Gravitational particle production during cosmic-string formation in the sudden approximation

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(Received 14 March 1989)

The production of particles due to the changing gravitational field during formation of cosmic strings is studied in the sudden approximation for massless scalar fields with arbitrary curvature coupling. A family of continuous spacetimes which describe sudden cosmic-string formation is constructed by joining together an initial spacetime and a final conical, cosmic-string spacetime along a spacelike hypersurface. Two initial spacetimes are examined: Minkowski space and the steady-state portion of de Sitter space. In the Minkowski case, it is possible to match the induced three-metrics on a one-parameter family of spacelike hypersurfaces. We find that, as the matching surface is varied, the particle production ranges from zero to a maximum which is linearly proportional to α (where α is the deficit angle of the string). Over a broad domain of matching surfaces, the particle production is proportional to α^2, in agreement with the earlier calculation of Parker. The amount of particle production thus seems to be significantly sensitive to the details of the model of the formation process. In the initially de Sitter case (which would model strings forming at the end of inflation) only one matching seems possible, and the particle production is found to be linearly proportional to α. Since α is small (typically $\alpha \approx 8\pi \times 10^{-6}$ for grand-unified-theory strings) the particle production (and created energy density) in this case is significantly larger than earlier estimates. Some general issues concerning the implementation of the sudden approximation in curved spacetimes are also considered.

I. INTRODUCTION

In the early Universe, phase transitions may result in the formation of cosmic strings according to many field theories, including a wide variety of grand unified theories (GUT's). The time-varying gravitational field associated with the formation of cosmic strings in the early Universe will create particles. These particles will contribute to the average energy density of the Universe, possibly producing cosmologically significant effects. If an inflationary scenario is correct and if the strings form before inflation these effects will be inflated away, but if strings form at the end of (or after) inflation the particle creation associated with string formation could contribute significantly to the reheating of the Universe. The gravitational creation of particles due to the formation of strings could be especially important for conformally coupled fields, as such particles are not produced in the isotropic expansion of the Universe. Many other interesting properties and cosmological consequences of strings (such as their possible role in galaxy formation) are discussed in the excellent review by Vilenkin.[1]

The purpose of this paper is to study the particle production due to the changing gravitational field during the string's formation, extending the earlier work of Parker.[2] Parker considered the creation of massless scalar field particles minimally coupled to the scalar curvature in a spacetime in which the initial geometry (for all $t<0$) was described by the Minkowski metric, and the final geometry (for all $t>0$) was described by the conical cosmic-string metric. We extend this model by allowing arbitrary coupling of the scalar field to the scalar curvature and by considering a number of different models of string formation within the sudden approximation. The essence of the sudden approximation is to restrict all evolution of the background fields (including the spacetime metric) to within an infinitesimally thin spacelike hypersurface. Our models then consist of an initial spacetime segment, a final spacetime segment, and the spacelike hypersurface on which the initial and final segments are joined (using the Israel formalism[3]) to form the complete spacetime. The initial spacetime segments are pieces of either Minkowski space or de Sitter space. In the case of an initially Minkowski space, we examine a one-parameter family of matching surfaces. The initially de Sitter model describes cosmic-string formation at the end of an inflationary epoch. The final spacetime segment is in all cases taken to be the conical flat spacetime of a cosmic string.[4-9] Parker's model is similar to ours in spirit; however, as we show in Sec. II, there is no continuous (C^0) spacetime which represents his model.

One might wonder to what extent the final results of these calculations depend on the use of the sudden approximation. A definitive answer would require a detailed model of the string's formation, which is too complicated to consider at this point. Instead, we investigate a simpler (but related) question within the scope of the sudden approximation for the models which are initially Minkowski space. We determine how the number of particles produced depends on the choice of matching surface.

As the choice of matching surface is varied particle production varies smoothly from a maximum which is linearly proportional to α (the deficit angle of the string), through a large domain where production is roughly proportional to α^2 (which is what Parker found), and finally approaches zero as the matching surface approaches a null surface. Although the results do seem to be sensitive to the choice of matching surface, for the vast majority of the choices of matching parameters the particle production is of the same order as in Parker's model,[2] roughly proportional to α^2.

In the alternative case where the initial spacetime segment is a piece of de Sitter space, there seems to be only one matching surface possible. This matching leads to particle production which is linearly proportional to α, leading to a dramatic increase in the average energy density due to the produced particles compared to the earlier calculations.

