

## Squark and squarkonium production by gauge-boson fusion at TeV $e^+e^-$ colliders

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We explore the production of squarkonium as well as squark and slepton pairs by gauge-boson fusion at TeV  $e^+e^-$  colliders. Although squarkonium production rates are found to be too small to be observable, gauge-boson fusion contributions to squark and slepton pair production can be sizable for a reasonable range of model parameters.

### I. INTRODUCTION

The construction of  $e^+e^-$  colliders in the TeV energy range will open a new window in the search for physics beyond the standard model<sup>1</sup> (SM). One possibility for such new physics is supersymmetry (SUSY); production rates of and signals for SUSY partner production at  $e^+e^-$  colliders via “conventional”  $s$ - and  $t$ -channel exchange have been thoroughly studied in the literature.<sup>2</sup> In this paper we examine, instead, the production cross sections for pairs of squarks and sleptons, as well as squark-antisquark bound states (i.e., squarkonium) via  $\gamma\gamma$  and gauge-boson fusion mechanisms using the effective gauge-boson approximation.<sup>3</sup> As we will see, our estimates show that for a reasonable range of model parameters  $\gamma\gamma$  and  $W^+W^-$  fusion yield significant production rates for squark and slepton pair production which are comparable in magnitude to those obtained via the usual  $s$ -channel  $\gamma$  and  $Z$  exchanges. Squarkonium production by this mechanism will, however, be shown to be too small to be observable for any foreseeable integrated luminosity if the wave function at the origin is approximately determined by the Coulomb short-distance part of the potential. Alterations in the potential, e.g., due to Higgs-boson exchange may alter these conclusions drastically as we will see below. Since the signatures for the production and decay of SUSY particles at  $e^+e^-$  colliders have been extensively discussed in the literature,<sup>2</sup> we will confine our attention to the estimates of production rates for these particles using the gauge-boson fusion mechanism at these new TeV  $e^+e^-$  colliders.

Section II contains a discussion of squark and slepton pair production while squarkonium production is discussed in Sec. III. Our results and conclusions are given in Sec. V.

### II. SQUARK AND SLEPTON PAIR PRODUCTION

The usual mechanism for the production of squark and slepton pairs in  $e^+e^-$  collisions is via  $s$ -channel  $\gamma$  and  $Z$  exchange ( $e^+e^- \rightarrow \tilde{f}\tilde{f}^*$ ) with a differential cross section given by ( $N_c = 1$  for sleptons,  $N_c = 3$  for squarks)

$$\frac{d\sigma}{dz} = N_c \frac{3\pi\alpha^2}{4} \beta^3 s (1-z^2) \sum_{i,j=\gamma,Z} P_{ij} c_i^f c_j^f (v_i v_j + a_i a_j)_e, \quad (2.1)$$

where  $z = \cos\theta$  ( $\theta$  being the  $e^- \tilde{f}$  scattering angle),  $\beta = (1 - 4M_s^2/s)^{1/2}$ , and

$$P_{ij} = \frac{(s - M_i^2)(s - M_j^2) + (\Gamma_i M_i)(\Gamma_j M_j)}{[(s - M_i^2)^2 + (\Gamma_i M_i)^2][(s - M_j^2)^2 + (\Gamma_j M_j)^2]} \quad (2.2)$$

with the couplings normalized as ( $X_i = \gamma, Z$ )

$$\mathcal{L} = e[\bar{e}\gamma_\mu(v_{ie} - a_{ie}\gamma_5)e + c_i^f(p_1 - p_2)_\mu \tilde{f}^* \tilde{f}] X_i^\mu. \quad (2.3)$$

In the above expression for the cross section we have neglected any possible  $t$ -channel exchange contributions which occurs for  $\tilde{e}$  production or when leptoquarks also exist in the theory as in  $E_6$  superstring-inspired models.<sup>4</sup>

As discussed above, at high-energy  $e^+e^-$  colliders there are additional production mechanisms which may lead to potentially sizable cross sections for SUSY partner production. One possibility is  $\gamma\gamma$  collisions which leads to  $\tilde{f}\tilde{f}^*$  final states via  $e^+e^- \rightarrow e^+e^- \gamma\gamma \rightarrow e^+e^- \tilde{f}\tilde{f}^*$ . The production cross section in the Weizacker-Williams-effective-gauge-boson approximation<sup>5</sup> is given by

$$\begin{aligned} \sigma(e^+e^- \rightarrow e^+e^- \tilde{f}\tilde{f}^*) \\ = \int_{2M_s^2/\sqrt{s}}^{\sqrt{s}/2} \frac{dw_1}{w_1} \int_{M_s^2/w_1}^{\sqrt{s}/2} \frac{dw_2}{w_2} N(w_1)N(w_2) \hat{\sigma}_{\gamma\gamma}, \end{aligned} \quad (2.4)$$

where (with  $E' = E - w$  and  $E = \sqrt{s}/2$ )

$$\begin{aligned} N(w) = \frac{\alpha}{\pi} \left[ \frac{E^2 + E'^2}{E^2} \ln \left[ \frac{E}{m_e} - \frac{1}{2} \right] \right. \\ \left. + \frac{(E - E')^2}{2E^2} \left[ \ln \frac{2E'}{E - E'} + 1 \right] \right. \\ \left. + \frac{(E + E')^2}{2E^2} \ln \frac{2E'}{E + E'} \right] \end{aligned} \quad (2.5)$$

and the subprocess cross section is

$$\hat{\sigma}_{\gamma\gamma} = \frac{2\pi\alpha^2}{\hat{s}} N_c Q^4 \left[ \hat{\beta}(2-\hat{\beta}^2) - \frac{1}{2}(1-\hat{\beta}^4) \ln \frac{1+\hat{\beta}}{1-\hat{\beta}} \right]. \quad (2.6)$$

