PHYSICAL REVIEW D VOLUME 40, NUMBER 9 1 NOVEMBER 1989

Are there really no experimental limits on a light Higgs boson from kaon decay?

Hai-Yang Cheng and Hoi-Lai Yu
Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, Republic of China
(Received 11 May 1989)

We reexamine the theoretical estimates of the decay $K \to \pi H$ and the experimental constraints on the existence of a light Higgs boson from this process. We find that (i) pole diagrams generated from the Higgs-boson–gluon coupling via a loop of heavy quarks do contribute to $K \to \pi H$, (ii) there is an additional contribution to the $K \to \pi H$ amplitude coming from the effective KHW and πHW couplings, (iii) even if B, the unknown parameter in the chiral-Lagrangian description of $K - \pi H$ transitions, is nonzero and even if the real part of the $K \to \pi H$ amplitude is canceled accidentally, the imaginary contribution alone suffices to rule out a Higgs boson lighter than $2m_\pi$, and (iv) whether Higgs bosons in the mass range $2m_\pi < m_H < 350$ MeV are excluded by the imaginary part of the $K \to \pi H$ amplitude depends on the branching ratio of $H \to \mu^+\mu^-$ and the top-quark mass. Decay modes $K_L \to \pi^+\pi^- H$ and $K^+ \to l^+ \nu H$ are briefly discussed.

I. INTRODUCTION

In the last decade, particle physics has harvested the marvelous successes of the standard model (SM). However, the Higgs bosons, which are the indisputable building blocks of the SM, are still missing. The search for them or their surrogates in the current or future high-energy experiments has become one of the top-priority issues. Unfortunately, the problem is obscured by the absence of any theoretical constraint on the number or masses of the Higgs bosons within the framework of the SM. For the minimal model, there is a vacuum stability bound,[1] $m_H > 7$ GeV, but this argument fails when there exist heavy fermions, or when there are more than one doublet of Higgs bosons. Recently, the ARGUS and CLEO Collaborations[2] have reported a large B_d^0-$\bar{B}_d^0$ mixing. This, when combining with the null signal of the top-quark search by the UA1 Collaboration,[3] indicates that the top quark is likely to be heavy. Thus, it is important to consider possible experimental limits on the existence of the light Higgs bosons.

Within the minimal model, it has been pointed out earlier[4] that the branching ratio for $B \to HX$ could be as large as several percents for a light Higgs boson (i.e., $m_H \lesssim 4.5$ GeV) and a heavy top quark. This provides a good laboratory for the light-Higgs-boson search. However, results from various experimental groups[5] seem to rule out the existence of a light Higgs boson within the mass range of $0.3 \lesssim m_H \lesssim 5$ GeV. But, as pointed out by the authors of Refs. 6 and 7, even though the branching ratio for $B \to HX$ is fairly large, the theoretical uncertainties in the $H \to \mu^+\mu^-$ branching ratio may ruin any definite conclusions.

In this paper, we update and fill up loopholes in arguments[6–13] of using various K decays to exclude light Higgs bosons with mass $m_H \lesssim 360$ MeV. In an earlier paper, Vainshtein, Zakharov, and Shifman[8] have pointed out that $m_H \lesssim 350$ MeV is ruled out by the $K^\pm \to \pi^\pm H$ mode. However, their calculation assumed a momentum-independent $K\pi$ transition amplitude which is inconsistent with the underlying chiral symmetry. Later, Willey and one of us[9] (H.L.Y.) presented a quark-model calculation which includes a one-loop $s \to d + H$ transition, but neglected the contributions from the nonspectator diagram, Fig. 1(e). Pham and Sutherland[10] advocated that both the nonspectator contributions and $\Delta I = \frac{1}{2}$ enhancement are important and may partially cancel the spectator term; therefore, this raises doubt in the conclusion of Willey and Yu. Willey,[11] in a later paper, has made a detailed reanalysis of this problem by using Fermi statistics and Bethe-Salpeter equation techniques to demonstrate that the spectator and nonspectator contributions are in fact constructive and hence rule out a Higgs boson of a mass between $50 \lesssim m_H \lesssim 211$ MeV. On the other hand, Chivukula and Manohar[13] (CM) used chiral perturbative theory and vacuum insertion to obtain an expression which indicates a destructive interference between the spectator and nonspectator diagrams and which depends on an unknown parameter B in the

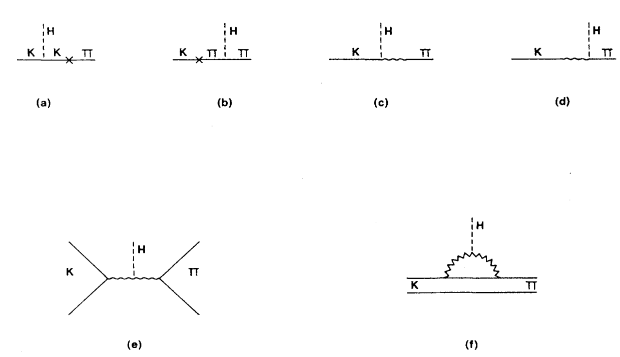

FIG. 1. Contributions to $K \to \pi H$. Shown are (a) and (b) pole diagrams, (c)–(e) nonspectator diagrams, and (f) the one-loop spectator diagram.

effective chiral Lagrangian. Despite the above complications, CM still managed to conclude that $m_H < 360$ MeV from the $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ decay modes is ruled out. Taking a rather conservative attitude, Raby, West, and Hoffman[7] argued that because B is unknown, many of the claims in the literature excluding Higgs bosons from kaon decays are not valid.

