

## Massive-top-quark decay with emission of a photon or a gluon

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We consider the decay of a massive top quark accompanied by the emission of a “hard” photon or gluon. We find that the hard-photon bremsstrahlung increases the width of the top quark by 1% or so while the hard-gluon contribution can be as large as 30%. We also consider the possibility of using the first process to measure precisely the mass of the top quark or some Kobayashi-Maskawa mixing-matrix parameters.

### I. INTRODUCTION

It is now believed that the elusive top quark has a mass larger than 56 GeV (Ref. 1). Current experiments at the Fermilab Tevatron will probe up to the 100-GeV range and possibly beyond in the near future. This large value is in no conflict with theoretical estimates,<sup>2</sup> which indeed could accommodate a mass of a few hundred GeV. The only bound that can still be considered as strong is the radiative correction to the  $\rho$  parameter<sup>3</sup> which requires the isospin-doublet mass splitting to be less than  $\sim 200$  GeV. Now that we are faced with a rather massive top quark, we must consider the possibility of a top quark decaying to a  $W$  boson and a  $b$  quark. Different production rates and possible decay signatures of such a massive top quark have been studied in great detail.<sup>4</sup> Most important to us, it has been observed that the finite width of the  $W$  boson has a very large effect near threshold (i.e., when  $m_t \sim M_W$ ). The decay of a massive quark to a  $W$  boson, a light quark, and a Higgs boson has been calculated<sup>5</sup> both in the framework of the standard model (SM) and beyond. These processes could be sources of Higgs bosons and their signatures might signal a new quark. The decay of a massive gauge boson to either  $t\bar{b}$  in the case of the  $W$  boson or  $t\bar{t}$  in the case of a  $Z$  boson has been studied extensively<sup>6</sup> since it could have been a copious source of  $t$  quarks. Although the photon and gluon bremsstrahlung processes have been considered for the  $W$  decay modes,<sup>7</sup> it seems that these processes have been overlooked for the top-quark-decay modes. In this paper, we want to remedy this situation and we consider the processes

$$t \rightarrow bW\gamma \text{ and } t \rightarrow bWg.$$

In Fig. 1, we present the Feynman diagrams relevant to the first process; Figs. 1(a) and 1(b) only contribute to the second one.

Considering that the top quark is one of the most important missing links of the SM, we feel it is important to have as many handles on it as possible. Contrary to the other quarks, the top quark (above 100 GeV) has a width in the range of several hundreds MeV, maybe even in the GeV range. Therefore, a higher-order calculation can have significant effects. In the following, we will consider

the emission of a real  $W$  boson only. According to Gilman and Kauffman<sup>4</sup> this implies that we will have to consider top-quark masses larger than  $\sim 100$  GeV in order to neglect  $W$ -boson-width effects. Note also that in spite of the fact that the Feynman diagrams of Fig. 1 are part of a radiative correction, we are not exactly doing a radiative correction since we require observation of the outgoing photon. Certainly when we let the minimum photon energy go to zero, we will observe a logarithmic divergence. It is precisely this divergence that would cancel the infrared divergence coming from the other set of Feynman diagrams of a complete radiative correction calculation.

The paper is organized as follows: In the next section we discuss the process  $t \rightarrow bW\gamma$ . After writing down the amplitude, we will give the doubly differentiated decay rate. Then, we will focus mainly on how this process could lead to a precise determination of  $m_t$  or a precise measurement of the Kobayashi-Maskawa (KM) mixing-matrix parameters. In the third section, we use the previous result and look at the process  $t \rightarrow bWg$ . As we will see, there are some indications that radiative corrections from gluon exchange could be in the 20% range.

### II. $t \rightarrow bW^+\gamma$

Since we will compare our results with the “bare width” ( $\Gamma_0$ ) of the  $t$  quark ( $t \rightarrow bW$ ), it is useful to give this expression first. The calculation is very straightforward (for the decay to a real  $W$ ) and we simply give the result here:

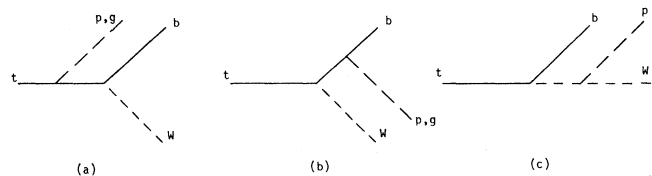


FIG. 1. Feynman diagrams relevant to the processes  $t \rightarrow bW\gamma$  and  $t \rightarrow bWg$ . Only (a) and (b) contribute to the second process.