Here,  $Q$  is the electric charge of  $\tilde{f}$  with  $\hat{\beta} = (1 - 4M_s^2/\hat{s})^{1/2}$  and  $\hat{s} = 4w_1w_2 \leq s$ . The diagrams contributing to this process are shown in Figs. 1(A)–1(C) with  $V = \gamma$  and  $\tilde{f}' = \tilde{f}$ . Figures 2(a)–2(c) show a comparison of the  $\gamma\gamma$  contribution to  $\tilde{f}\tilde{f}$  production to that for ordinary  $s$ -channel  $\gamma$  and  $Z$  exchange for different choices of  $f$ . In our analysis below, we will work in the weak-eigenstate basis. Note that  $\sigma(e^+e^- \rightarrow e^+e^-\tilde{f}\tilde{f})$  is the same for  $\tilde{f}_L$  and  $\tilde{f}_R$  since it only depends on  $Q$  and  $N_c$  while  $\sigma(e^+e^- \rightarrow \tilde{f}\tilde{f})$  also depends on  $T_3$  (the third component of weak isopin). As one might expect, for  $Q = -\frac{1}{3}$ , the  $\gamma\gamma$  contribution is quite small in comparison to the usual  $s$ -channel  $\gamma$  and  $Z$  terms unless  $M_s \lesssim 40$  (100) GeV for  $T_3 = -\frac{1}{2}$  (0) at  $\sqrt{s} = 1$  (2) TeV but is somewhat more significant when  $Q = \frac{2}{3}$ . In this case one finds sizable  $\gamma\gamma$  contributions for  $M_s \lesssim 70$  (150) GeV for  $T_3 = \frac{1}{2}$  (0) at  $\sqrt{s} = 1$  (2) TeV  $e^+e^-$  colliders as shown in Fig. 2(b). The situation is even better for  $Q = -1$  where sizable  $\gamma\gamma$  contributions occur for  $M_s \lesssim 90$  (200) GeV with  $T_3 = -\frac{1}{2}$  (0) for  $\sqrt{s} = 1$  (2) TeV colliders. Thus we see that for relatively light squark and slepton masses (in comparison to  $\sqrt{s}$ ) the  $\gamma\gamma$  contribution to the cross section for  $\tilde{f}\tilde{f}$  production can be quite significant. It should be noted that while  $\tilde{f}\tilde{f}$  production is peaked somewhat forward, due to the large value of  $M_s/\sqrt{s}$  the average  $p_T$  of the  $\tilde{f}/\tilde{f}$  is  $\approx M_s$  so that reasonably large scattering an-

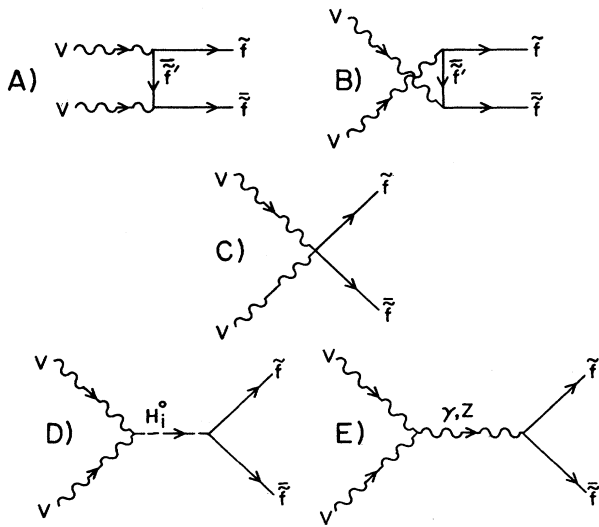


FIG. 1. Various subprocesses which contribute to the  $VV \rightarrow \tilde{f}\tilde{f}$  production process. The top (bottom) solid curve corresponds to  $\sqrt{s} = 1$  (2) TeV while for the  $s$ -channel process the  $\sqrt{s} = 2$  TeV curves extend further to the right.

gles for  $\tilde{f}\tilde{f}$  result. In addition, since  $\tilde{f}$ 's are highly unstable (as well as quite massive) their decay products will appear at even larger angles. Since planned detectors<sup>1</sup> should go down to within a few degrees of the beam,  $\tilde{f}$ 's