In light of the above-mentioned confusing status we shall present in this paper an updated version of the calculation of the $K \to \pi H$ decay amplitude.[14] We claim that (1) pole diagrams which include the Higgs-boson interaction with gluons via a triangular loop of heavy quarks do contribute to $K \to \pi H$, (2) there is an additional contribution to the $K \to \pi H$ amplitude coming from the effective KHW and πHW couplings, and (3) the imaginary contributions alone to $K \to \pi H$ suffice to rule out a Higgs boson lighter than 270 MeV. Finally, to complete our discussion we also calculate the branching ratio of $K_L \to \pi^+ \pi^- H$ and $K^+ \to e^+ \nu H$, though the present experimental limits on the corresponding $K_L \to \pi^+ \pi^- e^+ e^-$ and $K^+ \to e^+ \nu e^+ e^-$ decay modes cannot give any conclusion on the Higgs-boson masses.

II. THEORETICAL EVALUATION OF $K \to \pi H$ AMPLITUDES

We shall reexamine in this section the theoretical estimate of the $K \to \pi H$ rate. To begin with we write down the relevant effective Lagrangian for $K \to \pi H$ (Ref. 13):

$$\mathcal{L} = i \sum_i \bar{q}_i \gamma_\mu \partial^\mu q - \left[1 + \frac{H}{v} \right] \sum_i m_i \bar{q}_i q_i + \left[1 + \frac{H}{v} \right]^{-2} \mathcal{L}^{\Delta S=1} + \mathcal{L}_{1\text{ loop}} - \frac{n_h \alpha_s}{12\pi} H G^a_{\mu\nu} G^{a\mu\nu} , \tag{2.1}$$

where we have included effective Higgs-boson–gluon interactions via a heavy-quark triangle diagram for later purposes, n_h is the number of heavy quarks, $v = 1/(\sqrt{2} G_F)^{1/2} = 246$ GeV, $\mathcal{L}^{\Delta S=1}$ is the effective $\Delta S = 1$ weak Lagrangian

$$\mathcal{L}^{\Delta S=1} = -\sqrt{2} G_F V_{us} V^*_{ud} \sum_{i=1}^{6} c_i(\mu) O_i(\mu) , \tag{2.2}$$

where V_{ij} are the Kobayashi-Maskawa mixing matrix elements, O_i are four-quark operators [we follow the notation of Eq. (3.9) of Ref. 15], and $c_i(\mu)$ are Wilson coefficient functions in which perturbative QCD corrections from M_W down to the renormalization scale, typically chosen at 1 GeV, are taken into account. The characteristic values of Wilson coefficients are[16]

$$\begin{aligned} &c_1 = -2.11, \quad c_2 = 0.12, \quad c_3 = 0.09 , \\ &c_4 = 0.45, \quad c_5 = -0.025, \quad c_6 = -0.003 . \end{aligned} \tag{2.3}$$

The term $\mathcal{L}_{1\text{ loop}}$ in Eq. (2.1) is the flavor-changing $\Delta S = 1$ two-quark interaction induced at the one-loop level:

$$\mathcal{L}_{1\text{ loop}} = \frac{3\alpha}{32\pi \sin^2\theta_W} \left[\sum_i V_{is} V^*_{id} \frac{m_i^2}{M_W^2} \right] \times \frac{H}{v} [m_s \bar{d}(1+\gamma_5) s + m_d \bar{d}(1-\gamma_5) s] + \text{H.c.} , \tag{2.4}$$

which was first obtained by Willey and Yu.[9]

We first consider the pole diagrams, Figs. 1(a) and 1(b). At the quark level, pole diagrams corresponds to Higgs-boson emission from the bound-state quarks of initial and final mesons. It has been argued in the literature[10] that diagrams 1(a) and 1(b) compensate as the Higgs-boson coupling is proportional to the meson mass squared, i.e., $g_{H\pi\pi}/g_{HKK} = m_\pi^2/m_K^2$. However, as we shall see shortly, the Higgs-boson—meson coupling does not vanish even in the chiral limit, and hence the pole contributions are not necessarily zero. To see this, let us consider the matrix element

$$\langle \pi^+ \pi^- | H \rangle = \frac{1}{v} \left\langle \pi^+ \pi^- \left| \sum_i m_i \bar{q}_i q_i \right| 0 \right\rangle . \tag{2.5}$$

It is well known that heavy quarks contribute indirectly to the Higgs-boson—pion coupling by virtue of the triangle diagram[17] [i.e., the last term in Eq. (2.1)]

$$\langle \pi^+ \pi^- | H \rangle = \frac{1}{v} \left\langle \pi^+ \pi^- \left| \sum_l m_l \bar{q}_l q_l - \frac{n_h \alpha_s}{12\pi} G^a_{\mu\nu} G^{a\mu\nu} \right| 0 \right\rangle , \tag{2.6}$$

where the subscript l denotes light quarks. The momentum dependence of the matrix element $\langle \pi^+(p_1) \pi^-(p_2) | H \rangle$ is easily seen from the approach of chiral Lagrangian. In the chiral limit the general chiral representation for the HGG interaction given in Eq. (2.1) reads[6]

$$\frac{f_\pi^2}{16} \frac{H}{v} (a \, \text{Tr} \partial_\mu U \partial^\mu U^\dagger + b \, \text{Tr} U \Box U^\dagger) + \text{H.c.} \tag{2.7}$$