$$\begin{aligned} \Gamma_0 &\equiv \Gamma(t \rightarrow bW) \\ &= \frac{\alpha [m_2^2 - (m_1 + M_W)^2]^{1/2} [m_2^2 - (m_1 - M_W)^2]^{1/2}}{16m_2^3 M_W^2 \sin^2 \theta_W} \\ &\quad \times [(m_2^2 - m_1^2)^2 + M_W^2(m_1^2 + m_2^2) - 2M_W^4] \quad (1) \end{aligned}$$

with  $m_2 \equiv m_t$ ,  $m_1 \equiv m_b$ , and we use  $|V_{tb}|^2 = 1$ . This is a standard result and for  $m_2 \gg M_W$  we recover the well-known result that  $\Gamma_0 \sim m_t^3$ . In the following we will neglect the contributions of possible Higgs bosons. The charged-Higgs-boson contribution is comparable to  $\Gamma_0$

$$\begin{aligned} M &= \frac{-ieg}{2\sqrt{2}} \epsilon_\gamma^\nu \epsilon_W^\mu \bar{u}(\bar{p}) \left[ Q_i \gamma^\nu \frac{1}{l_1 - m_1} \gamma^\mu (1 - \gamma_5) + Q_j \gamma^\mu (1 - \gamma_5) \frac{1}{l_2 - m_2} \gamma^\nu \right. \\ &\quad \left. + \frac{Q(g^{\alpha\beta} - h^\alpha h^\beta / M_W^2)}{h^2 - M_W^2} [(h - 2q)^\beta g^{\mu\nu} + 2q^\nu g^{\mu\beta} - (2h - q)^\mu g^{\beta\nu}] \gamma^\alpha (1 - \gamma_5) \right] u(p), \quad (2) \end{aligned}$$

where we have defined  $h \equiv p - \bar{p} = q + k$ ,  $l_1 \equiv \bar{p} + k = p - q$ ,  $l_2 \equiv \bar{p} + q = p - k$ ,  $Q_i = Q_b = -\frac{1}{3}$ ,  $Q_j = Q_t = +\frac{2}{3}$ ,  $Q = Q_W = +1$ . It is easy to check at this point that this amplitude is gauge invariant (i.e.,  $M \rightarrow 0$  if  $\epsilon_\gamma^\mu \rightarrow k^\mu$ ) if  $Q_i - Q_j + Q = 0$ .

Using  $k^\nu \epsilon_\gamma^\nu = 0$  and  $q^\mu \epsilon_W^\mu = 0$ , we rewrite the amplitude as

$$M = \frac{-ieg}{2\sqrt{2}} \epsilon_\gamma^\nu \epsilon_W^\mu \bar{u}(\bar{p}) \left[ Q_i - \frac{Q(\bar{p} \cdot k)}{q \cdot k} \right] \left[ \gamma^\nu \frac{1}{l_1 - m_1} \gamma^\mu (1 - \gamma_5) + (1 + \gamma_5) \gamma^\mu \frac{1}{l_2 - m_2} \gamma^\nu \right] u(p). \quad (3)$$

The factorization that we see here has been observed elsewhere, in the process  $W \rightarrow tb\gamma$ , for example.<sup>7</sup> One can easily see that the Abelian part (second set of large parentheses) represents the sum of the amplitudes when the photon is emitted from the quark legs. It will also apply when we will attach a gluon to the quark legs. We see here that the effect of introducing a trilinear gauge-boson vertex [Fig. 1(c)] is to bring in a multiplicative factor. It is nontrivial that such a factorization is possible. For example, a nonstandard value of the magnetic moment of the  $W$  boson<sup>8</sup> would change the form of the vertex in Eq. (2) and spoil this factorization by adding some extra terms. Note though that Eq. (3) is valid for any values of the electric charges as long as conservation of the electric charge is respected at every vertex. We also see that the process under consideration will *not* lead to *radiative zeros*,<sup>9</sup> in contrast with the  $W \rightarrow tb\gamma$  decay. This was expected since the same charge requirement for the occurrence of radiative zeros is not met here, while it obvi-

ously is for the  $W$  decay. The remaining steps are tedious but straightforward: we square the amplitude, we average over the initial and sum over final spin states, and we sum over quark colors. We define the variables

$$x \equiv p \cdot k = m_2 E_\gamma \quad \text{and} \quad y \equiv \bar{p} \cdot k = \Delta - m_2 E_W,$$

where  $E_\gamma, E_W$  are the energies of the photon and  $W$  boson, respectively. We also introduce the mass combinations

$$\begin{aligned} \Delta &\equiv \frac{1}{2}(m_2^2 + M_W^2 - m_1^2), \\ \rho &\equiv \frac{1}{2}(m_2^2 - M_W^2 - m_1^2), \\ \beta &\equiv \frac{1}{2}(m_2^2 - M_W^2 + m_1^2). \end{aligned} \quad (4)$$

