Superheavy-Higgs-scalar effects in effective gauge theories from SO(10) grand unification with low-mass right-handed gauge bosons

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We investigate possible modifications of SO(10) predictions due to superheavy components of Higgs scalars, needed for the spontaneous symmetry breaking of the grand unified theory to effective gauge theories, where parity and $SU(2)_R$ breakings are decoupled. Interesting modifications with low-mass $W_R^{\pm}$ gauge bosons are found to be possible if the superheavy masses are nondegenerate, but satisfy the Coleman-Weinberg constraint. With the single intermediate symmetry, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, even a factor of 10 nondegeneracy is found to lower the $W_R^{\pm}$ and Z_R mass prediction by 4 orders, compared to earlier results, yielding $M_R \simeq 100$ TeV for $\sin^2\theta_W \simeq 0.235$. In the presence of the second intermediate symmetry, $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$, which could survive down to $M_{Z_R} \simeq 500$ GeV, we obtain 1 TeV $< M_{W_R} <$ 80 TeV for $0.238 > \sin^2\theta_W > 0.231$. The values of $\sin^2\theta_W$ can be lowered further if the nondegeneracy factor is allowed to be larger.

I. INTRODUCTION

Several attempts have been made[1-4] during the past years to obtain a low right-handed scale (M_R) in grand unified theories[5,6] (GUT's) corresponding to the spontaneous symmetry breaking (SSB) of the intermediate gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ $(=G_{2213})$. Since SO(10) has several attractive features compared to many other GUT's (Ref. 6), we confine ourselves to this model in the present paper. In the conventional embeddings of left-right-symmetric $G_{2213}(g_{2L}=g_R)$ in SO(10), where the left-right discrete symmetry and $SU(2)_R \times U(1)_{B-L}$ break at the same scale, the value of M_R is high ($\sim 10^{12}$ GeV) for $\sin^2\theta_W \simeq 0.23-0.24$. This rules out any possibility of observing low-energy signatures of right-handed currents, including CP violation in $K \to 2\pi$ decays.[7] Several cosmological difficulties, such as the inadequate baryon asymmetry of the Universe,[8] and the presence of undesirable domain walls[9] have been noted, if the intermediate symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C \times D(g_{224P})$, or $G_{2213} \times D$ (D = left-right discrete symmetry) is allowed to survive sufficiently below the GUT scale ($\mu < 10^{12}$ GeV).

In a series of papers,[2] Chang, Mohapatra, and one of the authors (M.K.P.) have suggested a new approach to left-right-symmetric gauge theories, where the breakings of parity (P), and $SU(2)_R$ are decoupled from each other. The mechanism can be explained in the following manner. Let Σ_{ij} $(i,j=1,2,\ldots,10)$ represent the totally antisymmetric generators of SO(10), where $i,j=1,2,\ldots,6$ are the SO(6) indices, and $i,j=7,8,9,10$ are those of SO(4). There is an element of the SO(10) gauge group, called D parity, where $D=\Sigma_{23}\Sigma_{67}$, which plays the role similar to charge conjugation (C), or the parity (P) operator, as the case may be; but, in general, it cannot be identified with either. Only under special circumstances can D be identified with C or P. For instance, D changes a fermion (quark or lepton) $\Psi_L \xrightarrow{D} (\Psi^C)_L = (C\bar{\Psi}^T)_L$ which has charge opposite to Ψ_L, but it has also opposite helicity. This is clearly different from the usual C operation of transforming a particle to its antiparticle. Denoting $\Delta_L(3,1,10)$ and $\Delta_R^*(1,3,\overline{10})$ as the left- and right-handed triplets, respectively, contained in the Higgs representation $\mathbf{126} \subset SO(10)$, with the transformation properties under G_{224} specified against each of them, D can be identified with C under the special situation by demanding that $\Delta_L(3,1,10) \xrightarrow{C} \Delta_R^*(1,3,\overline{10})$. This property of D has been exploited to study the creation of strings, monopoles, and domain walls in the early Universe.[9] But, it has been noted in Ref. 2, that D can be identified with P that takes $\Psi_L \to \Psi_R$ when all the couplings in the SO(10) invariant Lagrangian are real. It has been found that certain Higgs representations, such as **210** and **45** of SO(10), contain singlets under G_{224} or G_{2213}, which are odd under D parity. When vacuum expectation values (VEV's) are assigned in the odd direction under D, the discrete symmetry D, and hence P, break spontaneously, but $SO(2)_R \times U(1)_{B-L}$, or $SU(2)_R \times SU(4)_C$ remain unbroken. When such a parity-breaking scale $M_P > 10^{12}$ GeV, cosmological domain walls do not cause a problem.[2] Adequate baryon asymmetry of the Universe is generated[2] if $M_P \sim M_U \gg M_R$ in the model with G_{2213} intermediate symmetry. Within the constraint of minimal fine-tuning of parameters, and $\sin^2\theta_W \simeq 0.22-0.24$, the scales M_R, corresponding to the

$SU(2)_R \times U(1)_{B-L}$ breaking, or M_C, corresponding to $SU(2)_R \times SU(4)_C$ breaking in SO(10), have been found to be significantly lower than conventional model predictions. The most profound symmetry-breaking chain has been noted[3] to be

$$SO(10) \xrightarrow[M_U]{54} G_{224P} \xrightarrow[M_P]{210} G_{224} \xrightarrow[M_R^+]{210} G_{2113} \xrightarrow[M_R^0]{126} G_{213} , \tag{1}$$

where $G_{2113}=SU(2)_{2L}\times U(1)_R \times U(1)_{B-L}\times SU(3)_C$, and $G_{213}=SU(2)_L\times U(1)_Y\times SU(3)_C$. With $M_R^0=M_{Z_R}=500-1000$ GeV, and $M_R^+=M_C=10^5$ GeV, this chain offers the possibility of detection of a low-mass z_R at the super colliders, measurement of signatures of quark-lepton unification through $K_L\to\mu\bar{e}$, n-$\bar{n}$ oscillations, and Majorana-neutrino masses ($m_{\nu_e}\sim 1$ eV) with proton lifetime barely within the reach of ongoing experiments. The chain (1) has three intermediate symmetries. Because of their minimal character, GUT predictions with one or two intermediate symmetries might be more appealing. In SO(10) with the decoupling mechanism, predictions have been made including two-loop corrections in the chains

$$\text{(a)}\quad SO(10) \xrightarrow[M_U=M_P]{45} G_{2213} \xrightarrow[M_R]{126} G_{213} ,$$

$$\text{(b)}\quad SO(10) \xrightarrow[M_U=M_P]{45_1} G_{2213} \xrightarrow[M_R^+]{45_2} G_{2113} \xrightarrow[M_R^0]{126} G_{213}$$

with $10^8 < M_R = M_R^+ < 10^{11}$ GeV for $0.24 > \sin^2\theta_W > 0.23$.

In this paper, we investigate possible modifications to these predictions caused by the superheavy components of Higgs representations used for SSB in cases (a) and (b). Such contributions have been estimated in SU(5) GUT for superheavy-component masses differing by a factor $10^{-4}-10^4$ from the GUT scale. Including Higgs representations **5**, **10**, **15**, **45**, and **50** of SU(5), and imposing the Coleman-Weinberg[10] mass constraint on the assumed nondegenerate components, Marciano[11] has obtained an increase in the proton lifetime (τ_p) for the $p\to e^+\pi^0$ mode by a large factor of 150 with the corresponding increase $\Delta\sin^2\theta_W(M_W)=0.001$ as compared to the minimal GUT predictions.

We find, in cases (a) and (b) of SO(10), that superheavy-component masses, compatible with the Coleman-Weinberg constraint, contained in the Higgs representations necessary for SSB, are capable of lowering the W_R-mass prediction by 4–5 orders of magnitude as compared to the earlier results. Such masses, besides being detected by the supercolliders, could manifest in the $V+A$ structure of neutral-current processes, neutrinoless double-beta decay, small Majorana neutrino masses, and CP violation in K decay. We also note that the right-handed scale could be lowered further if the nondegeneracy factor is permitted to be larger. Besides, we point out how the conventional matching equations for the coupling constants are modified at the intermediate scales.

The paper is planned in the following manner. In Sec. II we identify superheavy components of scalar representations while stating their transformation properties under G_{2213}. In Sec. III we obtain modifications to the renormalization-group equations. We also derive matching functions for the gauge coupling constants using the effective-gauge-theory approach. Our new solutions are presented in Sec. IV. A brief summary and conclusions of this work are stated in Sec. V.