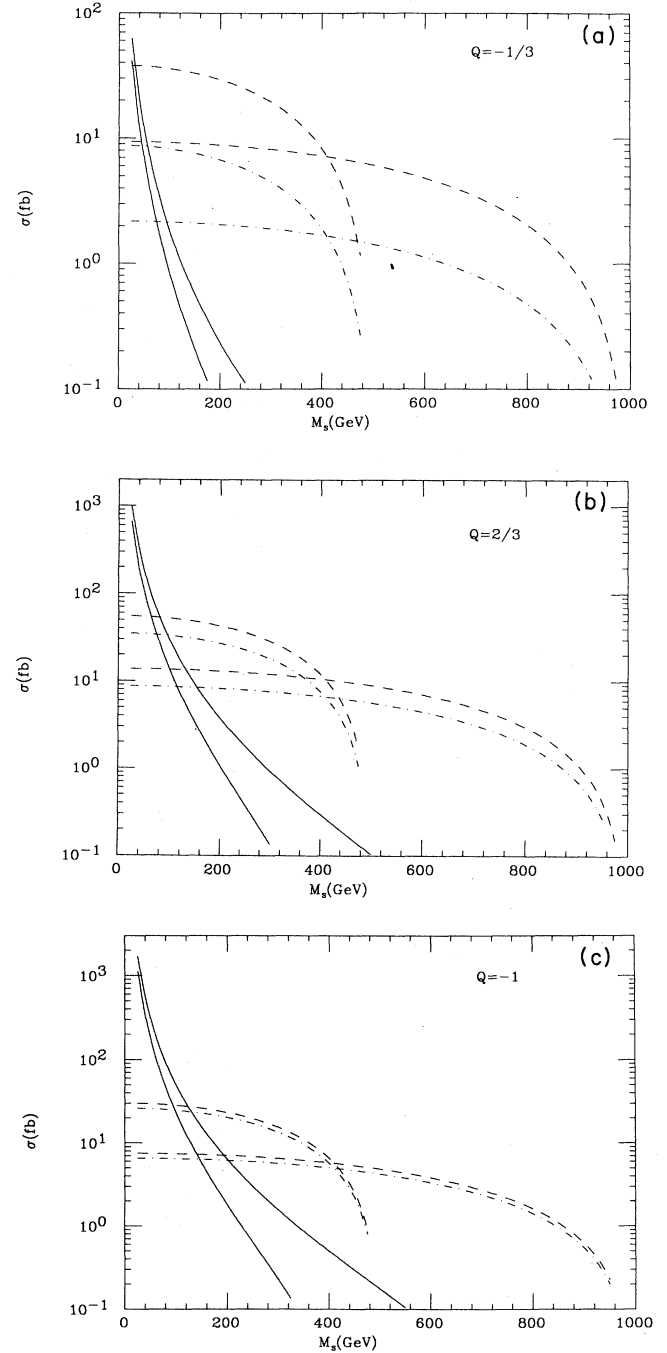


FIG. 2. A comparison of the  $\tilde{f}_L\tilde{f}_L$  (dashed) and  $\tilde{f}_R\tilde{f}_R$  (dash-dot)  $s$ -channel production cross section via  $\gamma$  and  $Z$  exchange with that for (solid)  $\gamma\gamma \rightarrow \tilde{f}\tilde{f}$  (for either  $\tilde{f}_L\tilde{f}_L$  or  $\tilde{f}_R\tilde{f}_R$ ) at  $\sqrt{s} = 1$  and 2 TeV  $e^+e^-$  colliders for (a)  $Q = -\frac{1}{3}$ , (b)  $Q = \frac{2}{3}$ , and (c)  $Q = -1$ .

produced in this manner should be easily observable.

Is it possible that gauge-boson fusion ( $V \neq \gamma$ ) can also yield a sizable contribution to  $\tilde{f} \bar{\tilde{f}}$  production? Although one might think *a priori* that their contribution is small, there may be significant enhancements in the effective couplings of longitudinal gauge bosons to SUSY partners as noted in our earlier work.<sup>6</sup> Thus, in our calculation below we will limit ourselves to cross-section estimates based solely on purely longitudinally polarized gauge bosons in the initial state. We first consider the  $Z_L Z_L \rightarrow \tilde{f} \bar{\tilde{f}}$  subprocess; in the absence of the model-dependent Higgs-boson-exchange contribution shown in Fig. 1(D) the matrix element would cancel in the longitudinal limit since  $\tilde{f}' = \tilde{f}$ . The Higgs-boson contribution is proportional to<sup>6</sup>

$$\sum_{i=1}^3 \frac{g_{iVV} g_{i\tilde{f}\tilde{f}}}{(\hat{s} - m_{H_i}^2) + i\Gamma_i m_{H_i}}, \quad (2.7)$$

where the sum extends over the three neutral Higgs fields with the dominant source of model dependence lying in  $g_{i\tilde{f}\tilde{f}}$  and  $m_{H_i}$ . In order to circumvent this difficulty we approximate the sum by a single Higgs-boson-exchange term and introduce a new mass scale parameter  $\mu$  and a dimensionless coefficient  $\lambda$ . Since Fig. 1(E) does not contribute in this limit, the effective action of the Higgs-boson-exchange term is to shift the contribution from Fig. 1(C), i.e., making the replacement

$$\hat{s} \rightarrow \hat{s} \left[ 1 + \lambda \frac{\mu^2}{\hat{s} - m_H^2 + i\Gamma_H m_H} \right] \quad (2.8)$$

so that the  $Z_L Z_L \rightarrow \tilde{f} \bar{\tilde{f}}$  subprocess cross section is given by [since the  $\hat{s}$  term in Fig. 1(C) cancels with the contributions from Figs. 1(A) and 1(B)]

$$\frac{d\sigma_{ZZ}}{dz} = \frac{G_F^2}{\pi \hat{s}} \hat{\beta} \frac{\lambda^2 \hat{s}^2 \mu^4}{(\hat{s} - m_H^2)^2 + m_H^2 \Gamma_H^2} N_c (T_3 - x_W Q)^4. \quad (2.9)$$