Using these, the doubly differentiated decay rate for the process  $t \rightarrow bW\gamma$  can be written as

$$\begin{aligned} \frac{d^2\Gamma}{dx dy} &= \frac{\alpha^2}{8\pi \sin^2 \theta_W m_2^3} \left[ Q_i - \frac{Qy}{x-y} \right]^2 \frac{1}{xy} |V_{tb}|^2 \\ &\quad \times \left[ x^2 + y^2 + \frac{2(x-y)}{M_W^2} [(\Delta - y)x + (\rho - x)y] - \frac{\beta - (x-y)}{M_W^2} [3M_W^4 + 2M_W^2(x-y) - 4(\Delta - y)(\rho - x)] \right. \\ &\quad \left. + \frac{m_1^2 + m_2^2}{M_W^2} \{2(\rho - x)(\Delta - y) + M_W^2[\beta - (x-y)]\} - \frac{8\beta}{xy} (m_1^2 x^2 + m_2^2 y^2) \right. \\ &\quad \left. + \frac{2m_1^2(\Delta - y)}{M_W^2} [\Delta + (x-y)] + \frac{2m_2^2(\rho - x)}{M_W^2} [\rho - (x-y)] - \frac{2}{xy M_W^2} [x^2 \rho(\Delta - y) m_1^2 + y^2 \Delta(\rho - x) m_2^2] \right], \quad (5) \end{aligned}$$

where we have integrated out the  $b$ -quark phase space. From now on, we will use  $\alpha = \frac{1}{128}$ ,  $\sin^2\theta_W = 0.215$ ,  $M_W = 82$  GeV,  $m_b = 5$  GeV in our numerical results.

As mentioned previously, we expect an infrared divergence in the photon energy. Therefore, we define a minimum photon energy:  $\delta$ . Kinematics considerations lead to the phase-space limits

$$\delta \leq E_\gamma \leq \frac{\beta - m_1^2 - m_1 M_W}{m_2}, \quad E_W^{\min} \leq E_W \leq E_W^{\max} \quad (6)$$

with

$$E_W^{\max, \min} = \frac{(m_2 - E_\gamma)(m_2^2 - m_2 E_\gamma - \beta) \pm E_\gamma [m_2 m_1^2 (2E_\gamma - m_2) + (\beta - m_2 E_\gamma)^2]^{1/2}}{m_2(m_2 - 2E_\gamma)}$$

A somewhat simpler form for the maximum energy of the photon can be obtained in the *approximation* that the  $b$  quark is generated at rest; one then gets  $E_\gamma^{\max} = [(m_2 - m_1)^2 - M_W^2]/2(m_2 - m_1)$ . This approximation gives bounds very similar to the exact ones. The phase space described by Eq. (6) is the complete phase space. Admittedly, it includes collinearity of the particles, which would make the identification of a hard photon very difficult. By imposing constraints on the phase space, one can eliminate collinearity. One could require, for example, a minimum angular separation between each particle, thereby defining a cone of acceptance. As this is highly detector dependent, we decided to not take these requirements into account. Typical phase-space plots (Dalitz plots) are shown in Fig. 2. One must remember that the largest fraction of the width arises from low photon energy.

In principle, the total width can be obtained analytically. Given the "size" of Eq. (5), it is doubtful that such a result would be particularly illuminating. We performed the integrations numerically.

We plot in Fig. 3(a)  $\Gamma(t \rightarrow bW\gamma)$  as a function of  $\delta$  for a top quark of 100 GeV. There is a drastic change in the width depending on which value of  $\delta$  we pick. Note that for  $m_2 = 100$  GeV,  $m_1 = 5$  GeV, and  $M_W = 82$  GeV, we get  $E_\gamma^{\max} = 12.155$  GeV, which is why the width drops sharply for  $\delta \geq 10$  GeV. This figure shows clearly that the partial width we are calculating here depends "critically" on what value we choose for  $\delta$ . We also know that we have an infrared divergence. The question then arises: What value of  $\delta$  is reasonable? In order to answer this question, we used Fig. 3(b), which is the same as Fig. 3(a) but on a larger energy scale. We clearly see the logarithmic divergence. We now extrapolate the straight line toward the high  $\delta$  and assume that when it crosses the  $x$