This chiral Lagrangian will contribute not only to the Higgs-boson—meson coupling but also to the effective $H\phi W$ vertex responsible for the nonspectator diagrams, Figs. 1(c) and 1(d), as we shall see later. It is straightforward to check that[18]

$$\langle \pi^+(p_1) \pi^-(p_2) | H \rangle = \frac{a-b}{v} p_1 \cdot p_2 . \tag{2.8}$$

The unknown coefficient $(a-b)$ in Eq. (2.8) is intimately related to the trace of energy-momentum tensor

$$\theta^\mu{}_\mu = \sum_l m_l \bar{q}_l q_l - \frac{(33-2n_l)\alpha_s}{24\pi} G^a_{\mu\nu} G^{a\mu\nu} , \tag{2.9}$$

where n_l is the number of light quarks, and use of heavy-quark operator-product expansion[17] has been made. Therefore,

$$\langle \pi^+\pi^-|H\rangle = \frac{1}{v}\frac{2n_h}{33-2n_l}\langle \pi^+\pi^-|\theta^\mu{}_\mu|0\rangle + \frac{1}{v}\left[1-\frac{2n_h}{33-2n_l}\right]\left\langle \pi^+\pi^-\left|\sum m_l\bar{q}_l q_l\right|0\right\rangle . \quad (2.10)$$

The lowest-order chiral Lagrangian implies

$$\langle \pi^+(p_1)\pi^-(p_2)|\theta^\mu{}_\mu|0\rangle = 2p_1\cdot p_2 + 4m_\pi^2 . \quad (2.11)$$

Hence, $(a-b)$ is fixed to be

$$a-b=\frac{4n_h}{33-2n_l} \quad (2.12)$$

and

$$\langle \pi^+(p_1)\pi^-(p_2)|H\rangle = \frac{1}{v}\left[\frac{4n_h}{33-2n_l}p_1\cdot p_2 + \frac{6n_h}{33-2n_l}m_\pi^2 + m_\pi^2\right] \quad (2.13)$$

for small $(p_1+p_2)^2$. Likewise,

$$\langle K^+(p_1)K^-(p_2)|H\rangle = \frac{1}{v}\left[\frac{4n_h}{33-2n_l}p_1\cdot p_2 + \frac{6n_h}{33-2n_l}m_K^2 + m_K^2\right] . \quad (2.14)$$

Raby and West[19] have emphasized recently that the $H\to\pi\pi$ decay is further enhanced by final-state interactions via a possible resonance in the $\pi\pi$-scattering amplitude. However, this is not relevant for the pole contributions discussed here.

From Eqs. (2.13) and (2.14) it is easily seen that the pole diagrams with momentum-dependent Higgs-boson—pion couplings (i.e., the $p_1\cdot p_2$ terms) do not can- the pole amplitude to be

$$A(K^+\to\pi^+H)_{\rm pole} = \frac{1}{v}\left[\frac{4n_h}{33-2n_l}\right]p_K\cdot p_\pi \frac{\langle \pi^+(p_K)|\mathcal{L}^{\Delta S=1}|K^+(p_K)\rangle - \langle \pi^+(p_\pi)|\mathcal{L}^{\Delta S=1}|K^+(p_\pi)\rangle}{m_K^2-m_\pi^2} . \quad (2.15)$$

The K-π transition can be evaluated in several different methods (for a detailed discussion, see Secs. 6.1 and 7.3 of Ref. 15). Here we focus on the chiral-Lagrangian approach. The lowest-order chiral representation for $\mathcal{L}^{\Delta S=1}$ has the form (using the notation of Chap. 7 of Ref. 15)

$$\mathcal{L}^{\Delta S=1}_{\rm chiral} = g_8{\rm Tr}(\lambda_6\partial_\mu U\partial^\mu U^\dagger) + g_{27}^{(1/2)}\Theta^{(27,1/2)} + g_{27}^{(3/2)}\Theta^{(27,3/2)} , \quad (2.16)$$

where $U=\exp(2i\phi/f_\pi)$, $\phi=(1/\sqrt{2})\phi^a\lambda^a$, and ${\rm Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$. Total derivative terms such as ${\rm Tr}\partial^\mu(\lambda_6 U\partial_\mu U^\dagger)$ are prohibited because the $\Delta S=1$ weak Hamiltonian at the quark level respects an additional discrete *CPS* symmetry,[20] which is the product of ordinary *CP* with a switching symmetry *S*, which switches the *s* and *d* quarks.[21] The coupling constants are determined from the experimental $K\to\pi\pi$ rates to be[15]

$$|g_8+g_{27}^{(1/2)}| = 0.26\times10^{-8}m_K^2 , \quad |g_{27}^{(3/2)}| = 0.86\times10^{-10}m_K^2 \quad (2.17)$$

but with their signs undetermined. To fix the sign we see that a direct application of factorization yields [cf. Eq. (6.40) of Ref. 15]

$$\langle \pi^+(p_\pi)|\mathcal{L}^{\Delta S=1}|K^+(p_K)\rangle = \frac{\sqrt{2}}{6}G_F V_{us}V_{ud}^*\left[c_1-2c_2-2c_3-2c_4 + \tfrac{32}{3}(c_5+\tfrac{16}{3}c_6)\frac{\sigma^2}{\Lambda_\chi^2}\right]f_\pi^2(p_K\cdot p_\pi) , \quad (2.18)$$

where $\sigma = m_\pi^2/(m_u+m_d) = m_{K^+}^2/(m_u+m_s)$ characterizes the spontaneous breaking of chiral symmetry, and $\Lambda_\chi\sim 1$ GeV (Ref. 22) sets the scale of higher-order chiral terms. with the Wilson coefficients Eq. (2.3) it turns out that the sign of $\langle \pi^+|\mathcal{L}^{\Delta S=1}|K^+\rangle$ and hence g_8 is fixed to be negative. It follows from Eq. (2.16) that

$$\langle \pi^+(p_\pi)|\mathcal{L}^{\Delta S=1}_{\rm chiral}|K^+(p_K)\rangle = \frac{4}{f_\pi^2}(g_8+g_{27}^{(1/2)}+g_{27}^{(3/2)})p_K\cdot p_\pi . \quad (2.19)$$