II. IDENTIFICATION OF SUPERHEAVY COMPONENTS

In this section we mention superheavy components of Higgs representations and their transformation properties under G_{2213}. In the first stage of chain (a) or (b), when the neutral component of (1,1,15) under G_{224}, contained in $\mathbf{45}\subset SO(10)$, acquires VEV, parity is broken spontaneously. The trilinear coupling $\mathbf{45}\times\mathbf{126}\times\overline{\mathbf{126}}$ allows the right-handed triplet $\Delta_R(1,3,\sqrt{3/2},1)$ under G_{2213} contained in $\mathbf{126}\subset SO(10)$ to remain light, whereas the left-handed triplet $\Delta_L(3,1,\sqrt{3/2},1)$, and all other components in **126** acquire superheavy masses $\mu\sim M_U=M_P$. At the second stage in (a), the VEV of Δ_R breaks $G_{2213}\to G_{213}$. The right-handed neutrino receives a Majorana neutrino mass[12] of the order M_R. In the second stage of (b), a second representation $\mathbf{45}_2\subset SO(10)$ is used. Only its component $\chi_R(1,3,0,1)$ remains light with mass $\mu\sim M_R^+$, but all other components acquire superheavy masses $\mu\sim M_U$. In the third stage of (b), the component of Δ_R, singlet under G_{2113}, remains light with mass $\mu\sim M_R^0$, but all other components of Δ_R acquire mass $\mu\sim M_R^+$. Similarly, all other components of **126** acquire mass $\mu\sim M_U$. In both chains (a) and (b) the standard symmetry $G_{213}\to SU(3)_C\times U(1)_{em}$, when the left-right-symmetric (LRS) doublet $\phi(2,2,0,1)$ contained in $\mathbf{10}\subset SO(10)$ acquires appropriate VEV. The $SU(3)_C$ triplet and its conjugate, contained in **10**, acquire superheavy masses $\mu\sim M_U$. All superheavy masses can be specified more explicitly, along with their transformation properties under G_{2213}, as noted below:

$$\mathbf{45}_1=M_{S1}(1,3,0,1)+M_{S2}(3,1,0,1)+M_{S3}(1,1,0,8)+\cdots , \tag{2}$$

$$\mathbf{45}_2=M''_{S1}(3,1,0,1)+M''_{S2}(1,1,0,8)+M''_{S3}(2,2,\sqrt{3/2}\tfrac{1}{3},3)+M''_{S4}(2,2,-\sqrt{3/2}\tfrac{1}{3},\bar{3})$$
$$+M''_{S5}(1,1,-\sqrt{3/2}\tfrac{2}{3},3)+M''_{S6}(1,1,\sqrt{3/2}\tfrac{2}{3},3)+\cdots , \tag{3}$$

$$126=M'_{H1}(3,1,\sqrt{3/2}\tfrac{1}{3},6)+M'_{H2}(1,3,-\sqrt{3/2}\tfrac{1}{3},\bar{6})+M'_{H3}(3,1,\sqrt{3/2},1)+M'_{H4}(3,1,-\sqrt{3/2}\tfrac{1}{3},3)$$
$$+M'_{H5}(1,3,\sqrt{3/2}\tfrac{1}{3},\bar{3})+M'_{H6}(2,2,-\sqrt{3/2}\tfrac{1}{3},3)+M'_{H7}(2,2,\sqrt{3/2}\tfrac{1}{3},\bar{3})+M'_{H8}(2,2,0,8)$$
$$+M'_{H9}(2,2,0,1)+M'_{H10}(1,1,-\sqrt{3/2}\tfrac{1}{3},3)+M'_{H11}(1,1,\sqrt{3/2}\tfrac{1}{3},\bar{3})+\cdots\ , \tag{4}$$

$$10=M_{H1}(1,1,\sqrt{3/2}\tfrac{1}{3},\bar{3})+M_{H2}(1,1,-\sqrt{3/2}\tfrac{1}{3},3)+\cdots\ . \tag{5}$$

Among the components not explicitly mentioned in (2)–(5) or in (6) specified below, some are singlets under G_{2213} which do not contribute to the desired modifications. Others are either absorbed as would-be Goldstone components of appropriate gauge bosons, or they are light, and the corresponding contributions are included in one- and two-loop coefficients of the β function in the usual manner.[3]

Several years ago,[13] when it was not known that $(1,1,15)\subset$**45** contains a D-odd neutral component, it was noted that to break SO(10)$\rightarrow G_{2213}$, a **54** is also necessary. The SSB is understood by visualizing SO(10)$\rightarrow$SO(6)$\times$SO(4) by the VEV of **54**, and SO(6)$\times$SO(4)$\rightarrow G_{2213}$ by the VEV of **45** at the same scale. In all considerations of Refs. 2 and 3, where all superheavy Higgs-boson masses are taken to be M_U in the most nautral manner, the presence of **54** does not make any difference in the SO(10) predictions in cases (a) and (b). But when superheavy-component masses are different from M_U, the presence of **54** makes some difference as demonstrated in Secs. III and IV. The superheavy components of **54** can be specified in a manner similar to Eqs. (2)–(5):

$$54=M'_{S1}(3,3,0,1)+M'_{S2}(1,1,-\sqrt{3/2}\tfrac{2}{3},3)+M'_{S3}(1,1,\sqrt{3/2}\tfrac{2}{3},\bar{3})+M'_{S4}(1,1,0,1)$$
$$+M'_{S5}(1,1,0,8)+M'_{S6}(2,2,\sqrt{3/2}\tfrac{1}{3},3)+M'_{S7}(2,2,-\sqrt{3/2}\tfrac{1}{3},\bar{3})+\cdots\ . \tag{6}$$

Even if the superheavy-gauge bosons are assumed to possess the unification mass, there are nonvanishing threshold corrections to the coupling constants. In order to include such corrections and obtain the exact matching constraints at the intermediate scale, we also specify the masses of various gauge bosons contained in $\mathbf{45}_V$ SO(10) in case (a):

$$45_V=M_W(3,1,0,1)+M_R(1,3,0,1)+0(1,1,0,8)$$
$$+M_U(2,2,\sqrt{3/2}\tfrac{1}{3},3)+M_U(2,2,-\sqrt{3/2}\tfrac{1}{3},\bar{3})$$
$$+M_U(1,1,-\sqrt{3/2}\tfrac{2}{3},3)+M_U(1,1,\sqrt{3/2}\tfrac{2}{3},\bar{3})$$
$$+M_R(1,1,0,1)\ . \tag{7}$$

In the next section we show how the superheavy-Higgs-scalar contributions are taken into account by renormalization-group equations (RGE's) to modify GUT predictions.

III. RENORMALIZATION-GROUP CONSTRAINTS AND MATCHING FUNCTIONS

In this section we derive modifications to the RGE's including contributions of the superheavy-Higgs-scalar components. We also derive matching functions for $\ln M_U/M_W$, $\sin^2\theta_W$, and the fine-structure constant, noted to be important while computing heavy-particle effects in effective-gauge theories (EGT's) from GUT's. We also note changes in the matching equations of gauge-coupling constants at the intermediate scales from those conventionally adopted. For a general subgroup of the form $G_1\times G_2\times G_3\times\cdots$, emerging from the SSB of the GUT at the scale M_U, the usual RGE's between the scales $m<\mu<M_U$ are written as

$$\frac{1}{\alpha_i(m)}=\frac{1}{\alpha_i(M_U)}+\frac{a_i}{2\pi}\ln\frac{M_U}{m}+\frac{1}{4\pi}\sum_j B_{ij}\ln x_j,\quad i,j=1,2,\ldots, \tag{8}$$

where the second (third) terms are the one- (two-) loop contributions, $x_j=\alpha_j(M_U)/\alpha_j(m)$, $\alpha_j(\mu)=g_i^2(\mu)/4\pi$, and g_i is the gauge-coupling constant for G_i. In the old approach, all coupling constants were assumed to unify with the GUT coupling constant (α_G) at $\mu=M_U$; i.e., $\alpha_i(M_U)=\alpha_G$. But with the emergence of the effective-gauge-theory (EGT) approach,[14–16] it has been found that the gauge-copuling constants are prevented from merging together by superheavy-particle effects. Such threshold corrections arise because of contributions of heavy gauge bosons (V), Higgs scalars (S), and fermions (F) to the quantum corrections of lighter gauge-boson propagators. For μ near M_U,

$$\frac{1}{\alpha_i(\mu)}=\frac{1}{\alpha_G}-\frac{\lambda_i(\mu)}{12\pi},\quad i=1,2,3,\ldots, \tag{9}$$

$$\lambda_i(\mu)=C_i^V(\mu)+C_i^S(\mu)+C_i^F(\mu)\ , \tag{10}$$

$$C_i^S=\sum_n \mathrm{Tr}\left[t_{iSn}^2\Lambda_{Sn}\ln\frac{M_n^S}{\mu}\right], \tag{11a}$$

$$C_i^V=\sum_m \mathrm{Tr}(t_{iVm}^2)-21\sum_m \mathrm{Tr}\left[t_{iVm}^2\ln\frac{M_m^V}{\mu}\right], \tag{11b}$$

$$C_i^F=\sum_p 8\,\mathrm{Tr}\left[t_{iFp}^2\ln\frac{M_p^F}{\mu}\right], \tag{11c}$$

where the superscripts V, S, and F represent contributions due to superheavy gauge bosons, scalars, and fermions, if any; t_{iV}, t_{iS}, and t_{iF} denote the corresponding matrix representation of operators; and Λ_{Sm} is the projection operator that removes the Goldstone components from S. Although Eqs. (9)–(11c) have been stated near the unification mass, such formulas hold at the intermediate scales where a higher gauge symmetry undergoes SSB to a lower one, but with suitable replacement of M_U and α_G. In what follows we assume only three fermion generations that are light with the top-quark mass $m_t < 80$ GeV. We will also assume all superheavy-gauge bosons to have masses exactly at the grand unification mass. These assumptions imply $C_i^F=0$, and $C_i^V=\sum_n \mathrm{Tr}(t_{iVn}^2)$. Using the decomposition of $\mathbf{45}_V \subset \mathrm{SO}(10)$, under G_{2213} as shown in Eq. (7), we compute

$$C_{3c}^V(M_U)=5,\quad C_{2L}^V(M_U)=C_{2R}^V(M_U)=6\ ,\quad C_{BL}^V(M_U)=8\ . \tag{12}$$

