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Predictions for semileptonic decays of charm baryons. II. Nonrelativistic and MIT bag quark models

R. Pérez-Marcial,* R. Huerta, A. García, and M. Avila-Aoki†
Departamento de Física, Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, Apartado Postal 14-740, 07000 México, Distrito Federal, México

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Predictions for the semileptonic decays of charm baryons are given in two quark models: The nonrelativistic and the MIT bag models. We find that several processes have large branching ratios, of the order of few percent; these are very large in comparison with the strange-hyperon semileptonic-decay branching ratios. We also notice that the q^2 dependence overwhelms other corrections and should be considered in the future with more emphasis. Finally, the predictions found for the Λ_c^+ decay—when compared with the corresponding experimental value—leave ample room for the existence of a fourth generation.

I. INTRODUCTION

In the preceding paper[1]—which we shall refer to as I—we set up the formulas to calculate the rates of charm-baryon semileptonic decays (CBSD) and we applied them to obtain the SU(4)-symmetry-limit predictions for those decay rates. Since flavor SU(4) is expected to be badly broken, it is important to obtain other predictions which do involve symmetry-breaking contributions. In this paper we shall proceed to obtain symmetry-breaking predictions in two cases: the nonrelativistic quark model (NRQM) and the MIT bag model (MBM). In both approaches an important feature is that the induced pseudotensor form factor (weak electricity) g_2 comes out to be nonzero. This then allows that the complete set of four form factors that are relevant in electron-mode decays be in operation, and not three as in the SU(4)-symmetry limit.

We will not attempt very refined calculations; our main objective is to obtain predictions that may serve, on the one hand, to appreciate the size of theoretical uncertainties and that help us establish which are the more important ones and, on the other hand, to help build expectations for experimental work. We shall, nevertheless, consider several corrections to the NRQM and MBM predictions: namely, the q^2 dependence of the form factors, the hard-gluon QCD contributions, and the wave-function mismatch (WFM).

In this paper we will limit ourselves to study only two groups of decays: the $\Delta S=0$, $\Delta C=-1$ and the $\Delta S=\Delta C=-1$. The predictions of SU(4) for the third group, $\Delta C=0$, should be good enough to serve as guidance; their branching ratios are so small that there is little chance they will be measured in the foreseeable future and so we should not devote any more effort to them. In this paper, for short, we shall refer to the first group as $c\to d$ decays and to the second group as $c\to s$ decays.

In Sec. II we shall give the general expressions for the form factors in terms of covariants in the Breit frame, following the approach of Carlson *et al.*[2,3] In Sec. III we evaluate these covariants both in the NRQM and in the MBM. The expressions quoted there for these covariants are valid for any baryon semileptonic decay $[\frac{1}{2}^+\to\frac{1}{2}^+]$. Also, in this section the decay rates and branching ratios are evaluated. In Sec. IV we add to these predictions the corrections induced by the q^2 dependence of the form factors, the QCD corrections corresponding to hard-gluon effects, and the WFM corrections stemming from distortions of the orbital wave functions. Finally, in Sec. V we collect our results, discuss our findings, and compare them with the results of I.

II. BARE FORM FACTORS

In evaluating the form factors, we use a framework that permits us to obtain all of them simultaneously, in such a way that the effect of one form factor on another is consistently taken into account.

It is important to recall that we must consider apart from the usual form factors f_1, f_2, and g_1, the form factor g_2. In CBSD, because of the large momentum transfer, the frequently neglected pseudotensor second-class form factor g_2 cannot be ignored and must be included. In the electron-mode semileptonic decays the f_3 and g_3 form factors do not play a direct role. However, as it has been discussed originally by Kubodera *et al.*,[4] and in great detail by Carlson *et al.*,[2,3] these two form factors do play a very important indirect role in determining the other four form factors, and, in particular, in determining g_2. We shall follow the approach of Carlson *et al.*[3]

We shall work in the Breit frame. The matrix element of the hadronic weak current is given by Eq. (4) of I. In this frame, the decaying and emitted baryons have momenta $\mathbf{p}_1=-\mathbf{p}_2=\mathbf{q}/2$, and so $q_0=E_1-E_2$. Their corresponding spinors are given by

$$u_1=N_i\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{q}}{2(E_1+M_1)}\end{bmatrix}\chi_i\ , \tag{1a}$$

$$\bar{u}_f = N_f \chi_f^\dagger \left[1, \frac{\sigma\cdot\mathbf{q}}{2(E_2+M_2)} \right], \tag{1b}$$

where

$$N_{i,f} = [(E_{1,2}+M_{1,2})/2M_{1,2}]^{1/2}$$

and

$$E_{1,2} = M_{1,2}(1+\mathbf{q}^2/4M_{1,2}^2)^{1/2}$$

are the normalization factor and the energy, respectively, of the initial and final baryons. The $\chi_{i,f}$ are two-component Pauli spinors.

With this choice of Lorentz frame the left-hand side of the matrix element, Eq. (4) of I, is reduced to the two-component form

$$\langle B_f | J^\mu(q) | B_i \rangle \Big|_{q_0=E_1-E_2}^{\mathbf{p}_i=-\mathbf{p}_f=\mathbf{q}/2} \equiv V^\mu(\mathbf{q}) + A^\mu(\mathbf{q}) \tag{2}$$

in terms of the covariants ($\mu=0,i$ and $i=1,2,3$)

$$V^0(\mathbf{q}) \equiv \chi_f^\dagger \mathcal{V}_0(\mathbf{q}^2)\chi_i , \tag{3a}$$

$$V^i(\mathbf{q}) \equiv \chi_f^\dagger [q^i \mathcal{V}_v(\mathbf{q}^2) + i\epsilon^{ijk} q^j \sigma^k \mathcal{V}_M(\mathbf{q}^2)]\chi_i , \tag{3b}$$

$$A^0(\mathbf{q}) \equiv \chi_f^\dagger [\sigma\cdot\mathbf{q}\mathcal{A}_0(\mathbf{q}^2)]\chi_i , \tag{3c}$$

$$A^i(\mathbf{q}) \equiv \chi_f^\dagger [\sigma^i \mathcal{A}_s(\mathbf{q}^2) + (q^i q^j - \tfrac{1}{3}\mathbf{q}^2\delta^{ij})\sigma^j \mathcal{A}_T(\mathbf{q}^2)]\chi_i . \tag{3d}$$

The coefficients $\mathcal{V}_J$ ($J=0,V,M$) and $\mathcal{A}_J$ ($J=0,S,T$) are scalar functions of q^2. By expanding the right-hand side of the matrix element, Eq. (4) of I, in this frame we identify the scalar coefficients in terms of the six original form factors, i.e.,

$$\mathcal{V}_0(\mathbf{q}^2) = N\left[f_1 \left[1 - \frac{\mathbf{q}^2}{4M_1M_2\beta_1\beta_2} \right] - f_2 \left[\frac{M_1\beta_1+M_2\beta_2}{2M_1M_2\beta_1\beta_2} \right] \mathbf{q}^2 \Big/ M_1 + f_3 \left[1 + \frac{\mathbf{q}^2}{4M_1M_2\beta_1\beta_2} \right] q_0 \Big/ M_1 \right], \tag{4a}$$

$$\mathcal{V}_V(\mathbf{q}^2) = N\left[-f_1 \left[\frac{M_1\beta_1-M_2\beta_2}{2M_1M_2\beta_1\beta_2} \right] - f_2 \left[\frac{M_1\beta_1+M_2\beta_2}{2M_1M_2\beta_1\beta_2} \right] q_0 \Big/ M_1 + f_3 \left[1 + \frac{\mathbf{q}^2}{4M_1M_2\beta_1\beta_2} \right] \Big/ M_1 \right], \tag{4b}$$

$$\mathcal{V}_M(\mathbf{q}^2) = N\left[f_1 \left[\frac{M_1\beta_1+M_2\beta_2}{2M_1M_2\beta_1\beta_2} \right] + f_2 \left[1 + \frac{2q_0(M_1\beta_1-M_2\beta_2)-\mathbf{q}^2}{4M_1M_2\beta_1\beta_2} \right] \Big/ M_1 \right], \tag{4c}$$

$$\mathcal{A}_0(\mathbf{q}^2) = N\left[-g_1 \left[\frac{M_1\beta_1-M_2\beta_2}{2M_1M_2\beta_1\beta_2} \right] - g_2 \left[1 + \frac{\mathbf{q}^2}{4M_1M_2\beta_1\beta_2} \right] \Big/ M_1 + g_3 \left[\frac{M_1\beta_1+M_2\beta_2}{2M_1M_2\beta_1\beta_2} \right] q_0 \Big/ M_1 \right], \tag{4d}$$

$$\mathcal{A}_S(\mathbf{q}^2) = N\left[g_1 \left[1 + \frac{\mathbf{q}^2}{12M_1M_2\beta_1\beta_2} \right] - g_2 \left[q_0 - \frac{\mathbf{q}^2[q_0 - 4M(M_1\beta_1-M_2\beta_2)]}{12M_1M_2\beta_1\beta_2} \right] \Big/ M_1 + g_3 \left[\frac{\mathbf{q}(M_1\beta_1+M_2\beta_2)}{6M_1M_2\beta_1\beta_2} \right] \Big/ M_1 \right], \tag{4e}$$

$$\mathcal{A}_T(\mathbf{q}^2) = N\left[g_3 \left[\frac{M_1\beta_1+M_2\beta_2}{2M_{12}} \right] - \tfrac{1}{2}[g_1 - g_2(M_1\beta_1 - M_2\beta_2 - q_0)/M_1] \right] \Big/ (M_1M_2\beta_1\beta_2) , \tag{4f}$$

where $N \equiv (\beta_1\beta_2/4)^{1/2}$, $q_0 \equiv M_1\beta_1 - M_2\beta_2 - \Delta M$, $\beta_i \equiv 1 + (1+\mathbf{q}^2/M_i^2)^{1/2}$, and $\Delta M \equiv M_1 - M_2$. The above expressions are valid for an arbitrary $\mathbf{q}^2$.