The final result for the pole contribution has the form

$$A(K^+\to\pi^+H)_{\rm pole} = \frac{1}{v}\left[\frac{8n_h}{33-2n_l}\right]\frac{g_8}{f_\pi^2}(m_K^2+m_\pi^2-m_H^2) . \quad (2.20)$$

We turn next to the nonspectator diagram, Fig. 1(e), in which the spectator u quark in K^+ participates directly in the Higgs-boson production. The amplitude of Fig. 1(e) governed by the interaction $-2(H/v)\mathcal{L}^{\Delta S=1}$ reads

$$A(K^+\to\pi^+H)_e = \frac{2}{v}\langle \pi^+(p_\pi)H|H\mathcal{L}^{\Delta S=1}|K^+(p_K)\rangle . \quad (2.21)$$

Because of the Higgs-boson—gluon interaction arising from the heavy-quark triangle diagram, there are two additional contributions to the $K\to\pi H$ decay: namely, Figs. 1(c) and 1(d). At the quark level, the effective $H\phi W$ vertex is depicted in Fig. 2. The $H\phi W$ coupling can be obtained by coupling the chiral Lagrangian for *HGG* interactions [Eq. (2.7)] to external gauge fields A_μ^L and A_μ^R:

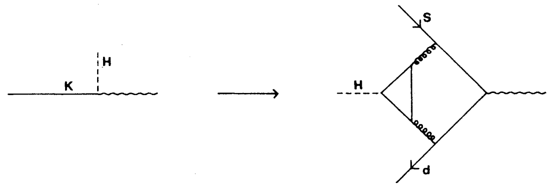

FIG. 2. The effective HKW vertex at the quark level.

$$\partial_\mu U \to D_\mu U = \partial_\mu U + A_\mu^L U - U A_\mu^R \ . \tag{2.22}$$

In the present case the external fields are identified with left-handed $W_\mu^\pm$ boson fields

$$A_\mu^L = -i\frac{g}{\sqrt{2}} W_\mu \Gamma, \quad A_\mu^R = 0 \ , \tag{2.23}$$

where Γ is identical to the KM matrix element V_{ij} for the pseudoscalar meson constructed from $\bar{q}_i q_j$ and vanishes otherwise. For example, $\Gamma_{ij}=0$ except for $\Gamma_{12}=V_{ud}$ for $H\pi^+W$ coupling. It is easily seen from Eqs. (2.7), (2.22), and (2.23) that

$$\mathcal{L}_{H\phi W} = -\left[\frac{4n_h}{33-2n_l}\right]\frac{gf_\pi}{2\sqrt{2}}\frac{H}{v} W^\mu \mathrm{Tr}(\Gamma \partial_\mu \phi) \ . \tag{2.24}$$

Recalling that

$$\mathcal{L}_{\phi W} = -\frac{g}{2\sqrt{2}} f_\pi W^\mu \mathrm{Tr}(\Gamma \partial_\mu \phi) \tag{2.25}$$

we find[23]

$$A(K^+ \to \pi^+ H)_c = A(K^+ \to \pi^+ H)_d = -\frac{1}{v}\left[\frac{4n_h}{33-n_l}\right]\langle \pi^+ H | H \mathcal{L}^{\Delta S=1} | K^+ \rangle . \tag{2.26}$$

Summing over the nonspectator amplitude of Figs. 1(c)–1(e) we obtain

$$A(K^+ \to \pi^+ H)_{\rm NS} = \frac{2}{v}\left[2 - \frac{4n_h}{33-2n_l}\right]\langle \pi^+ H | H \mathcal{L}^{\Delta S=1} | K^+ \rangle \ . \tag{2.27}$$

To evaluate the K-πH matrix elements we note that in the chiral Lagrangian approach the flavor-changing $\Delta S=1$ interactions involving Higgs bosons are given by[13,25]

$$\left[1-2\frac{H}{v}\right]\mathcal{L}^{\Delta S=1} = g_8[\mathrm{Tr}(\lambda_6 \partial_\mu U \partial^\mu U^\dagger) - B\,\mathrm{Tr}\,\partial^\mu(\lambda_6 U \partial_\mu U^\dagger)]\left[1-2\frac{H}{v}\right] + C\left[1-2\frac{H}{v}\right]\mathrm{Tr}\left[\lambda_6 \mathrm{M}^\dagger\left[1+\frac{H}{v}\right]\right] + \mathrm{H.c.} \ , \tag{2.28}$$

where we have neglected the 27-plet contributions (recall that $g_{27}^{(1/2)} \approx g_{27}^{(3/2)}/5$). As noticed in Ref 24, CPS symmetry eliminates the B term as before, but it does not remove away the $C\,\mathrm{Tr}(\lambda_6 \mathrm{M}^\dagger)H$ contribution. The chiral rotation which diagonalizes the quark mass only deletes the Higgs-boson-independent piece of the C term. To eliminate the remaining C term via a Higgs-boson-dependent chiral rotation will reintroduce the B term. This means that the K-πH transitions receives an additional contribution depending on the unknown parameter B. We find the nonspectator amplitude to be

$$A(K^+ \to \pi^+ H)_{\rm NS} = \frac{4}{v}\left[2 - \frac{4n_h}{33-2n_l}\right]\frac{g_8}{f_\pi^2}[m_K^2 - m_\pi^2 - m_H^2 + \tfrac{1}{2}B(m_K^2 - m_\pi^2)] \ . \tag{2.29}$$