Thus the  $Z_L Z_L$  contribution to  $\tilde{f} \bar{\tilde{f}}$  production is given by ( $\tau \equiv \hat{s}/s$ )

$$\sigma_{ZZ} = \int_{4M_s^2/s}^1 d\tau \int_{\tau}^1 F_L^Z(x) F_L^Z(\tau/x) \int_{-1}^1 dz \left[ \frac{d\hat{\sigma}_{ZZ}}{dz} \right]. \quad (2.10)$$

In performing our calculation, we take the longitudinal  $Z$  distribution functions ( $F_L^Z$ ) given in Ref. 7 in the leading-log approximation. In order to show the maximum size of the  $Z_L Z_L$  contribution we will take  $\lambda = 1$ ,  $\mu = 500$  GeV and consider the case  $\tilde{f} = \tilde{d}_L$  since this leads to a value of  $N_c (T_3 - x_W Q)^4$  which is the largest among the various possible choices. More realistic choices of parameters will, of course, yield smaller cross sections from this subprocess. Figure 3 shows the  $Z_L Z_L$  contribution to  $\tilde{d}_L \bar{\tilde{d}}_L$  production with the above values of  $\mu$  and  $\lambda$ , as a func-

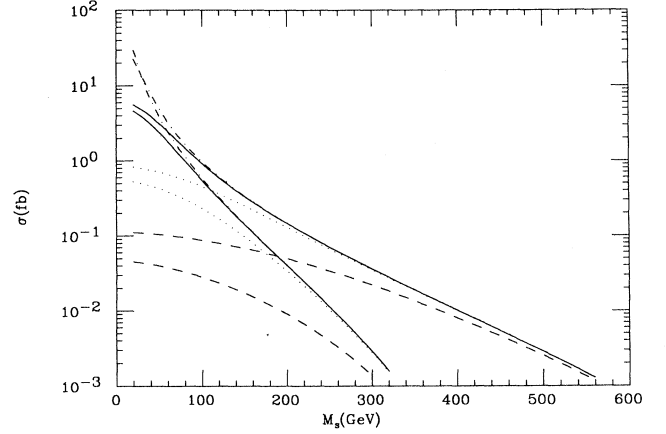


FIG. 3.  $\tilde{f}_L \bar{\tilde{f}}_L$  production cross section from  $Z_L Z_L$  fusion as a function of the squark mass for  $\sqrt{s} = 1$  and 2 TeV  $e^+e^-$  colliders different values of  $m_H$  and  $\Gamma_H/m_H$ :  $m_H = 100$  GeV,  $\Gamma_H/m_H = 0.1$  (dash-dot);  $m_H = 300$  GeV,  $\Gamma_H/m_H = 0.2$  (solid);  $m_H = 600$  GeV,  $\Gamma_H/m_H = 0.3$  (dot);  $m_H = 1$  TeV,  $\Gamma_H/m_H = 0.5$  (dash).  $\mu = 500$  GeV and  $\lambda = 1$  have been assumed and the  $\sqrt{s} = 2$  TeV curves extend further to the right.

tion of  $M_s$ , for  $\sqrt{s} = 1$  and 2 TeV and several choices of  $m_H$  and  $\Gamma_H$ . It is clear that even for these relatively extreme choices of the parameters the  $Z_L Z_L$  contribution is not significant for SUSY-partner masses  $\gtrsim 80$  GeV. For other values of  $\mu$  and  $\lambda$ ,  $\sigma$  will scale as ( $\sigma_0$  being the cross section shown in Fig. 3)

$$\frac{\sigma}{\sigma_0} = \lambda^2 \left[ \frac{\mu}{500 \text{ GeV}} \right]^4 \quad (2.11)$$

so, e.g., for  $\lambda^2 = 0.3$  and  $\mu = 300$  GeV, the resulting value of  $\sigma$  is suppressed by a factor  $\simeq 25$  compared to that in Fig. 3. We thus conclude that  $Z_L Z_L$  fusion does not make a significant contribution to  $\tilde{f} \bar{\tilde{f}}$  production. It should be noted that these conclusions may be softened to some extent if  $\tilde{f}_L$  and  $\tilde{f}_R$  mix and form nondegenerate mass eigenstates. Then the mass of the  $u$ - and  $t$ -channel-exchange particle can be different from that produced in the final state. Equation (2.9) now becomes

$$\begin{aligned} \frac{d\hat{\sigma}_{ZZ}}{dz} = & \frac{G_F^2}{\pi \hat{s}} \hat{\beta} \left[ \frac{(t - M'^2)^2}{t - M_s^2} + \frac{(u - M'^2)^2}{u - M_s^2} \right. \\ & \left. + \hat{s} \left[ 1 + \frac{\lambda \mu^2}{(\hat{s} - m_H^2) + i\Gamma_H m_H} \right] \right]^2 \\ & \times N_c (T_3 - x_W Q)^4 \sin^2 \phi \cos^2 \phi \end{aligned} \quad (2.9')$$

with  $\phi$  being the  $\tilde{f}_L$ - $\tilde{f}_R$  mixing angle. If  $M' - M_s$  is sufficiently large and  $\phi$  is not too small one may be able to enhance the  $Z_L Z_L$  contribution to the cross section.