Specifically focusing attention on GUT strings, we find that the expectation value of the number operator for the created particles in the lowest modes of the field is less than one (but not zero) after formation of a single string segment. Since the string spacetime is flat, particle number has a well-defined physical meaning; we interpret our results to mean that most string segments will not emit any particles during formation. In the initially de Sitter model, about one GUT string segment in 5×10^4 will emit one (or a few) very energetic particles. When averaged over a background network of strings these particles contribute an average energy density of roughly 6×10^{88} ergs/cm^3, an increase by a factor of 6×10^4 over previous estimates.[10]

The paper is organized as follows. In Sec. II we construct the first set of model spacetimes by joining together Minkowski space and the conical string space along a spacelike hypersurface. We derive the intrinsic three-metric matching conditions on the spacelike hypersurface, and calculate the surface stress energy. We also discuss in Sec. II the relationship of these spacetimes to the "sudden approximation" as used in Parker's calculation. In Sec. III we then compute the gravitational particle creation in these model spacetimes. In Sec. IV we construct an alternative model of string formation in which the initial spacetime is a portion of de Sitter space, rather than Minkowski space. The gravitational particle creation is calculated in this model spacetime. In Sec. V the results of these calculations are summarized and discussed.

We use "natural" units, with $\hbar=c=G=1$, except where otherwise indicated. We have generally attempted to follow the notational conventions of Parker[2] in order to facilitate comparison of our results to his.

II. MINKOWSKI SPACE AND COSMIC-STRING SPACE: JUNCTION CONDITIONS AND THE SUDDEN APPROXIMATION

In this section we describe how to create a spacetime which models the formation of a cosmic string by joining together a portion of Minkowski space (to represent the spacetime before the formation of the string) and a portion of the conical cosmic-string spacetime. The two spacetime segments are joined together along a spacelike hypersurface. We believe that this is an appropriate way to implement the "sudden" approximation in a general spacetime. An alternative approach, followed by Parker,[2] is to simply define the initial and final spacetime states, and then match solutions to the wave equation across the constant time surface $t=0$ (or any other constant). While this procedure is equivalent to ours in a Robertson-Walker cosmology, it is inequivalent when the spacetime has less symmetry (as in the case here). We will show that it is not possible to join together segments of Minkowski space and the cosmic-string space if both are cut along such a constant time surface.

The spacetime metric before the string forms is taken to be the Minkowski metric, given in cylindrical coordinates by

$$ds^2=dt^2-dr^2-dz^2-r^2d\theta^2 \ . \qquad (1)$$

The final-state spacetime metric, assuming a static string has formed along the z axis, is then given by[4–9]

$$ds^2=d\tilde{t}^2-d\tilde{r}^2-d\tilde{z}^2-\tilde{r}^2a^2d\tilde{\theta}^2 \ , \qquad (2)$$

where

$$a\equiv1-\alpha/2\pi \ . \qquad (3)$$

The geometry outside the string is flat, but with a conical singularity present along the string on the z axis. The deficit angle of the cone is given by α. This in turn is related to the mass per unit length of the string, μ, by $\alpha=8\pi\mu$.

We now consider the most general possible way in which the two spacetimes described by the metrics of Eqs. (1) and (2) can be sliced along spacelike hypersurfaces and joined together on a common surface we will denote by Σ. The spacelike hypersurfaces along which the spacetimes are cut can be labeled by functions $t(r,\theta,z)$ and $\tilde{t}(\tilde{r},\tilde{\theta},\tilde{z})$. Since both spacetimes possess translational Killing vector fields in the direction of the basis vectors $\partial/\partial z$ and $\partial/\partial\tilde{z}$, and rotational Killing vector fields $\partial/\partial\theta$ and $\partial/\partial\tilde{\theta}$, we choose the surface Σ to be tangent to these Killing vector fields. This implies that the functions t and $\tilde{t}$ defining Σ are independent of z, $\tilde{z}$, θ, and $\tilde{\theta}$. The remaining freedom in choosing Σ thus lies in choosing functions $t(r)$ and $\tilde{t}(\tilde{r})$. The intrinsic three-metric induced on Σ by the Minkowski-space region is then

$$d\sigma_-^2=[1-(dt/dr)^2]dr^2+dz^2+r^2d\theta^2 \qquad (4)$$

and the three-metric induced on Σ by the cosmic-string region is

$$d\sigma_+^2=[1-(d\tilde{t}/d\tilde{r})^2]d\tilde{r}^2+d\tilde{z}^2+\tilde{r}^2a^2d\tilde{\theta}^2 \ . \qquad (5)$$

The functions $t(r)$ and $\tilde{t}(\tilde{r})$ must obviously be constrained by $(dt/dr)^2<1$ and $(d\tilde{t}/d\tilde{r})^2<1$ everywhere so that Σ will be globally spacelike.

If the four-dimensional spacetime regions are to be joined together on Σ, they must induce the same intrinsic metric on Σ. The coordinates $\tilde{z}$ and z can be set equal to each other at all points on Σ, as can the coordinates $\tilde{\theta}$ and θ, without loss of generality, owing to the existence of Killing vector fields in these directions, and the fact that θ and $\tilde{\theta}$ are both identified with the same period, 2π. The induced intrinsic metrics will then be equal if and only if we set

$$\tilde{r}=r/a \tag{6}$$

and

$$[1-(dt/dr)^2]a^2=1-(d\tilde{t}/d\tilde{r})^2 . \tag{7}$$

Any functions $t(r)$ and $\tilde{t}(\tilde{r})$ which satisfy Eq. (7) define a possible joining hypersurface Σ.