We now turn to the spectator diagram, Fig. 1(f), in which $s \to dH$ occurs at the one-loop level. The hadronic matrix element of the $\bar{s}d$ density is given by[26]

$$\langle \pi^+(p_\pi) | \bar{s}d | K^+(p_K) \rangle = \sigma \tag{2.30}$$

with σ being defined in Eq. (2.18). It follows from Eq. (2.4) that

$$A(K^+ \to \pi^+ H)_{1\ \rm loop} = \sum_i^{u,c,t} \frac{m_K^2}{v}\frac{3\alpha}{32\pi \sin^2\theta_W} V_{id} V_{is}^\dagger \frac{m_i^2}{M_W^2} \equiv \sum_i^{u,c,t} h_i \ . \tag{2.31}$$

Note that the sign of h_i relative to $A(K^+ \to \pi^+ H)_{\rm NS}$ is unambiguously fixed and is independent of the quark phase convention. For example, h_c is of opposite sign to the nonspectator amplitude due to the negativity of the coupling g_8, in agreement with Ref. 13. To evaluate the t-quark contribution we recast the KM matrix element $V_{td}V_{ts}^*$ in terms of the Wolfenstein parametrization[27]

$$V_{td}V_{ts}^* = -\lambda^5 A^2(1-\rho-i\eta) \ , \tag{2.32}$$

where $\lambda = |V_{us}| = 0.22$, A is close to unity, and η measures CP violation. The spectrum of leptons in semileptonic B decay implies $(\rho^2+\eta^2)^{1/2} \le 0.9$. The recent ARGUS and CLEO observations of B_d^0-$\bar{B}_d^0$ mixing[2] strongly suggest a negative ρ and a heavy top quark: $m_t \ge 60$ GeV. The parameter η can be determined from the recent NA31 measurement[28] of ϵ'/ϵ. In the standard KM model ϵ'/ϵ has the expression[29]

$$\frac{\epsilon'}{\epsilon}=3.3\left[\frac{300\text{ MeV}}{m_s}\right]^2 B_K'\mathrm{Im}(V_{td}V_{ts}^*) , \tag{2.33}$$

where

$$B_K'\equiv\frac{\langle\pi^+\pi^-|O_5|K^0\rangle}{\langle\pi^+\pi^-|O_5|K^0\rangle^{(1/N_c)}}=\frac{\langle\pi^+\pi^-|O_5|K^0\rangle}{0.055\text{ GeV}^3} \tag{2.34}$$

measures the deviation of the K-$\pi\pi$ penguin matrix element from the $1/N_c$ calculation (for a detailed discussion, see Ref. 29), analogous to the parameter B_K defined in K^0-$\bar{K}^0$ mixing. Using $m_s=150$ MeV, $B_K'=1$, and the NA31 result[28] $\epsilon'/\epsilon=(3.3\pm1.1)\times10^{-3}$, we find

$$\eta=0.57\pm0.19 . \tag{2.35}$$

Now we have

$$h_t=4.3\times10^{-10}\text{ GeV}\left[\frac{m_t}{M_W}\right]^2(1-\rho-i\eta) . \tag{2.36}$$

Because $\rho<0$ inferred from the B_d^0-$\bar{B}_d^0$ mixing data, a conservative lower bound for the real part of h_t can be set by putting $\rho=0$. Since $h_c=0.73\times10^{-10}$ GeV for $m_c=1.5$ GeV, it is evident that the dominant contribution to the spectator amplitude arises from the top quark. As we shall see in the next section, even the imaginary part of h_t alone suffices to rule out light Higgs bosons within certain mass ranges.

Summing over the contributions from Figs. 1(a)–1(f), we find the amplitude for $K\to\pi H$ to be [for three generations, i.e., $n_l=n_h=3$ (Ref. 30)]

$$\begin{aligned}A(K^+\to\pi^+H)=&\Bigg[-1.5\times10^{-10}(1-\tfrac{2}{9})\left[1+\frac{m_\pi^2-m_H^2}{m_K^2}\right]\\&+0.73\times10^{-10}+h_t\\&+B(0.39\times10^{-10})\Bigg]\text{ GeV} ,\\A(K_L\to\pi^0H)=&\mathrm{Re}\,A(K^+\to\pi^+H) .\end{aligned} \tag{2.37}$$

From Eq. (2.37) it is obvious that the $K\to\pi H$ amplitude is at least of order 10^{-10} GeV and is dominated by the t-quark contribution. The branching ratios are then given by

$$\begin{aligned}B(K^+\to\pi^+H)&=7.57\times10^{-6}\left[\frac{2p_H}{m_K}\right]\left|\frac{A}{10^{-10}\text{ GeV}}\right|^2 ,\\B(K_L\to\pi^0H)&=3.15\times10^{-5}\left[\frac{2p_H}{m_K}\right]\left|\frac{A}{10^{-10}\text{ GeV}}\right|^2 ,\end{aligned} \tag{2.38}$$

where p_H is the momentum of the Higgs boson.