The  $W_L^+ W_L^- \rightarrow \tilde{f}_L \bar{\tilde{f}}_L$  situation is somewhat different since  $\tilde{f}' \neq \tilde{f}$  always holds and there is a potentially large splitting between the masses of  $\tilde{f}'$  and  $\tilde{f}$ . One finds the

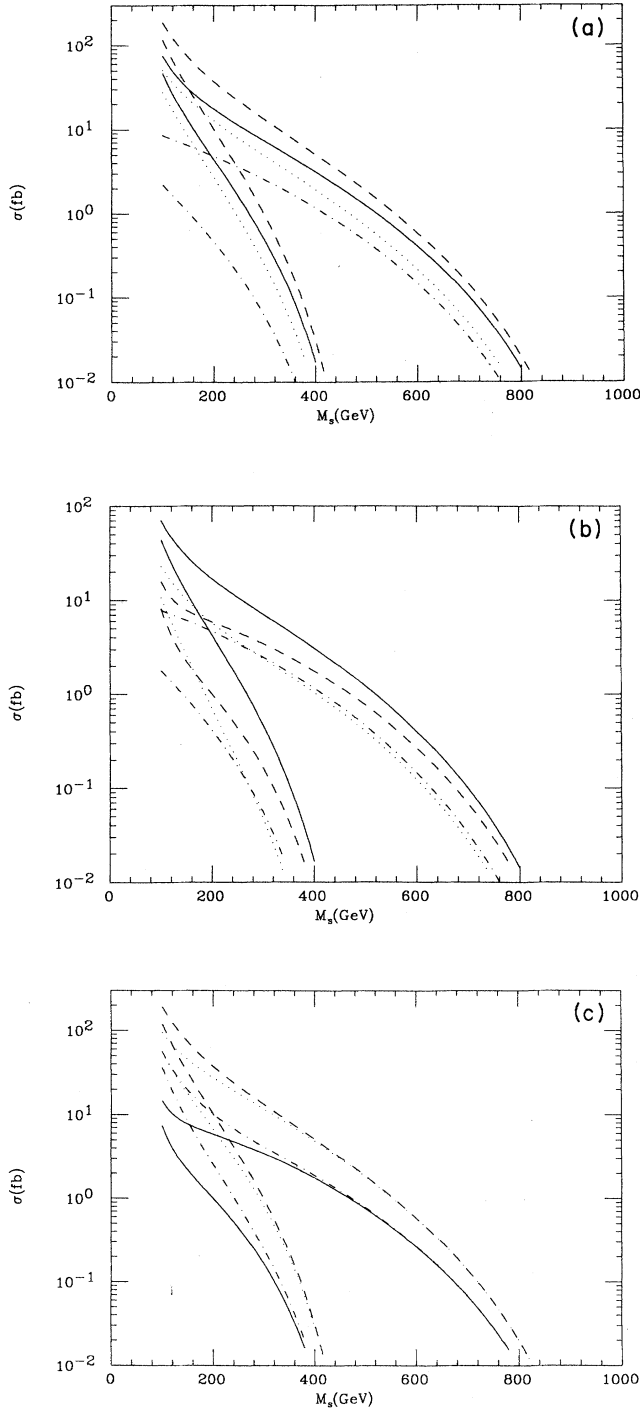


FIG. 4. Cross section for  $W^+W^- \rightarrow \tilde{f}\tilde{f}^*$  as a function of the squark mass at  $\sqrt{s}=1$  and 2 TeV  $e^+e^-$  colliders. In (a)  $\delta M=150$  GeV,  $\mu=250$  GeV,  $\lambda=0.1$  (solid);  $\delta M=50$ ,  $\mu=250$  GeV,  $\lambda=0.1$  (dash-dot);  $\delta M=150$  GeV,  $\mu=500$  GeV,  $\lambda=1$  (dash);  $\delta M=50$  GeV,  $\mu=500$  GeV,  $\lambda=1$ . (b) is the same as (a) but with  $\lambda \rightarrow -\lambda$ .  $m_H=300$  GeV and  $\Gamma_H/m_H=0.2$  are assumed in both (a) and (b). (c)  $\mu=500$  GeV and  $\delta M=150$  GeV with  $m_H=100$  GeV,  $\Gamma_H/m_H=0.1$ ,  $\lambda \equiv \pm 1$  (solid, dash) and  $m_H=1$  TeV,  $\Gamma_H/m_H=0.5$ ,  $\lambda = \pm 1$  (dash-dot, dot). The  $\sqrt{s}=2$  TeV curves extend further to the right.

subprocess cross section to be (neglecting intergenerational mixing)

$$\frac{d\hat{\sigma}_{WW}}{dz} = N_c \frac{G_F^2 \hat{\beta}}{4\pi \hat{s}} \left| \frac{(t-M'^2)^2}{t-M_s^2} + \frac{1}{2} \hat{s} \left[ 1 + \frac{\lambda \mu^2}{(\hat{s}-m_H^2) + i\Gamma_H m_H} \right] \right|^2, \quad (2.12)$$