axis (at  $\bar{\delta} \sim 3$  GeV) the contribution of the logarithmic part of the amplitude is negligible. We take this width ( $\Gamma_\gamma$ ) as an acceptable width for the process  $t \rightarrow bW\gamma$ . Short of a complete radiative correction calculation, this procedure seems the most reasonable. We can now interpret  $\Gamma_\gamma$  as the increase in the width of the top quark due to the emission of a hard photon. In Table I we give  $\Gamma_\gamma$  for different sets of parameters. These values of  $\bar{\delta}$  are also reasonable from an experimental point of view. Al-

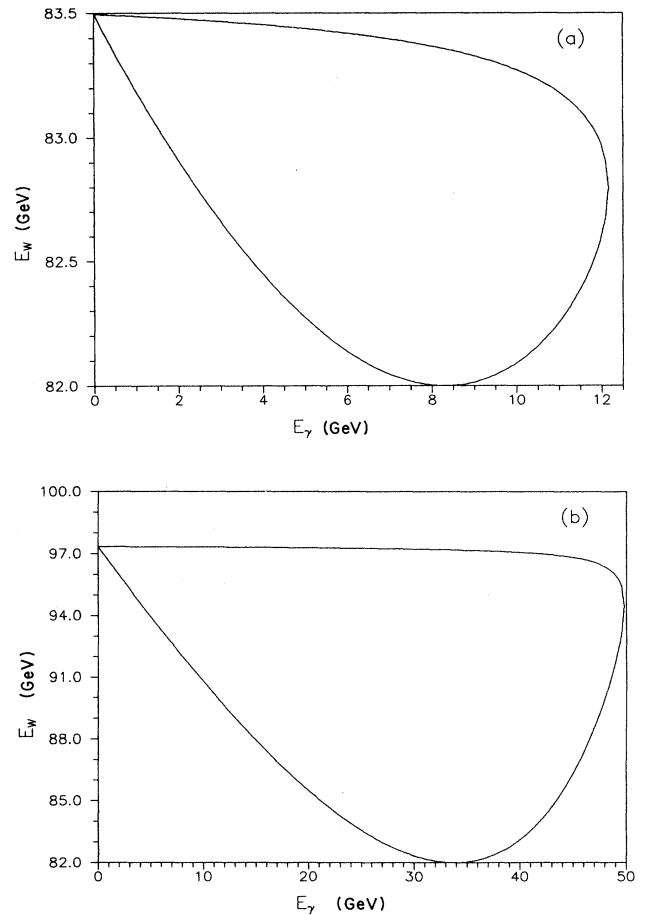


FIG. 2. Typical Dalitz plots for the process  $t \rightarrow bW\gamma$ . The  $b$ -quark phase space has been integrated out. (a)  $m_t = 100$  GeV, (b)  $m_t = 150$  GeV.

TABLE I.  $\bar{\delta}$  and  $\Gamma_\gamma$  for different values of  $m_t$ . Figures similar to Fig. 3(b) have been used to obtain the table.  $\Gamma_0$  is given by Eq. (1).

$m_t$ (GeV)	$\bar{\delta}$ (GeV)	$\Gamma_0$ (MeV)	$\Gamma_\gamma$ (MeV)	$\Gamma_\gamma/\Gamma_0$ (%)
100	3	80	0.13	0.16
125	7.5	400	1.3	0.33
150	10	900	6	0.67
200	15	$2.5 \times 10^3$	28	1.1

though a 3-GeV photon would be difficult to observe, it is likely that a 10-GeV photon would be easy to pick out. Note that  $\Gamma_\gamma$  is always small compared to  $\Gamma_0$ , which is to be expected since we calculated electromagnetic “correction” to the bare width. We plot in Fig. 4 the ratio  $\Gamma(t \rightarrow bW\gamma)/\Gamma_0$  as a function of  $m_t$  for three different values of  $\delta$ . We see that for a heavy top quark, this ratio can be up to  $\sim 5\%$  (this value is in the “soft-photon region” for a 200-GeV top quark, according to our previous definition), while it is well below 1% for a light top quark. This particular process will certainly be difficult to observe.