Note that the simplest possible choice for these functions, taking t and $\tilde{t}$ to be constant on Σ, is not a solution of Eq. (7). This is effectively the choice Parker made in his calculation (although it should be emphasized that his approach to the implementation of the sudden approximation was different than that followed here). In such an approach, one must match modes across boundaries on which it is not physically possible to join the spacetimes together (i.e., the "spacetime," if one considers it as such, on which one is doing the calculation, is not even C^0). We will see to what extent this affects the results of the particle creation calculation in Sec. III.

The usual surface-layer formalism of Israel[3] may now be used to compute the surface stress energy associated with Σ. We may define an orthornormal triad in the surface by

$$e_1=\left[1-\left[\frac{dt}{dr}\right]^2\right]^{-1/2} \times\left[\left[\frac{dt}{dr}\right]\left[\frac{\partial}{\partial t}\right]+\left[\frac{\partial}{\partial r}\right]\right] , \tag{8}$$

$$e_2=\frac{1}{r}\frac{\partial}{\partial\theta} , \tag{9}$$

$$e_3=\frac{\partial}{\partial z} , \tag{10}$$

where we choose to use the Minkowski-space coordinates on the surface. The components of the surface stress-energy tensor are then

$$8\pi S_{11}=-\frac{1}{r}\left[1-\left[\frac{dt}{dr}\right]^2\right]^{-1/2} \times\left\{\left[1-a^2+a^2\left[\frac{dt}{dr}\right]^2\right]^{1/2}-\left[\frac{dt}{dr}\right]\right\} , \tag{11}$$

$$8\pi S_{22}=-\left[\frac{d^2t}{dr^2}\right]\left[1-\left[\frac{dt}{dr}\right]^2\right]^{-3/2} \times\left\{\left[1-a^2+a^2\left[\frac{dt}{dr}\right]^2\right]^{-1/2}\left[\frac{dt}{dr}\right]-1\right\} , \tag{12}$$

$$8\pi S_{33}=8\pi(S_{11}+S_{22}) , \tag{13}$$

where the intrinsic metric matching conditions [Eqs. (6) and (7)] have been used to eliminate $\tilde{t}$ and $\tilde{r}$ in favor of t and r, and we have assumed that the sign of dt/dr is the same as the sign of $d\tilde{t}/d\tilde{r}$ (if the signs are opposite, it greatly increases the surface stress energy).

We will now restrict our attention to the simple case where the matching hypersurfaces are cones, i.e.,

$$t(r)=Cr , \tag{14}$$

$$\tilde{t}(\tilde{r})=C'\tilde{r} , \tag{15}$$

where C and C' are constants, related [from Eq. (7)] by

$$(1-C^2)a^2=1-C'^2 , \tag{16}$$

and we assume that C and C' have the same sign, as discussed above. Furthermore, we restrict C by $-1\le C\le 1$. This ensures the matching surface is spacelike (and thus a Cauchy surface) upon which we can match modes. This choice of functions has the immediate effect of causing the azimuthal surface stress S_{22} to vanish. The nonzero surface stress-energy components are minimized by allowing the surface to approach a null surface; as $C\rightarrow 1$, the surface stress energy approaches zero. We will not restrict attention to this special case, but will consider the full one-parameter family of surfaces defined by Eqs. (14)–(16).

III. GRAVITATIONAL PARTICLE CREATION

We will consider in this section only the simplest possible field theory, a massless scalar field with arbitrary coupling to the scalar curvature. Although the scalar curvature is zero in both the initial Minkowski and final cosmic-string spacetimes (and hence the scalar field modes take on the same form in those regions for any choice of curvature coupling), the scalar curvature R will generally contain a δ function on the spacelike hypersurface where the geometries are joined. The discontinuity in the normal derivative of the scalar field at the spacelike hypersurface will depend on the magnitude of the δ function in the scalar curvature if the curvature coupling is nonzero.

The field equation for the arbitrarily coupled scalar field is

$$\Box\phi+\xi R\phi=0 . \tag{17}$$

To calculate the energy in particles created by the formation of a system of strings it seems reasonable to simplify the situation by treating only the gravitational particle production due to a short straight building block of string. This is done (following Parker) by enclosing the infinite string in our models in a cylinder, and then imposing appropriate boundary conditions. This simplification can be justified on cosmological grounds. In the simplest case (the Abelian Higgs model) the strings which form must be either simple closed loops (i.e., no vertices) or infinitely long. The statistical distribution of such strings was studied by Vachaspati and Vilenkin[11] using a Monte Carlo simulation. They showed the strings lie along Brownian trajectories, with 80% of the total length of string in any closed volume due to segments of infinite string. More complicated situations can occur if, for example, the broken symmetry after the phase transition is a product of a continuous group and the discreet group Z_n, or if the strings are noncommuting. In these cases (with $n>2$ for Z_n strings) a stable network of strings can form, which eventually dominates the Universe. In all cases, however, at the time of formation the loops and segments of infinite string are built of pieces with a characteristic length on the average equal to the correlation length of the Higgs field λ. This would also be the average distance between neighboring strings. Since causality requires that the correlation length cannot be greater than the cosmological horizon size (which roughly equals the cosmic time t for noninflationary models) we have $\lambda=ft$, with $f\le 1$. However, expansion of the Universe (and friction with the surrounding medium) eventually smoothes out strings on all scales smaller than the horizon.