For the particular case in the Breit frame in which $\mathbf{q}=0$ (and $q^2=\Delta M^2$) Eqs. (4a)–(4f) take a simple form, and then inverting them we get, for the form factors,

$$f_1 = \mathcal{V}_0 - \mathcal{V}_M \Delta M^2/M_{12} - \mathcal{V}_V \Delta M , \tag{5a}$$

$$f_2 = (-\mathcal{V}_0 + \mathcal{V}_M M_{12} + \mathcal{M}_V \Delta M) M_1/M_{12} , \tag{5b}$$

$$f_3 = \mathcal{V}_V M_1 + \mathcal{V}_M M_1 \Delta M/M_{12} , \tag{5c}$$

$$g_1 = (1 - \Delta M^2/2M_{12}^2)\mathcal{A}_s + (\mathcal{A}_T \Delta M - \mathcal{A}_0) 4M_1M_2\Delta M/M_{12}^2 , \tag{5d}$$

$$g_2 = (\mathcal{A}_T \Delta M - \mathcal{A}_s \Delta M/8M_1M_2 - \mathcal{A}_0) 4M_1^2M_2/M_{12}^2 , \tag{5e}$$

$$g_3 = (\mathcal{A}_s/2 + \mathcal{A}_T 4M_1M_2) M_1/M_{12} , \tag{5f}$$

$M_{12} = M_1 + M_2$. We must note that the coefficients $\mathcal{A}_J$ and $\mathcal{V}_J$ are evaluated at $\mathbf{q}=0$ and the form factors at $q^2 = q_0^2 = (\Delta M)^2$. Choosing $\mathbf{q}=0$ is not an approximation; it is a particular way to compute the form factors. In principle, the knowledge of the functional q^2 dependence of the form factors should correct for this particular choice. The reason to choose this value is that for it both baryons are static in the Breit frame, and thus one may expect that static-bag and quark-model wave functions best resemble the compound hadron states. It is so, as discussed in detail by Carlson *et al.*, that the calculation is carried on more consistently.

In order to calculate the scalars $\mathcal{A}_J$ and $\mathcal{V}_J$ we write the left-hand side of Eq. (4) of I in terms of the Fourier transform of the space-time wave functions of the quark or bag model to be chosen. We have that, in the Breit frame and in the absence of vertex corrections,[3]

$$2\pi\delta(E_1 - E_2 - q_0)\langle B_f | J^\mu(\mathbf{q}) | B_i \rangle = \sum_c \int d^4y\, e^{iq_0y_0 - i\mathbf{q}\cdot\mathbf{y}} \times \langle B_f | :\bar{\psi}_f^c(y)\gamma^\mu(1+\gamma_5)\psi_i^c(y): | B_i \rangle , \tag{6}$$

where c is the color index. The canonical quark field operator $\psi^c(y)$ is given by

$$\psi^c(y)=\sum_N a_N^c q_N(\mathbf{y})e^{-i\omega_N y_0}\ , \tag{7}$$

where $q_N(\mathbf{y})$ are static orbital quark wave functions of energy ω_N. The flavor and spin wave functions are contained in $|B_i\rangle$ and $|B_f\rangle$ (see Table I), a_N^c are the usual annihilation quark operators, and N denotes all other quantum numbers. We will assume quarks in the ground state and with zero angular momentum.

In baryon semileptonic decay the quark operators will annihilate a quark $q^{c,i}$ of $|B_i\rangle$ and will create a quark $q^{c,f}$ to form $|B_f\rangle$. The values of the matrix elements of the quark operators between the spin-flavor part of the baryon wave functions are given by

$$\sum_c \langle B_f|a_m^{\dagger c,f}a_{m'}^{c,i}|B_i\rangle = A\delta_{m\uparrow}\delta_{m'\uparrow}+B\delta_{m\downarrow}\delta_{m'\downarrow}\ , \tag{8}$$

where A and B are real numbers, and m' and m denote the spin projections of annihilated and created quarks, respectively. In the determination of A and B it suffices to take both initial and final baryons states with spin up. Notice that A and B are independent of the orbital wave function. The results are given in Table II. We notice that these coefficients vanish for the $c\to s$ processes $\Lambda_c^+\to\Sigma^0$ and $\Sigma_c^+\to\Lambda$ because of isospin selection rules, as was discussed in I.

So, in general the left-hand side of Eq. (4) of I is given by

$$\langle B_f|J_\mu(\mathbf{q})|B_i\rangle = V_\mu(\mathbf{q})+A_\mu(\mathbf{q})=\sum_{mm'}(A\delta_{m\uparrow}\delta_{m'\uparrow}+B\delta_{m\downarrow}\delta_{m'\downarrow})\int d^3\mathbf{y}\, e^{-i\mathbf{q}\cdot\mathbf{y}}[\bar{q}^{\,f}_{jlmn}(\mathbf{y})\gamma_\mu(1+\gamma_5)q^i_{j'l'm'n'}(\mathbf{y})] \tag{9}$$

with the orbital indices $j=j'=\frac{1}{2}$, $l=l'=0$, $n=n'=0$ corresponding to the ground state. In the quark orbital wave function $q(\mathbf{y})$, i and f denote flavor indices.

Equation (9) can be evaluated only when a particular model is chosen. We shall need an explicit expression for the quark orbital wave function $q(\mathbf{y})$. Once we have such wave functions we can get explicit expressions for the $\mathcal{A}_J$ and $\mathcal{V}_J$, and, along with them, the form factors can be evaluated using Eqs. (5a)–(5f). In the next section we shall study two specific models: the NRQM and MBM.

III. DECAY RATES AND BRANCHING RATIOS

In this section we shall determine both the NRQM and the MBM predictions for the decay rates and branching ratios of CBSD.

The main difference between these two models consists of how we handle Eq. (9). For the MBM the integral is over the bag space, while for the NRQM we perform the integration over the whole space. It must be mentioned that although the MBM approach is more refined than

TABLE I. Flavor-spin wave functions for the 20-plet $J^P=\frac{1}{2}^+$ baryons. Here $\chi_s\equiv(2\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)/\sqrt{6}$ and $\chi_A\equiv(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)/\sqrt{2}$. (ij) means we must permute the quark in place i with the quark in place j. $abc\chi_s$ and $abc\chi_A$ mean $abc\chi_s=(2a_\uparrow b_\uparrow c_\downarrow-a_\uparrow b_\downarrow c_\uparrow-a_\downarrow b_\uparrow c_\uparrow)/\sqrt{6}$ and $abc\chi_A=(a_\uparrow b_\downarrow c_\uparrow-a_\downarrow b_\uparrow c_\uparrow)/\sqrt{2}$. For example, the permutation (23) of $abc\chi_s$ is $(2a_\uparrow b_\downarrow c_\uparrow-a_\uparrow b_\uparrow a_\downarrow-a_\downarrow b_\uparrow c_\uparrow)/\sqrt{6}$.

Particle	Flavor-spin wave function	SU(3)
Ω_{cc}^+	$-[ccs\chi_s+(13)+(23)]/\sqrt{3}$	3
Ξ_{cc}^{++}	$-[ccu\chi_s+(13)+(23)]/\sqrt{3}$	3
Ξ_{cc}^+	$-[ccd\chi_s+(13)+(23)]/\sqrt{3}$	3
Ω_c^0	$[ssc\chi_s+(13)+(23)]/\sqrt{3}$	6
Ξ_c^+	$[(usc+suc)\chi_s+(13)+(23)]/\sqrt{6}$	6
Ξ_c^0	$[(dsc+sdc)\chi_s+(13)+(23)]/\sqrt{6}$	6
Σ_c^{++}	$[uuc\chi_s+(13)+(23)]/\sqrt{3}$	6
Σ_c^+	$[(udc+duc)\chi_s+(13)+(23)]/\sqrt{6}$	6
Σ_c^0	$[ddc\chi_s+(13)+(23)]/\sqrt{3}$	6
$\Xi_c^{A\,+}$	$[(usc-suc)\chi_A+(13)+(23)]/\sqrt{6}$	3*
$\Xi_c^{A\,0}$	$[(dsc-sdc)\chi_A+(13)+(23)]/\sqrt{6}$	3*
Λ_c^+	$[(udc-duc)\chi_A+(13)+(23)]/\sqrt{6}$	3*
Ξ^0	$-[ssu\chi_s+(13)+(23)]/\sqrt{3}$	8
Ξ^-	$-[ssd\chi_s+(13)+(23)]/\sqrt{3}$	8
Σ^+	$[uus\chi_s+(13)+(23)]/\sqrt{3}$	8
Σ^0	$[(uds+dus)\chi_s+(13)+(23)]/\sqrt{6}$	8
Σ^-	$[dds\chi_s+(13)+(23)]/\sqrt{3}$	8
Λ	$[(uds-dus)\chi_A+(13)+(23)]/\sqrt{6}$	8
p	$[uud\chi_s+(13)+(23)]/\sqrt{3}$	8
n	$-[ddu\chi_s+(13)+(23)]/\sqrt{3}$	8

TABLE II. Coefficients coming from the flavor-spin part of the baryon wave functions for the $\Delta C \neq 0$ decays, see Eq. (8).