We now ask what one could learn from the top quark using this process. We first consider a precise determination of the mass. An obvious answer is to say that the maximum photon energy is a “direct” measurement of the mass of the top quark. The problems here are the energy resolution which becomes critical and the fact that the maximum photon energy is at the end of phase space which lead to a very small rate. It seems wiser to consider the minimum photon energy one can accept and see how this “trigger” can be used to measure the top-quark

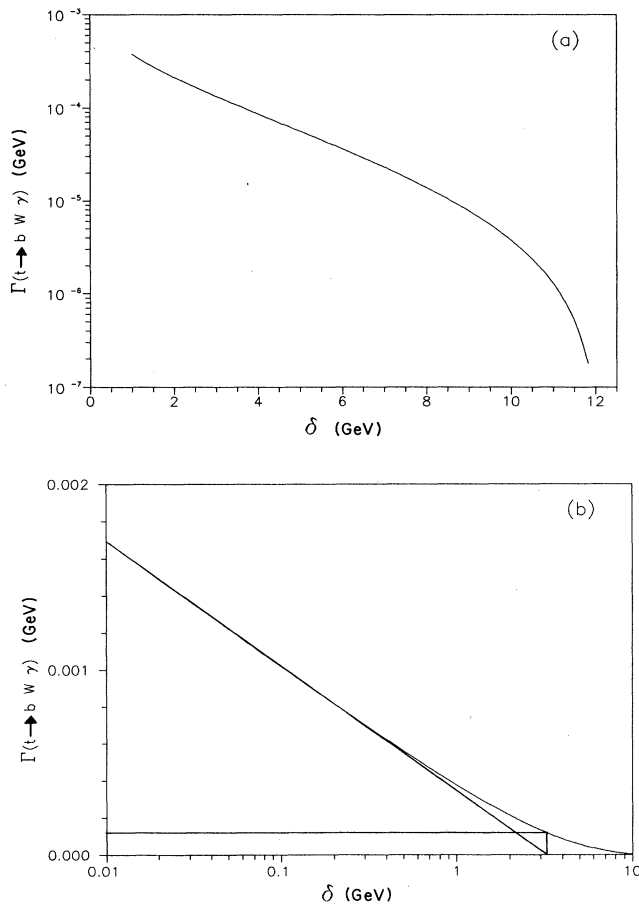


FIG. 3.  $\Gamma(t \rightarrow bW\gamma)$  as a function of  $\delta$  for  $m_t = 100$  GeV. The KM matrix parameter is *not* included. (a) Hard-photon range. (b) Infrared divergence. As explained in the text, we use this plot to obtain  $\bar{\delta}$  and  $\Gamma_\gamma$ .

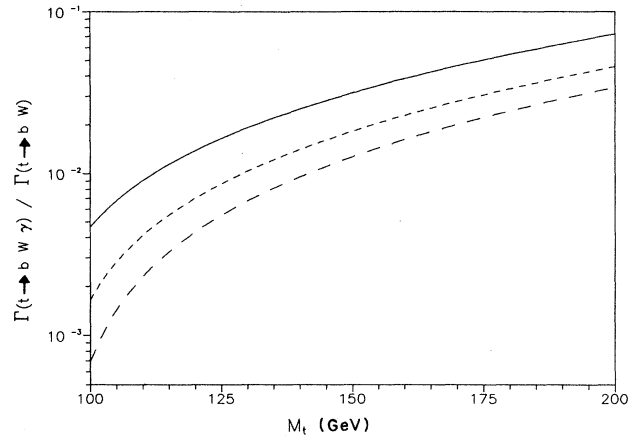


FIG. 4.  $\Gamma(t \rightarrow bW\gamma)/\Gamma(t \rightarrow bW)$  as a function of  $m_t$  for three different values of  $\delta$ . Solid line,  $\delta = 1$  GeV; short-dashed line,  $\delta = 3$  GeV; long-dashed line,  $\delta = 5$  GeV.

mass. This is shown in Fig. 5 where we consider three close values of  $m_t$  centered on 120 GeV. For a value of  $\delta \sim 16$  GeV, the ratios  $\Gamma_\gamma/\Gamma_0$  for  $m_t = 122, 120, 118$  GeV are  $6 \times 10^{-4}, 4.7 \times 10^{-4}, 3.2 \times 10^{-4}$ , respectively. For a  $4\sigma$  deviation, one would require approximately  $7 \times 10^5$  top quarks, while a  $1\sigma$  deviation requires only  $4 \times 10^4$  top quarks. This certainly closes this process for CERN LEP II where the pair-production rate of a 100-GeV top quark is more like 3000/yr. This measurement would most likely be relevant at the Superconducting Super Collider (SSC), where one expects to be able to reconstruct  $\sim 10^8$  (Ref. 10) top decays per year. A similar analysis for masses centered on  $m_t = 200$  GeV leads to the following: with  $\delta \sim 74$  GeV,  $3 \times 10^5$  top quarks identified would lead to a  $1\sigma$  deviation while  $5 \times 10^6$  events would lead to a  $4\sigma$  deviation. Clearly, this cannot be achieved in today's accelerators but is relevant for the SSC. Admit-