We thus choose the length of the straight string segment, and the radius of the surrounding cylinder, to be $\approx\lambda$. The values of the radial coordinates at the cylinder are defined to be r_0 and $\tilde{r}_0$. We then impose on ϕ vanishing boundary conditions at $r=r_0$ and $\tilde{r}=\tilde{r}_0$, and periodic boundary conditions in z with period L.

In the initial Minkowski space, the positive-frequency modes are given by the usual solution to the wave equation in cylindrical coordinates:

$$f_{n,m,s}=N_1\exp(-iw_s t)\exp(ikz)\times\exp(im\theta)J_{|m|}((w_s^2-k^2)^{1/2}r)\ , \tag{18}$$

with

$$N_1=(2\pi r_0^2 L w_s)^{-1/2}[J'_{|m|}((w_s^2-k^2)^{1/2}r_0)]^{-1}\ ,$$

$k=2\pi n/L$, and J'_m denoting the derivative of the Bessel function J_m. The frequencies w_s are determined from the boundary condition $J_{|m|}((w_s^2-k^2)^{1/2}r_0)=0$. In the conical cosmic-string spacetime region, the modes are given by

$$g_{n,m,s}=N_2\exp(iW_s\tilde{t})\exp(ik\tilde{z})\times\exp(im\tilde{\theta})J_{|m/a|}((W_s^2-k^2)^{1/2}\tilde{r})\ , \tag{19}$$

with

$$N_2=(2\pi a\tilde{r}_0^2 L W_s)^{-1/2}[J'_{|m/a|}((W_s^2-k^2)^{1/2}\tilde{r}_0)]^{-1}\ ,$$

$k=2\pi n/L$, and the frequencies W_s determined by the boundary condition $J_{|m/a|}((W_s^2-k^2)^{1/2}\tilde{r}_0)=0$. In both cases we have $n,m=0,\pm1,\pm2,\ldots$ and $s=1,2,3,\ldots$.

The scalar field can then be expanded in terms of creation and annihilation operators in the usual way. In the initial spacetime segment we have

$$\phi_-(t,r,z,\theta)=\sum_{n,m,s}(A_{n,m,s}f_{n,m,s}+A^\dagger_{n,m,s}f^*_{n,m,s}), \tag{20}$$

while in the final spacetime segment

$$\phi_+(\tilde{t},\tilde{r},\tilde{z},\tilde{\theta})=\sum_{n,m,s}(B_{n,m,s}g_{n,m,s}+B^\dagger_{n,m,s}g^*_{n,m,s})\ . \tag{21}$$

We now require that the field evolve in a continuous manner across the matching surface:

$$\phi_+|_\Sigma=\phi_-|_\Sigma\ , \tag{22}$$

where the subscript Σ denotes that the quantities are to be evaluated on the matching surface. To find the relation between the normal derivatives of ϕ evaluated at the matching surface Σ we first use Eqs. (11)–(16) and the trace of the Einstein equations, $R=-8\pi S\delta(t-Cr)$, to find the scalar curvature on the matching surface:

$$R=\frac{2(C'-C)}{r}(1-C^2)^{-1/2}\delta(t-Cr)\ . \tag{23}$$

Substituting this result into Eq. (17) and integrating we find

$$n^\mu_+\partial_\mu\phi_+|_\Sigma=\left[n^\mu_-\partial_\mu\phi_- -\frac{2\xi(C'-C)}{r}(1-C^2)^{-1/2}\phi_-\right]_\Sigma\ . \tag{24}$$

These conditions along with the scalar inner product can be used to derive a relation between the final and initial creation and annihilation operators given in general by[12]

$$B_j=\sum_i(A_i\alpha_{ij}+A_i^\dagger\beta_{ij}^\dagger)\ , \tag{25}$$

where from this point on we will simplify the mode notation by letting the index i represent (n',m',s') and j represent (n,m,s). For the specific case at hand, β_{ij} is found to be

$$\beta_{ij}=i\int\left[f_i n^\mu_+\partial_\mu g_j-(n^\mu_-\partial_\mu f_i)g_j+\frac{2\xi(C'-C)}{r}(1-C^2)^{-1/2}f_i g_j\right]d\Sigma\ , \tag{26}$$

where $d\Sigma$ is the three-volume element in the surface Σ. The integration over z and θ is trivial, allowing us to write

$$\beta_{ij}=\beta_{s',s}\delta_{-n',n}\delta_{-m',m}\ . \tag{27}$$

The average number of particles produced in the mode n,m,s is given by

$$\langle 0_i|N_{n,m,s}|0_i\rangle=\sum_{s'}\beta^*_{s',s}\beta_{s',s}\ , \tag{28}$$

where $|0_i\rangle$ is the vacuum state of the initial spacetime.