Equation (12) implies that, even if all superheavy-gauge- and Higgs-boson masses are identical to M_U, the coupling constants do not meet at $\mu=M_U$, and satisfy the relation

$$\begin{aligned}\frac{1}{\alpha_G}=\frac{1}{\alpha_{2L}(M_U)}+\frac{1}{2\pi}&=\frac{1}{\alpha_{2R}(M_U)}+\frac{1}{2\pi}\\ &=\frac{1}{\alpha_{3C}(M_U)}+\frac{5}{12\pi}\\ &=\frac{1}{\alpha_{BL}(M_U)}+\frac{2}{3\pi}\ .\end{aligned} \tag{13}$$

Using Eqs. (8)–(11), we obtain expressions for $\sin^2\theta_W$ and $\ln M_U/M_W$ in cases (a) and (b).

Very recently,[17] it has been noted that, while heavy-particle effects are taken into account in the usual manner, as described above, the fine-structure constant extrapolated from the low-energy theory based upon $\mathrm{SU}(3)_C\times\mathrm{U}(1)_{\mathrm{em}}$ does not match the GUT predictions at $\mu=M_W$. A method for GUT predictions, by exactly matching the fine-structure constant, has been also found.[17] Following this method, we compute the matching functions corresponding to the relation

$$\frac{1}{\alpha(M_W)}=\frac{5}{3}\frac{1}{\alpha_Y(M_W)}+\frac{1}{\alpha_{2L}(M_W)}\ . \tag{14}$$

For the given values of M_U, M_R, and superheavy masses, iterative convergence approach is used to solve RGE's with variation of α_G. For a certain value of α_G, the fine-structure constant is found to match exactly. We now compute matching functions for different gauge-coupling constants $\ln M_U/M_W$ and $\sin^2\theta_W$ in the two cases along with the matching constraints at the intermediate scales.

A. Matching functions with G_{2213} intermediate symmetry

Before obtaining matching functions for $\mu\sim M_U$, we note that the usual matching constraint at $\mu=M_R$, namely,

$$\frac{1}{\alpha_Y(M_R)}=\frac{3}{5}\frac{1}{\alpha_{2R}(M_R)}+\frac{2}{5}\frac{1}{\alpha_{BL}(M_R)}\ ,$$

is modified in the EGT approach. The right-handed gauge bosons $M_R(1,3,0,1)$ under G_{2213} carrying $B-L=0$ are decomposed under G_{213} as $M_R(1,\sqrt{3/5},1)+M_R(1,-\sqrt{3/5},1)+M_R(1,0,1)$. The Y-boson propagator is modified by the quantum corrections due to the heavy gauge bosons $M_R(1,\pm\sqrt{3/5},1)$ in the loop, as shown in Fig. 1(a), giving rise to

$$\frac{1}{\alpha_Y(\mu)}=\frac{3}{5}\frac{1}{\alpha_{2R}(M_R)}+\frac{2}{5}\frac{1}{\alpha_{BL}(M_R)}-\frac{C_Y^V(\mu)}{12\pi} \tag{15}$$

for $\mu\sim M_R$, with

$$C_Y^V(\mu)=\tfrac{6}{5}-\tfrac{126}{5}\ln\frac{M_R}{\mu}\ . \tag{16}$$

Here we have taken the $W_R^\pm$ gauge-boson mass as M_R and neglected possible heavy scalar contributions to $C_Y^S(\mu)$. Equations (15) and (16) now yield the modified matching condition

$$\frac{1}{\alpha_Y(M_R)}=\frac{3}{5}\frac{1}{\alpha_{2R}(M_R)}+\frac{2}{5}\frac{1}{\alpha_{BL}(M_R)}-\frac{1}{10\pi}\ . \tag{17}$$

We use the following values of the a_i and B_{ij}, occurring in Eq. (8), in the respective mass ranges.[2,3]

$M_W<\mu<M_R$: $a_Y=\frac{41}{10}$, $a_{2L}=-\frac{19}{6}$, $a_{3C}=-7$,

$B_{ij}=$

	Y	$2L$	$3C$
Y	$\frac{199}{205}$	$-\frac{81}{95}$	$-\frac{44}{35}$
$2L$	$\frac{9}{41}$	$-\frac{35}{19}$	$-\frac{12}{7}$
$3C$	$\frac{11}{41}$	$-\frac{27}{19}$	$\frac{26}{7}$

(18)

$M_R<\mu<M_U$: $a_{BL}=\frac{11}{2}$, $a_{2L}=-3$, $a_{2R}=-\frac{7}{3}$, $a_{3C}=-7$,

$B_{ij}=$

	BL	$2L$	$2R$	$3C$
BL	$\frac{61}{11}$	$-\frac{3}{2}$	$-\frac{243}{14}$	$-\frac{4}{7}$
$2L$	$\frac{3}{11}$	$-\frac{8}{3}$	$-\frac{9}{7}$	$-\frac{12}{7}$
$2R$	$\frac{27}{11}$	-1	$-\frac{80}{7}$	$-\frac{12}{7}$
$3C$	$\frac{1}{11}$	$-\frac{3}{2}$	$-\frac{27}{14}$	$\frac{26}{7}$

(19)

Using the combinations $e^{-2}(M_W)-\frac{8}{3}g_{2L}^{-2}(M_W)$, $e^{-2}(M_W)-\frac{8}{3}g_3^{-2}(M_W)$, and $\alpha^{-1}(M_W)=\frac{5}{3}\alpha_Y^{-1}(M_W)+\alpha_{2L}^{-1}(M_W)$, and relations (8) and (17), we obtain

$$\ln\frac{M_U}{M_W}=\frac{2\pi}{17}\left[\frac{1}{\alpha}-\frac{8}{3\alpha_s}\right]-\frac{16}{51}\ln\frac{M_R}{M_W}-\tfrac{1}{34}(\tfrac{46}{41}\ln\chi_Y^R+\tfrac{10}{19}\ln\chi_{2L}^R-\tfrac{96}{7}\ln\chi_{3C}^R+\tfrac{68}{11}\ln\chi_{BL}^U-\tfrac{2}{3}\ln\chi_{2L}^U-\tfrac{134}{7}\ln\chi_{2R}^U-\tfrac{96}{7}\ln\chi_{3C}^U)$$
$$+\tfrac{1}{102}(\lambda_{2L}+\lambda_{2R}+\tfrac{2}{3}\lambda_{BL}-\tfrac{8}{3}\lambda_{3C}),\tag{20}$$

$$\sin^2\theta_W=\frac{12+19\alpha/\alpha_s}{51}-\frac{\alpha}{34\pi}\left[\frac{145}{6}\ln\frac{M_R}{M_W}\right]$$
$$-\frac{\alpha}{136\pi}(\tfrac{436}{11}\ln\chi_Y^R+\tfrac{352}{19}\ln\chi_{2L}^R+\tfrac{524}{7}\ln\chi_{3C}^R-\tfrac{476}{11}\ln\chi_{BL}^U+\tfrac{103}{3}\ln\chi_{2L}^U-175\ln\chi_{2R}^U+\tfrac{524}{7}\ln\chi_{3C}^U)$$
$$-\frac{\alpha}{34\pi}[\tfrac{13}{6}\lambda_{2L}-\tfrac{2}{3}(\lambda_{2R}+\tfrac{2}{3}\lambda_{BL})-\tfrac{19}{18}\lambda_{3C}],\tag{21}$$

$$\frac{1}{\alpha(M_W)}=\frac{17}{7\alpha_G}+\frac{5}{21\alpha_s}+\frac{1}{14\pi}\left[\frac{112}{3}\ln\frac{M_R}{M_W}+\tfrac{509}{82}\ln\chi_Y^R-\tfrac{389}{38}\ln\chi_{2L}^R-\tfrac{345}{21}\ln\chi_{3C}^R+\tfrac{493}{22}\ln\chi_{BL}^U-\tfrac{181}{12}\ln\chi_{2L}^U\right.$$
$$\left.-\tfrac{2335}{28}\ln\chi_{2R}^U-\tfrac{345}{21}\ln\chi_{3C}^U\right]$$
$$-\frac{1}{12\pi}(\lambda_{2L}+\lambda_{2R}+\tfrac{2}{3}\lambda_{BL}-\tfrac{5}{21}\lambda_{3C}).\tag{22}$$