where

$$t \equiv M_s^2 - \frac{1}{2} \hat{s} (1 - \hat{\beta} z). \quad (2.13)$$

In this case we find that the integrated cross section, given by Eq. (2.10) with the replacement  $Z \rightarrow W$ , is quite sensitive to variations in  $\delta M$  ( $\equiv M' - M_s$ ) as well as  $\lambda$  and  $\mu$  but not extremely sensitive to variations in  $m_H$  and  $\Gamma_H$ . This is shown explicitly in Figs. 4(a)–4(c) where  $N_c=3$  has been assumed. In Figs. 4(a) and 4(b),  $m_H$  and  $\Gamma_H$  are held fixed while  $\mu$ ,  $\lambda$ , and  $\delta M$  are varied whereas in Fig. 4(c)  $\mu$ ,  $\lambda$ , and  $\delta M$  are held fixed while  $m_H$  and  $\Gamma_H$  are varied for  $\lambda = \pm 1$  with very little effect on  $\sigma$  for either case except in the lower- $M_s$  region. Note the effect on the cross section of changing the sign of  $\lambda$ . It is clear from these figures that for a reasonable range of parameters  $W^+W^-$  fusion can make a potentially significant contribution to  $\tilde{f}_L \tilde{f}_L^*$  production ( $\approx 10$  fb) for  $M_s \lesssim 200$  (400) GeV at  $\sqrt{s}=1$  (2) TeV colliders. If one includes contributions from transverse gauge-boson degrees of freedom, these cross sections can only be enhanced so that this conclusion will only be strengthened.

### III. SQUARKONIUM PRODUCTION

The possibility that squarks can form color-singlet bound states leads to interesting phenomenology discussed by a number of authors.<sup>4,6,8</sup> The  $S$  wave,  $0^{++}$  squarkonium states (which should be the lowest lying) cannot be made directly in ordinary  $e^+e^-$  annihilation in the  $s$  channel unless new Yukawa interactions, which can occur in some  $E_6$  superstring-inspired models,<sup>4</sup> are also present. However, since  $0^{++}$  squarkonium ( $\tilde{S}$ ) has a large decay rate to pairs of gauge bosons, especially when longitudinal coupling enhancement are taken into account, one might hope that the process  $VV \rightarrow \tilde{S}$  may have a significant cross section. First, we consider  $\tilde{S}$  production by  $\gamma\gamma$  fusion for which we find<sup>5</sup>

$$\sigma(e^+e^- \rightarrow e^+e^-\tilde{S}) = \frac{16\alpha^2}{M^3} \ln^2 \left[ \frac{\sqrt{s}}{m_e} \right] \Gamma(\tilde{S} \rightarrow \gamma\gamma) f(M/\sqrt{s}), \quad (3.1)$$

where  $M$  is the mass of  $\tilde{S}$  and the function  $f$  is given by

$$f(x) = (2+x^2)^2 \ln(1/x) - (1-x^2)(3+x^2) \quad (3.2)$$

and

$$\Gamma(\tilde{S} \rightarrow \gamma\gamma) = \frac{24\pi\alpha^2}{M^2} \alpha^4 |\psi(0)|^2. \quad (3.3)$$

If one assumes that for large  $M$  the finding is essentially Coulombic then

$$|\psi(0)|^2 = \frac{1}{\pi} \left[ \frac{1}{3} \alpha_s(M) M \right]^3 \quad (3.4)$$

although the actual value of  $|\psi(0)|^2$  may be significantly large due to Higgs-boson exchange. Figure 5 shows the  $\gamma\gamma$  fusion cross sections for  $Q = \frac{2}{3}$  and  $-\frac{1}{3}$  with  $\sqrt{s} = 1$  and 2 TeV as functions of  $M$  assuming Coulombic wave functions with  $\alpha_s$  running with  $\Lambda = 200$  MeV. Clearly  $\sigma$  is far too small for  $\tilde{S}$  to be observed if this were the only production mechanism.

The  $Z_L Z_L$  fusion contribution to  $\tilde{S}$  production can be written as

$$\begin{aligned} \sigma_Z(e^+e^- \rightarrow e^+e^-\tilde{S}) \\ = \frac{2G_F^2 M_Z^4}{\pi^2} (v_e^2 + a_e^2)^2 \frac{\Gamma(\tilde{S} \rightarrow Z_L Z_L)}{M^3} g(M/\sqrt{s}) \end{aligned} \quad (3.5)$$

while that from  $W_L^+ W_L^-$  fusion is given by

$$\sigma_W(e^+e^- \rightarrow \nu\bar{\nu}\tilde{S}) = \frac{G_F^2 M_W^4}{2\pi^2} \frac{\Gamma(\tilde{S} \rightarrow W_L^+ W_L^-)}{M^3} g(M/\sqrt{s}), \quad (3.6)$$

where  $v_e = -\frac{1}{2} + 2x_W$ ,  $a_e = -\frac{1}{2}$  ( $x_W = \sin^2\theta_W \approx 0.230$ ) and the function  $g$  is given by

$$g(x) = 2(1+x^2)\ln(1/x) - 2(1-x^2). \quad (3.7)$$

The  $\tilde{S} \rightarrow VV$  width can be calculated using the results of Ref. 6, i.e.,

$$\Gamma(\tilde{S} \rightarrow VV) = |\psi(0)|^2 \frac{3}{32\pi M_s^2} \left[ 1 - \frac{M_V^2}{M_s^2} \right]^{1/2} |T|^2 \quad (3.8)$$