The expression for $\beta_{s',s}$ for arbitrary n and m is somewhat complicated. However, if we take Δt to be the actual time of formation for the string, production of particles in modes with frequencies that are large compared with $1/\Delta t$ will be suppressed. The most conservative estimate (that which results in the least particle production) is obtained by choosing the largest time scale around; namely, the cosmic time t. For strings forming at the GUT time this is thought to be a reasonable approximation. Under these assumptions particle production is suppressed in the higher modes and we may estimate a lower bound to the energy output by considering only the number of particles produced in the lowest mode: $n=0$, $m=0$, or $s=1$. In this case, after some simplification including an integration by parts, $\beta_{s',s}$ is given by

$$\beta_{s',s}(n=0,m=0)=\frac{2i\xi(C'-C)}{(1-C^2)^{1/2}}D'\int_0^1 \exp(-i\chi_1 x)J_0(z_{s'}x)J_0(z_s x)dx$$
$$-iD(C'-C)\int_0^1 \exp(-i\chi_1 x)[J_0(z_{s'}x)J_1(z_s x)+J_1(z_{s'}x)J_0(z_s x)]x\,dx\;, \quad (29)$$

where $D=(z_s/z_{s'})^{1/2}[(1+z_s/z_{s'})J_1(z_{s'})J_1(z_s)]^{-1}$, $D'=[(z_{s'}z_s)^{1/2}J_1(z_{s'})J_1(z_s)]^{-1}$, $\chi_1=z_{s'}C+z_sC'$, and z_s is the sth root of $J_0(x)$. The integrals have been put into dimensionless form using $x=r/r_0$ and $z_s=w_sr_0$.

Using $C'=C+[(1-C^2)/2\pi C]\alpha+O(\alpha^2)$, it is easy to expand this out to lowest order in the deficit angle α. We find

$$\beta_{s',s}(n=0,m=0)=\frac{i\xi D'(1-C^2)^{1/2}\alpha}{\pi C}\int_0^1 \exp(-i\chi_2 x)J_0(z_{s'}x)J_0(z_s x)dx$$
$$-\frac{iD(1-C^2)\alpha}{2\pi C}\int_0^1 \exp(-i\chi_2 x)[J_0(z_{s'}x)J_1(z_s x)+J_1(z_{s'}x)J_0(z_s x)]x\,dx+O(\alpha^2)\;, \quad (30)$$

where $\chi_2=(z_{s'}+z_s)C$.

It is apparent that Eq. (30) cannot be valid when $C=0$. The reason is that $C'=[\alpha/\pi(1-\alpha/4\pi)]^{1/2}$ for this special case, and thus the first term in the series expansion of β is proportional to $\alpha^{1/2}$ instead of α. Thus, for $C=0$ we find

$$\beta_{s',s}(n=0,m=0)=2i\xi D'(\alpha/\pi)^{1/2}\int_0^1 J_0(z_{s'}x)J_0(z_s x)dx$$
$$-iD(\alpha/\pi)^{1/2}\int_0^1 x[J_0(z_{s'}x)J_1(z_s x)+J_1(z_{s'}x)J_0(z_s x)]dx+O(\alpha)\;. \quad (31)$$

This corresponds to particle production greater by roughly a factor of $1/\alpha$ than indicated by the previous calculation of Parker.[2] Does this mean there is a sudden jump in production at $C=0$? To find out, we numerically integrated Eq. (29) (which gives $\beta_{s',s}$ to all orders in α), for $s=1$ and the first few values of s', and then used the result to sum the first few terms of Eq. (28). [Approximations for the integrals in Eq. (29) show for large s' the series given by Eq. (28) converges like $(1/s')^3$, justifying keeping only the leading terms in the sum.] The results are shown in Fig. 1 for minimal ($\xi=0$) and conformal ($\xi=\frac{1}{6}$) coupling, where we have plotted the base 10 logarithm of the number of particles produced in the lowest mode versus C, the slope of the matching surface. We see that the particle production varies smoothly as C is varied; it approaches zero as C approaches 1, and smoothly rises to a maximum as C approaches 0. The results are unchanged if both C and C' are negative. Thus, there is no nonzero minimum particle creation; as the matching surface approaches a null surface ($C\to 1$), the particle production approaches zero, independent of the curvature coupling. We note, however, that for a wide domain of values of C, the particle production is roughly proportional to α^2, as was the case in Parker's calculations.[2] It is not clear to us whether nature follows a path which minimizes particle production (the special case $C\to 1$), or whether the more typical case (production proportional to α^2) is actually the more likely to occur. Depending on how the string formation process actually arranges itself, the average energy density due to particles produced in the lowest mode could vary from zero, to the value given by Parker, up to a maximum value (when $C=0$). For completeness and comparative purposes, we will compute the average energy density for this latter (maximum) case. The calculation will be illustrative, even if for a special case, since the arguments presented here will be used again in the next section.