III. LIMITS FROM $K\to\pi H$ DECAYS

To set a limit on a light Higgs bosons from existing experimental data, it is important to take the experimental situation into consideration, for instance, the experimental decay vertex requirement. For this purpose, we follow the analysis of Raby, West, and Hoffman[7] (RWH) to summarize the available data for $K\to\pi H$:

$$\begin{aligned}&(1)\text{ Ref. 31, } B(K^+\to\pi^+H)B(H\to e^+e^-)<2.7\times10^{-7}\text{ for }100\text{ MeV}<m_H<2m_\mu ,\\&(2)\text{ Ref. 32, } B(K^+\to\pi^+H)B(H\to e^+e^-)<3.5\times10^{-7}\text{ for }140\text{ MeV}<m_H<2m_\mu ,\\&(3)\text{ Ref. 33, } B(K^+\to\pi^+H)B(H\to\mu^+\mu^-)<1.5\times10^{-7}\text{ for }220\text{ MeV}<m_H<320\text{ MeV} ,\\&(4)\text{ Ref. 34, } B(K^+\to\pi^+H)<1.5\times10^{-6}\text{ for }0<m_H<80\text{ MeV} ,\\&(5)\text{ Ref. 35, } B(K^+\to\pi^+X)<1.4\times10^{-6}\text{ for }5\text{ MeV}<m_H<100\text{ MeV} ,\\&(6)\text{ Ref. 36, } B(K_L\to\pi^0H)B(H\to e^+e^-)<2.3\times10^{-6}\text{ for }80\text{ MeV}<m_H<2m_\mu ,\\&(7)\text{ Ref. 37, } B(K_L\to\pi^0H)B(H\to e^+e^-)<2.0\times10^{-7}\text{ for }10\text{ MeV}<m_H<2m_\mu ,\\&(8)\text{ Ref. 36, } B(K_L\to\pi^0H)B(H\to\mu^+\mu^-)<1.2\times10^{-6}\text{ for }2m_\mu<m_H<360\text{ MeV} .\end{aligned} \tag{3.1}$$

When $m_H<2m_\mu$, $H\to e^+e^-$ is the dominant decay mode, while for $2m_\mu<m_H<2m_\pi$, the Higgs boson will predominately decay into $\mu^+\mu^-$. It was argued recently by RWH (Ref. 7) that because of the undetermined B parameter in the chiral Lagrangian (2.28), no unambiguous and definite limits on the existence of light Higgs bosons can be drawn from kaon decays. However, we have shown in Sec. II that the imaginary contribution to the $K^+\to\pi^+H$ amplitude coming from the intermediate top quark in the $s\to dH$ loop is large and suffices to rule out a Higgs boson lighter than $2m_\pi$ as long as $m_t>45$ GeV. Hence, *even if B is nonzero and even if the real part is canceled accidentally, the imaginary contribution alone is enough to exclude Higgs bosons with* $m_H<2m_\pi$.

For $2m_\pi<m_H<360$ GeV, the branching ratio of $H\to\mu^+\mu^-$ is suppressed due to the existence of the decay mode $H\to\pi\pi$. An estimate of $H\to\pi\pi$ rates by Eq. (2.13) leads to a branching ratio of 40% for $H\to\mu^+\mu^-$ at $m_H=300$ MeV. Raby and West[19] claimed a large enhancement of $H\to\pi\pi$ by final-state interactions and they concluded that $B(H\to\mu^+\mu^-)\approx\frac{1}{24}$ at the same Higgs-boson mass. A recent reanalysis of this issue by Truong and Willey,[38] found, however, no large enhancement of $H\to\pi\pi$ for $m_H<950$ MeV; they estimated $B(H\to\mu^+\mu^-)$ to be of 30% at $m_H=300$ MeV. As a result, whether Higgs bosons in this mass range can be excluded by the imaginary contribution to the $K^+\to\pi^+H$ amplitude depends strongly on the branching ratio of $H\to\mu^+\mu^-$ and the top-quark mass. For $(H\to\mu^+\mu^-)\approx 0.3$, we find that light Higgs bosons with $2m_\pi<m_H<350$ MeV do not exist if $m_t>65$ GeV. (Recall that $m_t\geq 80$ GeV is required to evade the Linde-Weinberg constraint.[1]) But if $B(H\to\mu^+\mu^-)\approx\frac{1}{24}$, the top quark must be heavier than 105 GeV in order to implement the job.

IV. $K_L\to\pi^+\pi^-H$ AND $K^+\to l^+\nu H$ DECAYS

Processes, e.g., $K\to\pi\pi H$ and $K\to l\nu H$ can in principle be used to constrain the existence of a light Higgs boson. Here we confine our attention to the decay modes $K_L\to\pi^+\pi^-H$ and $K^+\to l^+\nu H$ because of the availability of the experimental measurements of $K_L\to\pi^+\pi^-e^+e^-$ and $K^+\to e^+\nu e^+e^-$.