In Eqs. (20)–(22) $X_i^R=\alpha_i(M_R)/\alpha_i(M_W)$, $\chi_i^U=\alpha_i(M_U)/\alpha_i(M_R)$. It is clear that the matching function contributions to $\ln M_U/M_W$, $\sin^2\theta_W$, and $\alpha^{-1}(M_W)$ are given by f_M, f_θ, and f_α, respectively:

$$f_M=\tfrac{1}{102}(\lambda_{2L}+\lambda_{2R}+\tfrac{2}{3}\lambda_{BL}-\tfrac{8}{3}\lambda_{3C}),\tag{23}$$

$$f_\theta=-\frac{\alpha}{34\pi}[\tfrac{13}{6}\lambda_{2L}-\tfrac{2}{3}(\lambda_{2R}+\tfrac{2}{3}\lambda_{BL})-\tfrac{19}{18}\lambda_{3C}],\tag{24}$$

$$f_\alpha=-\frac{1}{12\pi}(\lambda_{2L}+\lambda_{2R}+\tfrac{2}{3}\lambda_{BL}-\tfrac{5}{21}\lambda_{3C}).\tag{25}$$

Assuming all superheavy-gauge bosons to have masses at $\mu=M_U$, and using Eq. (12),

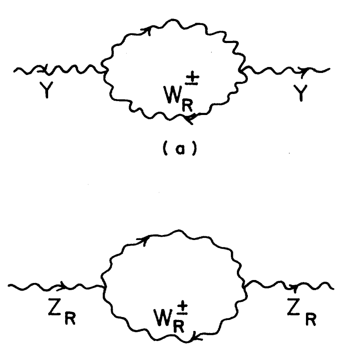

FIG. 1. (a) One-loop diagram in the effective-gauge theory that modifies the coupling-constant-matching condition at the intermediate scale M_R in case (a) of SO(10). (b) One-loop diagram in the effective-gauge theory that modifies the coupling-constant-matching condition at $\mu=M_R^0$ in case (b) of SO(10).

$$\lambda_{2L}=6+C_{2L}^{(S)}(M_U),\quad \lambda_{2R}=6+C_{2R}^{(S)}(M_U),$$
$$\lambda_{BL}=8+C_{BL}^{(S)}(M_U),\quad \text{and}\quad \lambda_{3C}=5+C_{3C}^{(S)}(M_U),$$

leading to

$$f_M=\tfrac{2}{51}+f_M^{(S)},\quad f_\theta=-\frac{\alpha}{204\pi}+f_\theta^{(S)},$$
$$f_\alpha=-\frac{337}{216\pi}+f_\alpha^{(S)},\tag{26}$$

where

$$f_M^{(S)}=\tfrac{1}{102}(C_{2L}^{(S)}+C_{2R}^{(S)}+\tfrac{2}{3}C_{BL}^{(S)}-\tfrac{8}{3}C_{3C}^{(S)}),$$
$$f_e^{(S)}=-\frac{\alpha}{34\pi}[\tfrac{13}{6}C_{2L}^{(S)}-\tfrac{2}{3}(C_{2R}^{(S)}+\tfrac{2}{3}C_{BL}^{(S)})-\tfrac{19}{18}C_{3C}^{(S)}],\tag{27}$$
$$f_\alpha^{(S)}=-\frac{1}{12\pi}(C_{2L}^{(S)}+C_{2R}^{(S)}+\tfrac{2}{3}C_{BL}^{(S)}-\tfrac{5}{21}C_{3C}^{(S)}).$$

Here $C_i^{(S)}$ represent superheavy-Higgs-scalar contributions at $\mu=M_U$. Using decomposition of $(\mathbf{45})_1$, **54**, **126**, and **10**, given by Eqs. (2)–(6), we compute

$$C_{2L}^{(10)}=C_{2R}^{(10)}=0,\quad C_{BL}^{(10)}=C_{3C}^{(10)}=\eta_{H1}+\eta_{H2},$$
$$C_{2L}^{(45)}=2\eta_{S2},\quad C_{2R}^{(45)}=2\eta_{S1},\quad C_{3C}^{(45)}=3\eta_{S3},$$
$$C_{BL}^{(45)}=0,\quad C_{2L}^{(54)}=C_{2R}^{(54)}=12\eta'_{S1}+6\eta'_{S6}+6\eta'_{S7},$$
$$C_{3C}^{(54)}=5\eta'_{S2}+5\eta'_{S3}+6\eta'_{S5}+4\eta'_{S6}+4\eta'_{S7},$$
$$C_{BL}^{(54)}=8\eta'_{S2}+8\eta'_{S3}+4\eta'_{S6}+4\eta'_{S7},\tag{28}$$
$$C_{2L}^{(126)}=24\eta'_{H1}+4\eta'_{H3}+12\eta'_{H4}$$
$$+6\eta'_{H6}+6\eta'_{H7}+16\eta'_{H8}+2\eta'_{H9},$$

$$C_{2R}^{(126)}=24\eta'_{H2}+12\eta'_{H5}+6\eta'_{H6}+6\eta'_{H7}+16\eta'_{H8}+2\eta'_{H9}\ ,$$

$$C_{3C}^{(126)}=15\eta'_{H1}+15\eta'_{H2}+3\eta'_{H4}+3\eta'_{H5}+4\eta'_{H6}+4\eta'_{H7}+24\eta'_{H8}+\eta'_{H10}+\eta'_{H11}\ ,$$

$$C_{BL}^{(126)}=6\eta'_{H1}+6\eta'_{H2}+9\eta'_{H3}+3\eta'_{H4}+3\eta'_{H5}+4\eta'_{H6}+4\eta'_{H7}+\eta'_{H10}+\eta'_{H11}\ ,$$

where $\eta_i=\ln M_i/M_U$ and $\eta'_i=\ln M'_i/M_U$. Using (28) in (26) and (27), we obtain

$$f_M^{(S)}=\tfrac{1}{102}[2(\eta_{S1}+\eta_{S2})-8\eta_{S3}-2(\eta_{H1}+\eta_{H2})+24\eta'_{S1}-8(\eta'_{S2}+\eta'_{S3})-16\eta'_{S5}+4(\eta'_{S6}+\eta'_{S7})-12(\eta'_{H1}+\eta'_{H2})+10\eta'_{H3}+6\eta'_{H4}+6\eta'_{H5}+4\eta'_{H6}+4(\eta'_{H7}+\eta'_{H9})-32\eta'_{H8}-2(\eta'_{H10}+\eta'_{H11})]\ , \tag{29}$$

$$f_\theta^{(S)}=\frac{\alpha}{612\pi}[-78\eta_{S2}+24\eta_{S1}+57\eta_{S3}+27(\eta_{H1}+\eta_{H2})-324\eta'_{S1}+159(\eta'_{S2}+\eta'_{S3})+114\eta'_{S5}-54(\eta'_{S6}+\eta'_{S7})-603\eta'_{H1}+621\eta'_{H2}-84\eta'_{H3}-387\eta'_{H4}+225\eta'_{H5}-54(\eta'_{H6}+\eta'_{H7}+\tfrac{35}{27}\eta'_{H9})+24\eta'_{H8}+27(\eta'_{H10}+\eta'_{H11})]\ , \tag{30}$$

$$f_\alpha^{(S)}=-\frac{1}{12\pi}[2(\eta_{S1}+_{S2})-\tfrac{5}{7}\eta_{S3}+\tfrac{3}{7}(\eta_{H1}+\eta_{H2})+24\eta'_{S1}+\tfrac{29}{7}(\eta'_{S2}+\eta'_{S3})-\tfrac{10}{7}\eta'_{S5}+\tfrac{96}{7}(\eta'_{S6}+\eta'_{S7})+\tfrac{171}{7}\eta'_{H1}+\tfrac{171}{7}\eta'_{H2}+10\eta'_{H3}+\tfrac{93}{7}\eta'_{H4}+\tfrac{93}{7}\eta'_{H5}+\tfrac{96}{7}\eta'_{H6}+\tfrac{96}{7}\eta'_{S7}+\tfrac{184}{7}\eta'_{H8}+4\eta'_{H9}+\tfrac{3}{7}(\eta'_{H10}+\eta'_{H11})]\ . \tag{31}$$

B. Matching functions with G_{2113} and G_{2213} intermediate symmetries

At the intermediate scale $\mu=M_R^+$, the usual matching condition $\alpha_{1R}^{-1}(M_R^+)=\alpha_{2R}^{-1}(M_R^+)$ is modified in the EGT approach due to the quantum corrections of the Z_R-boson propagator by the $W_R^\pm$ bosons in the loop, as shown in Fig. 1(b). Noting that

$$C_{1R}^V(\mu)=2-42\ln\frac{M_R^+}{\mu} \tag{32}$$

for $\mu\sim M_R^+$, the modified matching constraint is

$$\frac{1}{\alpha_{1R}(M_R^+)}=\frac{1}{\alpha_{2R}(M_R^+)}-\frac{1}{6\pi}\ . \tag{33}$$

Although the EGT modifications at the intermediate scales given by Eqs. (17) and (33) are not important, numerically we have derived them here for completeness. The coefficients a_i and B_{ij} occurring in one- and two-loop contributions to RGE's in different mass ranges are given below.[2,3]

$M_W<\mu<M_R^0$: Same as Eq. (18).