Note that the average number of particles produced in the lowest mode is less than one, even in the case where the particle production is maximized (where $C=0$). We can see from Fig. 1 that the maximum value for minimally or conformally coupled particles is roughly $\langle N_{0,0,1}\rangle\approx 1\times 10^{-6}$. However, recall that we have enclosed the string in a cylinder with dimensions of order the correlation length λ. Thus, the quantum-mechanical expectation value $\langle N_{0,0,1}\rangle$ can be interpreted as corre-

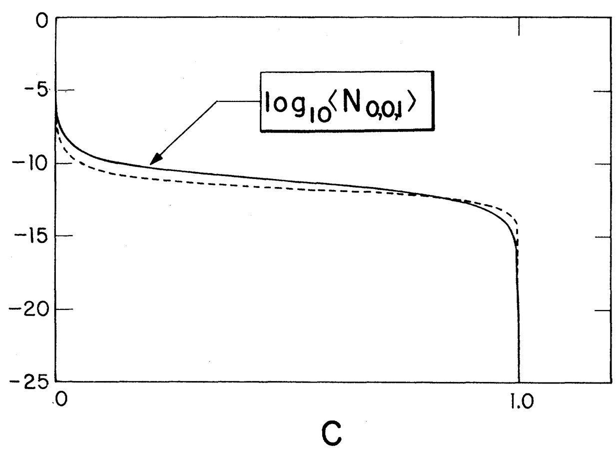

FIG. 1. The logarithm (base 10) of the number of particles created in the lowest mode of a scalar field is plotted against the slope of the spacelike cone C on which Minkowski space is joined to the conical string spacetime. Results are plotted for both minimal curvature coupling $\xi=0$ (solid curve), and conformal coupling $\xi=\frac{1}{6}$ (dashed curve). The particle production is seen to vary smoothly from zero, when the joining surface is null, to a maximum of about 10^{-6} when the slope is zero. Note that over most of the possible range for C the number of particles produced is between 10^{-10} and 10^{-13}, which agrees fairly well with the earlier calculation of Parker. Note also that the number of particles created depends only weakly on the value of the curvature coupling.

sponding to an average over an ensemble of forming string segments. Our results must then be construed to mean that in the string formation process, most string segments produce no particles due to the gross change in the gravitational field. Assuming the existence of about 100 different species of particles, even in the case of maximum particle creation only about one string segment in about ten thousand ($=10^6/100$) will eject one (or a few) very energetic particles.

The amount of energy carried by the particle in these rare instances, in the present calculation, is given by $E=W_1$, where $W_1=2.40/r_0+O(\alpha)$. Using $r_0 \approx L \approx ft$, the energy emitted per unit length of the string segment becomes

$$E/L \approx 2.4 f^{-2} t^{-2} . \tag{32}$$

For a GUT string, we have roughly $t \approx 10^{-38}$ sec for the cosmic time. Choosing $f=1$, which minimizes Eq. (32), and returning to conventional units, we find the energy emitted per unit length, when a string segment does emit a particle, is given by

$$E/L \approx 8\times 10^{38} \text{ ergs cm}^{-1} , \tag{33}$$

which is still small compared to the string's mass per unit length, $\mu \approx 10^{42}$ ergs/cm. As noted by Parker, a smaller value for f increases the ratio E/L but also makes W_1 large with respect to $1/t$, suppressing particle production. Thus, the result may be approximately valid for smaller values of f also. On the other hand, if the correlation length is smaller, then it seems likely that the formation time will be likewise reduced, resulting in distinctly larger values for E/L.

If the early Universe contained an extensive network of strings, and we take $\pi r_0^2 L$ to be approximately equal to the volume of space per segment of string, then the energy from the particles emitted by roughly one in 10^4 segments (our maximum estimate), viewed on a large enough scale to be treated as smeared over the entire background of strings, leads to an average density of

$$\rho \approx 10^{-4}\times E/(\pi r_0{}^2 L) \approx 3\times 10^{89} \text{ ergs cm}^{-3} , \tag{34}$$

an increase by a factor of 3×10^5 over the calculation by Parker[2,10] [for any other value of C, the density may be easily found by multiplying the right-hand side of Eq. (34) by $10^6\langle N\rangle$]. It should be remembered, however, that this number represents a maximum density in gravitationally created particles; over a large domain of matching surfaces, the particle production is in agreement with Parker's result, and, as the matching surface approaches a null surface, the created energy density (and number of particles produced) drops to zero. Thus, the amount of particle production can be strongly model dependent. In the next section we investigate particle creation associated with string formation from an initially de Sitter space. This may be a more physically appropriate model if strings form near the end of an inflationary era; also, as we shall see, there is less ambiguity about the amount of particle creation in this case, as the nonzero curvature of de Sitter space only allows one choice of matching surface.