$B_1 \to B_2 + e + \nu_e$	A	B
	$c \to d$ decays	
$\Xi_{cc}^{++} \to \Lambda_c^+$	$2/\sqrt{6}$	$1/\sqrt{6}$
$\Xi_{cc}^{++} \to \Sigma_c^+$	$4/3\sqrt{2}$	$-1/3\sqrt{2}$
$\Xi_{cc}^{+} \to \Sigma_c^0$	$4/3$	$-1/3$
$\Omega_{cc}^{+} \to \Xi_c^0$	$4/3\sqrt{2}$	$-1/3\sqrt{2}$
$\Omega_{cc}^{+} \to \Xi_c^{A\,0}$	$-2/\sqrt{6}$	$-1/\sqrt{6}$
$\Xi_c^0 \to \Sigma^-$	$-1/3\sqrt{2}$	$-2/3\sqrt{2}$
$\Omega_c^0 \to \Xi^-$	$-1/3$	$-2/3$
$\Xi_c^{A\,0} \to \Sigma^-$	$\sqrt{3/2}$	0
$\Lambda_c^+ \to n$	$\sqrt{3/2}$	0
$\Xi_c^{A\,+} \to \Sigma^0$	$\sqrt{3}/2$	0
$\Xi_c^{A\,+} \to \Lambda$	$1/2$	0
$\Sigma_c^+ \to n$	$1/3\sqrt{2}$	$2/3\sqrt{2}$
$\Xi_c^+ \to \Sigma^0$	$-1/6$	$-1/3$
$\Xi_c^+ \to \Lambda$	$1/2\sqrt{3}$	$1/\sqrt{3}$
$\Sigma_c^{++} \to p$	$1/3$	$2/3$
	$c \to s$ decays	
$\Xi_{cc}^{++} \to \Xi_c^{A\,+}$	$2/\sqrt{6}$	$1/\sqrt{6}$
$\Xi_{cc}^{++} \to \Xi_c^+$	$4/3\sqrt{2}$	$-1/3\sqrt{2}$
$\Xi_{cc}^{+} \to \Xi_c^0$	$4/3\sqrt{2}$	$-1/3\sqrt{2}$
$\Omega_{cc}^{+} \to \Omega_c^0$	$4/3$	$-1/3$
$\Xi_{cc}^{+} \to \Xi_c^{A\,0}$	$2/\sqrt{6}$	$1/\sqrt{6}$
$\Sigma_c^0 \to \Sigma^-$	$1/3$	$2/3$
$\Xi_c^0 \to \Xi^-$	$1/3\sqrt{2}$	$2/3\sqrt{2}$
$\Xi_c^{A\,0} \to \Xi^-$	$\sqrt{3/2}$	0
$\Lambda_c^+ \to \Lambda$	1	0
$\Xi_c^{A\,+} \to \Xi^0$	$\sqrt{3/2}$	0
$\Sigma_c^+ \to \Sigma^0$	$1/3$	$2/3$
$\Xi_c^+ \to \Xi^0$	$1/3\sqrt{2}$	$2/3\sqrt{2}$
$\Sigma_c^{++} \to \Sigma^+$	$1/3$	$2/3$
$\Sigma_c^+ \to \Lambda$	0	0
$\Lambda_c^+ \to \Sigma^0$	0	0

the NRQM, it has the problem of introducing several parameters such as the bag radius and the masses of the quarks in the bag. The NRQM does not depend on these parameters. Nevertheless, the MBM is expected to be more realistic than the NRQM.

A. Nonrelativistic quark model

For this case we take for $q(\mathbf{y})$ the plane wave functions described by Kokkedee.[5] With them we evaluate Eq. (9) and obtain

$$V_\mu + A_\mu = \sum_{mm'} (A\delta_{m\uparrow}\delta_{m'\uparrow} + B\delta_{m\downarrow}\delta_{m'\downarrow}) \times \bar{u}_m^f(\mathbf{q})\gamma_\mu(1+\gamma_5)u_{m'}^i(\mathbf{q}) \tag{10}$$

where the $u^{i,f}$ are free particle solutions of the Dirac equation, not restricted to be confined in a bag. Then, evaluating Eq. (9), identifying the covariants $\mathcal{A}_J$ and $\mathcal{V}_J$ in it at $\mathbf{q}=0$ and using Eqs. (5a)–(5f) we obtain the NRQM form factors at $q^2 = \Delta M^2$:

$$f_1 = \left[1 - \frac{\Delta M^2}{4M_1M_2}\right]\alpha_1 - \frac{\Delta M^2}{4M_1M_2}\alpha_2 , \tag{11a}$$

$$f_2 = -\frac{M_1}{M_{12}}\left[1 + \frac{\Delta M^2}{4M_1M_2}\right](\alpha_1 - \alpha_2) , \tag{11b}$$

$$f_3 = \frac{\Delta M}{4M_2}(-\alpha_1 + \alpha_2) , \tag{11c}$$

$$g_1 = \left[1 - \frac{\Delta M^2}{4M_{12}^2}\right]\alpha_2 , \tag{11d}$$

$$g_2 = -\frac{1}{4}\frac{M_1}{M_{12}^2}\Delta M\alpha_2 , \tag{11e}$$

$$g_3 = -\frac{1}{4}\frac{M_1}{M_{12}}\alpha_2 , \tag{11f}$$

with

$$\alpha_1 \equiv \sum_m (A\delta_{m\uparrow} + B\delta_{m\downarrow}) = A + B , \tag{12a}$$

$$\alpha_2 \equiv \sum_m m(A\delta_{m\uparrow} + B\delta_{m\downarrow}) = A - B . \tag{12b}$$

Let us recall that A and B are given in Table II.

B. MIT bag model

The equations and boundary conditions that define the MBM and determine both the field motion and the motion of the surface are difficult to solve in general. For simplicity, we shall take a spherical boundary, and since we have taken $\mathbf{q}=0$, the cavity of the bag will be at rest. Also the $\frac{1}{2}^+$ charm baryons are believed to be in the ground state of the orbital excitation. We therefore take the solutions for the orbital wave functions given by Chodos *et al.*,[6] for a static spherical boundary appropriate to particles at rest. Direct substitution of these orbital wave functions together with an expansion in powers of $\mathbf{q}$ yields, after some algebra, the following expressions, within the MBM:

$$\chi_\uparrow^\dagger \chi_\uparrow \mathcal{V}_0(0) = \sum_{m,m'} C(m,m')\chi_m^\dagger \chi_{m'} N^i N^f \int_0^R dr\, r^2 (W_+^i W_+^f j_0^i j_0^f + W_-^i W_-^f j_1^i j_1^f) , \tag{13a}$$

$$\chi_\uparrow^\dagger \mathbf{q} \chi_\uparrow \mathcal{V}_v(0) = \sum_{m,m'} C(m,m')\chi_m^\dagger \mathbf{q} \chi_{m'} \tfrac{1}{3} N^i N^f \int_0^R dr\, r^3 (W_-^i W_+^f j_1^i j_0^f - W_+^i W_-^f j_0^i j_1^f) , \tag{13b}$$

$$\chi_\uparrow^\dagger i(\mathbf{q}\times\boldsymbol{\sigma}) \chi_\uparrow \mathcal{V}_M(0) = \sum_{m,m'} C(m,m')\chi_m^\dagger i(\mathbf{q}\times\boldsymbol{\sigma}) \chi_{m'} \tfrac{1}{3} N^i N^f \int_0^R dr\, r^3 (W_-^i W_+^f j_1^i j_0^f + W_+^i W_-^f j_0^i j_1^f) , \tag{13c}$$

$$\chi_\uparrow^\dagger \mathbf{q}\cdot\boldsymbol{\sigma}\chi_\uparrow \mathcal{A}_0(0)=\sum_{m,m'} C(m,m')\chi_m^\dagger \mathbf{q}\cdot\boldsymbol{\sigma}\chi_{m'}\tfrac{1}{3}N^i N^f \int_0^R dr\, r^3(W_-^i W_+^f j_1^i j_0^f - W_+^i W_-^f j_0^i j_1^f) , \tag{13d}$$

$$\chi_\uparrow^\dagger \boldsymbol{\sigma}\chi_\uparrow \mathcal{A}_s(0)=\sum_{m,m'} C(m,m')\chi_m^\dagger \boldsymbol{\sigma}\chi_{m'} N^i N^f \int_0^R dr\, r^2(W_+^i W_+^f j_0^i j_0^f - \tfrac{1}{3} W_-^i W_-^f j_1^i j_1^f) , \tag{13e}$$

$$\chi_\uparrow^\dagger (\mathbf{q}\mathbf{q}\cdot\boldsymbol{\sigma} - \tfrac{1}{3}\mathbf{q}^2\boldsymbol{\sigma})\chi_\uparrow \mathcal{A}_T(0)=\sum_{m,m'} C(m,m')\chi_m^\dagger (\mathbf{q}\mathbf{q}\cdot\boldsymbol{\sigma} - \tfrac{1}{3}\mathbf{q}^2\boldsymbol{\sigma})\chi_{m'}(-\tfrac{2}{15})N^i N^f \int_0^R dr\, r^4 W_-^i W_-^f j_1^i j_1^f , \tag{13f}$$

where $C(m,m')\equiv A\delta_{m\uparrow}\delta_{m'\uparrow}+B\delta_{m\downarrow}\delta_{m'\downarrow}$;

$$N^q = N^q(x) = \left\{ \frac{\omega^q(x)[\omega^q(x)-m_q]}{R^3 j_0^{2q}(x)\left[2\omega^q(x)\left[\omega^q(x)-\frac{1}{R}\right]+m_q/R\right]} \right\}^{1/2} \tag{14}$$

is the quark normalization factor, the index q corresponds to the quark flavor indices i and f. Here,

$$\omega^q(x)\equiv(x_0^{q2}/R^2+m_q^2)^{1/2} , \tag{15}$$

where x_0^q is the lowest root of the transcendental equation,

$$\tan(x)=\frac{x}{1-m_q R-[x^2+(m_q R)^2]^{1/2}} , \tag{16}$$

which represents the boundary condition that prevents current flux of quark q through the bag surface.