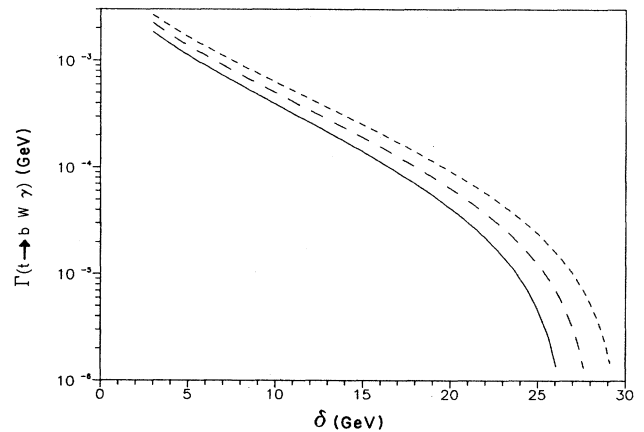


FIG. 5.  $\Gamma(t \rightarrow bW\gamma)$  as a function of  $\delta$  for three different values of  $m_t$ . The KM matrix parameter is *not* included. Solid line,  $m_t = 118$  GeV; long-dashed line,  $m_t = 120$  GeV; short-dashed line,  $m_t = 122$  GeV.

tedly, we have not included energy resolution here. This should not introduce drastic changes in our analysis since all curves are more or less the same, within a uniform shift. Indeed, as can be seen from Fig. 5, the curves are simply shifted and the 200-GeV curves can be obtained, roughly from the 100-GeV ones through a shift toward higher  $\delta$ . (16- and 74-GeV  $\delta$ 's both correspond to a partial width of  $\sim 1 \times 10^{-4}$  GeV for masses of 118 and 198 GeV, respectively.)

The conclusion here is that the knowledge of the  $\Gamma(t \rightarrow bW\gamma)/\Gamma(t \rightarrow bW)$  branching ratio can help in the determination of the top-quark mass. It would require excellent statistics and very good energy resolution but a 2% measurement is not totally out of reach.

The process at hand could also be used to measure the KM mass matrix parameters. Up to now, we have considered only  $t \rightarrow bW\gamma$  but the process  $t \rightarrow sW\gamma$  will also take place. Here, one would use the fact that the  $b$  quark is rather massive (5 GeV) while the  $s$  quark is rather light (0.5 GeV). Therefore, the maximum photon energy will