$M_R^0<\mu<M_R^+$: $a_{BL}=\frac{9}{2}$, $a_{1R}=\frac{9}{2}$, $a_{2L}=-\frac{19}{6}$, $a_{3C}=-7$,

$B_{ij}=$

	BL	$1R$	$2L$	$3C$
BL	$\frac{25}{9}$	$\frac{5}{3}$	$-\frac{27}{19}$	$-\frac{4}{7}$
$1R$	$\frac{5}{3}$	$\frac{5}{3}$	$-\frac{9}{19}$	$-\frac{12}{7}$
$2L$	$\frac{1}{3}$	$\frac{1}{9}$	$-\frac{35}{19}$	$-\frac{12}{7}$
$3C$	$\frac{1}{9}$	$\frac{1}{3}$	$-\frac{27}{19}$	$\frac{26}{7}$

(34)

$M_R^+<\mu<M_U$: $a_{BL}=\frac{11}{2}$, $a_{2L}=-3$, $a_{2R}=-2$, $a_{3C}=-7$,

$B_{ij}=$

	BL	$2L$	$2R$	$3C$
BL	$\frac{61}{11}$	$-\frac{3}{2}$	$-\frac{81}{4}$	$-\frac{4}{7}$
$2L$	$\frac{3}{11}$	$-\frac{8}{3}$	$-\frac{3}{2}$	$-\frac{12}{7}$
$2R$	$\frac{27}{11}$	-1	-18	$-\frac{12}{7}$
$3C$	$\frac{1}{11}$	$-\frac{3}{2}$	$-\frac{9}{4}$	$\frac{26}{7}$

(35)

Proceeding in the same manner as in case (a), and using Eqs. (8), (18), and (33)–(35), we obtain

$$\ln\frac{M_U}{M_W}=\frac{3\pi}{26}\left[\frac{1}{\alpha}-\frac{8}{3\alpha_s}\right]+\frac{1}{26}\ln\frac{M_R^0}{M_W}-\frac{17}{52}\ln\frac{M_R^+}{M_W}$$
$$-\tfrac{3}{52}(\tfrac{69}{123}\ln X_Y+\tfrac{5}{19}\ln X_{2L}^0-\tfrac{44}{7}\ln X_{3C}^0+\tfrac{16}{9}\ln X_{BL}^+ +\ln X_{1R}^+ +\tfrac{5}{19}\ln X_{2L}^+ -\tfrac{148}{21}\ln X_{3C}^+$$
$$+\tfrac{34}{11}\ln X_{BL}^U-\tfrac{1}{3}\ln X_{2L}^U-\tfrac{27}{2}\ln X_{2R}^U-\tfrac{140}{21}\ln X_{3C}^U)+f_M\ , \tag{36}$$

$$\sin^2\theta_W=\frac{3}{13}+\frac{5\alpha}{13\alpha_s}-\frac{3\alpha}{104\pi}\left[\frac{1046}{9}\ln\frac{M_R^0}{M_W}+\frac{230}{9}\ln\frac{M_R^+}{M_R^0}\right]$$
$$-\frac{9\alpha}{1664\pi}(\tfrac{5248}{369}\ln X_Y^0+\tfrac{1429}{57}\ln X_{2L}^0+\tfrac{6592}{63}\ln X_{3C}^0+\tfrac{2240}{81}\ln X_{BL}^+ +\tfrac{2560}{81}\ln X_{1R}^+ +\tfrac{4288}{171}\ln X_{2L}^+$$
$$+\tfrac{6592}{63}\ln X_{3C}^+ +\tfrac{5696}{63}\ln X_{BL}^U+\tfrac{464}{27}\ln X_{2L}^U+\tfrac{968}{3}\ln X_{2L}^U+\tfrac{6592}{63}\ln X_{3C}^U)+f_\theta\ , \tag{37}$$

$$\frac{1}{\alpha(M_W)}=\frac{52}{21\alpha_G}+\frac{4}{21\alpha_s}+\frac{7}{6\pi}\left[\ln\frac{M_R^0}{M_W}+\frac{17}{7}\ln\frac{M_R^+}{M_R^0}+\tfrac{6}{49}(\tfrac{139}{41}\ln X_Y^0-\tfrac{199}{38}\ln X_{2L}^0-\tfrac{166}{19}\ln X_{3C}^0+\tfrac{181}{27}\ln X_{BL}^+\right.$$
$$+\tfrac{89}{18}\ln X_{1R}^+ -\tfrac{199}{38}\ln X_{2L}^+ +\tfrac{166}{21}\ln X_{3C}^+ +\tfrac{370}{33}\ln X_{BL}^U$$
$$\left.-\tfrac{23}{3}\ln X_{2L}^U-57\ln X_{2R}^U-\tfrac{166}{21}\ln X_{3C}^U)\right]+f_\alpha\ , \tag{38}$$

where $X_i^0=\alpha_i(M_R^0)/\alpha_i(M_W)$, $X_i^+=\alpha_i(M_R^+)/\alpha_i(M_R^0)$, $X_i^U=\alpha_i(M_U)\alpha_i(M_R^+)$, and

$$f_M=\tfrac{1}{104}(\lambda_{2L}+\lambda_{2R}+\tfrac{2}{3}\lambda_{BL}-\tfrac{8}{3}\lambda_{3C})=\tfrac{1}{26}+f_M^{(S)}\ , \tag{39a}$$

$$f_\theta=\frac{\alpha}{52\pi}(\tfrac{5}{3}\lambda_{3C}+\lambda_{2R}+\tfrac{2}{3}\lambda_{BL}-\tfrac{10}{3}\lambda_{2L})=\frac{\alpha}{156\pi}+f_\theta^{(S)}\ , \tag{39b}$$

$$f_\alpha=\frac{1}{84\pi}[\tfrac{4}{3}\lambda_{3C}-7(\lambda_{2L}+\lambda_{2R}+\tfrac{2}{3}\lambda_{BL})]=-\frac{86}{63\pi}+f_\alpha^{(S)} \tag{39c}$$

with

$$f_M^{(S)}=\tfrac{1}{104}(C_{2L}^{(S)}+C_{2R}^{(S)}+\tfrac{2}{3}C_{BL}^{(S)}-\tfrac{8}{3}C_{3C}^{(S)})\ , \tag{40a}$$

$$f_\theta^{(S)}=\frac{\alpha}{52\pi}(\tfrac{5}{3}C_{3C}^{(S)}+C_{2R}^{(S)}+\tfrac{2}{3}C_{BL}^{(S)}-\tfrac{10}{3}C_{2L}^{(S)})\ , \tag{40b}$$

$$f_\alpha^{(S)}=\frac{1}{12\pi}(\tfrac{4}{21}C_{3C}^{(S)}-C_{2L}^{(S)}-C_{2R}^{(S)}-\tfrac{2}{3}C_{BL}^{(S)})\ . \tag{40c}$$

Compared to case (a), there are additional superheavy-Higgs-scalar components arising from $(\mathbf{45})_2$, used at the second stage of SSB. Thus, instead of Eq. (28), the contribution of **45**'s to $C_i^{(S)}$ is given by

$$C_{2L}^{(45)}=2\eta_{S1}+2\eta''_{S1}+3\eta''_{S3}+3\eta''_{S4},\quad C_{2R}^{(45)}=2\eta_{S2}+3\eta''_{S3}+3\eta''_{S4}\ ,$$
$$C_{3C}^{(45)}=3\eta_{S3}+3\eta''_{S2}+2\eta''_{S3}+2\eta''_{S4}+\eta''_{S5}/2+\eta''_{S6}/2,\quad C_{BL}^{(45)}=2\eta''_{S3}+2\eta''_{S4}+2\eta''_{S5}+2\eta''_{S6}\ . \tag{41}$$