$W_\pm^q$ are defined as

TABLE III. NRQM predictions for the bare form factors evaluated at $q^2=0$ of the $c\to d$ decays. The upper value corresponds to monopole approximation and the lower one to dipole approximation.

$B_1\to B_2+e+\nu_e$	f_1	f_2	f_3	g_1	g_2	g_3
$\Xi_{cc}^{++}\to\Lambda_c^+$	0.64	−0.25	0.13	0.28	−0.01	−0.04
	0.36	−0.14	0.08	0.20	−0.01	−0.03
$\Xi_{cc}^{++}\to\Sigma_c^+$	0.43	0.22	0.15	0.91	−0.03	−0.14
	0.28	0.14	0.10	0.70	−0.02	−0.10
$\Xi_{cc}^{+}\to\Sigma_c^0$	0.61	0.31	0.21	1.29	−0.04	−0.19
	0.41	0.21	0.14	1.00	−0.03	−0.15
$\Omega_{cc}^{+}\to\Xi_c^0$	0.40	0.20	0.14	0.87	−0.03	−0.13
	0.25	0.13	0.09	0.65	−0.02	−0.10
$\Omega_{cc}^{+}\to\Xi_c^{A\,0}$	−0.65	0.26	−0.12	−0.29	0.01	0.04
	−0.37	0.15	−0.07	−0.20	0.01	0.03
$\Xi_c^0\to\Sigma^-$	−0.34	0.32	−0.07	0.16	−0.01	−0.03
	−0.19	0.17	−0.04	0.11	−0.01	−0.02
$\Omega_c^0\to\Xi^-$	−0.46	0.42	−0.09	0.22	−0.01	−0.04
	−0.23	0.21	−0.05	0.14	−0.01	−0.02
$\Xi_c^{A\,0}\to\Sigma^-$	0.53	0.14	0.39	0.88	−0.05	−0.15
	0.32	0.08	0.23	0.64	−0.04	−0.11
$\Lambda_c^+\to n$	0.39	0.20	0.48	0.84	−0.06	−0.15
	0.22	0.11	0.27	0.58	−0.04	−0.10
$\Xi_c^{A\,+}\to\Sigma^0$	0.37	0.10	0.28	0.62	−0.04	−0.11
	0.22	0.06	0.16	0.45	−0.03	−0.08
$\Xi_c^{A\,+}\to\Lambda$	0.18	0.06	0.17	0.34	−0.02	−0.06
	0.10	0.03	0.09	0.23	−0.02	−0.04
$\Sigma_c^+\to n$	0.26	−0.26	0.08	−0.14	0.01	0.03
	0.11	−0.11	0.04	−0.09	0.01	0.02
$\Xi_c^+\to\Sigma^0$	−0.27	0.25	−0.05	0.12	−0.01	−0.02
	−0.17	0.15	−0.03	0.09	−0.01	−0.01
$\Xi_c^+\to\Lambda$	0.43	−0.40	0.10	−0.20	0.01	0.03
	0.24	−0.22	0.05	−0.14	0.01	0.02
$\Sigma_c^{++}\to p$	0.36	−0.37	0.12	−0.20	0.02	0.04
	0.16	−0.16	0.05	−0.12	0.01	0.02

$$W_{\pm}^{q} \equiv \left[\frac{\omega^{q} \pm m_{q}}{\omega^{q}} \right]^{1/2} . \tag{17}$$

Above m_q is the mass of the quark with flavor q in the bag and R is the bag radius. The spherical Bessel functions $j_{0,1}^{q}$ have argument $x_0^q r/R$.

Before performing any calculation we must know x_0^q, and this requires the knowledge of m_q and R. The masses m_u, m_d, and m_s in the MBM are taken from DeGrand *et al.*,[7] $m_{u,d}=0.005$ GeV, and $m_s=0.280$ GeV. In this reference they fix the parameters of the bag (zero-point energy, the coupling of quarks and gluons, and energy density in the bag) by fitting the masses of the proton and the delta baryons and the ω meson, and find that the bag radius of the noncharm baryons is close to 1 fm. To determine m_c for charm baryons we proceed as in Ref. 7. We fit the Λ_c^+ mass taking the same values of the bag parameters mentioned above and also a bag radius R equal to 1 fm. The value obtained for m_c is 1.5 GeV, which is the generally accepted one. Anyway, in our final calculations we allowed m_c and R to change 10–20 % around these values. We found that the variations of the predictions for the decay rates were between 4% and 7%. Changes in the value of m_s gave the same results. So, within the accuracy of our predictions we can reasonably take the above values for m_q and R as fixed.

Let us introduce the integrals

$$I_{nn} \equiv \int_0^1 dt\, t^2 j_n(tx_0^i) j_n(tx_0^f), \quad n=0,1 , \tag{18a}$$

$$I_{nm} \equiv \int_0^1 dt\, t^3 j_n(tx_0^i) j_m(tx_0^f), \quad n,m=0,1 \; n \neq m , \tag{18b}$$

$$J_{11} \equiv \int_0^1 dt\, t^4 j_1(tx_0^i) j_1(tx_0^f) . \tag{18c}$$

With them and performing the Pauli-matrix calculations, in Eqs. (13a)–(13f), we obtain for $\mathcal{A}_J$ and $\mathcal{V}_J$ the expressions

$$\mathcal{V}_0=(A+B)N^iN^fR^3(W_+^iW_+^fI_{00}+W_-^iW_-^fI_{11}) , \tag{19a}$$

$$\mathcal{V}_v=(A+B)N^iN^fR^3(W_-^iW_+^fI_{10}-W_+^iW_-^fI_{01})(R/3) , \tag{19b}$$

$$\mathcal{V}_M=(A-B)N^iN^fR^3(W_-^iW_+^fI_{10}+W_+^iW_-^fI_{01})(R/3) , \tag{19c}$$

$$\mathcal{A}_0=(A-B)N^iN^fR^3(W_-^iW_+^fI_{10}-W_+^iW_-^fI_{01})(R/3) , \tag{19d}$$

$$\mathcal{A}_s=(A-B)N^iN^fR^3(W_+^iW_+^fI_{00}-\tfrac{1}{3}W_-^iW_-^fI_{11}) , \tag{19e}$$

$$\mathcal{A}_T=(A-B)N^iN^fR^3W_-^iW_-^fJ_{11}(-2R/15) . \tag{19f}$$

Using the values of m_q and R discussed before we

TABLE IV. NRQM predictions for the bare form factors evaluated at $q^2=0$ of the $c \to s$ decays. The upper value corresponds to monopole approximation and the lower one to dipole approximation.

$B_1 \to B_2+e+\nu_e$	f_1	f_2	f_3	g_1	g_2	g_3
$\Xi_{cc}^{++} \to \Xi_c^{A+}$	0.83	−0.32	0.13	0.32	−0.01	−0.05
	0.58	−0.23	0.09	0.26	−0.01	−0.04
$\Xi_{cc}^{++} \to \Xi_c^{+}$	0.45	0.23	0.16	0.93	−0.03	−0.14
	0.31	0.16	0.11	0.73	−0.02	−0.11
$\Xi_{cc}^{+} \to \Xi_c^{0}$	0.49	0.23	0.15	0.97	−0.02	−0.14
	0.37	0.17	0.11	0.80	−0.02	−0.12
$\Omega_{cc}^{+} \to \Omega_c^{0}$	0.70	0.32	0.19	1.37	−0.03	−0.20
	0.53	0.24	0.14	1.13	−0.03	−0.16
$\Xi_{cc}^{+} \to \Xi_c^{A0}$	0.83	−0.32	0.13	0.32	−0.01	−0.05
	0.58	−0.23	0.09	0.26	−0.01	−0.04
$\Sigma_c^{0} \to \Sigma^{-}$	0.58	−0.54	0.11	−0.25	0.01	0.04
	0.38	−0.34	0.07	−0.18	0.01	0.03
$\Xi_c^{0} \to \Xi^{-}$	0.43	−0.38	0.07	−0.18	0.01	0.03
	0.28	−0.25	0.05	−0.13	0.01	0.02
$\Xi_c^{A0} \to \Xi^{-}$	0.69	0.11	0.37	0.96	−0.05	−0.16
	0.48	0.08	0.26	0.76	−0.04	−0.12
$\Lambda_c^{+} \to \Lambda$	0.51	0.12	0.36	0.78	−0.05	−0.13
	0.35	0.09	0.25	0.61	−0.04	−0.10
$\Xi_c^{A+} \to \Xi^{0}$	0.68	0.11	0.38	0.96	−0.05	−0.16
	0.48	0.08	0.26	0.76	−0.04	−0.12
$\Sigma_c^{+} \to \Sigma^{0}$	0.58	−0.54	0.11	−0.25	0.01	0.04
	0.38	−0.35	0.07	−0.18	0.01	0.03
$\Xi_c^{+} \to \Xi^{0}$	0.47	−0.41	0.07	−0.19	0.01	0.03
	0.33	−0.29	0.05	−0.15	0.01	0.02
$\Sigma_c^{++} \to \Sigma^{+}$	0.59	−0.54	0.11	−0.25	0.01	0.04
	0.38	−0.35	0.07	−0.19	0.01	0.03

determine x_0^g from Eq. (16). Once x_0^g is known $N^{i,f}$ are determined with Eq. (14), $W_{\pm}^{i,f}$ are determined using Eq. (17), and then the integrals of Eqs. (18a)–(18c) can be computed. The values of A and B were already tabulated in Table II. Thus, the left-hand side of Eqs. (19a)–(19f) are evaluated. Finally, the bare form factors are obtained using Eqs. (5a)–(5f). The next step is to make predictions for the decay rates and branching ratios of CBSD, using the formulas in I.