with

$$|T|^2 = [A(M_s^2/M_V^2 - 1) + C] + 2C^2 \quad (3.9)$$

and

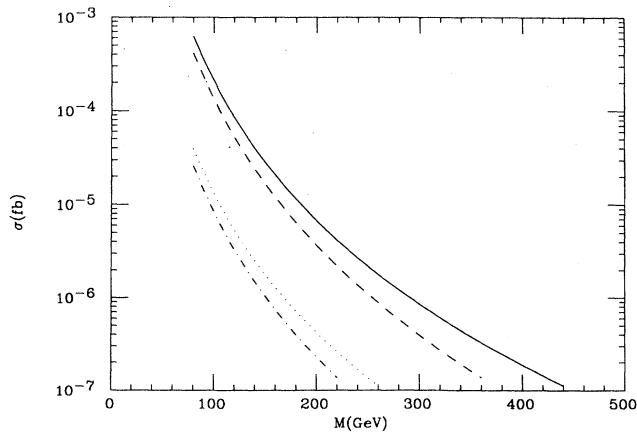


FIG. 5. Cross section for  $\gamma\gamma$  production of squarkonium with  $Q = \frac{2}{3}$ ,  $-\frac{1}{3}$  at  $\sqrt{s} = 1$  TeV (dash, dash-dot) and at  $\sqrt{s} = 2$  TeV (solid, dot).

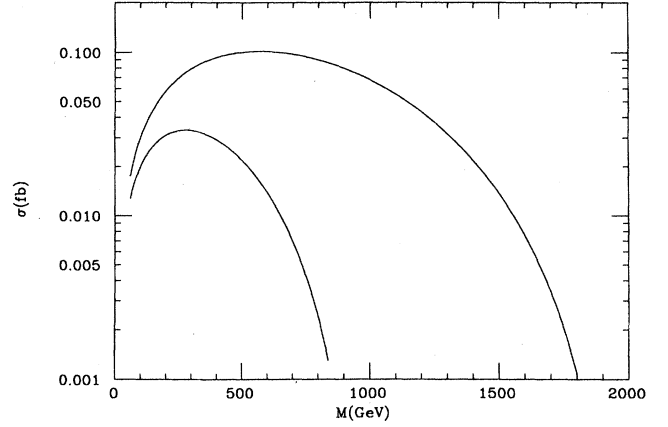


FIG. 6.  $Z_L Z_L$  contribution to squarkonium production for  $\lambda' = 1$  at  $\sqrt{s} = 1$  and 2 TeV colliders, with the  $\sqrt{s} = 2$  TeV curves extending further to the right.

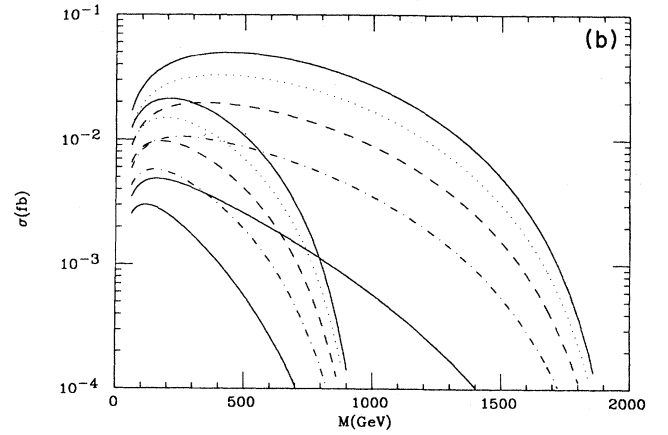
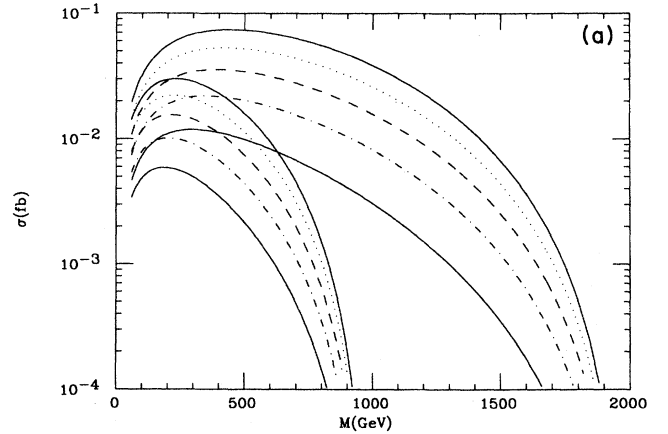


FIG. 7.  $W_L^+ W_L^-$  contribution to squarkonium production at  $\sqrt{s} = 1$  and 2 TeV colliders for (a)  $\delta M = 150$  GeV and (b) 50 GeV:  $\lambda' = 0$  (lower solid);  $\lambda' = 0.25$  (dash-dot);  $\lambda' = 0.5$  (dash);  $\lambda' = 0.75$  (dot);  $\lambda' = 1$  (upper solid). The  $\sqrt{s} = 2$  TeV curves extend further to the right.

