as treatment of average quantities is concerned. Further, the agreement shows that the NKG function, i.e., the most well-approximated expression of the total number of electrons under approximation B, one-dimensional Nishimura-Kamata-Greisen function, utilized in the hybrid method is a reasonable approximation to the Nishimura and Kamata function, i.e., the analytical solution under approximation B. Some minor differences between the two results in Fig. 8 at greater depth are due to fluctuations in the small number of electrons which arise from the Monte Carlo part of the hybrid method.

C. Comparison with the calculations of Dedenko, Matsushko, Stern, and Zheleznykh

Dedenko, Matsushko, Stern, and Zheleznykh[17] calculated LPM showers including ionization loss by a hybrid method and a numerical method. Their results in lead are compared with our results in Fig. 9. Although the general tendency of their cascade curve is similar to ours, there are big differences in absolute values.

V. CONCLUSIONS

Our conclusions can be summarized as follows.

(1) We have obtained basic quantities which characterize LPM showers: the electron transition curves, track length, fractional dissipated energies, positions of shower maximum, the electron numbers at shower maximum, and the full width at half maximum in electron numbers.

We have extracted the characteristics of LPM showers, comparing these quantities with the corresponding ones of BH showers. These physical quantities which charac-

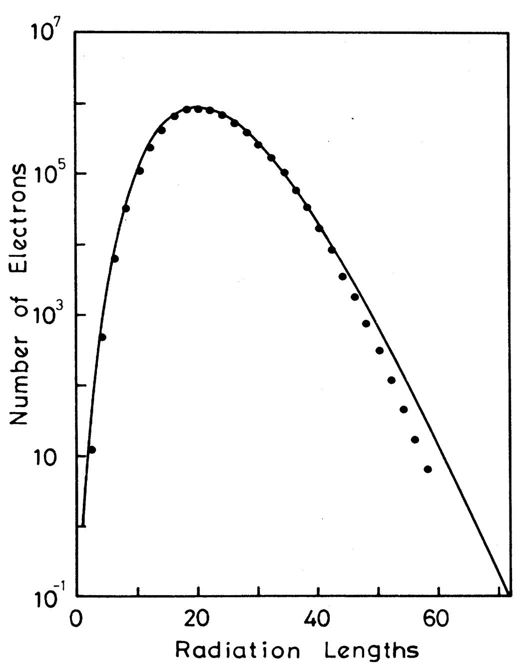

FIG. 8. Comparison between the present result and the result of Stanev *et al.* for total number of electrons in lead. The primary energy is 10^{14} eV, and the calculation by Stanev *et al.* was carried out by a hybrid method.

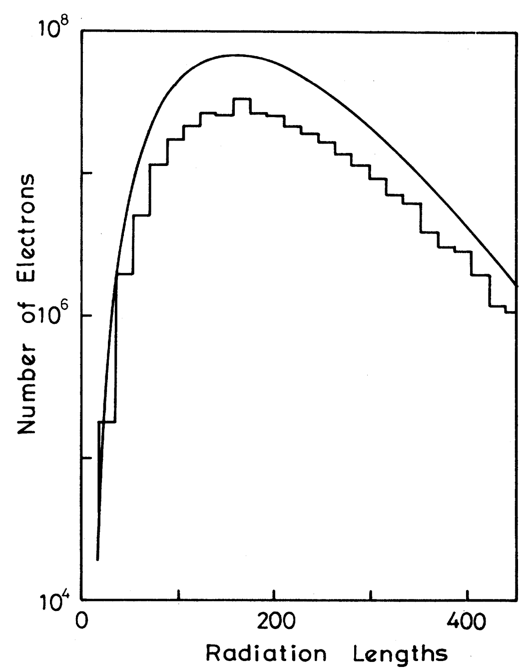

FIG. 9. Comparison of the results of Dedenko, Matsushko, Stern, and Zheleznykh with our results in lead. The primary energy is 10^{17} eV and their results were obtained by both a hybrid method and a numerical method.

terize the behavior of LPM showers strongly deviate from those of BH showers at higher energies.

(2) Especially, the characteristics of the development of LPM showers on average are as follows: They develop more slowly, reach the shower maximum at greater depth, and attain a smaller electron number there in comparison with BH showers, both in the cases with and without ionization losses. Differences in these quantities between LPM showers and BH showers become remarkably large as the primary energies increase. In the case where we utilize the electron number at shower maximum as a measure of the primary energy of the cascade shower, we may underestimate the primary energy remarkably, say, an underestimation in one order magnitude at 10^{17} eV, if we do not allow for the presence of the LPM effect. As a whole, it is extremely difficult to determine the energy of the cascade showers by the transition curves at higher energy, which have been frequently utilized in the emulsion-chamber studies, because LPM showers at higher energies fluctuate greatly so that we cannot apply the transition curves method to them.[4,5] Probably, we are obliged to introduce some method related to the calorimetric method for their energy determination. In order to absorb LPM showers completely in the material, we need about ten times more material at an incident energy of 10^{17} eV in the case of lead in comparison with BH showers.

(3) Because of the elongation of LPM showers, we have to pay much attention to the discrimination of electromagnetic cascade showers from nuclear cascade showers at higher energies, because even if the cascade showers are purely of electromagnetic origin, they may show "strange" behavior due to the LPM effect.[4,5] As nuclear cascade showers in the extremely-high-energy region have complex structures which consist of various LPM showers of different primary energies and with different starting points, the existence of the LPM effect makes interpretation of these showers extremely difficult. For the discrimination between purely LPM showers and nuclear cascade showers in the presence of the LPM effect, extensive calculations on the structure of LPM showers are absolutely necessary not only on their average behavior but also on their individual behavior. Particularly, in emulsion-chamber studies, where we should pay much attention to their fluctuation, detailed information on three-dimensional LPM showers is required, because their threshold energies are so high that large fluctuations may occur.

(4) The LPM effect appears notably in the early stage of development of cascade showers. BH showers with higher primary energies always develop more rapidly than BH showers with lower primary energies. Contrary to BH showers, LPM showers with higher primary energies develop more slowly than LPM showers with lower primary energy. From this strong contrast, we should pay more attention to examining the early development of the cascade showers at higher energies.

(5) Stanev *et al.*[14] introduced the concept E_{LPM} above which the LPM effect becomes significant in the cross sections of bremsstrahlung and pair production. It is shown in this paper that E_{LPM} is also a good measure for the primary energy of the cascade showers above which LPM showers can be distinguished from BH showers. The value of E_{LPM} in lead is 3.5×10^{13} eV. The concept of E_{LPM} also holds in the case with ionization loss.

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