IV. STRING FORMATION FROM AN INITIAL DE SITTER SPACE

In the early Universe, the energy density of the vacuum is thought to have been quite large and may have dominated at the time just before GUT strings form (with the subsequent phase transition, of course, corresponding to a large downward shift in the vacuum energy density). It is at present not definitely known whether one should associate cosmic-string formation with the end of a de Sitter (inflationary) stage. Thus, it may be more appropriate to approximate the spacetime metric before the string form by a portion of de Sitter space, here written in steady-state cylindrical coordinates:

$$ds^2=dt^2-\exp(2\kappa t)(dr^2+dz^2+r^2 d\theta^2) , \tag{35}$$

where $\kappa=(8\pi\rho_v/3)^{1/2}$, and ρ_v is the energy density of the vacuum. In this section we will compute the gravitational particle creation associated with the formation of a cosmic string from a spacetime initially described by the de Sitter metric of Eq. (35).

We will also slightly modify the field theory under consideration by now restricting attention to a conformally coupled massless scalar field (though the result for arbitrary coupling will be discussed briefly). The vacuum state in the initial de Sitter space will be chosen to be the conformal vacuum. As we shall see, conformal coupling has the advantage of eliminating any particle production independent of α, i.e., that due to the phase transition itself rather than the formation of the string (for a discussion of particle creation associated with the phase transition itself, see Ref. 13). A scalar field coupled nonconformally to the scalar curvature will produce particles even if a string is not formed in the phase transition (e.g., as de Sitter space is transformed into Minkowski space in the "sudden" model considered here). The particle production associated with a conformally coupled field will be entirely due to the formation of the string and the nonzero deficit angle of its conical geometry.

Using the value $R=12\kappa^2$ for de Sitter space in Eq. (17), and imposing the same cylindrical boundary conditions as in Sec. III, the initial positive-norm modes are given by

$$\begin{aligned} f_{n,m,s} &= N_1 \exp[-\kappa t+(iw_s/\kappa)(e^{-\kappa t}-1)]\exp(ikz) \\ &\quad \times \exp(im\theta) J_{|m|}((w_s^2-k^2)^{1/2} r) , \end{aligned} \tag{36}$$

where N_1, k, and w_s are given by expressions identical to those given for the modes in Minkowski space, and we have chosen the phase of the modes in such a way as to allow the limit $\kappa \to 0$ to be easily taken. Since $R=0$ in the string spacetime, the "out" modes are unchanged.

We now determine the shape of the joining surface Σ by matching intrinsic metrics. By the same arguments as in Sec. II, the equations for the surface Σ should be independent of the θ, $\tilde{\theta}$, z, and $\tilde{z}$ coordinates, as there exist Killing vectors in these directions in both four-geometries. Simple examination of the metrics in Eqs. (2) and (35) then reveals that the intrinsic metrics induced on Σ by the two four-geometries will be the same only if Σ is taken to be a $t=$const surface. For simplicity we choose

$$t=0\ , \tag{37}$$

$$\tilde{t}=[\alpha/\pi(1-\alpha/4\pi)]^{1/2}\tilde{r} \tag{38}$$

as the surfaces to be joined on Σ. This corresponds to the choice $C=0$ in the language of the last section.

To find the matching condition on the normal derivative of the field care must be taken to include the δ-functional curvature of the matching surface when integrating Eq. (17) across Σ. As in Sec. III above, the contribution of the spacelike hypersurface Σ to the scalar curvature is easily found using the Israel thin-shell formalism.[3] The final result for the scalar curvature of the entire spacetime is

$$R=12\kappa^2H(-t)-6\kappa\delta(t)+2[(\alpha/\pi)(1-\alpha/4\pi)]^{1/2}\delta(t)/r\ , \tag{39}$$

where $H(t)$ is the Heaviside step function.

Integrating Eq. (17) using Eq. (39) for R, we find that the matching conditions for the conformally coupled scalar field on Σ become

$$n_{+}^{\mu}\,\partial_\mu\phi_+|_\Sigma=\left[n_{-}^{\mu}\,\partial_\mu\phi_-+\kappa\phi_- -\left[\frac{1}{3r}\right][(\alpha/\pi)(1-\alpha/4\pi)]^{1/2}\phi_-\right]_\Sigma \tag{40}$$

and

$$\phi_+|_\Sigma=\phi_-|_\Sigma\ . \tag{41}$$

These can then be used as in Sec. III to derive the Bogoliubov coefficient β_{ij}:

$$\beta_{ij}=i\int\left[f_i n_{+}^{\mu}\,\partial_\mu g_j-(n_{-}^{\mu}\,\partial_\mu f_i)g_j-\kappa f_i g_j+\left[\frac{1}{3r}\right][(\alpha/\pi)(1-\alpha/4\pi)]^{1/2}f_i g_j\right]d\Sigma\ . \tag{42}$$