C. Predictions

In order to obtain the NRQM and MBM predictions for decay rates and branching ratios we need the form factors evaluated at $q^2=0$. We go from $q^2=(\Delta M)^2$ to $q^2=0$ according to the assumed monopole or dipole q^2 dependence of the form factors discussed earlier in Eqs. (6) and (7) of I.

The values of the form factors at $q^2=0$ for the NRQM are listed in Tables III and IV, and those for the MBM are listed in Tables V and VI. It is clear that these values of the form factors depend on the values we have used for M_A, M_V, and the form of q^2 dependence we have adopted.

Once we have the bare form factors at $q^2=0$, we can combine them with those values of the transition-rate coefficients obtained when we assume no q^2 dependence of the form factors (Tables VII and X of I) to obtain the corresponding predictions for the decay rates Γ_0. Also as we mentioned in I, we use the Λ_c^+ lifetime to estimate the corresponding branching ratios B_0. We obtain these Γ_0 and B_0 predictions as a starting point. To them we shall add several corrections which we shall consider in the next section. The predictions Γ_0 and B_0 for the decay rates and branching ratios in the NRQM and MBM are listed in Tables VII and VIII.

In these predictions, for definiteness and in accordance with I, we have taken $|V_{cd}|=0.22$ and $|V_{cs}|=0.9748$. These values for the Kobayashi-Maskawa (KM) matrix elements come from the three-generation current ranges for them:[8]

$$0.217<|V_{cd}|<0.223 \tag{20a}$$

and

$$0.9733<|V_{cs}|<0.9754\ . \tag{20b}$$

It is not that we are ignoring the q^2 dependence. Obviously we cannot, because the predictions for the form fac-

TABLE V. MBM predictions for the bare form factors evaluated at $q^2=0$ of the $c\to d$ decays. The upper value corresponds to monopole approximation and the lower one to dipole approximation.

$B_1\to B_2+e+\nu_e$	f_1	f_2	f_3	g_1	g_2	g_3
$\Xi_{cc}^{++}\to\Lambda_c^+$	0.80	−0.02	−0.50	0.21	−0.02	−1.00
	0.45	−0.01	−0.28	0.15	−0.01	−0.70
$\Xi_{cc}^{++}\to\Sigma_c^+$	0.45	1.36	−0.10	0.72	0.06	−3.36
	0.30	0.91	−0.07	0.56	0.05	−2.59
$\Xi_{cc}^{+}\to\Sigma_c^0$	0.64	1.93	−0.15	1.02	0.09	−4.78
	0.43	1.30	−0.10	0.79	0.07	−3.70
$\Omega_{cc}^{+}\to\Xi_c^0$	0.42	1.34	−0.09	0.68	0.02	−3.57
	0.26	0.84	−0.06	0.50	0.01	−2.65
$\Omega_{cc}^{+}\to\Xi_c^{A\,0}$	−0.81	−0.01	0.54	−0.22	0.01	1.14
	−0.46	−0.01	0.31	−0.15	0.01	0.80
$\Xi_c^0\to\Sigma^-$	−0.50	0.51	0.31	0.11	−0.02	−0.21
	−0.27	0.28	0.17	0.08	−0.01	−0.14
$\Omega_c^0\to\Xi^-$	−0.67	0.69	0.44	0.15	−0.03	−0.34
	−0.34	0.35	0.22	0.10	−0.02	−0.22
$\Xi_c^{A\,0}\to\Sigma^-$	0.69	0.48	−0.08	0.65	−0.06	−1.08
	0.41	0.29	−0.05	0.47	−0.04	−0.78
$\Lambda_c^+\to n$	0.60	0.33	−0.00	0.59	−0.10	−0.73
	0.33	0.18	−0.00	0.41	−0.07	−0.50
$\Xi_c^{A\,+}\to\Sigma^0$	0.48	0.34	−0.05	0.46	−0.04	−0.76
	0.29	0.20	−0.03	0.33	−0.03	−0.55
$\Xi_c^{A\,+}\to\Lambda$	0.25	0.17	−0.02	0.24	−0.04	−0.39
	0.14	0.09	−0.01	0.17	−0.03	−0.27
$\Sigma_c^+\to n$	0.43	−0.43	−0.25	−0.09	0.03	0.14
	0.19	−0.19	−0.11	−0.06	0.02	0.08
$\Xi_c^+\to\Sigma^0$	−0.39	0.39	0.23	0.09	−0.01	−0.15
	−0.23	0.23	0.14	0.06	−0.01	−0.11
$\Xi_c^+\to\Lambda$	0.63	−0.63	−0.38	−0.14	0.02	0.23
	0.35	−0.35	−0.21	−0.10	0.01	0.16
$\Sigma_c^{++}\to p$	0.61	−0.61	−0.36	−0.13	0.04	0.20
	0.26	−0.26	−0.16	−0.08	0.02	0.12

tors were made at $q^2=(\Delta M)^2$. What we are doing is to make a rational and systematic separation of the many assumptions that are involved in making predictions for the decay rates. In CBSD the q^2 dependence turns out to be a very important and delicate assumption. At this stage, it should be clear that the q^2 dependence is playing a double role, one in getting the form factors at $q^2=0$, and another in adding to the kinematical coefficients of the decay-rate formulas the corrections induced by the q^2 dependence of the several form factors. We want to emphasize this point, because this double role leads to drastic changes in the predictions,[9] up to an order of magnitude.

The corrections induced by the q^2 dependence of the form factors in the transition rate coefficients were computed in I. We shall consider them in the next section, where we shall also include the QCD one-gluon-exchange contributions and the orbital wave-function distortions that lead to the WFM corrections.

IV. CORRECTIONS

A. q^2 dependence corrections

Given the predictions for Γ_0 and B_0 of Tables VII and VIII, the q^2 dependence of the form factors amounts to a correction to these predictions. As such, this is an important correction in CBSD whenever the mass differences involved are very large, as is the case in $c \rightarrow d$ and $c \rightarrow s$ decays.

We take into account the corrections due to the q^2 dependence of the form factors through the transition-rate coefficients. For consistency, we must assume the same q^2 dependence as in going from $q^2=\Delta M^2$ to $q^2=0$ when computing the form factors: i.e., we must use Eqs. (6) and (7) of I. These new coefficients are given in Tables VIII, IX, XI, and XII of I. The predictions with the q^2 dependence are then obtained going back to Eq. (5) of I and using the values of the form factors collected in Tables III–VI. The results for the decay rates and branching ratios (Γ_{q^2} and B_{q^2}) are given in Tables VII and VIII.

B. QCD corrections

The QCD corrections are normally divided in two parts, one that comes from soft gluons and one which is due to hard-gluon effects. It is generally believed that the soft part is taken into account in the wave function which is responsible for keeping the quarks confined within the hadrons; therefore, in the MBM they have already been included. In the NRQM it is assumed that they can be ignored. On the other hand, the hard-gluon effects are responsible for the corrections at the quark vertex. The latter are considered by many authors[10] in a similar way

TABLE VI. MBM predictions for the bare form factors evaluated at $q^2=0$ of the $c \rightarrow s$ decays. The upper value corresponds to monopole approximation and the lower one to dipole approximation.