Using the contributions of **54**, **126**, and **10**, from Eq. (28), and that of **45** from (41), we find

$$f_M^{(S)}=\tfrac{1}{104}[2(\eta_{S1}+\eta_{S2})-8\eta_{S3}-2(\eta_{H1}+\eta_{H2})+24\eta'_{S1}-8(\eta'_{S2}+\eta'_{S3})-16\eta'_{S5}$$
$$+4(\eta'_{S6}+\eta'_{S7})+2\eta''_{S1}-8\eta''_{S2}+2(\eta''_{S3}+\eta''_{S4})-12(\eta'_{H1}+\eta'_{H2})+10\eta'_{H3}+6\eta'_{H4}$$
$$+6\eta'_{H5}+4(\eta'_{H6}+\eta'_{H7}+\eta'_{H9})-32\eta'_{H8}-2(\eta'_{H10}+\eta'_{H11})]\ , \tag{42a}$$

$$f_\theta^{(S)}=\frac{\alpha}{9984\pi}[-1282\eta_{S1}+384\eta_{S2}+960\eta_{S3}+448(\eta_{H1}+\eta_{H2})-5388\eta'_{S1}+2624(\eta'_{S2}+\eta'_{S3})$$
$$+1920\eta'_{S5}-902\eta'_{S6}-902\eta'_{S7}-1282\eta''_{S1}+960\eta''_{S2}-451(\eta''_{S3}+\eta''_{S4})$$
$$+416(\eta''_{S5}+\eta''_{S6})-9792\eta'_{H1}+10176\eta'_{H2}-1412\eta'_{H3}-6348\eta'_{H4}+3648\eta'_{H5}$$
$$-902(\eta'_{H6}+\eta'_{H7})+496\eta'_{H8}-898\eta'_{H9}+448(\eta'_{H10}+\eta'_{H11})]\ , \tag{42b}$$

$$f_\alpha^{(S)}=\frac{1}{12\pi}[\tfrac{10}{21}(\eta_{H1}+\eta_{H2})-2(\eta_{S1}+\eta_{S2})+\tfrac{4}{7}\eta_{S3}-24\eta'_{S1}-\tfrac{92}{21}(\eta'_{S2}+\eta'_{S3})+\tfrac{8}{7}\eta'_{S5}$$
$$-\tfrac{292}{21}(\eta'_{S6}+\eta'_{S7})-2\eta''_{S1}+\tfrac{4}{7}\eta''_{S2}-\tfrac{146}{21}(\eta''_{S3}+\eta''_{S4})-\tfrac{26}{21}(\eta''_{S5}+\eta''_{S6})-\tfrac{176}{7}\eta'_{H1}$$
$$-\tfrac{176}{7}\eta'_{H2}-10\eta'_{H3}-\tfrac{94}{7}(\eta'_{H4}+\eta'_{H5})-\tfrac{292}{21}\eta'_{H6}-\tfrac{292}{21}\eta'_{H7}-\tfrac{192}{7}\eta'_{H8}-4\eta'_{H9}-\tfrac{10}{21}(\eta'_{H10}+\eta'_{H11})] \ . \quad (42c)$$

In the next section we specify the method for obtaining new solutions to the RGE's including superheavy-scalar effects.

IV. NEW PREDICTIONS WITH SUPERHEAVY HIGGS SCALARS

Given the RGE's and the corresponding equations for $\ln M_U/M_W$ and $\sin^2\theta_W$, the latter two can be predicted for a certain value of M_R, or M_R^+ and M_R^0, provided the superheavy masses are known. One of the most natural assumptions, consistent with extended survival hypothesis, is that every superheavy component has mass $\mu=M_U$, for which SO(10) predictions have been investigated. Also, we note that no interesting modifications are possible if all superheavy components are taken to be degenerate at mass $M\neq M_U$. If the Higgs-scalar masses are taken to be arbitrarily nondegenerate, the predictive power of the model is lost. A more natural constraint on nondegenerate Higgs scalars is due to the Coleman-Weinberg[10] mechanism by which a factor of 10 mass difference can be generated among different nondegenerate components. Besides the Coleman-Weinberg mass constraints, the other constraint, which divides the nondegenerate scalars into two different classes with unequal masses, is due to the minimization of the GUT prediction of $\sin^2\theta_W$, as illustrated below, for the two cases.

A. Predictions with G_{2213} intermediate symmetry

For values of $M_R<10^{10}$ GeV, this model predicts $\sin^2\theta_W>0.23$ when the superheavy-Higgs-scalar contribution is neglected. We expect the superheavy-Higgs-scalar contributions to reduce the values of $\sin^2\theta_W$ so as to make them compatible with the currently accepted world average, $\sin^2\theta_W=0.23\pm0.005$.

Using Eq. (30), we first calculate the maximum decrease in $\sin^2\theta_W$, excluding and including the contributions of **54**. Excluding **54** ($\eta'_{Si}=0$), maximum decrease in $\sin^2\theta_W$ is possible under the Coleman-Weinberg mass constraint for

$$\eta_{H1}=\eta_{H2}=\eta_{S1}=\eta_{S3}=\eta'_{H2}=\eta'_{H5}=\eta'_{H8}=\eta'_{H10}=\eta'_{H11}=\eta^{(+)} ,$$
$$\eta_{S2}=\eta'_{H1}=\eta'_{H3}=\eta'_{H4}=\eta'_{H6}=\eta'_{H7}=\eta'_{H9}=\eta^{(-)} \quad (43)$$

with

$$\eta^{(+)}=\ln\rho^{(+)},\quad \eta^{(-)}=\ln\rho^{(-)},\quad \rho^{(\pm)}=\frac{M^{(\pm)}}{M_U}$$

leading to

$$f_\theta^{(S)}=\frac{\alpha}{612\pi}(1059\eta^{(+)}-1330\eta^{(-)}) ,$$
$$f_M^{(S)}=\tfrac{1}{102}(18\eta^{(-)}-52\eta^{(+)}) , \quad (44)$$
$$f_\alpha^{(S)}=-\frac{1}{12\pi}(82.7\eta^{(+)}+129.2\eta^{(-)}) ,$$

where $\eta^{(+)}$ and $\eta^{(-)}$ can be related to the maximum nondegeneracy factor 10 allowed by the Coleman-Weinberg mass constraint:

$$|\eta^{(+)}-\eta^{(-)}|=\ln\left[\frac{M^{(+)}}{M^{(-)}}\right]=\ln 10 \ . \quad (45)$$

When **54** is included, the following new constraints, in addition to (43), are satisfied by the superheavy components for obtaining minimum values of $\sin^2\theta_W$:

$$\eta'_{S2}=\eta'_{S3}=\eta'_{S5}=\eta^{(+)} ,$$
$$\eta'_{S1}=\eta'_{S6}=\eta'_{S7}=\eta^{(-)} \ . \quad (46)$$

Using (43) and (46) gives

$$f_\theta^{(S)}=\frac{\alpha}{612\pi}(1491\eta^{(+)}-1762\eta^{(-)}) ,$$
$$f_M^{(S)}=\tfrac{1}{102}(50\eta^{(-)}-84\eta^{(+)}) , \quad (47)$$
$$f_\alpha^{(S)}=-\frac{1}{12\pi}(\tfrac{517}{7}\eta^{(+)}+\tfrac{928}{7}\eta^{(-)}) \ .$$

Having obtained the matching function corrections as shown in Eqs. (44) and (47), it is now easier to estimate the superheavy masses $M^{(+)}$ and $M^{(-)}$ for which $\sin^2\theta_W$ is significantly less compared to the earlier predictions.[3] For example, the combination $(M^{(+)}/M_U, M^{(-)}/M_U)$ =(1,10), (2,20), (5,50), and (15,150) gives with $\eta^{(+)}=-\ln 10+\eta^{(-)}$, $(f_\theta,f_M)=(-0.012,0.406)$, $(-0.013,-0.175)$, $(-0.014,-0.13)$, and $(-0.0155,-0.49)$, respectively, when the contribution of **54** is neglected

TABLE I. Acceptable solutions for right-handed gauge-boson mass M_R, M_U, and $\sin^2\theta_W$ for case (a) of SO(10), excluding the contribution of **54**, as a function of superheavy-Higgs-scalar masses, $M^{(+)}$ and $M^{(-)}$, defined in the text, with $\rho^{(\pm)}=M^{(\pm)}/M_U$.