$$A = \frac{4M_s^2(c_a^2 + c_b^2)}{M_V^2 - M_s^2 - M'^2} + 2C, \quad (3.10)$$

where one defines the couplings

$$\begin{aligned} c_a^Z &= c_b^Z = \frac{g}{\cos\theta_W} (T_3 - x_W Q), \\ c_a^W &= g/\sqrt{2}, \quad c_b^W = 0 \quad (Q = \frac{2}{3}), \\ c_a^W &= 0, \quad c_b^W = g/\sqrt{2} \quad (Q = -\frac{1}{3}), \end{aligned} \quad (3.11)$$

so that  $C = c_a^2 + c_b^2 + \delta c$ .  $\delta c$  is the Higgs-boson-exchange contribution analogous to Eq. (3.7),

$$\delta c = - \sum_i \frac{g_{iVV} g_{i\tilde{f}\tilde{f}}}{M^2 - M_i^2}, \quad (3.12)$$

and is simply a constant for a fixed value of  $M$ . Assuming Coulombic wave function we find [using  $\alpha_s = \alpha_s(M)$ ]

$$\begin{aligned} \Gamma(\tilde{S} \rightarrow Z_L Z_L) &= \frac{128 G_F^2 M_Z^4}{9\pi^2} \alpha_s^3 M (T_3 - x_W Q)^4 \left[ \frac{\lambda' M^2}{4M_Z^2} \right]^2, \\ \Gamma(\tilde{S} \rightarrow W_L^+ W_L^-) &= \frac{8 G_F^2 M_W^4}{9\pi^2} \alpha_s^3 M \left\{ 1 + \lambda' - \left[ 1 + 2 \left[ \frac{\delta M}{M} \right]^2 \right]^{-1} \right\}^2 \\ &\quad \times \left[ \frac{\lambda' M^2}{4M_W^2} \right]^2, \end{aligned} \quad (3.13)$$

and  $\delta M$  is defined above with  $\lambda' \equiv \delta c / (c_a^2 + c_b^2)$ . We first turn to the  $Z_L Z_L$  case; to maximize the  $\tilde{S} \rightarrow Z_L Z_L$  width we use  $\lambda' = 1$  and take  $\tilde{S}$  to be a  $\tilde{d}_L \bar{\tilde{d}}_L$  bound state which given the largest value of  $(T_3 - x_W Q)^4$ . Figure 6 shows the cross section for  $e^+ e^- \rightarrow e^+ e^- \tilde{S}$  via  $Z_L Z_L$  fusion as a function of  $M$  for  $\sqrt{s} = 1$  and 2 TeV. Although the  $Z_L Z_L$  contribution to  $\tilde{S}$  production is significantly larger than that for  $\gamma\gamma$ , it is still far too small to be observable for any reasonable value of the integrated luminosity. Figures 7(a) and 7(b) summarize the situation for the  $W_L^+ W_L^-$  contribution to  $\tilde{S}$  production for different  $\lambda'$  values and two different values of  $\delta M$ . The production rate is relatively sensitive to both of these parameters but one always finds cross sections which are more than an order of magnitude too small to allow  $\tilde{S}$  production by this mechanism.

It should be pointed out that it may be possible, especially for heavy squarks, that Higgs-boson exchange between  $\tilde{f}$  and  $\bar{\tilde{f}}$  may include an additional attractive potential over and above that from gluon exchange.<sup>9</sup> For a certain range of parameters for which this Higgs-boson exchange dominates,  $|\psi(0)|^2$  may be enhanced by more than 1–2 orders of magnitude which may allow for the observation of this production mode. Without this possi-

ble enhancement these cross sections are far too small to be seen at  $e^+ e^-$  colliders in the TeV energy range.

#### IV. DISCUSSIONS AND CONCLUSIONS

We have considered the production of squark and slepton pairs and squarkonium bound states at TeV  $e^+ e^-$  colliders via  $\gamma\gamma$  and longitudinal-gauge-boson fusion. We have found that the  $\gamma\gamma$  subprocess can make a reasonably sizable contribution to  $\tilde{l}\tilde{l}$  and  $\tilde{u}\tilde{u}$  production for light partner masses up to  $\approx 200$  GeV at  $\sqrt{s} = 2$  TeV but is quite small for the case of  $\tilde{d}\tilde{d}$  production. Correspondingly,  $Z_L Z_L$  fusion was found to yield cross sections which were quite small ( $\lesssim 1$  fb) for sfermion masses above  $\approx 100$  GeV even for a very optimistically chosen set of parameters. On the other hand,  $W_L^+ W_L^-$  fusion was found to yield significant cross sections for pair production over a reasonable range of parameter space for masses at high at 400 GeV at  $\sqrt{s} = 2$  TeV. Thus  $\gamma\gamma$  and gauge-boson fusion may yield significant contributions to  $\tilde{f}\tilde{f}$  production at least for a limited mass range. This conclusion is only strengthened if one includes the contributions due to transversely polarized gauge bosons in the initial state since this only results in an increase in our cross-section estimates.

Squarkonium production via  $\gamma\gamma$  was found to have a very tiny cross section ( $\lesssim 10^{-3} - 10^{-4}$  fb) and  $Z_L Z_L$  and  $W_L^+ W_L^-$  cross sections in the  $10^{-2} - 10^{-1}$ -fb range were obtained assuming purely Coulombic wave functions. If Higgs-boson exchange dominates ordinary gluon exchange in the binding of the squarkonium then cross sections in the few-fb range could be obtained making squarkonium production visible with integrated luminosities in the  $10\text{-fm}^{-1}$  range. Without such enhancements squarkonium production by  $\gamma\gamma$  and gauge-boson fusion mechanisms would be unobservably small.

A further detailed analysis of squark slepton production by  $\gamma\gamma$  and gauge-boson fusion may lead to even further interesting results. After this work was completed, our attention was brought to Ref. 10 where the possibility of  $W_L^+ W_L^- \rightarrow \tilde{l}\tilde{l}$  being enhanced was first discussed.

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