$B_1 \rightarrow B_2 + e + \nu_e$	f_1	f_2	f_3	g_1	g_2	g_3
$\Xi_{cc}^{++} \rightarrow \Xi_c^{A+}$	0.97	−0.06	−0.45	0.28	−0.01	−0.92
	0.68	−0.04	−0.32	0.22	−0.00	−0.73
$\Xi_{cc}^{++} \rightarrow \Xi_c^{+}$	0.48	1.19	−0.02	0.79	−0.02	−2.64
	0.34	0.84	−0.02	0.63	−0.02	−2.08
$\Xi_{cc}^{+} \rightarrow \Xi_c^{0}$	0.52	1.29	−0.06	0.84	0.03	−2.83
	0.39	0.97	−0.04	0.70	0.03	−2.34
$\Omega_{cc}^{+} \rightarrow \Omega_c^{0}$	0.75	1.94	−0.11	1.20	0.06	−4.51
	0.56	1.46	−0.08	0.99	0.05	−3.72
$\Xi_{cc}^{+} \rightarrow \Xi_c^{A0}$	0.97	−0.06	−0.45	0.28	−0.01	−0.92
	0.68	−0.04	−0.32	0.22	−0.00	−0.73
$\Sigma_c^{0} \rightarrow \Sigma^-$	0.80	−0.78	−0.37	−0.20	0.03	0.22
	0.51	−0.50	−0.24	−0.15	0.02	0.16
$\Xi_c^{0} \rightarrow \Xi^-$	0.57	−0.56	−0.27	−0.14	0.02	0.18
	0.38	−0.37	−0.18	−0.11	0.01	0.14
$\Xi_c^{A0} \rightarrow \Xi^-$	0.84	0.44	−0.04	0.80	−0.07	−0.93
	0.59	0.31	−0.03	0.63	−0.05	−0.74
$\Lambda_c^{+} \rightarrow \Lambda$	0.66	0.27	0.00	0.64	−0.07	−0.56
	0.46	0.19	0.00	0.50	−0.05	−0.44
$\Xi_c^{A+} \rightarrow \Xi^0$	0.84	0.44	−0.04	0.80	−0.07	−0.93
	0.58	0.31	−0.03	0.63	−0.05	−0.73
$\Sigma_c^{+} \rightarrow \Sigma^0$	0.80	−0.78	−0.37	−0.20	0.03	0.22
	0.52	−0.50	−0.24	−0.15	0.02	0.16
$\Xi_c^{+} \rightarrow \Xi^0$	0.61	−0.58	−0.28	−0.15	0.01	0.18
	0.43	−0.41	−0.20	−0.12	0.01	0.14
$\Sigma_c^{++} \rightarrow \Sigma^+$	0.80	−0.78	−0.37	−0.20	0.03	0.21
	0.52	−0.50	−0.24	−0.15	0.02	0.16

as the QED corrections are made to the muon decay. Also, as shown by Carlson *et al.*[3] these hard-gluon corrections can be improved by recalculating them, keeping in mind that even at the elementary level the quarks are confined within the bag. We shall not go into these refinements. For definiteness, we shall apply the corrections calculated by Ali and Pietarinen.[10] The result of these authors for the one-gluon-corrected decay rate $\Gamma_{\rm QCD}$, which includes both virtual and bremsstrahlung contributions, is

$$\Gamma_{\rm QCD}=f_{\rm QCD}\Gamma_{q^2}\equiv\left[1-\frac{2}{3}\frac{\alpha_s(m_Q^2)}{\pi}f(r)\right]\Gamma_{q^2}\,, \qquad (21)$$

with $r\equiv m_q/m_Q$, and

$$\alpha_s\equiv\frac{12\pi}{(33-2n_f)\ln(m_Q^2/\Lambda^2)}\,. \qquad (22)$$

Γ_{q^2} is the decay rate to zeroth order in QCD evaluated in Sec. IV A. α_s is the strong running coupling constant and m_Q and m_q are the masses of the decaying and emitted quarks, respectively. Λ is the renormalization point, the value we take for it is $\Lambda\simeq 500$ MeV, and $f(r)$ is determined from Fig. 1 of Ref. 10. In order to evaluate $\Gamma_{\rm QCD}$ and α_s from Eqs. (21) and (22) we must know the masses of the d, s, and c quarks. Since these masses are model dependent, the values for $\Gamma_{\rm QCD}$ and α_s will be model dependent too. In the NRQM we use $m_{u,d}=350$ MeV, $m_s=500$ MeV, and $m_c=1500$ MeV, taken from Flamm and Schöberl.[11] For the MBM we use, for consistency, the masses of the preceding section. For both models we obtain $\alpha_s(m_c^2)=0.686$. The values for $f(r)$ are $f(m_d/m_c)=2.75$ and $f(m_s/m_c)=2.4$ for the NRQM, and $f(m_d/m_c)=3.61$ and $f(m_s/m_c)=2.91$ for the MBM. The $f_{\rm QCD}$ correction factor is then $f_{\rm QCD}=0.599$ for $c\to d$ and $f_{\rm QCD}=0.651$ for $c\to s$ for the NRQM. For the MBM the values are $f_{\rm QCD}=0.474$ for $c\to d$ and $f_{\rm QCD}=0.576$ for $c\to s$.

C. Wave-function mismatch (WFM) corrections

Another correction to the decay rates is the WFM. The WFM corrections arise because of distortions of the

TABLE VII. Decay rates (branching ratios) in units of 10^{11} sec^{-1} (percentage) for the $c\to d$ processes. The upper values correspond to the NRQM predictions and the lower ones correspond to the MBM. Γ_0 (B_0) corresponds to no q^2 dependence of the form factors when evaluating the transition rate coefficients; Γ_{q^2} (B_{q^2}) contains in its coefficients the q^2 dependence of the form factors; $\Gamma_{\rm cor}$ ($B_{\rm cor}$) includes the q^2 dependence, the QCD, and the WFM corrections.

			Monopole			Dipole	
$B_1\to B_2$	I^2	Γ_0 (B_0)	Γ_{q^2} (B_{q^2})	$\Gamma_{\rm cor}$ ($B_{\rm cor}$)	Γ_0 (B_0)	Γ_{q^2} (B_{q^2})	$\Gamma_{\rm cor}$ ($B_{\rm cor}$)
$\Xi_{cc}^{++}\to\Lambda_c^+$	0.99	0.08 (0.14)	0.11 (0.19)	0.06 (0.11)	0.03 (0.05)	0.06 (0.10)	0.03 (0.06)
		0.10 (0.17)	0.13 (0.24)	0.05 (0.09)	0.03 (0.06)	0.07 (0.12)	0.02 (0.04)
$\Xi_{cc}^{++}\to\Sigma_c^+$	0.99	0.17 (0.31)	0.22 (0.40)	0.13 (0.24)	0.10 (0.18)	0.17 (0.30)	0.10 (0.18)
		0.12 (0.22)	0.16 (0.28)	0.06 (0.11)	0.07 (0.12)	0.12 (0.21)	0.04 (0.08)
$\Xi_{cc}^{+}\to\Sigma_c^0$	0.99	0.34 (0.61)	0.43 (0.78)	0.26 (0.46)	0.20 (0.36)	0.33 (0.59)	0.20 (0.35)
		0.24 (0.44)	0.31 (0.56)	0.12 (0.21)	0.14 (0.25)	0.23 (0.41)	0.09 (0.15)
$\Omega_{cc}^{+}\to\Xi_c^0$	0.99	0.22 (0.39)	0.28 (0.51)	0.17 (0.30)	0.12 (0.21)	0.21 (0.37)	0.12 (0.22)
		0.15 (0.27)	0.20 (0.36)	0.08 (0.14)	0.08 (0.14)	0.14 (0.26)	0.05 (0.10)
$\Omega_{cc}^{+}\to\Xi_c^{A0}$	0.99	0.08 (0.14)	0.11 (0.20)	0.07 (0.12)	0.03 (0.06)	0.06 (0.11)	0.04 (0.06)
		0.10 (0.18)	0.14 (0.24)	0.05 (0.09)	0.04 (0.06)	0.07 (0.12)	0.03 (0.05)
$\Xi_c^0\to\Sigma^-$	0.97	0.02 (0.03)	0.02 (0.04)	0.01 (0.03)	0.01 (0.01)	0.01 (0.02)	0.01 (0.01)
		0.03 (0.05)	0.04 (0.07)	0.01 (0.02)	0.01 (0.02)	0.02 (0.03)	0.01 (0.01)
$\Omega_c^0\to\Xi^-$	0.98	0.04 (0.07)	0.06 (0.10)	0.03 (0.06)	0.01 (0.02)	0.03 (0.05)	0.02 (0.03)
		0.06 (0.11)	0.09 (0.16)	0.03 (0.06)	0.02 (0.03)	0.04 (0.07)	0.01 (0.02)
$\Xi_c^{A0}\to\Sigma^-$	0.97	0.19 (0.33)	0.25 (0.45)	0.15 (0.26)	0.09 (0.17)	0.17 (0.31)	0.10 (0.18)
		0.14 (0.25)	0.19 (0.34)	0.07 (0.13)	0.07 (0.12)	0.12 (0.22)	0.05 (0.08)
$\Lambda_c^+\to n$	0.96	0.18 (0.32)	0.25 (0.45)	0.14 (0.26)	0.08 (0.15)	0.17 (0.30)	0.10 (0.17)
		0.13 (0.23)	0.18 (0.33)	0.07 (0.12)	0.06 (0.10)	0.11 (0.20)	0.04 (0.08)
$\Xi_c^{A+}\to\Sigma^0$	0.97	0.09 (0.17)	0.13 (0.23)	0.07 (0.13)	0.05 (0.08)	0.09 (0.16)	0.05 (0.09)
		0.07 (0.13)	0.10 (0.17)	0.04 (0.06)	0.03 (0.06)	0.06 (0.11)	0.02 (0.04)
$\Xi_c^{A+}\to\Lambda$	0.97	0.03 (0.06)	0.05 (0.09)	0.03 (0.05)	0.02 (0.03)	0.03 (0.06)	0.02 (0.03)
		0.03 (0.05)	0.04 (0.06)	0.01 (0.02)	0.01 (0.02)	0.02 (0.04)	0.01 (0.01)
$\Sigma_c^+\to n$	0.97	0.02 (0.03)	0.02 (0.04)	0.01 (0.02)	<0.01 (0.01)	0.01 (0.02)	0.01 (0.01)
		0.03 (0.05)	0.04 (0.07)	0.01 (0.03)	0.01 (0.01)	0.01 (0.03)	0.01 (0.01)
$\Xi_c^+\to\Sigma^0$	0.97	0.01 (0.01)	0.01 (0.02)	0.01 (0.01)	<0.01 (0.01)	0.01 (0.01)	<0.01 (0.01)
		0.01 (0.02)	0.02 (0.03)	0.01 (0.01)	<0.01 (0.01)	0.01 (0.01)	<0.01 (0.01)
$\Xi_c^+\to\Lambda$	0.97	0.02 (0.04)	0.03 (0.06)	0.02 (0.03)	0.01 (0.02)	0.02 (0.03)	0.01 (0.02)
		0.04 (0.07)	0.05 (0.09)	0.02 (0.03)	0.01 (0.02)	0.02 (0.04)	0.01 (0.02)
$\Sigma_c^{++}\to p$	0.97	0.03 (0.06)	0.05 (0.08)	0.03 (0.05)	0.01 (0.02)	0.02 (0.04)	0.01 (0.02)
		0.05 (0.09)	0.08 (0.14)	0.03 (0.05)	0.01 (0.02)	0.03 (0.05)	0.01 (0.02)