From Eq. (2.1) it is clear that the decay $K_L\to\pi\pi H$ is prohibited in the limit of CP symmetry. Since the imaginary and real parts of the KM matrix element $V_{td}V_{ts}^*$ are comparable, the dominant contribution to the CP-violating $K_L\to\pi\pi H$ obviously arises from the t-quark loop diagram. From the chiral representation of the quark density[39]

$$\bar{q}_{Rj}q_{Li}=-\tfrac{1}{4}f_\pi^2\sigma U_{ij} \tag{4.1}$$

it follows[40]

$$\langle\pi^+\pi^-|\bar{s}\gamma_5 d|K^0\rangle=-\langle\pi^+\pi^-|\bar{d}\gamma_5 s|\bar{K}^0\rangle=-i\frac{2}{3}\frac{\sigma}{f_\pi}\ . \tag{4.2}$$

Consequently,

$$A(K_L\to\pi^+\pi^-H)\simeq i\frac{\sqrt{2}\alpha}{16\pi\sin^2\theta_W}\left[\frac{m_K^2-m_\pi^2}{f_\pi v}\right]\left[\frac{m_t}{M_W}\right]^2\times\mathrm{Im}(V_{td}V_{ts}^*)\approx i(1.64\pm0.55)\times10^{-9}\left[\frac{m_t}{M_W}\right]^2, \tag{4.3}$$

where use of Eq. (2.33) has been made. After integrating over the phase space we find

$$B(K_L\to\pi^+\pi^-H)\lesssim(1.7\pm1.2)\times10^{-6}\left[\frac{m_t}{m_W}\right]^2 \tag{4.4}$$

for $m_H\neq 0$. Consequently, the current experimental upper bound[41] $B(K_L\to\pi^+\pi^-e^+e^-)<2.5\times10^{-6}$ is not strong enough to provide evidence against the existence of light Higgs bosons unless $m_t\gtrsim 100$ GeV.

We have also calculated the branching ratio for the process $K^+\to e^+\nu H$. Since the pole contribution vanishes in the limit $m_e\to 0$ due to vector-current conservation, Higgs-boson production comes from the emission from the virtual W boson and from the K-W vertex. We find[41]

$$B(K^+\to e^+\nu H)=\frac{\sqrt{2}G_F m_K^4}{96\pi^2 m_\mu^2\left[1-\frac{m_\mu^2}{m_K^2}\right]^2}\times B(K^+\to\mu^+\nu)_{\mathrm{expt}}f(x)=6.4\times10^{-8}f(x)\ , \tag{4.5}$$

where

$$f(x)=[(1-8x+x^2)(1-x^2)-12x^2\ln x]\times\left[1-\frac{2n_h}{33-2n_l}\right]^2 \tag{4.6}$$

with $x=m_H^2/m_K^2$. The available experimental measurement[42] $B(K^+\to e^+\nu e^+e^-)=(2.1^{+2.1}_{-1.1})\times10^{-7}$ indicates that nothing can be learned from this decay mode. Similarly, for $\pi^+\to e^+\nu H$ we obtain

$$B(\pi^+\to e^+\nu H)=3.3\times10^{-9}f(y) \tag{4.7}$$

with $y=m_H^2/m_\pi^2$. It seems to us that the branching ratio obtained in Ref. 7 is too large by a factor of 2. Recently, the SINDRUM Collaboration[43] has obtained upper limits on the branching ratio $B(\pi^+\to e^+\nu H)$ ranging from 10^{-9} to 10^{-11} depending on the mass and the lifetime of the Higgs boson. A mass range $10<m_H<100$ MeV is clearly ruled out.[43]

V. CONCLUSIONS

We have explored the Higgs-boson production in the process $K\to\pi H$. Effects such as heavy-quark contributions to the pole and nonspectator diagrams via the triangle diagram with external gluons and CP-violating effects on the $K\to\pi H$ decay mode, which were not considered by previous calculations, are elaborated on.

We find the $K\to\pi H$ amplitude to be dominated by the t-quark contribution in the $s\to dH$ loop. Unfortunately, there is an undetermined parameter B in the chiral-Lagrangian description of the $K\to\pi H$ transition. Nevertheless, the imaginary contribution to the $K^+\to\pi^+H$ amplitude coming from the imaginary part of the Kobayashi-Maskawa mixing angles in $s\to dH$ is large. We conclude that even if B is nonzero and even if the real part of the $K^+\to\pi^+H$ amplitude is canceled accidentally, the imaginary contribution alone with $m_t\geq 45$ GeV will suffice to rule out a Higgs boson lighter than $2m_\pi$. Whether the Higgs bosons with $2m_\pi<m_H<350$ MeV can be excluded by the CP-violating contribution depends strongly on the branching ratio of $H\to\mu^+\mu^-$. For

$B(H \to \mu^+\mu^-) \approx 30\%$, a top quark heavier than 65 GeV will rule out a light Higgs boson in the above-mentioned mass range. But if $B(H \to \mu^+\mu^-) \approx \frac{1}{24}$, the top quark should not be lighter than 105 GeV in order to put constraints on the light Higgs boson in the mass range $2m_\pi < m_H < 350$ MeV.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Council of the Republic of China. We wish to thank R. S. Chivukula, Y. C. Lin, and R. W. Willey for helpful discussions.

[1]A. D. Linde, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 73 (1976) [JETP Lett. **23**, 64 (1976)]; S. Weinberg, Phys. Rev. Lett. **36**, 294 (1976).

[2]ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **192**, 245 (1987); CLEO Collaboration, M. Artuso *et al.*, Phys. Rev. Lett. **62**, 2233 (1989).

[3]UA1 Collaboration, C. Albajar *et al.*, Phys. Lett. B **186**, 247 (1986).

[4]R. S. Willey and H. L. Yu, Phys. Rev. D **26**, 3086 (1982).