The integrals over z and θ, as well as the physical arguments concerning suppression of particle production in the higher modes, are the same as before. Thus, the relevant factor $\beta_{s',s}$ for the lowest, dominant mode is given by

$$\beta_{s',s}(n=0,m=0)=\left[\frac{\alpha}{\pi}\right]^{1/2}\left[\frac{iD'}{3}\int_0^1 J_0(z_{s'}x)J_0(z_s x)dx-iD\int_0^1[J_0(z_{s'}x)J_1(z_s x)+J_1(z_{s'}x)J_0(z_s x)]x\,dx\right]+O(\alpha)\ . \tag{43}$$

Notice that, as mentioned above, no term independent of α appears in Eq. (43). Consider the limit $\alpha\rightarrow 0$: the conformally flat steady-state Universe now goes to Minkowski space across the matching surface and the particle production vanishes. This is just an example of Parker's theorem[14] (no particle production by conformally coupled fields in conformally flat spacetimes) in the sudden approximation. In the case of arbitrary coupling to the scalar curvature it can be shown there is an additional term proportional to $(1-6\xi)\kappa r_0$, independent of α. This term represents particle creation by nonconformal fields associated with the phase transition rather than with formation of the string (since it persists even if the deficit angle is set equal to zero). Such production will generally dominate over that which is associated with string formation, unless there are many more conformally coupled fields than nonconformal fields in nature (as may be the case). As in the case $C=0$ examined in Sec. III above, the particle production which is associated with string formation will be proportional to α, a significant increase over previously studied models.[2]

If we calculate the number of particles produced in this case [the series converges as described in Sec. III and so only the first few terms of Eq. (28) need be summed] we find

$$\langle N_{0,0,1}\rangle\approx 2\times10^{-7}\ . \tag{44}$$

As was true in Sec. III, the number of particles produced is again less than one. Thus, using the same arguments of Sec. III, one segment of string in about 5×10^4 will emit a very energetic particle with the ejected particle's energy divided by the segment's length equal to $E/L\approx 8\times10^{38}$ ergs/cm. Also, the energy of the emitted particles, viewed on a scale large enough to be treated as smeared out into an average energy density, gives

$$\rho\approx 2\times10^{-5}E/(\pi r_0^2L)\approx 6\times10^{88}\ \text{ergs cm}^{-3}\ . \tag{45}$$

This result closely corresponds to the maximum limit derived for the initially Minkowski models in Sec. III.

V. DISCUSSION

Using the sudden approximation, we have calculated the average number of particles produced during formation of a cosmic-string segment for the lowest mode of a massless scalar field with arbitrary curvature coupling. We have assumed particle production in higher modes is suppressed (by arguing the actual formation time is roughly the longest possible, the cosmic time).

Before we discuss our results, we consider the validity of the approximations in our calculation. First, the "sudden" approximation will be valid for those modes whose periods are long compared to the formation time of the string segment. In our calculation the period of the lowest mode is roughly three times our (maximum) estimate of the actual formation time of the string segment. This assumption thus leads to a very conservative lower bound on the number of created particles (and their energy density), since even with the largest possible formation time, several (rather than only the lowest) modes will have a significant probability of particle creation. Second, we have attempted to determine how strongly the results of a particle creation calculation depend on the details of the (necessarily nonunique) implementation of the sudden approximation in the initially Minkowski-space case. We have seen that the amount of particle production depends strongly on the choice of matching hypersurface. The number of created particles varies from zero up to a maximum which is linearly proportional to the deficit angle of the string. On the other hand, independent of the curvature coupling, for a large number of our models in Sec. III we find the particle production is proportional to α^2, in agreement with the calculation by Parker. In other words, regardless of the details, for a wide range of the matching parameter C we find a consistent answer, as is evident by the relative flatness of the curves in Fig. 1, which over a large domain yield values roughly of order $\langle N_{0,0,1} \rangle \approx 10^{-11}$. However, the $C \rightarrow 1$ and $C \rightarrow 0$ end points of the particle creation curve in Fig. 1 provide a cautionary note, showing that the particle production can be strongly model dependent.

We have found results for two different initial spacetime segments; either portions of Minkowski space or de Sitter space. The Minkowski-space results are discussed above and in Sec. III. In the initially de Sitter model, which might represent string formation at the end of an inflationary epoch, we find particle production is necessarily linearly proportional to α. The resultant energy density is roughly 6×10^{88} ergs/cm^3, significantly larger than in most of the other cases examined. If strings form near the end of inflation the energy density produced in emitted particles could significantly contribute to the reheating of the Universe. As mentioned above, the energy density calculated here is to be interpreted as a lower bound on the actual energy density, since we have considered only the lowest mode and have made conservation assumptions about the relative size of the correlation length and the horizon size. It should be pointed out that in all our models the energy emitted per unit length of the string is always small compared to the string's energy density and so the back reaction can be ignored. However, if our calculations are a severe underestimate, back reaction would of course become important to the calculation.

ACKNOWLEDGMENTS

We wish to thank Paul Anderson for helpful discussions. This work was supported by National Science Foundation Grants Nos. PHY-8803234 and ISP80-11449.

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