orbital wave function.[12–14] The measure of the deformation is given by the square of the overlap integral

$$I=\int d\tau\, \psi_f^\dagger \psi_i \ , \tag{23}$$

which will be less than unity if the wave functions are not equal. This correction can be approximately put as an overall factor of I^2 in the decay rate.

Bracken *et al.*[13] estimate the WFM effect using a harmonic-oscillator confinement potential. They obtain $I^2=0.984$, neglecting the light-quark mass with respect to the heavy one. We shall estimate the I^2 number both for the NRQM and the MBM. For the NRQM following Refs. 13 and 14 we get

$$I=\left[\frac{2(3m_Q/M_1)^{1/4}}{1+(3m_Q/M_1)^{1/2}}\right]^{3/2} , \tag{24}$$

without neglecting any quark mass and taking the baryon mass as the sum of the quark masses. In the MBM the overlap integral becomes

$$I=\int d^3\mathbf{y}\, q_f^\dagger q_i \ ,$$

where q_i and q_f are the orbital ground-state wave functions discussed in the previous section. Explicitly, this takes the form

$$I=N^i N^f R^3 (W_+^i W_+^f I_{00}+W_-^i W_-^f I_{11}) \tag{25}$$

with N^q given in Eq. (14), $W_\pm$ given in Eq. (17), and I_{00} and I_{11} given in Eq. (18a).

In the MBM $I^2=0.786$ for $c\to d$ and $I^2=0.907$ for $c\to s$ decays. Since I depends on M_1 for the NRQM, we display it, or rather I^2, in Tables VII and VIII.

There are more corrections that can be considered, such as recoil effects, Fermi motion, etc. We shall not include them in this paper, but when precise measurements are made available better predictions will be required and, then, these other corrections should be included. In the next section the three corrections we have discussed here will be applied to Γ_0 and B_0.

V. RESULTS AND DISCUSSION

Combining all the corrections discussed in the previous section we get a final result that we shall denote by $\Gamma_{\rm cor}$:

$$\Gamma_{\rm cor}\equiv I^2 f_{\rm QCD}\Gamma_{q^2} \ .$$

We have tabulated $\Gamma_{\rm cor}$ and the corresponding branching ratios for the NRQM and the MBM in Tables VII

TABLE VIII. Decay rates (branching ratios) in units of 10^{11} sec^{-1} (percentage) for the $c\to s$ processes. The upper values correspond to the NRQM predictions and the lower ones correspond to the MBM. Γ_0 (B_0) corresponds to no q^2 dependence of the form factors when evaluating the transition rate coefficients; Γ_{q^2} (B_{q^2}) contains in its coefficients the q^2 dependence of the form factors; $\Gamma_{\rm cor}$ ($B_{\rm cor}$) includes the q^2 dependence, the QCD, and the WFM corrections.

$B_1\to B_2$	I^2	Monopole Γ_0 (B_0)	Monopole Γ_{q^2} (B_{q^2})	Monopole $\Gamma_{\rm cor}$ ($B_{\rm cor}$)	Dipole Γ_0 (B_0)	Dipole Γ_{q^2} (B_{q^2})	Dipole $\Gamma_{\rm cor}$ ($B_{\rm cor}$)
$\Xi_{cc}^{++}\to\Xi_c^{A+}$	0.99	1.19 (2.1)	1.45 (2.6)	0.94 (1.7)	0.64 (1.2)	0.98 (1.7)	0.63 (1.1)
		1.42 (2.5)	1.72 (3.1)	0.90 (1.6)	0.75 (1.3)	1.13 (2.0)	0.59 (1.1)
$\Xi_{cc}^{++}\to\Xi_c^{+}$	0.99	3.50 (6.3)	4.37 (7.8)	2.83 (5.1)	2.15 (3.8)	3.39 (6.1)	2.20 (3.9)
		2.82 (5.0)	3.53 (6.3)	1.84 (3.3)	1.70 (3.0)	2.70 (4.8)	1.41 (2.5)
$\Xi_{cc}^{+}\to\Xi_c^{0}$	0.99	2.56 (4.6)	3.07 (5.5)	1.99 (3.6)	1.72 (3.1)	2.49 (4.5)	1.61 (2.9)
		2.08 (3.7)	2.49 (4.5)	1.30 (2.3)	1.38 (2.5)	2.00 (3.6)	1.04 (1.9)
$\Omega_{cc}^{+}\to\Omega_c^{0}$	0.99	5.27 (9.4)	6.30 (11)	4.09 (7.3)	3.53 (6.3)	5.11 (9.2)	3.32 (5.9)
		4.28 (7.7)	5.13 (9.2)	2.68 (4.8)	2.83 (5.1)	4.11 (7.4)	2.14 (3.8)
$\Xi_{cc}^{+}\to\Xi_c^{A0}$	0.99	1.19 (2.1)	1.45 (2.6)	0.94 (1.7)	0.64 (1.2)	0.98 (1.7)	0.63 (1.1)
		1.42 (2.5)	1.72 (3.1)	0.90 (1.6)	0.75 (1.3)	1.13 (2.0)	0.59 (1.1)
$\Sigma_c^{0}\to\Sigma^-$	0.97	0.67 (1.2)	0.85 (1.5)	0.54 (0.96)	0.31 (0.56)	0.52 (0.94)	0.33 (0.59)
		0.97 (1.7)	1.23 (2.2)	0.64 (1.1)	0.42 (0.76)	0.70 (1.3)	0.37 (0.66)
$\Xi_c^{0}\to\Xi^-$	0.97	0.35 (0.62)	0.44 (0.78)	0.28 (0.50)	0.17 (0.30)	0.27 (0.49)	0.17 (0.31)
		0.49 (0.87)	0.61 (1.1)	0.32 (0.57)	0.22 (0.40)	0.36 (0.64)	0.19 (0.34)
$\Xi_c^{A0}\to\Xi^-$	0.97	3.01 (5.5)	3.78 (6.8)	2.38 (4.3)	1.84 (3.3)	2.88 (5.2)	1.81 (3.2)
		2.61 (4.7)	3.24 (5.8)	1.69 (3.0)	1.52 (2.7)	2.39 (4.3)	1.25 (2.2)
$\Lambda_c^{+}\to\Lambda$	0.96	1.90 (3.4)	2.37 (4.2)	1.48 (2.6)	1.13 (2.0)	1.80 (3.2)	1.12 (2.0)
		1.61 (2.9)	2.02 (3.6)	1.05 (1.9)	0.93 (1.7)	1.48 (2.6)	0.77 (1.4)
$\Xi_c^{A+}\to\Xi^0$	0.97	3.09 (5.5)	3.84 (6.9)	2.42 (4.3)	1.86 (3.3)	2.92 (5.2)	1.84 (3.3)
		2.64 (4.7)	3.29 (5.9)	1.72 (3.1)	1.54 (2.7)	2.42 (4.3)	1.27 (2.3)
$\Sigma_c^{+}\to\Sigma^0$	0.97	0.67 (1.2)	0.85 (1.5)	0.53 (0.96)	0.31 (0.56)	0.52 (0.93)	0.33 (0.59)
		0.96 (1.7)	1.22 (2.2)	0.64 (1.1)	0.42 (0.76)	0.70 (1.2)	0.36 (0.65)
$\Xi_c^{+}\to\Xi^0$	0.97	0.28 (0.50)	0.34 (0.61)	0.21 (0.38)	0.15 (0.27)	0.23 (0.41)	0.14 (0.26)
		0.38 (0.68)	0.46 (0.82)	0.24 (0.43)	0.20 (0.36)	0.30 (0.53)	0.15 (0.28)
$\Sigma_c^{++}\to\Sigma^+$	0.97	0.66 (1.2)	0.84 (1.5)	0.53 (0.95)	0.31 (0.56)	0.52 (0.93)	0.33 (0.58)
		0.96 (1.7)	1.21 (2.2)	0.63 (1.1)	0.42 (0.76)	0.70 (1.2)	0.36 (0.65)

and VIII.