[5]CLEO Collaboration, P. Avery *et al.*, Phys. Lett. B **183**, 429 (1987); Mark J. Collaboration, B. Adeva *et al.*, Phys. Rev. Lett. **50**, 799 (1983); TASSO Collaboration, M. Althoff *et al.*, Z. Phys. C **22**, 219 (1984); JADE Collaboration, W. Bartel *et al.*, Phys. Lett. **132B**, 241 (1983).

[6]B. Grinstein, L. Hall, and L. Randall, Phys. Lett. B **211**, 363 (1988).

[7]S. Raby, G. B. West, and C. M. Hoffman, Phys. Rev. D **39**, 828 (1989).

[8]A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Usp. Fiz. Nauk **131**, 537 (1980) [Sov. Phys. Usp. **23**, 429 (1980)].

[9]R. S. Willey and H. L. Yu, Phys. Rev. D **26**, 3287 (1982).

[10]T. N. Pham and D. G. Sutherland, Phys. Lett. **151B**, 444 (1985).

[11]R. S. Willey, Phys. Lett. B **173**, 480 (1986).

[12]R. Ruskov, Phys. Lett. B **187**, 165 (1987).

[13]R. S. Chivukula and A. V. Manohar, Phys. Lett. B **207**, 86 (1988); **217**, 568(E) (1989).

[14]See also R. S. Willey, Phys. Rev. D **39**, 2784 (1989), for a recent updated limit on the existence of a light Higgs boson implied by rare K decays.

[15]H. Y. Cheng, J. Mod. Phys. A **4**, 495 (1989).

[16]F. J. Gilman and M. B. Wise, Phys. Rev. D **20**, 2392 (1979); **27**, 1128 (1983).

[17]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. **78B**, 443 (1978).

[18]In the literature Eq. (2.8) is often written as $\frac{1}{2}(a-b)(p_1+p_2)^2$, which is valid only in both *chiral* and *soft-pion* limits. In the framework of current algebra, Eq. (2.8) is derived by applying the soft-pion theorem and the commutator relation $[\theta^\mu{}_\mu, Q_5^\pm]=0$ valid in the chiral limit.

[19]S. Raby and G. B. West, Phys. Rev. D **38**, 3488 (1988).

[20]C. Bernard, T. Draper, A. Soni, H. D. Politzer, and M. Wise, Phys. Rev. D **32**, 2343 (1985).

[21]Even without *CPS* symmetry, total derivative chiral terms do not contribute to momentum-conserving processes involving only pseudoscalar mesons.

[22]A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).

[23]To obtain Eq. (2.26) we note that the amplitude of Fig. 1(c) or 1(d) has the form

$$-\frac{1}{v}\left[\frac{4n_h}{33-2n_l}\right]\langle \pi^+ H|\mathcal{L}_{\rm tree}|K^+\rangle$$

with

$$\mathcal{L}_{\rm tree}=(G_F/\sqrt{2})V_{us}V_{ud}^* H\bar{s}\gamma^\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)d+\text{H.c.}$$

being the effective $\Delta S=1$ Lagrangian at the electroweak scale. The evolution of the $\Delta S=1$ weak Lagrangian from M_W down to the renormalization scale μ is given by Eq. (2.2). Notice that the wave-function renormalization done in Ref. 24 is actually our pole contribution.

[24]R. S. Chivukula and A. V. Manohar, Phys. Lett. B **217**, 568(E) (1989).

[25]We adopt the convention of Ref. 13 putting a minus sign in front of the parameter B.

[26]Contributions of higher-order chiral Lagrangians are of order $(p_K-p_\pi)^2/\Lambda_\chi^2=m_H^2/\Lambda_\chi^2$.

[27]L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1984).

[28]H. Burkhardt *et al.*, Phys. Lett. B **206**, 169 (1988).

[29]H. Y. Cheng, Report No. IP-ASTP-10-88 (unpublished).

[30]The first term in Eq. (2.37) is in agreement with Ref. 24 obtained in a different method.

[31]R. J. Cence *et al.*, Phys. Rev. D **10**, 776 (1974).

[32]P. Bloch *et al.*, Phys. Lett. **56B**, 201 (1975).

[33]M. S. Atiya *et al.*, Report No. BNL-43212, 1989 (unpublished).

[34]T. Yamazaki *et al.*, Phys. Rev. Lett. **52**, 1089 (1984).

[35]Y. Asano *et al.*, Phys. Lett. **107B**, 159 (1981); **113B**, 195 (1982).

[36]A. S. Carroll *et al.*, Phys. Rev. Lett. **44**, 525 (1980).

[37]NA31 Collaboration, H. G. Sander, in *Weak Interactions and Neutrinos,* proceedings of the Twelfth International Workship, Ginosar, Israel, 1989, edited by P. Singer and B. Gad Eilam [Nucl. Phys. B (Proc. Suppl.) (in press)].

[38]T. N. Truong and R. S. Willey, Phys. Rev. D (unpublished).

[39]See, e.g., H. Y. Cheng, Phys. Rev. D **36**, 2056 (1987).

[40]However, it should be stressed that $\langle \pi^+\pi^-|\bar{s}\gamma_5 d|K^0\rangle=0$ for the physical $K\to\pi^+\pi^-$ amplitude (see Sec. 7.1 of Ref. 15).

[41]Our result for $B(K^+\to e^+\nu H)$ is numerically different from the branching ratio obtained in Ref. 13 by a factor of 1.5.

[42]A. M. Diamant-Berger *et al.*, Phys. Lett. **62B**, 485 (1970).

[43]SINDRUM Collaboration, S. Egli *et al.*, Phys. Lett. B **222**, 533 (1989).