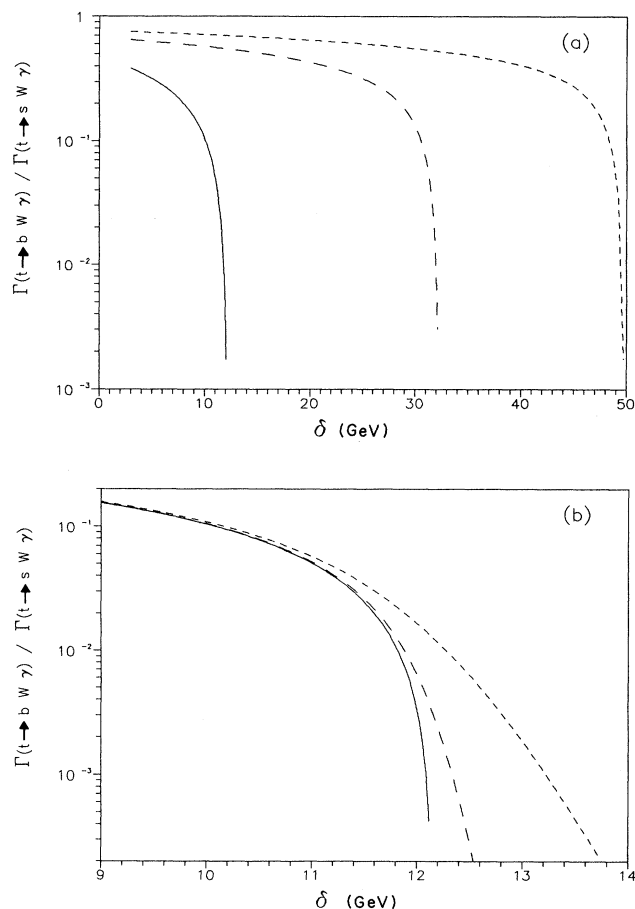


FIG. 6.  $\Gamma(t \rightarrow bW\gamma)/\Gamma(t \rightarrow sW\gamma)$  as a function of  $\delta$ . The KM matrix parameter is *not* included. (a) Solid line,  $m_t = 100$  GeV; long-dashed line,  $m_t = 125$  GeV; short-dashed line,  $m_t = 150$  GeV. (b)  $m_t = 100$  GeV. Solid line, perfect energy resolution; long-dashed line, 2% energy resolution; short-dashed line, 5% energy resolution.

be quite different in the two cases. By appropriate cut, one could enhance the  $sW\gamma$  final state with respect to the  $bW\gamma$  final state. For example, if the top quark is 100 GeV, the maximum photon energy is 12.2 GeV for the  $b$ -decay mode while it is 16 GeV for the  $s$ -decay mode. One could then select a large  $\delta$  in order to reduce the  $b$ -decay mode. Given that  $|V_{tb}| \sim 0.99$  and  $|V_{ts}| \leq 0.05$  one needs an enhancement of  $\sim 400$  to bring the two decay modes on equal footing. As shown in Fig. 6(a), very large enhancements can be achieved by selecting an appropriate value of  $\delta$ . However, it is clear that the energy resolution becomes critical in selecting  $\delta$ . In order to simulate this effect, we smeared the two widths with a resolution function and then took the ratio. The resolution function we used was the Gaussian  $\exp[-(E - E_0)^2/2\sigma^2]/\sqrt{2\pi}\sigma$ . This gives us Fig. 6(b) for a 100-GeV top quark. We plot two different energy resolutions (i.e.,  $\sigma/E_0$ ). As we are working in the top-quark frame, a 5% photon energy resolution seems reasonable while 2% seems rather optimistic. One can get an enhancement of 3 orders of magnitude by selecting  $\delta$  of 13 GeV or so. The question is then to know how many events we are left with. We calculated the ratio  $\Gamma(t \rightarrow sW\gamma)/\Gamma(t \rightarrow bW)$  as a function of  $\delta$  and found that requiring  $E_\gamma \geq 13$  GeV brings this ratio down to  $3 \times 10^{-7}$  (this includes all KM parameters). Such a branching ratio is at the extreme limit of the SSC. Backgrounds such as Higgs-boson bremsstrahlung now dominate by at least 1 order of magnitude. We also note that increasing the energy resolution will not help increase the branching ratio by much since the ratio  $\Gamma(t \rightarrow sW\gamma)/\Gamma(t \rightarrow bW)$  does not change much from  $\delta = 12$  GeV to 13 GeV. Furthermore, increasing the mass of the top quark would only make things worse as the two  $E_\gamma^{\max}$  become closer.

We could also turn the argument around and say that an observed branching ratio vastly different from the one previously mentioned would signal new physics either through new particles or unexpected forms of the KM mixing matrix parameters.

### III. $t \rightarrow bW^+g$

We now turn to the gluon bremsstrahlung process. Only Figs. 1(a) and 1(b) contribute to this process. The gluon bremsstrahlung will certainly be very difficult to observe because of the "nasty" environment of a hadron machine. The amplitude is given by

$$\begin{aligned}
 M = & \frac{-ig_s g}{2\sqrt{2}} \epsilon_g^\nu \epsilon_W^\mu T_{ij}^a \bar{u}_j(\bar{p}) \\
 & \times \left[ \gamma^\nu \frac{1}{\not{I}_1 - m_1} \gamma^\mu (1 - \gamma_5) \right. \\
 & \left. + (1 + \gamma_5) \gamma^\mu \frac{1}{\not{I}_2 - m_2} \gamma^\nu \right] u_i(p)
 \end{aligned} \quad (7)$$

with the same conventions as in Eq. (2). We also included the color indices here. The  $T_{ij}^a$  now gives a factor of 4 in the matrix elements squared. It is clear that the final width can be obtained from Eq. (5) by letting  $e \rightarrow g_s$ ,  $Q_i \rightarrow 1$ ,  $Q \rightarrow 0$ , and multiplying by  $\frac{4}{3}$ . This gives the doubly differentiated decay rate for the process  $t \rightarrow bWg$  as