Comparing the corresponding entries in Tables VII and VIII, we see that the predictions of the NRQM and the MBM are close to one another; sometimes the former's predictions are larger than the latter's and sometimes it is the other way around. Their differences are between 10% and 50%, and very frequently they are around 20%. In any case their predictions are certainly of the same order of magnitude. This should be contrasted with the SU(4)-symmetry-limit predictions, displayed in Tables XVII and XVIII of I. The NRQM and MBM predictions are around 1 order of magnitude smaller than the SU(4) predictions. This is a systematic effect, a factor of 5–50 in $c \to s$ decays and a factor 3–10 in $c \to d$ decays.

In order to better appreciate the results in Tables VII and VIII for the decay rates, one may look at the predictions for the branching ratios, which are easier and faster to grasp. Recall that since almost all the lifetimes of the charm baryons are still unknown, we have normalized all the branching ratios to the measured Λ_c^+ lifetime.[8] The corresponding numbers are displayed in parentheses in Tables VII and VIII. Clearly, these branching ratios should be understood only as approximate ones, but still they are very illustrative. For $c \to s$ decays the semileptonic branching ratios are very sizable, the NRQM and the MBM predict them to be around several percent and in some cases ($\Xi_c^{A\,+} \to \Xi^0$, $\Xi_c^{A\,0} \to \Xi^-$ $\Omega_{cc}^+ \to \Omega_c^0$, for example) at 5% or more. The SU(4) predictions of I are, so to speak, enormous, sometimes ($\Omega_{cc}^+ \to \Omega_c^0$, for example) as big as 20%. For $c \to d$ decays, despite the fact that they are more favored by phase space the smallness of V_{cd} overwhelms them, but they remain quite observable very often at a few tenths of a percent and sometimes at close to 1%, according to NRQM and the MBM. The SU(4) predictions are still very big, around a few percent. The differences of factors of 5–50 and of 3–10 mentioned before can also be easily seen in the branching ratios.

The WFM and the QCD corrections amount to decreases of around 6–10 % and 30–50 %, respectively. They are important, but not enough to explain the big differences between the SU(4) and the NRQM and MBM predictions. The reason for these differences lies in the correction due to the q^2 dependence of the form factors. It turns out to be so important that it deserves a detailed analysis.

It is to understand the role of the q^2 dependence of form factors that we have separated the Γ_0 and Γ_{q^2} from the $\Gamma_{\rm cor}$ predictions, throughout Tables VII and VIII of this paper and Tables XVII and XVIII of I. Because q^2 varies within $(m^2, \Delta M^2)$ (with m the mass of the charged lepton) and it is subtracted in the denominators of the form factors, its contributions make Γ_{q^2} always larger than Γ_0. This effect is bigger in the dipole case than in the monopole case. One observes frequently a 20–30 % increase in the first case and a 30–60 % increase in the second case. This is a systematic effect, as can be seen comparing Γ_{q^2} with Γ_0 in all the predictions of SU(4), the NRQM and the MBM. This effect corresponds to the increase in the coefficients of the decay-rate formula in Tables VII–XII of I. This is the direct effect of q^2. But, in the NRQM and MBM there is a second indirect effect of q^2, which is even more important. This second effect can already be seen comparing the columns under $\Gamma_0^{\rm mon}$ with the columns under $\Gamma_0^{\rm dip}$ in Tables VII and VIII; $\Gamma_0^{\rm mon}$ is 60% to over 100% bigger than $\Gamma_0^{\rm dip}$. The reason behind this reduction of $\Gamma_0^{\rm dip}$ with respect to $\Gamma_0^{\rm mon}$ lies in the suppression of all the form factors when going from their values at $q^2 = \Delta M^2$ to their values at $q^2 = 0$. Since the form factors at $q^2 = 0$ are to be used in Γ_0—which is where the big phase-space factors are—the effect of this suppression is very much amplified at the decay-rate level. This explains the enormous difference between the predictions of SU(4) and the NRQM and MBM. To better illustrate this point we can look at the $\Lambda_c^+ \to \Lambda$ decay. At $q^2 = \Delta M^2$ the form factors are $f_1 = 0.95$, $f_2 = 0.39$, $g_i = 0.81$, and $g_2 = -0.09$. If these values are used in Γ_0 instead of those of Table VI, we get $\Gamma_0(\Lambda_c^+ \to \Lambda) = 2.99 \times 10^{11}$ sec^{-1}. This number is 220% larger than $\Gamma_0^{\rm dip}(\Lambda_c^+ \to \Lambda)$ and 85% larger than $\Gamma_0^{\rm mon}(\Lambda_c^+ \to \Lambda)$. In contrast, it compares much better with the SU(4) prediction of 3.6×10^{11} sec^{-1}. Clearly, looked at from this point of view the NRQM and MBM predictions are not so different from the SU(4) ones.

From this discussion it becomes evident that in CBSD the order of importance of the assumptions and approximations, used in detailed theoretical calculations, is changed with respect to their order in semileptonic decays of light-quark baryons. In the former the assumptions regarding the q^2 dependence of form factors, whether it is monopole or dipole and to what extent can M_V and M_A be changed, is by far more important than other assumptions, WFM, QCD corrections in the bag, etc., while in the latter such q^2 dependence is of secondary importance. To summarize, we may say that calculations for heavy-quark semileptonic decays in the framework of quark and bag models must envisage the determination of the q^2 dependence of form factors as a priority. By the same token, experiments should make a priority to obtain precise determinations of such q^2 dependence and not just of the form factors at $q^2 = 0$. This task should be much eased by the favorable phase space and by the hopefully appreciable branching ratios of CBSD.

Next, we wish to devote some space to the determination of the KM matrix elements. The only available experimental branching ratio[8] is of $\Lambda_c^+ \to \Lambda$, from which we obtain $\Gamma_{\rm expt} = (0.61 \pm 0.45) \times 10^{11}$ sec^{-1}. This number is below the MBM, which is the most conservative one, although it agrees with it at one standard deviation. If we use the MBM result and this experimental number, we can determine V_{cs}, $|V_{cs}| = 0.87^{+0.27}_{-0.43}$ in the dipole case, and $|V_{cs}| = 0.74^{+0.24}_{-0.36}$ in the monopole case. Both predictions are compatible with three-generation range of Eqs. (20a) and (20b) within one standard deviation (actually, the upper bound 0.27 should be reduced to less than 0.11 as required by unitary). We should compare our present results with previous ones.[15] Earlier, the NRQM prediction was obtained, as is often done, at $q^2 = 0$. In this paper we have argued along with others[2,3] that it is at q^2 maximum where quark models should be more reliable.

This has introduced the double effect of the q^2 dependence of the form factors, as discussed above, leading to a net reduction of the predicted branching ratio. Also, the lifetime of Λ_c has decreased lately, which makes its semileptonic branching ratio larger, both in central value and in experimental error. All this has led to less disagreement between quark-model predictions and experiment in this particular decay. Nevertheless, theoretical predictions remain systematically above the experimental measurement. Thus, in all cases there remains ample room for an appreciable effect, through the lowering of V_{cs}, of the existence of a fourth generation.

In the future, the way to go about detecting the effects of a fourth generation in CBSD decays should be to separate the theoretical uncertainties that affect the three groups of such decays—$c \to s$, $c \to d$, and $\Delta S = -1, \Delta C = 0$ $(s,u \to d)$—from the effect of a reduction in V_{cs}. In collecting as many decay rates of the allowed 46 processes as possible, one should be able to observe a systematic reduction in the $c \to s$ group, which should not be observed in the other two groups. It would then remain to clearly discriminate it from a special suppression that could afflict this group only, and not the other ones. Of course, if it turns out that V_{cs} lies within the very narrow range of (20b), it would still not mean that a fourth generation is ruled out, because the nondiagonal KM matrix elements connecting the c and s quarks to the fourth-generation quarks could be very small.

At this stage, the length of this and the previous papers may obscure their contents. It is therefore convenient to attempt some concluding highlights. We shall do this in the manner of a listing.

(i) The main results obtained are collected in Tables XVI–XVIII of I and Tables VII and VIII of this paper.

(ii) The SU(4)-symmetry-limit predictions are systematically bigger than the quark-model predictions by typically an order of magnitude.

(iii) The reason for this can be traced to a double role played by the q^2 dependence of the form factors, which makes this correction of primary importance, far more important than the QCD, WFM, and other corrections.

(iv) It turns out that all predictions show that the most important CBSD are $\Xi_{cc}^{++} \to \Xi_c^{+}$, $\Xi_{cc}^{+} \to \Xi_c^{0}$, $\Omega_{cc}^{+} \to \Omega_c^{0}$, $\Xi_c^{A_0 0} \to \Xi^{-}$, $\Lambda_c^{+} \to \Lambda$, and $\Xi_c^{A+} \to \Xi^{0}$. Notice that their branching ratios are about 100 times larger than the typically 10^{-3} branching ratios of ordinary noncharm hyperon semileptonic decays.

(v) There is a host of other CBSD, around 23, that have smaller branching ratios, around 10^{-2} and 10^{-3}, which should also be fairly accessible to experiment.

All this points out that CBSD is a rich promising field that will significantly help to improve our understanding of theoretical modeling of hadronic matter.

*Present address: Escuela de Ciencias Físico-Matemáticas de la Universidad Autónoma de Puebla, Puebla, México.

†Present address: Departamento de Ciencias Básicas, Universidad Autónoma Metropolitana, Azcapotzalco, México, D.F., México.

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