$\rho^{(+)}$	$\rho^{(-)}$	M_R (GeV)	M_U (GeV)	$\sin^2\theta_W$
1	10	8×10^6	3.3×10^{15}	0.234
		8×10^5	8×10^{15}	0.238
		8×10^4	1.6×10^{16}	0.243
5	50	8×10^6	5.2×10^{15}	0.233
		8×10^5	10^{16}	0.237
		8×10^4	2×10^{16}	0.241

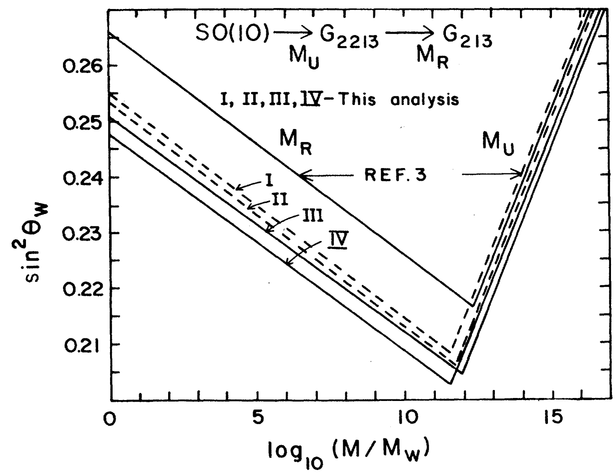

FIG. 2. New predictions in SO(10) for case (a) including superheavy-Higgs-scalar contributions. The dashed lines I and II have been obtained for $(\rho^{(+)},\rho^{(-)})=(1,10)$, and (5,50), respectively, excluding the contribution of **54**. The solid lines III and IV are the predictions for $(\rho^{(+)},\rho^{(-)})=(1,10)$, and (5,50), but including the contributions of **54**.

$(\eta'_{Si}=0)$. This results in decreasing the predicted values of $\sin^2\theta_W$ compared to those obtained in Ref. 3. Such a decrease in $\sin^2\theta_W$ now permits significantly lower values of M_R which were ruled out when[3] all superheavy masses were assumed to be the same as M_U.

Using iterative convergence procedure, solutions are obtained[17] by exactly matching $\alpha(M_W)$. In this method,[17] for every set of values of superheavy-Higgs-scalar masses, and assumed values of M_U and M_R, iterations are carried out for a given value of α_G. When iterations converge, values of $\alpha_S(M_W)$, $\alpha(M_W)$, and $\sin^2\theta_W$ are calculated, and the solutions are rejected (accepted) if the GUT prediction of $\alpha^{-1}(M_W)$ does not match (matches) with $\alpha^{-1}(M_W)=127.54$. If $\alpha(M_W)$ does not match iterations are carried out for a different α_G, but for the same M_U and M_R. It has been noted that the variation of α_G, for the same set of masses, makes it possible to match the fine-structure constant. The final solutions for which the GUT predictions of $\alpha(M_W)$ match with the low-energy extrapolation are presented in Table I for different combinations of $M^{(+)}/M_U$ and $M^{(-)}/M_U$. In Fig. 2 these solutions, excluding **54**, are shown as dashed lines and compared with the soutions to Ref. 3. Even without including the superheavy components of **54**, the decrease in $\sin^2\theta_W$ is found to be significant with $\delta\sin^2\theta_W=-0.0115$ (-0.0135) for $M^{(-)}=10(150)M_U$ and $M^{(+)}=2(15)M_U$.

TABLE II. Same as Table I, but including the contributions of **54**.

$\rho^{(+)}$	$\rho^{(-)}$	M_R (GeV)	M_U (GeV)	$\sin^2\theta_W$
1	10	8×10^6	6.6×10^{15}	0.231
		8×10^5	1.6×10^{16}	0.235
		8×10^4	3.3×10^{16}	0.239
		8×10^3	5.2×10^{16}	0.243
5	50	8×10^6	2.6×10^{15}	0.228
		8×10^5	6.6×10^{15}	0.232
		8×10^4	1.5×10^{16}	0.236
		8×10^3	2.6×10^{16}	0.240

Including the contributions of the superheavy components of **54** results in the values $(f_\theta,f_M)\simeq(-0.0165,1.1)$, $(-0.0173,0.9)$, $(-0.018,0.6)$, and $(-0.019,0.22)$ for $(M^{(-)}/M_U,M^{(+)}/M_U)=(10,1)$, (20,2), (50,5), and (150,15), respectively. Final solutions obtained, following the iterative convergence approach and fine-structure constant matching, are presented in Table II, and shown by solid lines in Fig. 2. For $\sin^2\theta_W=0.23$ (0.24), the two-loop prediction of Ref. 3 is $M_R\simeq10^{11}$ GeV (10^8 GeV). Excluding the contribution of **54**, the predictions are now modified as $M_R\simeq10^8$ (10^5) GeV, when the $M^{(-)}$ components are nearly 10 times heavier than M_U, but $M^{(+)}\simeq M_U$. Including **54**, we obtain $M_R\simeq10^7$ (10^5) GeV, and 10^6 (10^4) GeV for $(M^{(+)}/M_U, M^{(-)}/M_U)=(1,10)$, and (5,50), respectively, with the same value of $\sin^2\theta_W\simeq0.23$ (0.24). It is clear that for larger values of $M^{(-)}/M_U$, but with $M^{(-)}/M^{(+)}=10$, the decrease in M_R is larger. Also, we have noted that if the nondegeneracy factor is larger than 10 with

$$|\eta^{(+)}-\eta^{(-)}|>\ln 10\ ,$$

the decrease in M_R is still larger for the same $\sin^2\theta_W$.

B. Predictions with G_{2213} and G_{2113} intermediate symmetries

Compared to case (a), the Higgs representation due to a second $\mathbf{45}\subset$SO(10) contributes to the superheavy-Higgs-scalar effects. Excluding **54**, $(\eta'_{Si}=0)$, and confining to the Coleman-Weinberg constraint on the nondegeneracy factor, we obtain a minimum value of $\sin^2\theta_W$ for

$$\eta_{H1}=\eta_{H2}=\eta_{S1}=\eta_{S3}=\eta''_{S2}=\eta''_{S5}=\eta''_{S6}=\eta'_{H2}=\eta'_{H5}=\eta'_{H8}=\eta'_{H10}=\eta'_{H11}=\eta^{(+)}\ ,$$
$$\eta_{S2}=\eta''_{S1}=\eta''_{S3}=\eta''_{S4}=\eta'_{H1}=\eta'_{H3}=\eta'_{H4}=\eta'_{H6}=\eta'_{H7}=\eta'_{H9}=\eta^{(-)}\ , \tag{48}$$

TABLE III. Acceptable solutions for $W_R^\pm$ mass (M_R^+), M_U, and $\sin^2\theta_W$ for case (b), excluding the contribution of **54**, as a function of superheavy-Higgs-scalar masses, $M^{(+)}$ and $M^{(-)}$, defined in the text with $\rho^{(\pm)}=M^{(\pm)}/M_U$, and $M_R^0=500$ GeV.

$\rho^{(+)}$	$\rho^{(-)}$	$M_R^{(+)}$ (GeV)	M_U (GeV)	$\sin^2\theta_W$
1	10	8×10^6	1.5×10^{15}	0.228
		8×10^5	2.6×10^{15}	0.232
		8×10^4	5.3×10^{15}	0.237
		8×10^3	1.3×10^{16}	0.241
5	50	8×10^5	10^{15}	0.231
		8×10^4	2.6×10^{15}	0.235
		8×10^3	5×10^{15}	0.239

leading to

$$f_\theta^{(S)}=\frac{\alpha}{9984\pi}(19\,264\eta^{(+)}-23\,680\eta^{(-)})\ ,$$
$$f_M^{(S)}=\tfrac{1}{104}(-60\eta^{(+)}+24\eta^{(-)})\ , \quad (49)$$
$$f_\alpha^{(S)}=-\frac{1}{12\pi}(\tfrac{1496}{21}\eta^{(+)}+\tfrac{688}{7}\eta^{(-)})\ .$$

Including **54**, the superheavy components are subjected to the additional constraints, specified by Eq. (46), leading to

$$f_\theta^{(S)}=\frac{\alpha}{9984\pi}(26\,432\eta^{(+)}-30\,848\eta^{(-)})\ ,$$
$$f_M^{(S)}=\tfrac{1}{104}(-92\eta^{(+)}+56\eta^{(-)})\ , \quad (50)$$
$$f_\alpha^{(S)}=-\frac{\alpha}{12\pi}(\tfrac{552}{7}\eta^{(+)}+\tfrac{3152}{21}\eta^{(-)})\ .$$

With the constraint on masses specified by Eq. (45), we find that $(f_\theta,f_M)\simeq(-0.015,2)$, $(-0.017,1.24)$, $(-0.018,1)$, and $(-0.019,0.68)$, for $(M^{(-)}/M_U, M^{(+)}/M_U)=(1,0.1)$, (10,1), (20,2) and (50,5), respectively, when contributions of **54** are included. Excluding **54**, $(f_\theta,f_M)=(-0.011,1.3)$, $(-0.013,0.5)$, $(-0.014,0.3)$, and $(-0.015,-0.05)$, for the same sequential combinations of $(M^{(-)}/M_U,M^{(+)}/M_U)$. Using the iterative convergence procedure for obtaining solutions to RGE's, and

TABLE IV. Same as Table III, but including the contributions of **54**.