$$\begin{aligned}
\frac{d^2\Gamma}{dx dy} = & \frac{\alpha\alpha_s}{6\pi \sin^2\theta_w m_2^3} \frac{1}{xy} |V_{tb}|^2 \\
& \times \left[ x^2 + y^2 + \frac{2(x-y)}{M_W^2} [(\Delta-y)x + (\rho-x)y] - \frac{\beta-(x-y)}{M_W^2} [3M_W^4 + 2M_W^2(x-y) - 4(\Delta-y)(\rho-x)] \right. \\
& + \frac{m_1^2 + m_2^2}{M_W^2} \{2(\rho-x)(\Delta-y) + M_W^2[\beta-(x-y)]\} - \frac{8\beta}{xy} (m_1^2 x^2 + m_2^2 y^2) \\
& \left. + \frac{2m_1^2(\Delta-y)}{M_W^2} [\Delta+(x-y)] + \frac{2m_2^2(\rho-x)}{M_W^2} [\rho-(x-y)] - \frac{2}{xyM_W^2} [x^2\rho(\Delta-y)m_1^2 + y^2\Delta(\rho-x)m_2^2] \right]. \quad (8)
\end{aligned}$$

Again we are faced with a divergence when the minimum energy of the gluon goes to zero. We use the same procedure as described for the photon bremsstrahlung. This gives us Table II; we have used  $\alpha_s=0.1$ . Note that the scale is quite different and that the contribution of  $\Gamma_g$  is non-negligible compared to the bare width  $\Gamma_0$ . We interpret this as saying that the hard-gluon bremsstrahlung is of the order of 20%. This also suggests that radiative corrections from gluonic exchange could be of the order of 20% or so. This crude result is by no means to replace a complete radiative correction of the top-quark decay. It simply suggests that the radiative corrections could be of the order of 20%. It has been calculated in the past<sup>11</sup> that QCD radiative corrections to top-quark production can be quite large, as much as 50%, with a dependence on the mass of the top quark. It is not unreasonable that such corrections to the width of the particle would be of the order of 25%. This is quite substantial and should be observed given a large enough sample of top quarks. Note also that our ratio  $\Gamma_g/\Gamma_0$  increases with  $m_t$ . This is an interesting behavior that would further probe our understanding of QCD.

It should also be clear that our results can readily be applied to any particle that carries quantum numbers similar to those of the top quark; couplings can easily be adjusted via Eqs. (2) and (7).

#### IV. CONCLUSION

We have considered the two processes  $t \rightarrow bW\gamma$  and  $t \rightarrow bWg$ . We have seen that the emission of a "hard"

photon in the first decay increases the width of the  $t$  quark typically by 1% of the "bare" width ( $\Gamma_0 \equiv t \rightarrow bW$ ). This bremsstrahlung process gives us one more handle on the top quark and might prove useful in the precise determination of its mass. We have seen that a 2% measurement is not out of reach. In principle, this same process can be used in the determination of the  $|V_{ts}|$  parameter by reducing the  $|V_{tb}|$  "background." In order to bring the two competing processes on equal footing one has to impose photon energy cuts so tight that the production rate goes essentially to zero. Unless new physics sets in, this method does not seem very promising.

Furthermore, the process  $t \rightarrow bWg$  calculated here indicates that hard-gluon bremsstrahlung corrections to  $\Gamma_0$  could be substantial: in the 25% range. This would certainly call for a complete radiative correction calculation. Such a calculation would probe our understanding of QCD and could lead to effects easily observable with a large sample of top quarks.

*Note added.* When this work was completed, I became aware of a recent publication (see Ref. 12) where the authors consider the decay of a heavy quark to a light quark, a  $W$  boson, and a  $Z$  boson. By adjusting couplings, they calculate also the decay to a light quark, a  $W$  boson, and a photon. When they overlap, the two calculations agree. Furthermore, QCD corrections to semileptonic decays of heavy quarks have been calculated (see Ref. 13). Although the corrections are not as important as we expected, the same top-quark mass dependence is observed.

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TABLE II.  $\bar{\delta}$  and  $\Gamma_g$  for different values of  $m_t$ . Figures similar to Fig. 3(b) have been used to obtain the table.  $\Gamma_0$  is given by Eq. (1).

$m_t$ (GeV)	$\bar{\delta}$ (GeV)	$\Gamma_0$ (MeV)	$\Gamma_g$ (MeV)	$\Gamma_g/\Gamma_0$ (%)
100	3.2	80	11	13
125	8.5	400	90	22
150	12	900	250	26
200	16	$2.6 \times 10^3$	780	31

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