$\rho^{(+)}$	$\rho^{(-)}$	M_R^+ (GeV)	M_U (GeV)	$\sin^2\theta_W$
1	10	8×10^6	10^{15}	0.225
		8×10^5	2.6×10^{15}	0.228
		8×10^4	4.2×10^{15}	0.232
		8×10^3	1.3×10^{16}	0.237
		10^3	2.5×10^{16}	0.241
5	50	8×10^5	10^{15}	0.226
		8×10^4	2.5×10^{15}	0.231
		8×10^3	3.4×10^{15}	0.235
		10^3	1.2×10^{16}	0.238

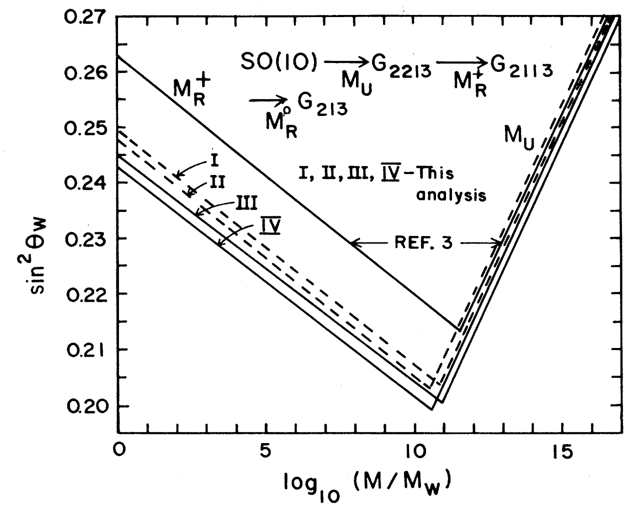

FIG. 3. Same as Fig. 2, but for case (b) of SO(10) grand unification.

adopting the method of fine-structure-constant matching by α_G variation,[17] final solutions excluding (including) **54** are presented in Table III (Table IV) and also plotted in Fig. 3. It is clear that with $\sin^2\theta_W=0.23$ (0.24), the allowed values of M_R^+ can now be brought down to $M_R^+=10^6$ (10^4) GeV excluding **54**, and to $M_R^+=10^5$ (10^3) GeV, including **54**, for $M^{(-)}$ ($M^{(+)}$)$=M_U$ ($M_U/10$). In this case, we have taken $M_R^0=M_{Z_R}\simeq500$ GeV, consistent with the available neutral-current data.

C. Predictions on neutrino masses

One of the major objectives in using the Higgs representation **126** is to generate the Majorana mass of the neutrino.[12] In case (a) where $G_{2213}\to G_{213}$ by the VEV of $\Delta_R(1,3,\sqrt{3/2},1)$, the neutrino masses generated by the seesaw mechanism are governed by the formula[12]

$$m_{\nu_i}=\frac{m_{l_i}^2}{M_R},\quad i=1,2,3\ldots, \quad (51)$$

where m_{ν_i} (m_{l_i}) is the neutrino (charged-lepton) mass of the ith generation. In case (b), where **126** is used to break $G_{2113}\to G_{213}$ at $\mu=M_R^0$, the Majorana-neutrino mass is governed by a different formula:[18]

$$m_{\nu_i}=\frac{m_{l_i}^2}{M_R^0},\quad i=1,2,3\ldots, \quad (52)$$

where M_R^0 (M_R^+) is the Z_R- ($W_R^\pm$-) boson mass, and the $W_R^\pm$ bosons decouple from the effective Lagrangian based upon G_{2113} gauge symmetry. In the absence of superheavy-Higgs-scalar contributions, $M_R=M_{W_R}\pm=M_{Z_R}=10^{10}$ GeV, and the predicted values of ν_e and ν_μ masses in case (a) are too small to be detected in the laboratory:

$$m_{\nu_e}\simeq10^{-8}\ \text{eV},\quad m_{\nu_\mu}\simeq10^{-3}\ \text{eV},\quad m_{\nu_\tau}\simeq1\ \text{eV}\ . \quad (53)$$

However, including superheavy-Higgs-scalar effects, the $W_R^{\pm}$ and Z_R masses have been brought down by 4–6 orders of magnitude, as demonstrated in Sec. IV A. This enhances ν_μ and ν_τ masses to values measurable in the laboratory, although ν_e mass is still smaller,

$$m_{\nu_e} \sim 10^{-5}-10^{-3}\ \text{eV}, \quad m_{\nu_\mu} \sim 1-100\ \text{eV}, \quad m_{\nu_\tau} \sim 1-100\ \text{keV}. \tag{54}$$

In case (b), since $W_R^{\pm}$ decouple from the Lagrangian, generating the neutrino mass, the mass predictions are the same as before[3,18] with a low-mass Z_R boson, i.e.,

$$m_{\nu_e} \sim 1\ \text{eV}, \quad m_{\nu_e}:m_{\nu_\mu}:m_{\nu_\tau} = m_e^2:m_\mu^2:m_\tau^2 .$$

V. DISCUSSION, SUMMARY, AND CONCLUSION

In the new SO(10) models,[2,3] where parity- and $SU(2)_R \times U(1)_{B-L}$-breaking scales are decoupled, although the $W_R^{\pm}$-mass prediction in cases (a) and (b) has been lowered by nearly 2–3 orders compared to the conventional models, the right-hand scale is too large to be detected directly by the supercolliders, or indirectly by low-energy experiments. In earlier computation of mass scales, all superheavy masses were assumed to be the same as M_U. In the present paper, using the method of effective-gauge theories,[14–16] we have computed the impact of superheavy Higgs components being nondegenerate, but constrained by the Coleman-Weinberg mechanism of mass generation, on the SO(10) predictions, where P and $SU(2)_R$ breakings are decoupled.[2,3] We find that even a factor-of-10 nondegeneracy can decrease the right-handed scale by 4–6 orders of magnitude compared to the earlier results.[2,3] Such significant effects on the predicted values of the intermediate scales (M_R or M_R^+), and $\sin^2\theta_W$ are due to two factors: (i) SO(10) contains larger representations, such as **54** and **126**, as compared to much smaller representations in SU(5); (ii) the mechanism of decoupling parity and $SU(2)_R$ breakings permits a freedom in choosing the superheavy masses within the Coleman-Weinberg constraint. For example, in the conventional approach, where parity is left unbroken down to M_R or M_R^+, the constraints on the nondegenerate superheavy masses, used in Secs. IV A and IV B, are not permitted.

We find that, if such a nondegeneracy among the superheavy component masses exists, it is possible to bring down the $W_R^{\pm}$- and Z_R-mass prediction in case (a) to 10–100 TeV, and the $W_R^{\pm}$ masses in case (b) to 1–10 TeV with $M_{Z_R} \simeq 500$ GeV. Such low-mass gauge bosons, in addition to being detected at the supercolliders in the near future, could manifest in low-energy experiments with detectable $V+A$ structure of charged and neutral currents, rare decays, CP violation in K^0-$\bar{K}^0$ and B^0-$\bar{B}^0$ mixings, neutrino masses, and a host of other processes. Since low-mass $W_R^{\pm}$ and Z_R gauge bosons correspond to larger values of neutrino masses, it is necessary to mention how to satisfy the cosmological bound $\sum_{i=e,\mu,\tau} m_{\nu_i} < 40$ eV, where the sum is over stable and light neutrinos. In such cases, the heavier neutrinos can be made unstable, with respect to decay into the lighter ones, by the emission of a Goldstone boson, called the Majoron, that appears in the theory as a result of spontaneous breaking of a global symmetry.[19] The introduction of such an additional global symmetry does not affect the GUT predictions as described in the present paper.

Significant effects of superheavy components of Higgs-scalar representations have been observed in the SU(5) model;[11,14,17,20] but, in those cases, additional Higgs representations, not needed for spontaneous symmetry breaking of the gauge symmetries, have been exploited. Also, the masses of the superheavy components have been taken to be 1–3 orders of magnitude different from the unification mass, in certain cases.[14,17] But, in the present analysis of the SO(10) predictions, only those Higgs representations needed for spontaneous symmetry breaking have been exploited, and the significant reduction of the right-hand scale is found to be possible within the Coleman-Weinberg constraint.

The computations of matching functions for $\ln M_U/M_W$, and $\sin^2\theta_W$ have been extensively studied for the SU(5) and SO(10) models with a grand desert.[11,14,17] For the first time, we have computed matching-function corrections due to superheavy Higgs scalars with one and two intermediate symmetries. Emphasizing that the fine-structure-constant matching[17] is important in such calculations, we have also computed the matching functions for $\alpha^{-1}(M_W)$. Although not very significant, numerically, we have derived analytically, for completeness, the exact matching relations for the coupling constants [e.g., Eqs. (17) and (33)] in the special case when the relevant heavy masses are equal to the intermediate scale. In some of our solutions, the superheavy-component masses are larger than the unification scale. Such a feature is specific to the EGT approach where the decoupling of heavy particles from the effective Lagrangian is not assumed, but their effects are computed explicitly. Thus, certain interesting new solutions, contained in this paper, would not have been obtained, had we followed the usual approach with Appelquist-Carrazone-type decoupling.

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